NAME: _____

VCE® General Mathematics

UNITS 3 & 4 Practice Written Examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
A	**14**	**14**	**60**
			Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, one bound reference, one approved technology.
- Students are NT permted to brin into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 26 pages.
- Additional space is available at the end of the book if you need extra paper to complete an answer.

Instructions
- Write your **student name** in the space provided above on this page.
- Unless otherwise indicated, diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions
Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, you should only round your answer when instructed to do so.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Data Analysis

Question 1 (9 marks)

The table below shows the results of a 100 m running race, including the lane number the runner was in, the nation the runner was from, the time it took to run 100 m, and the event type.

Lane	Nation	Time	Event
3	taly	9.80	Final
5	United States	9.84	Final
9	Canada	9.89	Final
4	China	9.83	Semi Final
6	United States	9.83	Semi Final
8	Great Britain	9.98	Semi Final
7	Nigeria	10.00	Semi Final
7	United States	9.96	Semi Final
6	Canada	9.98	Semi Final
1	Australia	10.01	Heat
5	Jamaica	10.06	Heat
3	France	10.24	Heat

Lane	Nation	Time	Event
4	South Africa	10.08	Heat
3	Ivory Coast	10.15	Heat
9	Canada	9.91	Heat
3	United States	9.97	Heat
2	South Africa	10.04	Heat
4	China	10.05	Heat
4	taly	9.94	Heat
9	Jamaica	10.04	Heat
6	Nigeria	9.98	Heat
3	Qatar	10.02	Heat
8	United States	10.03	Heat

a. Complete the table below for the type and sub type of the data.   1 mark

Variable	Type	Sub Type
Lane		
Nation	Categorical	Nominal
Time		
Event	Categorical	Ordinal

b. Complete the Frequency Table below to two decimal places.   1 mark

Time	Frequency	Percentage Frequency
9.80 < 9.9		
9.9 < 10.00		
10.00 < 10.10		
10.10 < 10.20		
10.20 < 10.30		
Total	23	

c. Show that there is an outlier in the data set.   2 marks

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d. Draw a boxplot of the winning times on the axis below.   1 mark
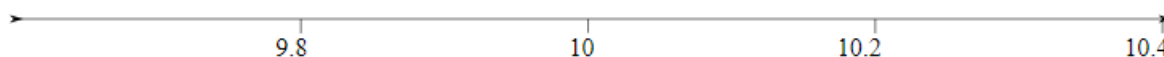
e. Find the **mean** and **standard deviation**, to four significant figures, for the data set.

2 marks

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f.   Usain Bolt holds the record of the fastest ever 100 m sprint.
Assuming that the data is normally distributed, what Z-score would his time of 9.58 seconds
have got in this event?   1 mark

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g.   If the data set is normally distributed as per the mean and standard deviation in
Part e, what is the percentage of results that fall between 9.8751 seconds
and 10.2947?

1 mark

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Question 2 (8 marks)

In analysing the 100 m sprint, an equation was found that relates the number of steps taken and the time it takes to complete the sprint. The equation is shown below.

$$Race\ Time = 6.81 + 0.07(\ number\ of\ steps\ )$$

a. For the equation of Race Time and Number of Steps, which is the explanatory variable?
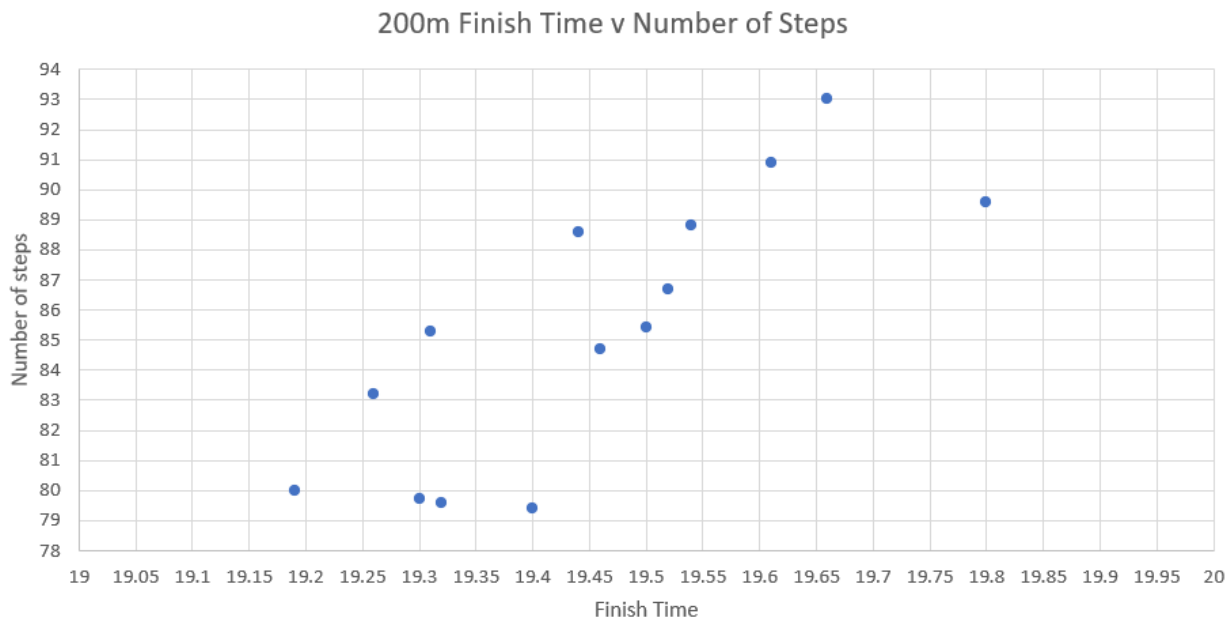
1 mark

