2004 Further Mathematics
Written Examination 1 (Facts, skills and applications)
Suggested answers and solutions

Multiple Choice Answers

SECTION A

Core: Statistics

1. **B**	2. **B**	3. **E**	4. **C**	5. **D**
6. **A**	7. **E**	8. **D**	9. **B**	10. **C**
11. **D**	12. **A**	13. **C**		

SECTION B

Module 1: Number patterns and applications

1. **E**	2. **D**	3. **A**	4. **E**	5. **C**
6. **C**	7. **B**	8. **C**	9. **B**	

Module 2: Geometry and trigonometry

1. **D**	2. **D**	3. **B**	4. **D**	5. **B**
6. **C**	7. **E**	8. **A**	9. **B**	

Module 3: Graphs and relations

1. **B**	2. **C**	3. **C**	4. **A**	5. **A**
6. **C**	7. **C**	8. **B**	9. **E**	

Module 4: Business-related mathematics

1. **B**	2. **D**	3. **C**	4. **C**	5. **D**
6. **E**	7. **C**	8. **E**	9. **E**	

Module 5: Networks and decision mathematics

1. **A**	2. **B**	3. **D**	4. **E**	5. **E**
6. **A**	7. **A**	8. **C**	9. **D**	

Core: Data analysis

Question 1 [B]

Examine this stem plot.

Stem	Leaf
15	0
16	
17	2 3 5
18	0 1 2 4 5 7 9
19	1 2 3 4 6 7 8 9 9

Using a graphics calculator and displaying as a boxplot, 150 is shown as an outlier.
The values are stretched towards the lower values so the distribution is considered as skewed to the negative.

Question 2 [B]

Using the 68-95-99.7% Rule, a study score of 44 is

$30 + 2 \times 7 = 44$

i.e. $\bar{x} + 2 \times s$

that is two standard deviations above the mean which puts the students in the top 2.5%.

2.5% of 22 000 = 550 students

Question 3 [E]

Continous numerical data involves data that is collected by measuring and not counting.

shoe size – ordinal data (not a true measure of size but sequential categorizing of size)

number of runs made by a cricket player – numerical but counted

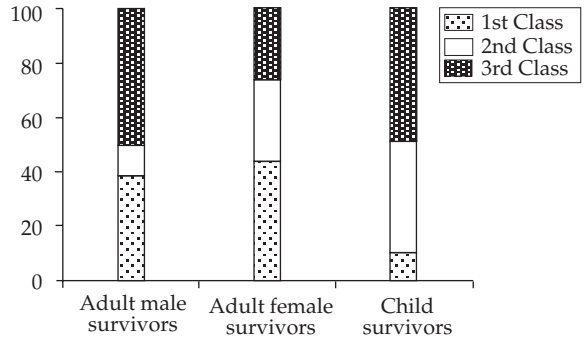
your favourite school year level in the past six years – ordinal data

Labour/Liberal prefrence of first 100 people surveyed at a shopping mall – categorical data

speed of a car captured by a speed camera – numerical and measured e.g. 60km/hr 60.4km/hr

Question 4 [C]

	Adult male survivors	Adult female survivors	Child survivors
1st Class	57	140	6
2nd Class	14	80	24
3rd Class	75	76	27
Total Survivors	146	296	57



The most appropriate visual display for a two-way frequency table to investigate the above data for relationship between survival by gender and class is the stacked bar chart percentaged for relative comparison of survival rate amongst 1st Class, 2nd Class and 3rd Class passengers.

Question 5 [D]

Coefficient of determination = r^2 and

Pearson's correlation coefficient = r

Given $r^2 = 0.49$ then $r = \sqrt{r^2} = \sqrt{0.49} = \pm 0.7$

As BAL decreases your driving ability

it is a negative relationship.

So $r = {}^{-}0.7$

Question 6 [A]

A residual is the vertical distance between a data value and the least squares regression line.
A residual plot is a visual display of each data point.

