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PROCESSING LABEL HERE

Write your **student number** in the boxes above. **Letter**

General Mathematics
Examination 2

Question and Answer Book

VCE (NHT) Examination – Friday 24 May 2024

- Reading time is **15 minutes**: 10.30 am to 10.45 am
- Writing time is **1 hour 30 minutes**:10.45 am to 12.15 pm

Approved materials

- One bound reference that may be annotated
- One approved CAS calculator or CAS software and one scientific calculator

Materials supplied

- Question and Answer Book of 28 pages
- Formula Sheet

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents pages

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

VICTORIA State Government

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, you should only round your answer when instructed to do so.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (8 marks)

Data was collected to investigate the behaviour of tides in Sydney Harbour.

There are usually two high tides and two low tides each day.

The variables in this study were:

- *Day*: the day number in the sample
- *LLT*: the height of the lowest low tide for that day (in metres)
- *HHT*: the height of the highest high tide for that day (in metres)

Table 1 displays the data collected for a sample of 14 consecutive days in February 2021.

Table 1

Day	*LLT* (m)	*HHT* (m)
1	0.43	1.65
2	0.49	1.55
3	0.55	1.44
4	0.61	1.42
5	0.68	1.42
6	0.73	1.42
7	0.72	1.42
8	0.65	1.47
9	0.57	1.55
10	0.48	1.64
11	0.39	1.74
12	0.30	1.83
13	0.25	1.90
14	0.22	1.92

Data based on: http://www.bom.gov.au/australia/tides/

a. For the *HHT* values in Table 1:

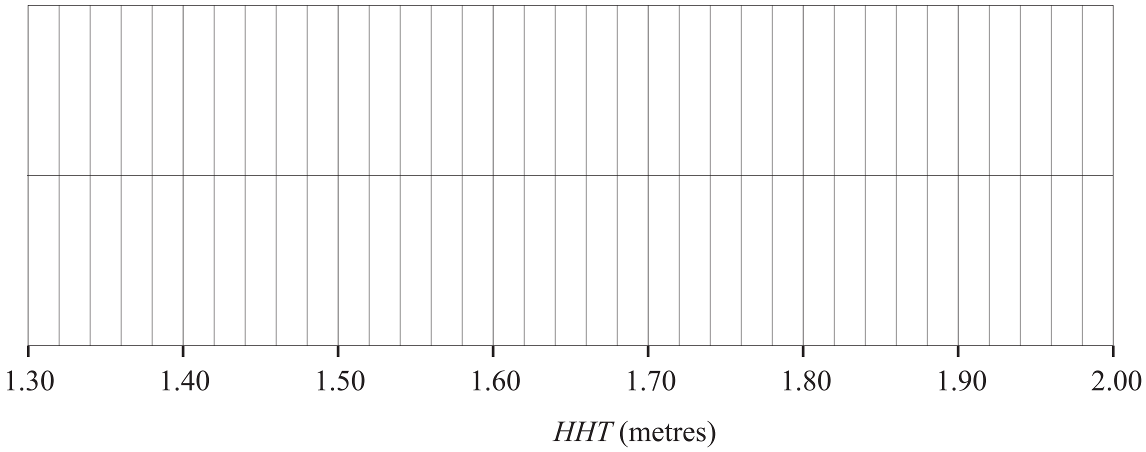
i. Calculate the mean, in metres.

Round your answer to one decimal place. 1 mark

ii. Calculate the standard deviation, in metres.

Round your answer to three decimal places. 1 mark

b. Use the *HHT* data from Table 1 to construct a boxplot on the grid below. 2 marks



c. The five-number summary of the *LLT* data is shown in Table 2 below.

Table 2

Minimum	Q1	Median	Q3	Maximum
0.22	0.39	0.52	0.65	0.73

Show that the minimum *LLT* value of 0.22 m is **not** an outlier. 2 marks

d. A least squares line can be used to model the association between *LLT* and *HHT*. In this model, *HHT* is the response variable.

Use the data from Table 1 to determine the equation of this least squares line.

Round the values of the intercept and slope to four significant figures.

Write your answers in the boxes provided. 2 marks

| | = | | + | | × | |

Question 2 (4 marks)

In the year 2021 there were 12 days where a new moon appeared in Sydney Harbour.