b. Explain the meaning of the intercept value in terms of the variables involved.   1 mark

Below is a graph and data for the number of steps taken to complete the 200 m race against the finish time. The data results in an equation of:

Steps to complete **200***m* = **- 334.3 + 21.58** *(finish time)*

200m Finish Time v Number of Steps

Finish Time	Steps
19.19	80
19.3	79.7
19.32	79.6
19.4	79.4
19.26	83.2
19.44	88.6
19.54	88.8

Finish Time	Steps
19.8	89.6
19.31	85.3
19.46	84.7
19.5	85.4
19.52	86.7
19.61	90.9
19.66	93

c. The graph can best be described with a strength and direction of: 1 mark

_____

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d. The correlation coefficient to four decimal places is: 1 mark

_____

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e. The percentage of variation of the number of steps caused by the variation in the finish time, to two decimal places, is: 1 mark

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f. On the graph on Page 6, plot the regression equation.   1 mark

g. Determine the amount of time, to two decimal places, it would take to finish the race f an athlete took 100 steps.   1 mark

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h. Explain the accuracy of the answer from **Part g.**   1 mark

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Question 3 (7 marks)

The results of the 100 metre male Olympic sprint times over the years are shown in the graph below.

100 meter finish time 1900-2020
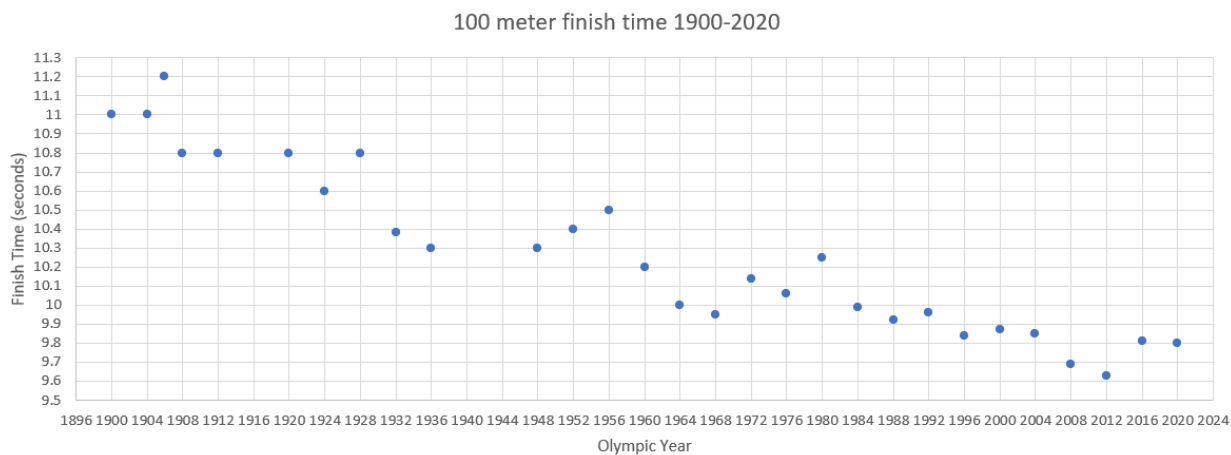
a. Describe any features that are evident on the graph above?   1 mark

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b. By using 5 mean smoothing, show that the smoothed value for 1980 is 10.07.   1 mark