For $x = 1$ the residual is 0 (on the regression line)

For $x = 2$ the residual is +5 (vertically 5 units above the regression line)

For $x = 3$ the residual is +10 (vertically 10 units above the regression line)

For $x = 4$ the residual is +5 (vertically 5 units above the regression line)

For $x = 5$ the residual is –5 (vertically 5 units below the regression line)

For $x = 6$ the residual is -5 (vertically 5 units below the regression line)

Question 7 [E]

Substitute 3 for x into the given equation

$y = -2.5 + 3.2x$

$y = -2.5 + 3.2 \times 3$

$y = -2.5 + 9.6$

$y = +7.1$

Question 8 [D]

The line is a straight line of the form $y = c + mx$. From the graph c is the y-intercept which is approximately 1. This eliminates options A, C and E.

m is the gradient and using two points **on the line** such as (0, 1) and (3, 7) gives

$$m = \frac{rise}{run} = \frac{7-1}{3-0} = \frac{6}{3} = 2$$

to give $c = 1$ and $m = 2$ so $y = 1 + 2x$

Question 9 [B]

For a negative correlation such as –0.75, causation is not implied and so there is only a tendancy for y to decrease as x increases.

Question 10 **[C]**

The height to which a ball bounces is up and down but the time between each highest or lowest point is changing so it is **cyclical** not seasonal. As well, with time, the height to which it rebounds is progressively getting smaller or a **downward cyclical trend**.

Question 11 **[D]**

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number of photos '000s	21	23	22	27	32	31	36	36	36	37	35	39

Using the 3-mean smoothing, the last point is for

Oct, Nov and Dec $= \frac{37 + 35 + 39}{3} = \frac{111}{3} = 37$

The second last point is for Sep, Oct and Nov

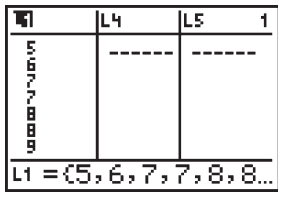
$= \frac{36 + 37 + 35}{3} = \frac{108}{3} = 36$

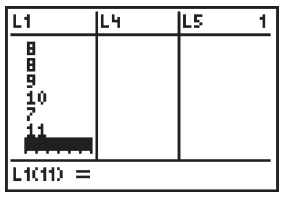
Question 12 **[A]**

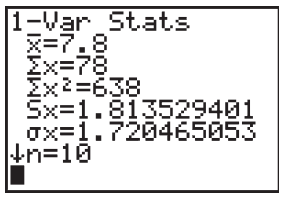
The sum of seasonal indices is equal to the number of seasons. So for quarterly data it is 4, monthly data it is 12, for days in a full week it is 7 and so on.

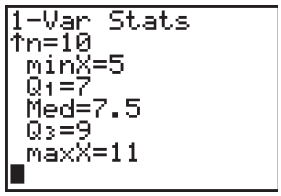
Question 13 **[C]**

Using 1-Variable Stats option of a graphics calculator and putting in the given eight data points along with the inclusion of the other two data points on a trial and error basis gives 7 and 11 as the missing values.









Module 1 : Number patterns and applications

Question 1 [E]

This sequence is arithmetic with $a = 3$ and $d = -2$

$$t_n = a + (n-1)d$$
$$-21 = 3 + (n-1)(-2)$$
$$-21 = -2n + 5$$
$$2n = 26 \Rightarrow n = 13$$

Question 2 [D]

To be geometric, a sequence must have a common ratio i.e. $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3}$ etc.

In option D, a common ratio of 2 exists.

Question 3 [A]

Since Vince has taken a pay cut his new salary is $100 - 10.5 = 89.5\%$ of the original value.

$$89.5\% = \frac{89.5}{100} = 0.895$$

Question 4 [E]

The gear ratio of A : B is 4 : 3

There is an inverse relationship between the gear ratio and the ratio of the number of cogs.

Therefore the ratio for the number of cogs is 3 : 4

Since gear A has 12 cogs, gear B will have

$\frac{4}{3} \times 12 = 16$ cogs.