For these 12 days the occurrence of the highest high tides (HHT) and lowest low tides (LLT) occurred before, on or after the appearance of the new moon.

These records are displayed in the table below.

Table 3

LLT	on	before	before	after	after	after
	after	after	after	after	after	after

HHT	on	before	before	before	after	after
	after	on	on	after	after	after

Data: http://www.bom.gov.au/australia/tides/

a. Use the data from Table 3 to complete the following frequency table. 2 marks

Table 4

	Frequency	
Occurrence	LLT	HHT
before		
on		
after		
Total	12	12

b. Does the data in Table 4 support the contention that the occurrence of the LLT is associated with the appearance of the new moon?

Explain your conclusion by comparing the values of appropriate percentages. 2 marks

Question 3 (2 marks)

The time difference between successive high tides and low tides is approximately normally distributed.

Analysis of the 2021 tide chart showed that

- 99.85% of the time differences are more than 4.88 hours
- 16% of the time differences are less than 5.76 hours.

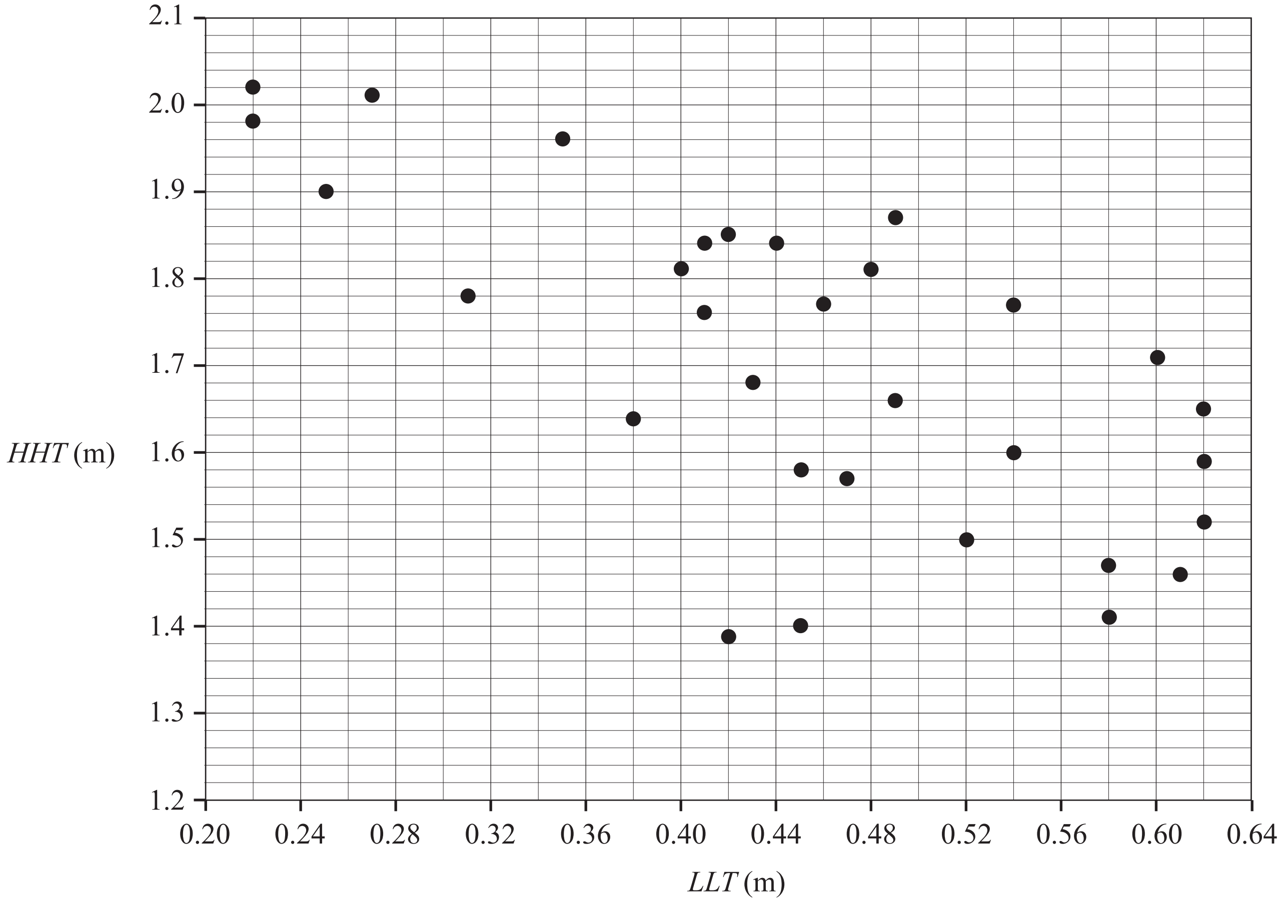
Use the 68-95-99.7% rule to determine the mean and standard deviation for this normal distribution.