Year	1964	1968	1972	1976	1980	1984	1988	1992	1996
Time	10	9.95	10.14	10.06	10.25	9.99	9.92	9.96	9.84

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c. Using the table of smoothed data below, determine the regression equation for the smoothed data to 3 decimal places. 1 mark

Year	Result	Smoothed Value
1906	11.2	10.96
1908	10.8	10.92
1912	10.8	10.84
1920	10.8	10.76
1924	10.6	10.676
1928	10.8	10.576
1932	10.38	10.476
1936	10.3	10.436
1948	10.3	10.376
1952	10.4	10.34
1956	10.5	10.28
1960	10.2	10.21
1964	10	10.158
1968	9.95	10.07
1972	10.14	10.08
1976	10.06	10.078
1980	10.25	10.072
1984	9.99	10.036
1988	9.92	9.992
1992	9.96	9.916
1996	9.84	9.888
2000	9.87	9.842
2004	9.85	9.776
2008	9.69	9.77
2012	9.63	9.756

d. Using the equation from **Part c.**, predict the finishing time of the 100 metre race in the 2024 Olympics?   1 mark

When looking at results for a running race, the wind speed is often mentioned. Depending on the direction of the wind this can aid or hamper a runner. The **seasonal indices** for both December and September are found to be **0.871 & 1.132** respectively.

e. What do these two seasonal indices mean?   2 marks

f. The actual wind speed recorded in September is 17.3 km/h.
What is the de-seasonalised value for this result?   1 mark

Recursion and Financial Modelling

Question 4 (3 marks)

A phone is purchased for $2899, and depreciates per year following the rule below.

$$V_0 = 2899, \quad V_{n+1} = V_n - 434.85$$

a. State the first **three** terms of the depreciation sequence.   1 mark

b. Determine the percentage the phone depreciates per year.   1 mark

c. Determine how many years, to two decimal places, it would take to reach the scrap value of $30.   1 mark

Question 5 (5 marks)

Beth was able to save $27 980 for a house. She invested this savings into an account that earnt her 5.8% pa interest, paid monthly.

a. Show that after one month, Beth will have earnt interest of $135.24.   1 mark

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b. State the recurrence relation the account will follow to five significant figures.   1 mark

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c. Beth hopes to put a deposit on a house in a year and a half.
How much will her account be worth at that time?   1 mark

Beth finds another bank offering her an account that pays 5.5% per annum, paid quarterly.

d. What is the effective interest rate of both account options?   1 mark

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e. Ideally Beth intends to use a deposit of $63 000. When will each option enable her to make that deposit?   1 mark

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Question 6 (4 marks)

Beth has found a loan for $630 000. She is required to make fortnightly repayments of $1817.92 as a minimum. During the first fortnight she is charged $1550.77 in interest.

a. What is the interest rate this loan would be charging her?   1 mark

b. State the recurrence relation the account will follow to seven significant figures.   1 mark

c. Complete the annuity table below for the third fortnight.   1 mark

Fortnight	Payment	Interest	Reduction	Balance
0	0	0	0	630 000
1	1817.92	1550.77	267.15	629 732.85
2	1817.92	1550.11	267.81	629 465.04
3				

d. What is the value of Beth's final payment?   1 mark

Matrices

Question 7 (2 marks)

Use matrix K below to answer the following questions.

$$K = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$$

a. What is the order of matrix K?   1 mark

b. State the determinant of matrix K.   1 mark

Question 8 (3 marks)

a. State the permutation matrix that is needed to convert matrix A below into matrix B.   1 mark

$$A = \begin{bmatrix} E \\ L \\ V \\ I \\ S \end{bmatrix} \qquad B = \begin{bmatrix} L \\ I \\ V \\ E \\ S \end{bmatrix}$$

b. The following are results from a round-robin competition between 4 teams: A, B, C and D. State the matrix that would display this information.

The first line implies that Team A beat Team B   1 mark

$A \rightarrow B$

$A \rightarrow D$

$B \rightarrow C$

$B \rightarrow D$

$C \rightarrow A$

$D \rightarrow C$

c. Determine the result order of the round-robin in **Part b.** by first finding the 2
step dominance matrix, and the sum of the results of the 1 and 2 step matrices.   1 mark

Question 9 (4 marks)

A local trout farm monitors its stock month to month, and categorises the stock as either J = Juvenile, Y = Young, A = Adult or D = dead. The stock appears to follow the transition matrix below.

$$\begin{array}{c} from \\ \begin{array}{cccc} J & Y & A & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.9 & 0.3 & 0 & 0 \\ 0 & 0.65 & 0.8 & 0 \\ 0.1 & 0.05 & 0.2 & 1 \end{bmatrix} \begin{array}{c} J \\ Y \\ A \\ D \end{array} \; to \end{array}$$

a. Interpret the bottom row of the transition matrix, in terms of the trout.   1 mark

b. If the farmer was to buy 500 Juveniles, what percentage will be left after two months?