Question 5 [C]

An arithmetic sequence is formed where $a = 5$, $n = 6$, $S_6 = 75$ and d is unknown.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$75 = 3[10 + 5d]$$
$$75 = 30 + 15d$$
$$15d = 45 \Rightarrow d = 3$$

Question 6 [C]

Substituting into the difference equation

$$t_2 = at_1 - 3$$
$$-5 = 2a - 3$$
$$2a = -2 \Rightarrow a = -1$$

Question 7 [B]

Since the sum of a large number of terms is approaching a given value, we use the formula for an infinite geometric series.

$$S_\infty = \frac{a}{1-r}$$
$$6 = \frac{4}{1-r}$$
$$1 - r = \frac{4}{6} = \frac{2}{3}$$
$$r = \frac{1}{3}$$

Question 8 [C]

If we divide Dick's share of 3 into 2 equal parts of 1.5 and give one each to Tom and Harry then the ratio for Tom and Harry becomes 6.5 : 3.5

Multiplying by 2 gives the simplest form of 13 : 7

Question 9 [B]

The points form a curved arrangement so the sequence must be geometric (arithmetic sequence would be a straight line). Since the terms are decreasing in magnitude the ratio must between 0 and 1.

N.B. The value of t_n is *increasing* in value.
As $n \to \infty$, $t_n \to 0$.

If r was greater than 1, the points would move further from the axis and if r was negative the terms would alternate either side of the axis.

Module 2 : Geometry and trigonometry

Question 1 [D]

Using Pythagoras' theorem

$x^2 = 5^2 - 3^2$ (half of the base is 3)

$x^2 = 16 \Rightarrow x = 4$

Question 2 [D]

$\cos a = \frac{3}{5}$

$a = 53.13° \Rightarrow 53°$ to the nearest degree

Question 3 [B]

For a 5 sided regular polygon the magnitude of each interior angle is

$\frac{(180 \times 5 - 360)°}{5} = 108°$

Each of the other angles in the given triangle is therefore $\frac{108°}{2} = 54°$

Angle a is $(180 - 54 - 54)° = 72°$

Question 4 [D]

Use the sine rule since we are given 2 sides and a non included angle.

$\frac{7.9}{\sin B} = \frac{3.2}{\sin 23°}$

$\sin B = \frac{7.9 \sin 23°}{3.2}$

$\sin B = 0.9646$

$B = 74°43'$

Question 5 [B]

Use the Cosine Rule since we are given 2 sides and the included angle.

$x^2 = 7^2 + 11^2 - 2 \times 7 \times 11 \times \cos 42°$

$x^2 = 55.556$

$x = 7.454 \Rightarrow 7.5$ to one decimal place

Question 6 [C]

The area of a triangle is half the product of 2 sides and the sine of the included angle.

Area $= \frac{1}{2} \times 7 \times 11 \times \sin 42°$

Question 7 [E]

The volume of a sphere is equal to $\frac{4}{3}\pi r^3$

For a sphere with radius of 6cm

volume $= \frac{4}{3} \times \pi \times 6^3$

$= 904.77 \text{ cm}^3 \approx 905$ to the nearest cubic centimetre

Question 8 [A]

For similar figures with a length ratio of $a : b$, the ratio of their volumes is $a^3 : b^3$

The volume ratio for the larger sphere to the smaller sphere is 8 : 1 therefore the

length ratio is $\sqrt[3]{8} : \sqrt[3]{1} = 2 : 1$

Therefore the radius of the second sphere will be $2 \times 6 = 12$ cm

Question 9 [B]

Let the height of the tree be x metres.

Using similar triangles $\frac{x}{1.5} = \frac{10}{2}$

Cross multiplying gives $2x = 15$, therefore $x = 7.5$ metres

Module 3 : Graphs and relations

Question 1 [B]

$4x - 9y + 15 = 0$ can be rearranged to give the equation in intercept form:

$4x - 9y = -15$

x-intercept, $y = 0$

$4x = -15$

$$x = -\frac{15}{4} = -3.75$$

y-intercept, $x = 0$

$-9y = -15$

$$y = \frac{-15}{-9} = 1\frac{2}{3}$$

Question 2 [C]

Rearrange the 2nd equation to give**:**

$4x + 5y = 7$

Multiply 1st equation by 2

$4x + 6y = 4$

Subtract : $y = -3$

Substitute : $2x + 3 \times {}^{-}3 = 2$

$$2x = 11$$

$$x = 5.5$$

Question 3 [C]

Sue and Roger are 120 km from home at approximately $1\frac{1}{3}$ hours from the start and $1\frac{1}{3}$ hours from the end of the journey.

$$11 - 2 \times \left(1\frac{1}{3}\right) = 8\frac{1}{3}$$

So Sue's phone is out of range for approximately 8.3 hours.