mean = ☐ standard deviation = ☐

Question 4 (7 marks)

In another study, the heights, in metres, of the highest high tide (HHT) for that day and the lowest low tide (LLT) for that day were recorded in Sydney Harbour for the 31 days of July 2021.

A scatterplot of this data is shown below.



Data: http://www.bom.gov.au/australia/tides/

When a least squares line is fitted to the scatterplot, the equation is found to be:

$$HHT = 2.19 - 1.08 \times LLT$$

The coefficient of determination is 0.4709

a. Draw the graph of the least squares line on the **scatterplot above**. 1 mark

(*Answer on the scatterplot above*.)

b. Determine the value of the correlation coefficient r.

Round your answer to three decimal places. 1 mark

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c. Describe the association between HHT and LLT in terms of form and direction. 1 mark

form	
direction	

d. Interpret the slope of the least squares line in terms of the variables HHT and LLT. 1 mark

e. In this investigation, the HHT value was 1.81 m for an LLT value of 0.40 m.

Show that when this least squares line is fitted to the scatterplot, the residual for this point is 0.052 2 marks

f. The mean of the HHT values for July 2021 is 1.70 m.

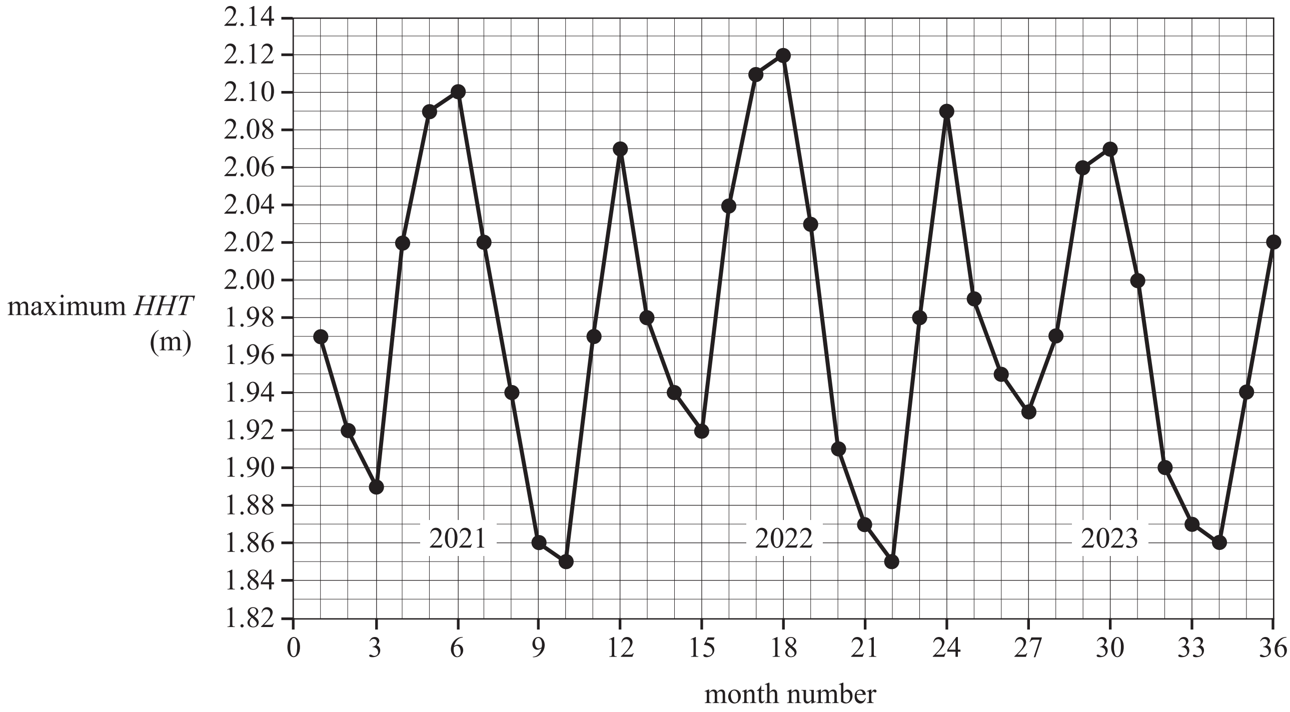
Calculate the mean of the LLT values.