1 mark

c. What is the long term outcome of the trout at this farm?   1 mark

d. To stay in business, the farmer has found that he needs to maintain a stock level of 300 babies, 1000 young and 3000 adults.

State the recursion relation needed for this if the initial state is $S_0 = \begin{bmatrix} 300 \\ 1000 \\ 3000 \\ 0 \end{bmatrix}$ 1 mark

Question 10 (3 marks)

The life cycle and breeding of a chipmunk is shown in the table below.

Age	0	1	2	3	4	5
Birth Rate	0	1.6	2.4	4.9	4.3	0.2
Survival Rate	0.91	0.88	0.73	0.65	0.62	0
Initial female Population	2014	345	194	131	104	62

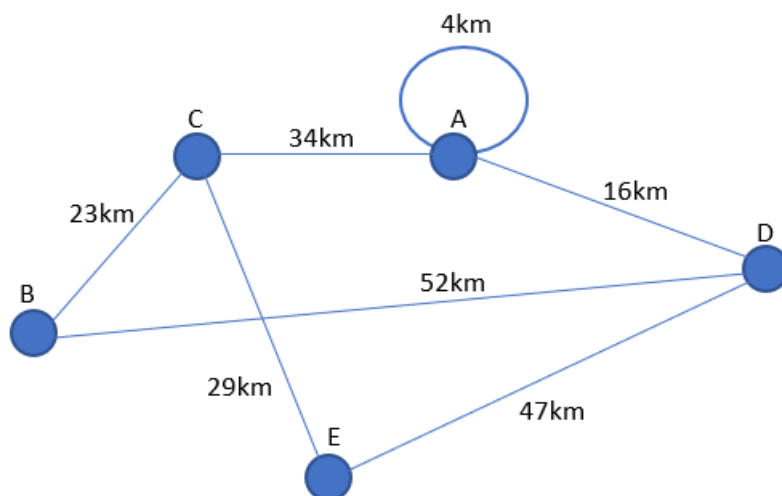**a.** Convert the table into a Leslie Matrix.   1 mark

b. Determine the number of female chipmunks in two years' time   1 mark

c. Determine the total number of chipmunks in eight years' time.   1 mark

Networks and Decision Mathematics

Question 11 (4 marks)

a.   Redraw the graph below as planar.   1 mark

b.   Show that the graph above, when in planar form, satisfies Euler's Formula.   1 mark

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**c.** If the graph above was showing roads between towns, and a gas line needed to be replaced, state the type of graph needed to do this.   1 mark

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d. Determine the length of the gas line that would be needed.   1 mark

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Question 12 (2 marks)

To replace an old gas line, **four** companies apply for the project roles to be completed. Each of the companies will be allocated one of the aspects of the job to make the project the most efficient. The proposals are as below.

Company	Excavation	Removal	Replace	Finish
Ace	$22 500	$8000	$7000	$11 200
Best	$19 200	$6250	$7300	$11 000
Con's	$23 000	$6000	$7220	$12 000
Dodgy	$19 250	$6900	$7000	$15 000

To achieve the minimum cost, which company should be allocated what task? Complete a reduction and then bipartite graph to show the allocations.

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Question 13 (3 marks)