Question 4 [A]

Average speed

$$= \frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{260}{4} \text{ km/hr} = 65 \text{ km/hr}$$

Question 5 [A]

Revenue = $96n$

Costs = $7200 + xn$; where x is the cost of materials for one of the articles.

At break-even point $n = 100$ and Revenue = Costs

$$96 \times 100 = 7200 + 100x$$

$$9600 - 7200 = 100x$$

$$x = \frac{2400}{100}$$

$$= 24$$

Question 6 [C]

A : True. \$9600 is the costs at the break-even point so he needs to spend more than this to make a profit.

B : True

C : **Not** true. Profit is the difference between Revenue and Costs and at 120 articles this can be seen from the graph to be about \$1500.

D : True. This can be seen from the graph.

E : True

Revenue = \$99 × n = 99n

Costs = \$7200 + \$24 × n = $7200 + 24n$

Revenue = Costs at the break-even point

$99n = 7200 + 24n$

$75n = 7200$

$n = 96$

Only 96 articles need to be made to break-even.

Question 7 [C]

On the original graph the point (2, 2) is on the graph of $y = kx^2$

Substituting gives $2 = k \times 2^2$

$$\text{giving } k = \frac{1}{2}$$

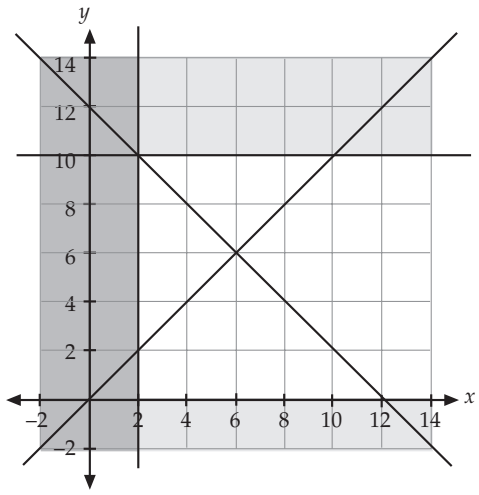
So $y = \frac{1}{2}x^2$ is the equation of the function.

The variable on the horizontal axis of the answers to A, B and C is x^2 so all these graphs are of the form $y = mx^2$

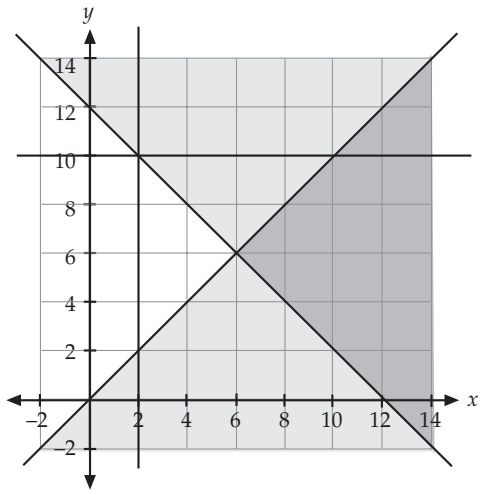
The gradient, m, for answer C is $\frac{1}{2}$

Question 8 [B]

The constraints $x \geq 2; y \geq 0$ and $y \leq 10$ are shaded-out below (unshaded region) :



Shading-out $x + y \leq 12$ and $y \geq x$:-

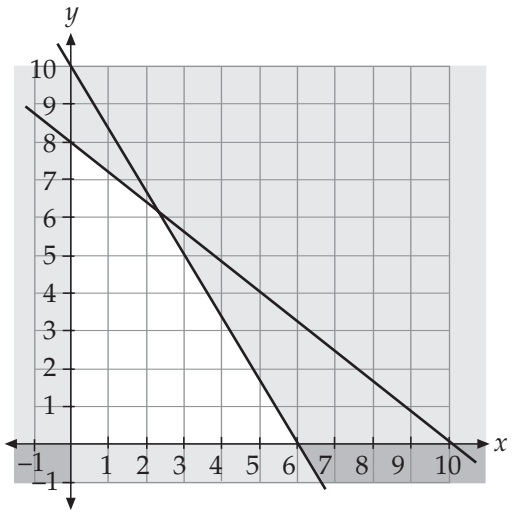


Combining these two unshaded regions gives answer B.