Round your answer to two decimal places. 1 mark

Question 5 (2 marks)

The height, in metres, of the maximum highest high tides (HHT) for Sydney Harbour change from month to month during the year. This is shown in the time series plot below for the years 2021, 2022 and 2023.

In this graph, month number 1 is January 2021, month number 2 is February 2021, and so on.



Data based on: http://www.bom.gov.au/australia/tides/

The average height, in metres, of the maximum HHT for each year, rounded to two decimal places, is given in the table below.

Table 5

Year	2021	2022	2023
Average height of the maximum *HHT* (m)	1.98	1.99	1.96

The three years of data shown in this graph and in Table 5 will be used to calculate seasonal indices.

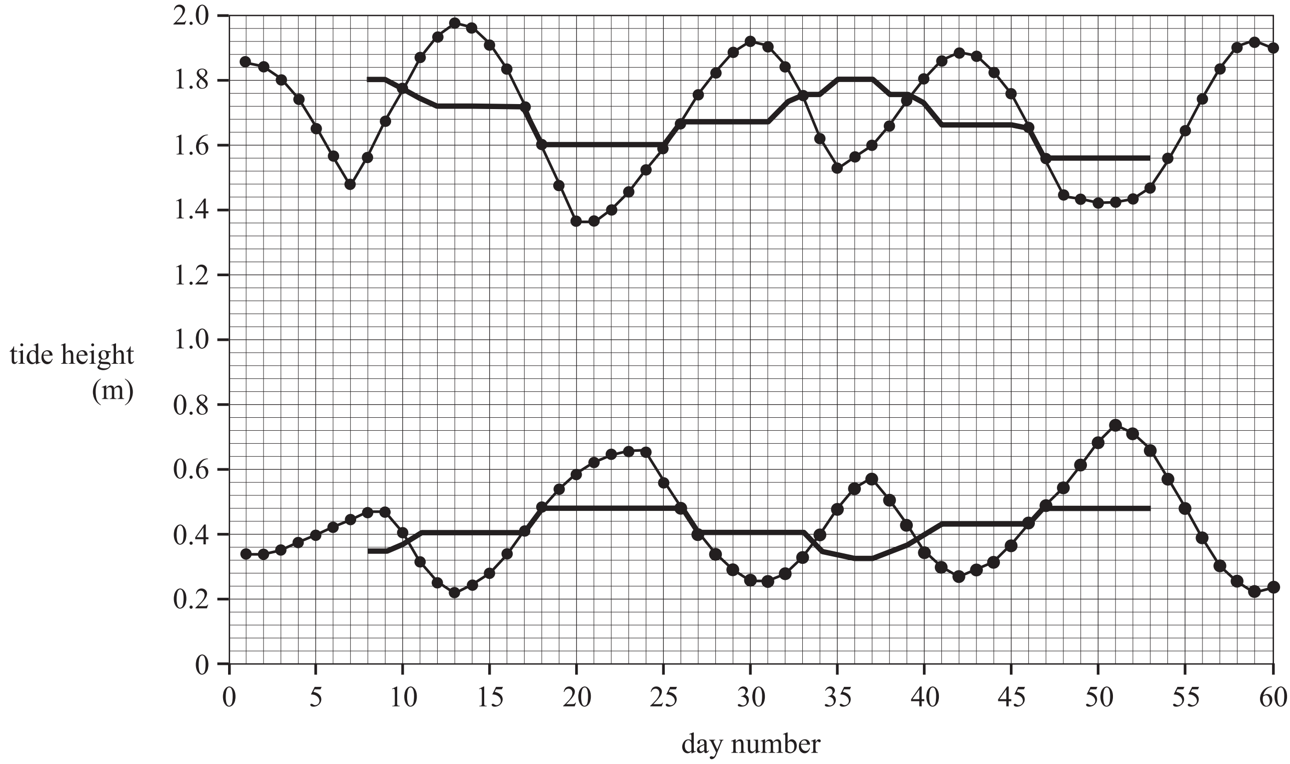
Determine the seasonal index for March.

