Further Mathematics Examination 2
Solutions

Section A

Core : Data analysis

Question 1

a. categorical **[A1]**

b.

Stem	Leaf
0	1 7 8 9
1	2 3 6 8 9 9
2	0 1 3 7
3	3

[A1]

c. males are negatively skewed, females are symmetrical **[A1]**

d. median = 18 **[A1]**

e. Interquartile range = 22 – 6

= 16 **[A1]**

f. 1.5 IQR = 24 **[M1]**

Upper 'fence' $= Q_3 + 24$

$= 46 \Rightarrow$ not an outlier **[A1]**

Question 2

a. Since gender may influence attitude to smoking, it is the independent variable. **[A1]**

b. 22 **[A1]**

c.

	Female	Male
For smoking	65.3	53.7
Against smoking	34.7	46.3
	100%	100%

[A1]

d. Based on this sample there appears to be a relationship. **[A1]**

A higher percentage of females (65.3%) than males (53.7%) are in favour of smoking in casinos. **[A1]**

Question 3

a. $M = 5.02 \times 2 - 2.65$ **[A1]**

$M = 7.39$

$M = \$7$ (to the nearest dollar)

b. As the number of hours gambled increases by 1, money lost increases by $5.02 (the gradient of the linear equation). **[A1]**

c. The predicted loss before you start gambling is $2.65. Therefore no practical significance can be applied to this result. **[M1]**

Total: 15 marks

Section B

Module 1 : Number patterns and applications

Question 1

a. The sequence is arithmetic with a = 1.8 and d = 0.6

4^{th} term = 3 + 0.6 = 3.6

5^{th} term = 3.6 +0.6 = 4.2 **[A1]**

b. a = 1.8, d = 0.6 **[A1]**

$L_n = a + (n-1)d$

$= 1.8 + (n-1) \times 0.6$ **[H1]**

$= 1.2 + 0.6n$

c. 1 litre / minute = 60 litres / hour

We need to find n when $L_n = 60$

Substituting in L_n

$60 = 1.2 + 0.6n$ **[M1]**

$58.8 = 0.6n$

$n = 58.8 \div 0.6$

$= 98$ **[A1]**

In the 98^{th} hour

d. Find the sum of the first 24 terms :

$S_{24} = \frac{24}{2}[2 \times 1.8 + 23 \times 0.6]$ **[M1]**

$= 208.8$

208.8 litres in total will leak from the crack in the first 24 hours. **[A1]**

Question 2

a. Increase = 7 – 5 = 2 litres

$\frac{2}{5} \times 100 = 40\%$ increase **[A1]**

b. The difference equation

$T_n = aT_{n-1}$ can be rearranged to give

$$\frac{T_n}{T_{n-1}} = a$$

Hence $a = \frac{T_2}{T_1} = \frac{7}{5} = 1.4$ **[A1]**

Alternatively, you can use the fact that a percentage increase of 40% gives a multiplying factor of 1.4

c. 1 litre/minute = 60 litres/hour

We need to find n such that $T_n = 60$

Recognising that this is a geometric sequence we can substitute in and solve

$T_n = a \times r^{n-1}$

$60 = 5 \times 1.4^{n-1}$ **[M1]**

$12 = 1.4^{n-1}$

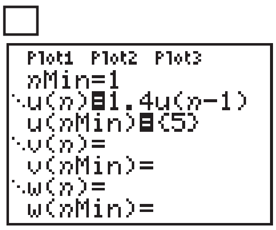
Using trial and error on the calculator:

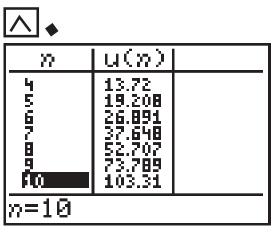
Hence $1.4^{n-1} = 12$ for a value of (n – 1) between 7 and 8

And so for a value of n between 8 and 9.

So the amount leaking will exceed 60 l/hour during the 9th hour. **[A1]**

Alternatively using **Seq**uence mode on the calculator :





Clearly the amount exceeds 60 l/hour in the 9th hour.