Question 9 [E]

Only whole number solutions are possible for x and y.

The feasible region (unshaded) is:



Substituting the points in the objective function gives:

A : 48

B : not whole numbers

C : 46

D : 51 but (2, 7) is not in the feasible region

E : 49

Module 4 : Business-related mathematics

Question 1 [B]

4.8% p.a. = 1.2% per 3-month period.

$$\text{Interest} = \$\frac{8400 \times 1.2 \times 1}{100} = \$100.80$$

Value of investment = \$(8400 +100.8)

= \$8500.80

Question 2 [D]

40% decrease = multiplying factor of 0.6

Value after year 1 = $6400 \times 0.6 = 3840$

Value after year 2 = $3840 \times 0.6 = 2340$

Value after year 3 = 1382.40

Value after year 4 = 829.44

Value after year 5 = 497.66

Question 3 [C]

Overall reduction of original price

= 25% + 50% of (75% of original price)

= 25% + 37.5%

= 62.5%

Question 4 [C]

For a reducing balance loan the earlier repayments contain a higher proportion of interest compared to principal being repaid.

The interest component of the repayments decreases with time but not at a constant rate, hence graph C.

Question 5 [D]

Amount to be repaid = \$6000 – \$600

= \$5400

Repayments = \$522 × 12 = \$6264

Interest paid = \$6264 – \$5400

= \$864

Interest as a percentage of amount to

be repaid = $\frac{864}{5400} \times 100 = 16\%$

Question 6 [E]

Monthly payments so $r = \frac{4.8}{12}\% = 0.4\%$

and $R = 1 + \frac{0.4}{100} = 1.004$

Number of monthly payments in 3 years:

$n = 36$

$P = 7200$ and $A = 20\,000$

An amount is being added to the principal of \$7200 so the "+" is used.

Question 7 [C]

$r = 8.5\%$ per annum gives

$$R = 1 + \frac{8.5}{100} = 1.085$$

Amount owing at the beginning of the 2nd year = \$6000 × 1.085 – \$1000

= \$5510

Amount owing at the end of the 2nd year = \$5510 × 1.085

= \$ 5978.35

Question 8 [E]

6.4% yearly = 1.6% quarterly

n years = $4n$ quarterly periods

Compounding quarterly so amount of investment after $4n$ quarters

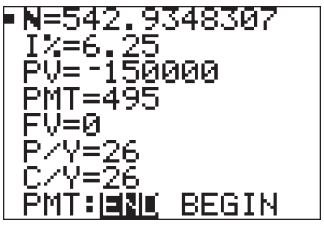
$= 25\,000 \times 1.016^{4n}$

Interest earned

$= 25\,000 \times 1.016^{4n} - 25\,000$

Question 9 [E]

Using the TVM Solver on the calculator to calculate the length of loan in fortnights:



The loan would take 543 fortnights to repay compared to 25 × 26 = 650 fortnights with the original loan.

Reduction in term = 650 – 543 = 107 fortnights.

107/26 = 4.12 years

Module 5 : Networks and decision mathematics

Question 1 [A]

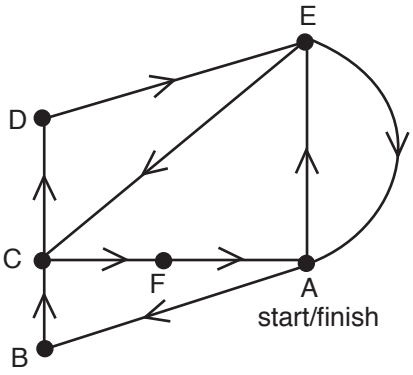
It is a complete graph as **each vertex is connected to every other vertex.**