Round your answer to two decimal places.

Question 6 (1 mark)

The time series plot below shows the height, in metres, of the highest high tides (HHT) and lowest low tides (LLT) for Sydney for the first 60 days of 2021.

The thick line for each tide type shows the results of smoothing using a moving median.



Data based on: http://www.bom.gov.au/australia/tides/

Complete the sentence below by entering a number in the box provided.

Both the HHT data and the LLT data have been smoothed using [ ] -median smoothing.

Recursion and financial modelling

Question 7 (5 marks)

Cleo took out a reducing balance loan to buy an apartment.

Interest on this loan is charged monthly and the loan is scheduled to be repaid in full with monthly repayments over 20 years.

The balance of Cleo's loan, in dollars, after n months, C_n, can be modelled by the recurrence relation

$$C_0 = 560\,000 \qquad C_{n+1} = 1.005C_n - 4012$$

a. What amount, in dollars, did Cleo borrow? 1 mark

b. Determine the total value, in dollars, of the repayments made by Cleo in the first year of the loan. 1 mark

c. The interest rate for Cleo's loan is 6% per annum.

Use this value in a calculation to show that the multiplication factor in the recurrence relation is 1.005 1 mark

d. Complete the next line in the amortisation table.

Write your answers in the spaces provided in the table below. 1 mark

Payment number	Repayment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	560 000.00
1				

e. The final monthly repayment required to fully repay the loan to the nearest cent will be slightly higher than all previous payments.

Determine the value of this final repayment.

Round your answer to the nearest cent. 1 mark

Question 8 (4 marks)

Cleo owns equipment that was purchased for $\$50\,000$.

She depreciates the value of the equipment using the unit cost method.

Let V_n be the value of the equipment, in dollars, after n units of use.

A recurrence relation that can model this value from one unit of use to the next is given by

$$V_0 = 50\,000, \qquad V_{n+1} = V_n - k$$

a. What does k represent in this recurrence relation? 1 mark

b. If $k = 12.50$, determine the value of the equipment after one year if it is used twice per day on all 365 days of the year. 1 mark

Another option for Cleo is to depreciate the value of the $\$50\,000$ equipment using the reducing balance method.

The value of the equipment, in dollars, after n months, V_n, can be modelled by a recurrence relation of the form

$$V_0 = 50\,000, \qquad V_{n+1} = RV_n$$

c. If the depreciation rate per month was 1.5%, what would be the value of R in this recurrence relation? 1 mark

d. For what value of R would the equipment be valued at $\$42\,868.75$ after three months? 1 mark

Question 9 (3 marks)

Cleo took out a loan of $\$35\,000$ to pay for an overseas holiday.

Interest is charged at the rate of 10% per annum compounding quarterly.

For the first year of this loan, Cleo made quarterly repayments of $\$1722$.

a. Let V_n be the balance of Cleo's loan, in dollars, after n quarters.

Write a recurrence relation in terms of V_0, V_{n+1} and V_n that can model the value of the loan from quarter to quarter for the first year. 1 mark

For the second year of the loan, Cleo increased her quarterly repayments to $\$2000$.

b. Determine the total amount of interest Cleo paid in the first two years of the loan.

Round your answer to the nearest cent. 2 marks

Matrices

Question 10 (3 marks)

An egg farmer has five barns on their property: I, J, K, L and M.

Matrix H below shows the available communication links between the five barns.

$$H = \text{sender} \begin{array}{c} \\ I \\ J \\ K \\ L \\ M \end{array} \overset{\text{receiver}}{\begin{array}{c} \begin{array}{ccccc} I & J & K & L & M \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}}$$

In this matrix:

- The '1' at element h_{25} indicates that barn J can directly communicate to barn M.
- The '0' at element h_{42} indicates that barn L cannot directly communicate to barn J.

a. Which barn(s) can directly communicate to barn L? 1 mark

b. State two communication paths for barn J to communicate to barn K. 1 mark

c. The farmer plans to install one new direct communication link in one barn to receive direct communication from barn I.