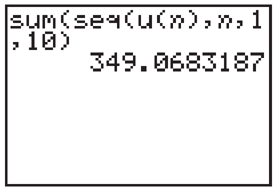
d. Substituting a = 5 and r = 1.4 in the equation for S_n for a geometric sequence :

$$S_{10} = \frac{5\left(1.4^{10} - 1\right)}{1.4 - 1}$$ **[M1]**

= 349.06 (349 l) **[A1]**

Or using the calculator:

sum(and **seq(** are accessed under LIST



Question 3

This is a geometric sequence with

$a = 20$ and $r = \frac{17}{20} = 0.85$

Hence we can find a sum to infinity because
$-1 < r < 1$ **[M1]**

$S_\infty = \frac{a}{1-r} = \frac{20}{1-0.85} = 133.33\ldots$ **[A1]**

133 l will leak from the crack from the time Deidre starts monitoring.

Total: 15 marks

Module 2 : Geometry and trigonometry

Question 1

a The triangle is an isosceles right-angled triangle, therefore the angle of decline is 45°.
Or using trigonometry

$\theta = \tan^{-1}\left(\frac{4}{4}\right) = \tan^{-1} 1 = 45°$ **[A1]**

b Lengths $\overline{AB}$ and $\overline{CD}$ are the same and using Pythagoras

Length $\overline{AB}$ or $\overline{CD}$

$c^2 = a^2 + b^2$

$c^2 = 4^2 + 4^2 = 32$

$c = \sqrt{32}$

Total Length

$= 2 \times \sqrt{32} + 6$

$= 17.3137..$ **[M1]**

≈ 17.3 metres **[A1]**

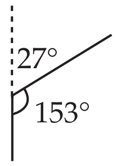
Question 2

a The complementary angle to N27°E is 63° and added to 90° (from east to south) gives a total angle for ∠OAB of 153° **[A1]**

b For Distance: Using Cosine Rule where

$a = x\,m$ $\quad b = 180\,m$ $\quad c = 140\,m$

and $\angle A = 153°$



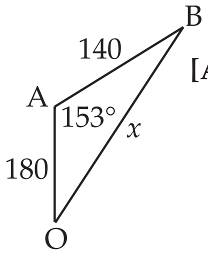
$a^2 = b^2 + c^2 - 2bc \times \cos A$

$a^2 = 180^2 + 140^2 - 2 \times 180 \times 140 \times \cos 153°$ **[M1]**

$a^2 = 96906.729$

$a = \sqrt{96906.729}$

$a = 311.298.. \approx 311$ metres **[A1]**



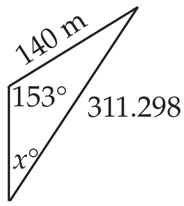
For Direction: Using the Sine Rule where

$a = 311.298$ m $\quad \angle A = 153°$

$c = 140$ m and $\angle C = x°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{311.298}{\sin 153°} = \frac{140}{\sin x°}$$



$$x° = \sin^{-1}(0.20417..) = 11.781° \approx 12°$$

[M1]

where the bearing is N 12° E **[A1]**

c G because the contour lines are closest together. **[A1]**

d Gradient $= \dfrac{rise}{run} = \dfrac{4}{15}$

or 1 in 3.75 **[A1]**

Question 3

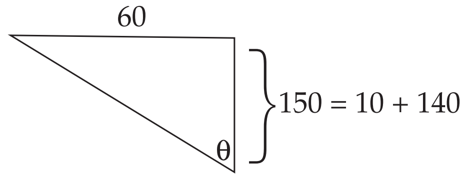
a

Clubhouse

	0	
Hole 1 20	40	
	140	5 Hole 5
Hole 2 60	390	
	400	80 Hole 4
	540	

Hole 3 **[2A]**

b Using trigonometry where

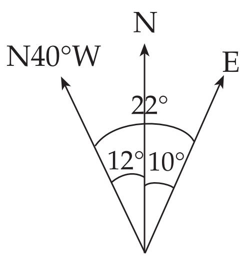


[M1]