Question 2 [B]

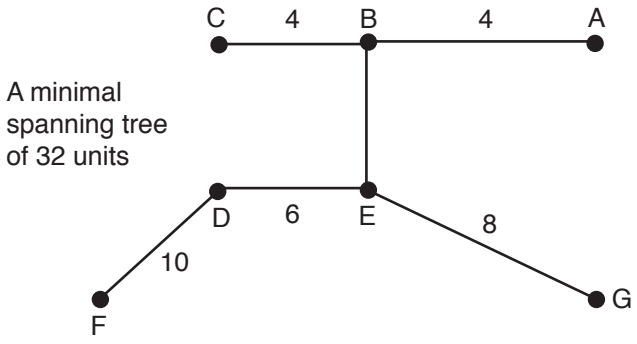
Count each edge once only.

Question 3 [D]

An Euler circuit has each edge used only once and has the same initial and final vertex.



Question 4 [E]



Question 5 [E]

A Hamiltonian path is a path that passes through each vertex exactly once. Beginning with A – G – E – B… the figure only needs C, D and F to complete the path and this can be done in four possible ways.

Path 1 continue path with C and then D and F

Path 2 continue path with C and then F and D

Path 3 continue path with D and then C and F

Path 4 continue path with D and then F and C

So the next possible vertex would be C or D.

Question 6 [A]

The maximum flow through the network:

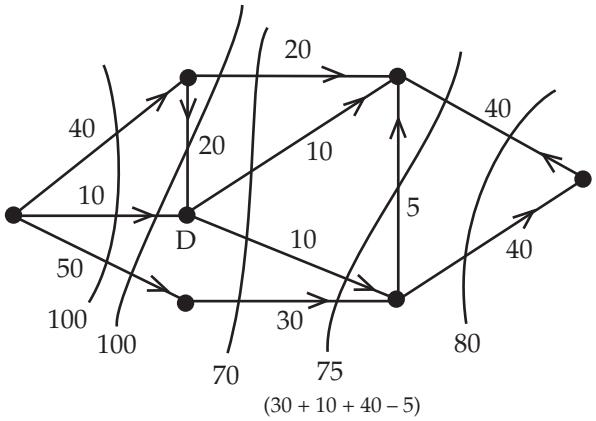
Maximum to vertex E is 20 from B,
10 from D and 5 from F
SUB TOTAL 35

Maximum to vertex F is 30 from C,
10 from D less 5 from F to E
SUB TOTAL 35

Therefore the Maximum Flow is 70

OR

Using cut method:



From the diagram, minimum value is 70

$\Rightarrow$ maximum flow is 70

Question 7 [A]

Using the Hungarian Algorithm

	Customer A	Customer B	Customer C	Customer D
Truck 1	20	10	20	15
Truck 2	5	15	10	25
Truck 3	15	10	5	10
Truck 4	10	10	20	20

STEP 1. Reduce each row by the smallest amount

Truck 1: 10 min
Truck 2: 5 min
Truck 3: 5 min
Truck 4: 10 min

	Customer A	Customer B	Customer C	Customer D
Truck 1	10	0	10	5
Truck 2	0	10	5	20
Truck 3	10	5	0	5
Truck 4	0	0	10	10

It only needs **3 lines** (do not use diagonals) to cover ALL ZEROS but need 4 allocations.

STEP 2. Reduce each column by the smallest amount
Customer A by 0
Customer B by 0
Customer C by 0
Customer D by 5

	Customer A	Customer B	Customer C	Customer D
Truck 1	10	0	10	0
Truck 2	0	10	5	15
Truck 3	10	5	0	0
Truck 4	0	0	10	5

It needs **4 lines** to cover ALL ZEROS so the 4 allocations can be made.

Truck 1 gets Customer D,

Truck 2 gets Customer A,

Truck 3 gets Customer C,

Truck 4 gets Customer B.

Question 8 [C]

Activity I only needs activities F and G to be completed to proceed as Activity H is not predecessor of Activity I but of Activity J.

Question 9 [D]

The critical path is B – F – H – J which has a total of 2+3+2+2 = 9 days .