In which barn should the new direct communication link be installed to ensure that barn I has either a one-step or a two-step communication link with every other barn? 1 mark

Question 11 (2 marks)

The farmer sells four different egg sizes (medium, large, extra large and jumbo) in cartons containing 12 eggs.

The table below shows the total egg weight, in grams, per carton for the different egg sizes.

egg size	medium	large	extra large	jumbo
total egg weight (grams)	504	600	696	804

This information is also displayed in Matrix E.

$$E = \begin{bmatrix} 504 & 600 & 696 & 804 \end{bmatrix}$$

a. Complete the following matrix equation that will calculate the average egg weight in a carton for each egg size. 1 mark

$$\boxed{} \times \begin{bmatrix} 504 & 600 & 696 & 804 \end{bmatrix} = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

b. On one day, the farmer sold 122 cartons of medium eggs, 148 cartons of large eggs, 80 cartons of extra large eggs, and 52 cartons of jumbo eggs as shown in Matrix Q.

$$Q = \begin{bmatrix} 122 & 148 & 80 & 52 \end{bmatrix}$$

In the space below, write a matrix calculation using Matrix Q and a column matrix to show that the total number of eggs sold on this day was 4824. 1 mark

(Answer in the space above.)

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Question 12 (4 marks)

Three types of eggs are sold at a local supermarket: cage eggs (C), free-range eggs (F) and barn-laid eggs (B).

Each week customers purchase eggs from one of the three types.

The supermarket is holding a 10-week campaign to encourage more shoppers to purchase free-range eggs.

Let S_n be the state matrix that shows the expected proportion of customers who purchased each type of egg n weeks after the campaign began.

The proportion of customers who purchased each type of egg prior to the beginning of the 10-week campaign is shown in S_0 below.

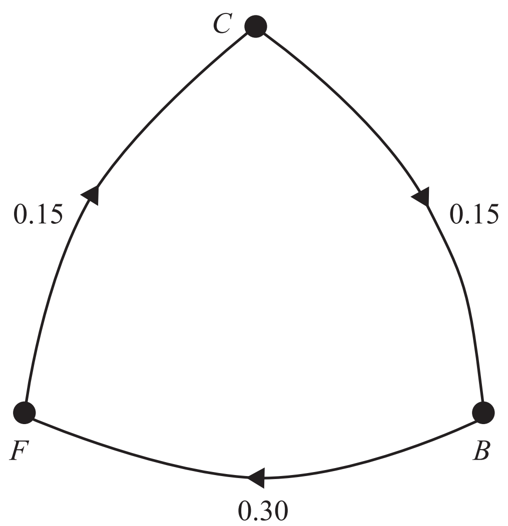
$$S_0 = \begin{bmatrix} 0.4 \\ 0.4 \\ 0.2 \end{bmatrix} \begin{matrix} C \\ F \\ B \end{matrix}$$

The recurrence relation that is used to calculate the expected proportion of customers who purchased each type of egg is given by

$$S_{n+1} = T \times S_n \qquad \text{where} \qquad \begin{matrix} & \textit{this week} \\ & \begin{matrix} C & \quad F & \quad B \end{matrix} \\ T = & \begin{bmatrix} 0.65 & 0.15 & 0.25 \\ 0.20 & 0.70 & 0.30 \\ 0.15 & 0.15 & 0.45 \end{bmatrix} \begin{matrix} C \\ F \\ B \end{matrix} \quad \textit{next week} \end{matrix}$$

a. An incomplete transition diagram for Matrix T is shown below.

Complete the transition diagram by adding the missing edges and the missing proportions. 1 mark



b. How many weeks after the campaign begins will more than 44% of the customers first be expected to purchase free-range eggs (F)? 1 mark

c. For the duration of the 10-week campaign, 2000 customers purchased eggs at this supermarket each week.

Identify the expected number of additional shoppers who purchased free-range eggs (F) at the end of the campaign compared to the start of the campaign.

Round your answer to the nearest whole number. 1 mark

d. The farmer believes that their free-range eggs are so superior that shoppers will eventually stop purchasing cage eggs and barn-laid eggs, and only buy their free-range eggs.