The flow of gas in a station is shown below, with the capacity of the pipes shown.
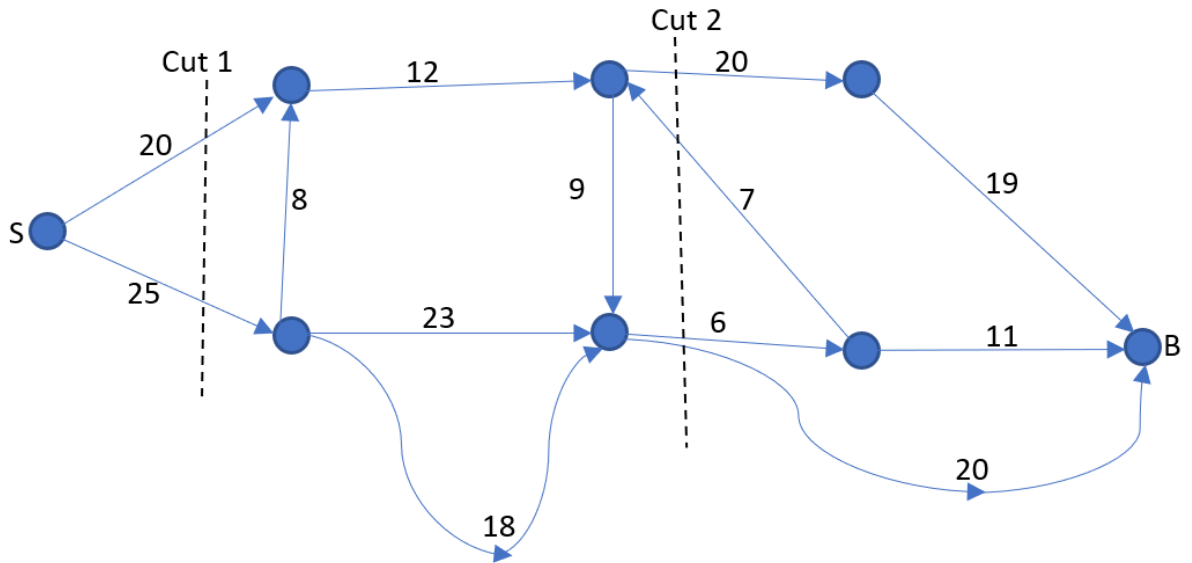
a.   The shortest path for the gas to flow from S to B is.   1 mark

b.   What is the capacity of each of Cut 1 and Cut 2?   1 mark

c.   Determine the maximum flow of gas through the internal pipe system shown above.   1 mark

Question 14 (3 marks)

Before the gas is released from the factory, there are **nine** steps that need to be completed as per the activity table below.

Activity	EST	Duration	Predecessor
A	0	5	-
B	0	7	-
C	5		A
D	7	6	
E	8	2	C
F		8	C
G	13	4	E, D
H	16	2	F
I	18	2	G, H

a. Complete the table above. 1 mark

b. Determine the critical path and duration. 1 mark

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c. Determine the float time for the activities not on the critical path 1 mark

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END OF EXAMINATION

Extra space if required.
Clearly number all responses in this space

VCE® General Mathematics

Unit 3 and 4 Practice Written Examination 2

ADVICE FOR TEACHERS

Solution Pathway

Below are sample answers. Please consider the merit of alternative responses.

Question 1 (9 marks)

a.   **1 mark**

Variable	Type	Sub Type
Lane	Categorical	Nominal
Nation	Categorical	Nominal
Time	Numerical	Continuous
Event	Categorical	Ordinal

- Mark only if all 4 values are correct. Lane is a marker not a measure and Time is a measure (continuous)

b.   **1 mark**

Time	Frequency	Percentage Frequency
9.80 < 9.9	5	21.74
9.9 < 10.00	7	30.43
10.00 < 10.10	9	39.13
10.10 < 10.20	1	4.35
10.20 < 10.30	1	4.35
Total	23	100

- Mark only if all values are correct to 2 decimal places.

$lower\ fence: Q_1 - 1.5(IQR) = 9.91 - 1.5(0.13) = 9.715$

$Upper\ fence: Q_3 + 1.5(IQR) = 10.04 + 1.5(0.13) = 10.235,$
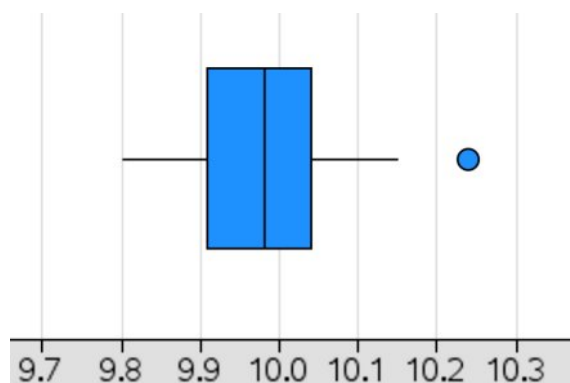
as 10.24 *is larger than this value it is an outlier*

c.   **2 marks**

- **1 mark** for both the upper and lower fence values correct.
- **1 mark** for the explanation as to why 10.24 is an outlier.

d.   **1 mark**

Must include the outlier and have the upper whisker finish at 10.15

e. $\bar{x} = 9.984$, $S_x = 0.1049$ **2 marks**

- **1 mark** for <u>each</u> correct value

f. $z = \dfrac{9.58 - 9.98}{0.1049} = -3.81$ **1 mark**

g. 9.8751 is 1 deviation below, 10.2987 is 3 deviations above, thus 83.85%   **1 mark**