$$\tan\theta = \frac{60}{150}$$

$\theta = \tan^{-1}(0.4) = 21.8° \approx 22°$ **[M1]**

and as a bearing 22° anticlockwise from N 10° E becomes N 12° W **[A1]**



Total: 15 marks

Module 3 : Graphs and relations

a $a = 40$, $b = 10000$ **[A1]**

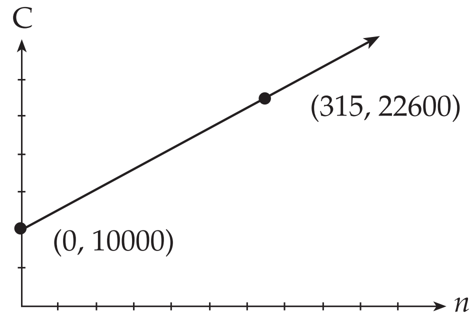
b $C = 40 \times 140 + 10000$

$= \$15600$ **[A1]**

c $22600 = 40n + 10000$

$n = 315$ **[A1]**

d



2 points correctly used **[M1]**

graph drawn correctly **[M1]**

e $R = 140n$ **[A1]**

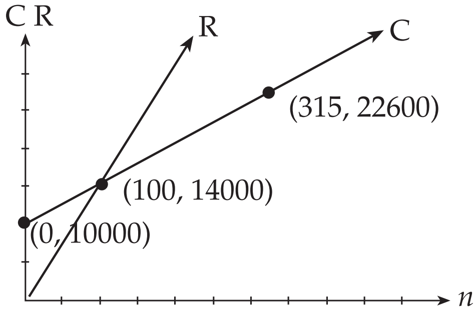
f Letting cost = revenue

$40n + 10000 = 140n$ **[M1]**

$100n = 10000$

$n = 100$ i.e. 100 calculators must be sold to break even **[A1]**

g

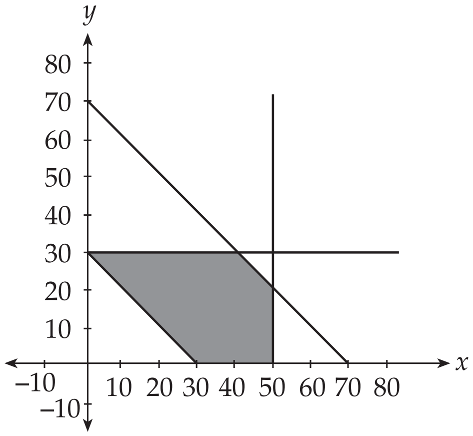


Correct line with point labelled **[M1]**

Question 2

a $x + y \leq 70$ **[A1]**

b



Vertical and horizontal lines **[M1]**

Diagonal lines **[M1]**

Correct region shaded **[A1]**

c From the graph, the maximum will occur at the point (50, 20) **[M1]**

Maximum profit = $(120 \times 50) + (80 \times 20)$

= $7600 **[A1]**

Total: 15 marks

Module 4 : Business related mathematics

Question 1

a Interest = $\frac{PRT}{100} = \frac{10000 \times 8 \times 3}{100}$ = $2400

[M1] [A1]

b Total of Investment = Principal + Interest
= $10 000 + $2 400 = $12 400 **[A1]**

c

Time (years)	Balance at start of the year ($)	Interest earned ($)	Balance at the end of the year ($)
1	10 000	800	10 800
2	10 800	$\frac{PRT}{100} = \frac{10800 \times 8 \times 1}{100} = 864$	11 664
3	11 664	$\frac{PRT}{100} = \frac{11664 \times 8 \times 1}{100} = 933.12$	1164 + 933.12 = 12 597.12
		[A1]	**[A1]**

d Compound interest bearing investment **[A1]**

Question 2

a A scrap value of $3 000 **[A1]**

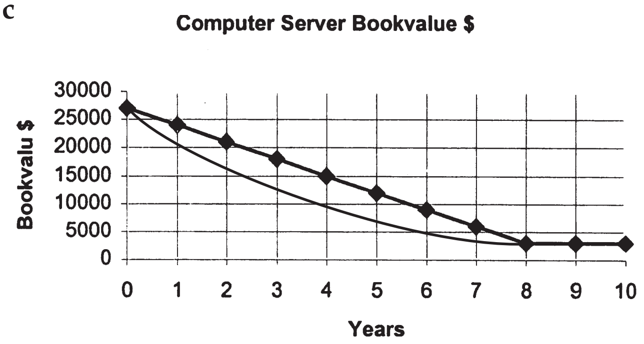
b $\text{Rate of Depreciation} = \frac{\text{Total Depreciation \$}}{\text{Years}}$