Complete column F in the following transition matrix for a scenario in which shoppers will eventually stop purchasing cage eggs and barn-laid eggs, and buy only free-range eggs. 1 mark

$$\begin{array}{c} \textit{this week} \\ \begin{array}{ccc} C & F & B \end{array} \end{array}$$

$$\begin{bmatrix} 0.65 & \_\_\_ & 0.25 \\ 0.20 & \_\_\_ & 0.30 \\ 0.15 & \_\_\_ & 0.45 \end{bmatrix} \begin{array}{l} C \\ F \\ B \end{array} \quad \textit{next week}$$

Question 13 (3 marks)

A farmer has four fields for chickens to roam in during the day.

The fields are named North (N), East (E), South (S) and West (W).

The following transition matrix is used to determine the expected proportion of chickens that will either change fields from one day to the next, or stay in the same field on consecutive days.

There are initially $10\,000$ chickens in each field.

$$T = \begin{array}{c} \\ \\ \end{array}\overset{\textit{this day}}{\begin{array}{cccc} N & E & S & W \end{array}}$$

$$T = \begin{bmatrix} 0.30 & 0.12 & 0.30 & 0.20 \\ 0.14 & 0.28 & 0.25 & 0.44 \\ 0.24 & 0.40 & 0.12 & 0.17 \\ 0.32 & 0.20 & 0.33 & 0.19 \end{bmatrix} \begin{matrix} N \\ E \\ S \\ W \end{matrix} \quad \textit{next day}$$

a. After one day, how many chickens are expected to remain in the same field that they were in initially? 1 mark

b. After two days, 8962 chickens are expected to be in North field.

How many of these 8962 chickens are expected to have been in North field initially? 1 mark

c. Of the chickens expected to be in East field after three days, what is the percentage of chickens that were also expected to be in East field after both one day and two days?

Round your answer to the nearest whole number. 1 mark

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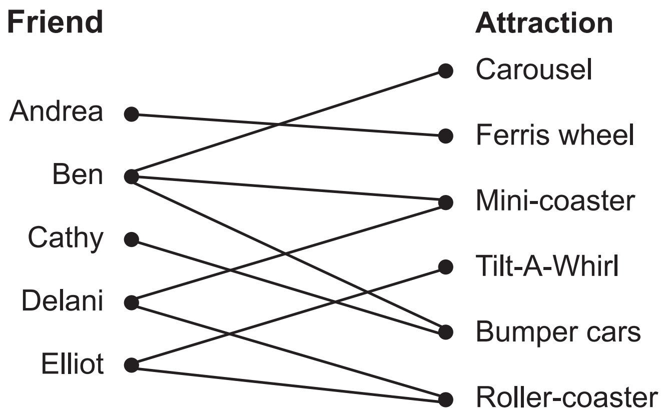
Networks and decision mathematics

Question 14 (3 marks)

Andrea, Ben, Cathy, Delani and Elliot are a group of friends.

They are visiting a theme park that has six attractions: a Carousel, a Ferris wheel, a Mini-coaster, a Tilt-A-Whirl, Bumper cars and a Roller-coaster.

The following bipartite graph illustrates the attractions each friend plans to visit.



a. Which friend plans to visit the most attractions? 1 mark

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b. How many attractions have only one friend planning to visit? 1 mark

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c. When the friends arrive at the theme park, there is only time for each friend to visit one of the attractions that they had planned to visit.

If the minimum number of different attractions are visited, list the names of the attractions that are visited. 1 mark

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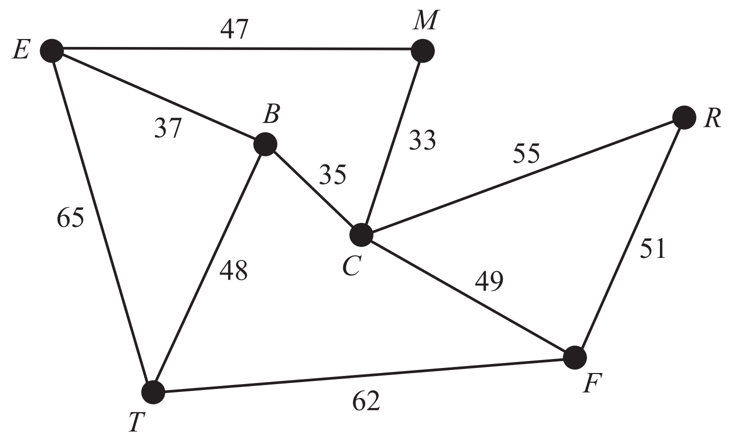
Question 15 (4 marks)

The theme park entrance (E), Roller-coaster (R), Mini-coaster (M), Bumper cars (B), Carousel (C), Tilt-A-Whirl (T), and Ferris wheel (F) are represented by the vertices of the graph below.