Question 2 (9 marks)

a. *Number of steps is the explanatory variable*   **1 mark**

b. *When the number of steps is 0, the Race Time is predicted to be 6.81 seconds.* **1 mark**
Similar responses can be accepted.

c. *Strong Positive* is the strength and direction   **1 mark**
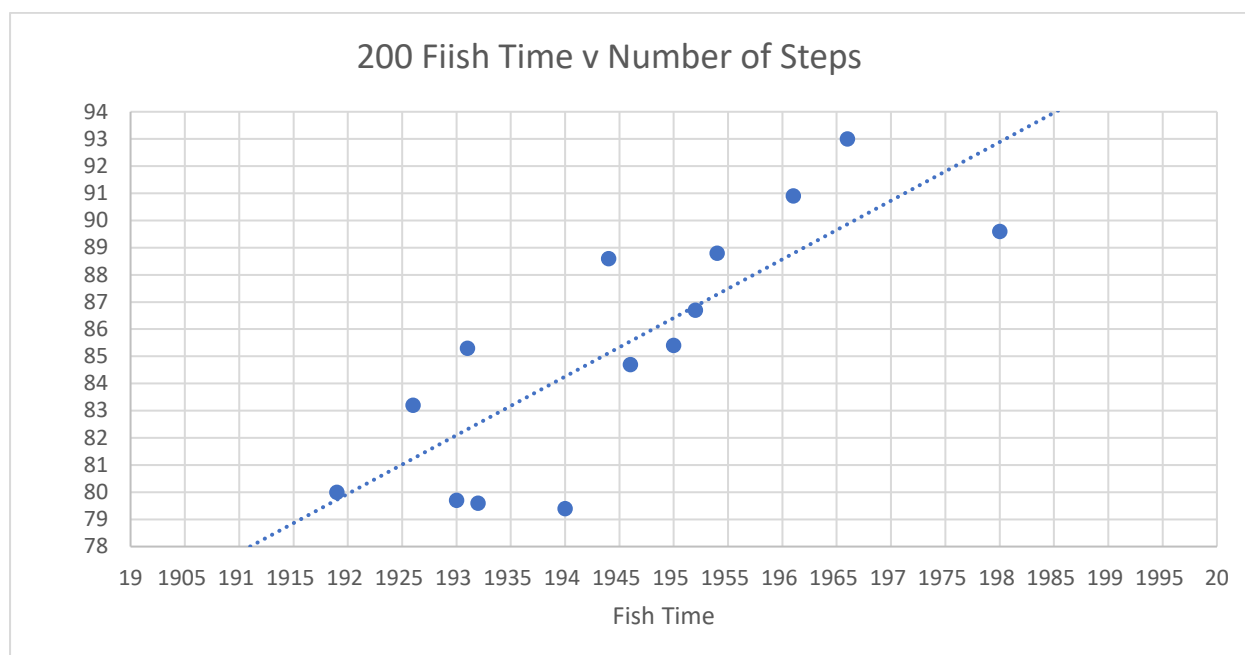
d. *0.8084*   **1 mark**

e. *65.35%*   **1 mark**

f.   *Graph as below, or very similar*   **1 mark**

g.   *2013 sec*   **1 mark**

$$100 = -334.3 + 21.58(Time)$$
$$Time = 20.13$$

h.   *Not accurate as the result is extrapolated outside the data set*   **1 mark**

Question 3 (7 marks)

a.   The graph shows a *decreasing trend* only.   **1 mark**

b.   Show that question, must show the steps.

$$\frac{10.14 + 10.06 + 10.25 + 9.99 + 9.92}{5} = 10.072 \cong 10.07$$ **1 mark**

c. $Time = 31.929 - 0.011(Year)$ **1 mark**

d.   *9.665 seconds.*

$$Time = 31.929 - 0.011(2024) = 9.665$$

Answer may be slightly different if different equation was formed in Part c. Award consequential marks for this.   **1 mark**

e. *0.871 means 12.9% below average.*

    1.132 means 13.2% above average   **2 marks**

- **1 mark** for <u>each</u> correct statement.