$= \frac{\$27000 - \$3000}{8 \text{ Years}}$

$= \$3000$ per year **[M1]**

$\text{Rate of Depreciation \%} = \frac{\$3000}{\$27000} \times \frac{100}{1} = 11.1\%$ **[A1]**

c



[A1]

Question 3

a Loan Amount = Purchase – Trade In = $35 000 – $3 000 =$32 000 **[M1]**

Interest charged =

$\frac{PRT}{100} = \frac{32000 \times 5 \times 5}{100} = \$8000$

Monthly Repayments = $\frac{\text{Total repayments}}{\text{Number of Instalments}}$

$= \frac{\$32000 + \$8000}{5 \times 12} = \frac{\$40000}{60} = \$666.67$ **[A1]**

b Effective interest rate $= \frac{2 \times n}{n+1} \times$ Flat Rat

$= \frac{2 \times 60}{60+1} \times 5\%$

$= 9.836\% = 9.8\%$ **[A1]**

c Loan Amount is the same as in **a.**

$$Q = \frac{PR^n(R-1)}{R^n - 1}$$

where $R = 1 + \frac{8.7/12}{100} = 1.00725$ **[M1]**

$$Q = \frac{32000 \times 1.00725^{60}(1.00725 - 1)}{1.00725^{60} - 1}$$

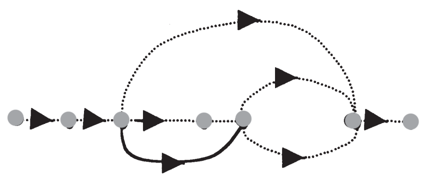
$= \$659.62$ **[A1]**

Total: 15 marks

Module 5 : Networks and decision mathematics

Question 1

a. Task D completes the network. **[A1]**



D (4)

b. Task C is not on the critical path so the latest it can start is 11days before the end of the project. (Task C takes 7 days and task I takes 4 days)

Hence the **LST for C is 26**. **[A1]**

Task F is on the critical path so the **EST is day 23** (A takes 5 days, B takes 12 and E takes 5; a total of 22 days). **[A1]**

c. The float (slack time) for task G is

LST – EST = 31 – 26
= 5 days **[A1]**

d. The critical path is ABEFHI.

Any task on this path, if delayed will delay the completion time of the project. **[A1]**

e. Completion time is 37 days **[A1]**

Question 2

a. It is sensible to shorten the tasks that are on the critical path.

Hence tasks B, E, H and I should be shortened. (A and F cannot be shortened). **[A1]**

b. B can be shortened 5 days, E can be shortened 1 day, H can be shortened 4 days and I can be shortened 2 days; a total of 12 days.

Hence the new completion time is

37 – 12 = 25 days. **[H1]**

c. The additional cost is

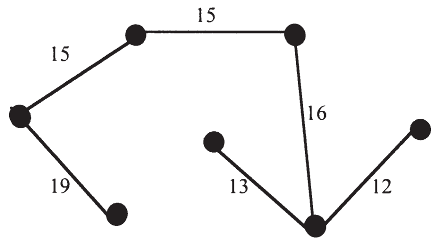
$5 \times \$100 + 1 \times \$200 + 4 \times \$150 + 2 \times \$150$

= \$1600

[H1]

Question 3

a. The minimal-length spanning-tree is :



Tree **[H1]**
Correct **[A1]**

b. The sum of the lengths on the **tree** above;
80 metres **[H1]**

Question 4

Using the Hungarian algorithm :

Subtract the minimum from each of the rows

	And.	Tom	Carl.	Eric
P	400	200	0	500
Q	160	250	0	400
R	1100	1500	0	1000
S	0	200	400	100

Subtract the minimum from each of the columns and cover the zeros with a minimum of lines : **[M1]**

	And.	Tom	Carl.	Eric
P	400	0	0	400
Q	160	50	0	300
R	1100	1300	0	900
S	0	0	400	0

There are only three lines (we need four for a solution)

Add the minimum uncovered number (160) to the elements at the intersections of the lines and subtract it from the elements not covered by the lines :

	And.	Tom	Carl.	Eric
P	240	0	0	240
Q	0	50	0	140
R	940	1300	0	740
S	0	160	560	0

We now have an additional line and an independent set of zeros

[M1] (circled)

Task P is allocated to Tom, Q to Andrew, R to Carlo and S to Eric **[A1]**

Total: 15 marks