The edges on the graph represent the paths within the theme park.

The numbers on each edge represent the length, in metres, of each path.



a. Rowan works for the theme park collecting money from the ticket booths at each of the attractions.

To do this, he visits each attraction once, starting and finishing at the entrance.

i. What is the mathematical term used to describe this route? 1 mark

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ii. Write down one possible route Rowan could take. 1 mark

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An inspector is called to check all the paths in the theme park. The inspector would like to walk a route that travels along all the paths once only, starting at the entrance.

b. With reference to the degrees of the vertices on the graph on page 22, explain why the inspector is not able to walk such a route. 1 mark

The inspector decides that the path between the Bumper cars (B) and the Tilt-A-Whirl (T) needs to be closed for maintenance.

The next day Rowan will again need to collect money from the ticket booths at each of the attractions, starting and finishing at the entrance. To do this, Rowan may need to visit one or more attractions twice.

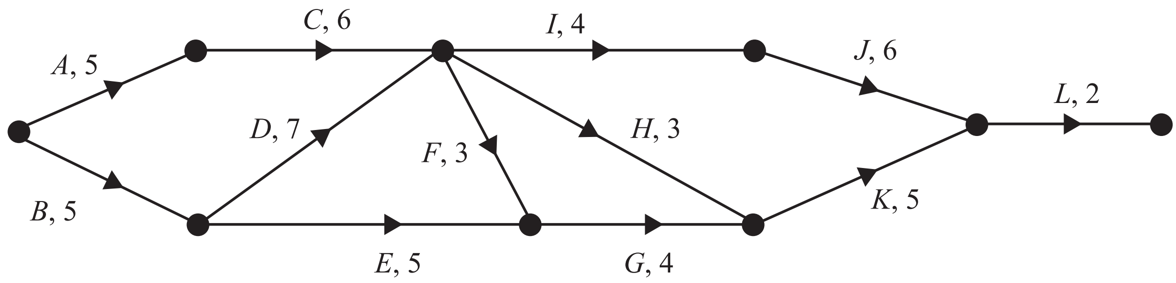
c. If Rowan visits each attraction, starting and finishing at the entrance, what is the length of the shortest path Rowan can take, in metres? 1 mark

Question 16 (5 marks)

The Tilt-A-Whirl attraction is being upgraded. It will be closed for the duration of the upgrade.

The upgrade involves 12 activities, A to L.

The directed network below shows the activities and their completion times, in days.



a. List the activities that have exactly one immediate predecessor. 1 mark

b. What is the minimum number of days the Tilt-A-Whirl will need to be closed to complete this project? 1 mark

c. What is the latest starting time, in days, for activity I? 1 mark

d. Which activity has the longest float time? 1 mark

The management of the theme park decides that the Tilt-A-Whirl attraction will be closed for too long to complete this project. They give the project manager a budget to reduce the overall completion time.

The project manager is able to hire extra people to reduce the time of some activities, represented in the table below. Each of the activities can be reduced by a maximum of two days.

Activity	Daily cost
A	$2000
B	$2000
D	$2500
E	$1000
G	$1500
H	$1200

e. Complete the table below, showing the reductions in individual activity times that would achieve the maximum reduction in completion time for the minimum cost. 1 mark

Activity	Reduction in completion time (0, 1 or 2 days)
A	
B	
D	
E	
G	
H	

End of examination questions

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2024

NHT

General Mathematics
Examination 2

Formula Sheet

You may keep this Formula Sheet.

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General Mathematics formulas

Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $\text{Q1} - 1.5 \times \text{IQR}$ upper $\text{Q3} + 1.5 \times \text{IQR}$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	$\text{residual value} = \text{actual value} - \text{predicted value}$
seasonal index	$\text{seasonal index} = \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = Ru_n + d$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	$S_0 = \text{initial state}$, $S_{n+1} = T\,S_n + B$
Leslie matrix recurrence relation	$S_0 = \text{initial state}$, $S_{n+1} = L\,S_n$

Networks and decision mathematics

Euler's formula	$v + f = e + 2$

End of Formula Sheet