f. *15.28km/h*

$$de-seasonalised = \frac{17.3}{1.132} = 15.28$$ **1 mark**

Question 4 (3 marks)

$$V_0 = 2899$$
$$V_1 = 2464.15$$
$$V_2 = 2029.30$$

a.   **1 mark**

b. *15%*

$$\frac{434.85}{2899}100 = 15$$ **1 mark**

c. *660 years*

$$30 = 2899 - 434.85n$$
$$n = 6.59768 \approx 6.60$$ **1 mark**

Question 5 (5 marks)

a. **"Show that" question: Students must show all working.**

$$\frac{27980 \times \frac{5.8}{12}}{100} = 135.2366 \cong 135.24$$ **1 mark**

b. $V_0 = 27980, \; V_{n+1} = 1.0048V_n$

    1.0048 is from $\dfrac{\frac{5.8}{12}}{100} + 1 = 1.0048$ **1 mark**

c. *$3051689*

    18 months is a year and a half, thus $1.0048^{18}(27980) = 30516.89$ **1 mark**

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d. *5.96% and 5.61%*

1 mark

First method: $\left[\left(1+\dfrac{5.8}{100(12)}\right)^{12}-1\right]\times100=5.956694\cong5.96$

Second method: $\left[\left(1+\dfrac{5.5}{100(4)}\right)^{4}-1\right]\times100=5.61448\cong5.61$

e. *169 months and 60 quarters*   **1 mark**

Using finance solving, method 1 is 169 months, or 14 years 1 month.

Method 2 will be 60 quarters or 15 years

Question 6 (4 marks)

a. *6.4%*   **1 mark**

$\dfrac{1550.77\times26}{100}=6.4$

b. $V_0=630000,\ V_{n+1}=1.00246V_n-1817.92$ **1 mark**

1.00246 is from $\dfrac{1550.77}{630000}+1=1.00246$

c. Table as below   **1 mark**

3	181792	154945	26847	62919657

d. *Final payment is $1813.71*   **1 mark**

Use finance solver to find 779 normal payments. Final payment will require $1809.26 before interest. Once interest is added increases to 1813.71356

Question 7 (2 marks)

a. *2x2*   **1 mark**

   2 rows and 2 columns

b. *Can be in equivalent form,* $\begin{bmatrix} \frac{5}{7} & -\frac{2}{7} \\ -\frac{4}{7} & \frac{3}{7} \end{bmatrix}$ *or* $\frac{1}{7}\begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}$ *or* $\begin{bmatrix} 0.71 & -0.29 \\ -0.57 & 0.43 \end{bmatrix}$ **1 mark**

Question 8 (3 marks)

a. $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$ **1 mark**

b.
$$\begin{array}{c} loss \\ A\ B\ C\ D \\ \begin{array}{c} A \\ win\ \ B \\ C \\ D \end{array} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \end{array}$$ **1 mark**

c. *A, B, C, D*   **1 mark**

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, summed = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \end{bmatrix}$$

Question 9 (4 marks)

a.   *10% of baby trout die*
5% of young die
20% of adults die
100% of dead fish remain dead   **1 mark**

Similar wording can be used.

b.   *85.5%*

$$T^2 \times \begin{bmatrix} 500 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 135 \\ 292.5 \\ 72.5 \end{bmatrix} so\ 427.5\ remain\ \frac{427.5}{500} = 85.5\%$$ **1 mark**

c.   *All the fish will die.*

Similar wording can be accepted.   **1 mark**

d. $S_0 = \begin{bmatrix} 300 \\ 1000 \\ 3000 \\ 0 \end{bmatrix}$, $S_{n+1} = T \times S_n + \begin{bmatrix} 300 \\ 430 \\ -50 \\ 0 \end{bmatrix}$ **1 mark**

Question 10 (3 marks)

a. $L = \begin{bmatrix} 0 & 1.6 & 2.4 & 4.9 & 4.3 & 0.2 \\ 0.91 & 0 & 0 & 0 & 0 & 0 \\ 0 & .88 & 0 & 0 & 0 & 0 \\ 0 & 0 & .73 & 0 & 0 & 0 \\ 0 & 0 & 0 & .65 & 0 & 0 \\ 0 & 0 & 0 & 0 & .62 & 0 \end{bmatrix}$ **1 mark**

b.   *8642*   **1 mark**

$$L^2 \times \begin{bmatrix} 2014 \\ 345 \\ 194 \\ 131 \\ 104 \\ 62 \end{bmatrix} = \begin{bmatrix} 4734 \\ 1928.38 \\ 1612.81 \\ 221.63 \\ 92.05 \\ 52.79 \end{bmatrix}$$ **which sums to 8642**
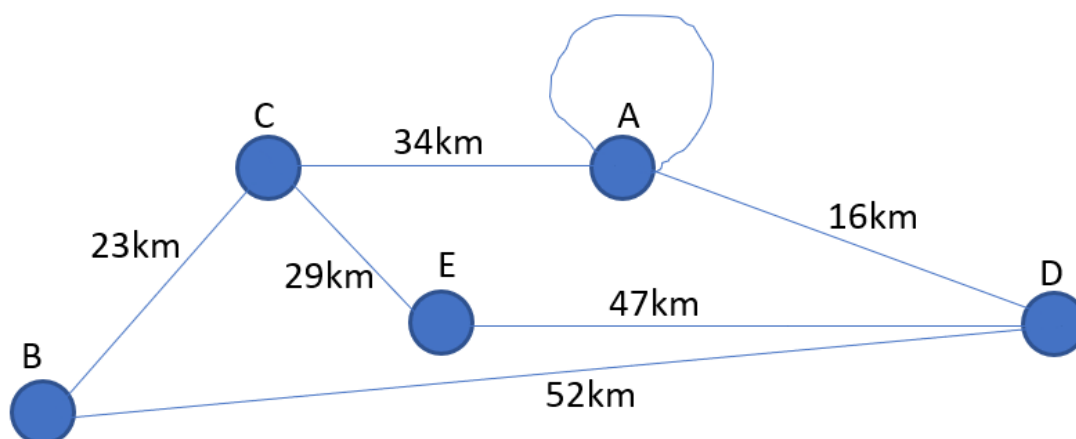
c. *764284*   **1 mark**

$$L^8 \times \begin{bmatrix} 2014 \\ 345 \\ 194 \\ 131 \\ 104 \\ 62 \end{bmatrix} = \begin{bmatrix} 208300.91 \\ 100302.85 \\ 46958.11 \\ 17992.36 \\ 6598.47 \\ 1990.36 \end{bmatrix}$$

the result is then summed and doubled to get total population and not just females.

Question 11 (4 marks)

a. Graph can be drawn many ways. As long as all correct data is included mark should be awarded.   **1 mark**

For example:

b. $V - E + f = 2$ **1 mark**
$5 - 7 + 4 = 2$ *as required*

c. *Minimum spanning tree*   **1 mark**

d. *102 km*   **1 mark**

16+34+23+29

Question 12 (2 marks)

a.   *Ace = Replace, Best = Finish, Con's = Remove, Dodgy = Excavate*   **2 marks**

1 mark for correct allocation.
1 mark for correct reduction to produce bipartite graph.

$$\begin{bmatrix} 22500 & 8000 & 7000 & 11200 \\ 19200 & 6250 & 7300 & 11000 \\ 23000 & 6000 & 7220 & 12000 \\ 19250 & 6900 & 7000 & 15000 \end{bmatrix} \text{column and row reduction}$$

$$\begin{bmatrix} 3150 & 1000 & 0 & 0 \\ 600 & 0 & 1050 & 550 \\ 4650 & 0 & 1220 & 1800 \\ 0 & 0 & 100 & 3900 \end{bmatrix} \text{cover 0's, add min to double covered, subtract from non covered}$$

$$\begin{bmatrix} 3250 & 1100 & 0 & 0 \\ 600 & 0 & 950 & 450 \\ 4650 & 0 & 1120 & 1700 \\ 0 & 0 & 0 & 3900 \end{bmatrix} \text{repeat}$$

$$\begin{bmatrix} 3250 & 1550 & 0 & 0 \\ 150 & 0 & 500 & 0 \\ 4200 & 0 & 670 & 1250 \\ 0 & 450 & 0 & 3900 \end{bmatrix} \text{final}$$

Allocation

Ace — Excavation
Best — Removal
Con's — Replace
Dodgy — Finish
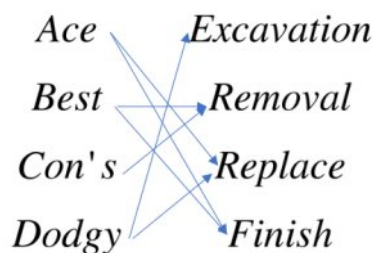
Question 13 (3 marks)

a. *58*   **1 mark**

  20+12+9+6+11 be aware of arrow direction

b. *Cut 1=45, Cut 2 =46*   **1 mark**

c. *37*   **1 mark**

  Cut across 25, 8 and 12. 8 is flowing into the back of the cut, thus not counted.

Question 14 (3 marks)

a. *Est = 8, Duration = 3, Predecessor = B*   **1 mark**

b. *A-C-F-H-F*   **1 mark**

c. *B=1, D=1, E=4, G=1*   **1 mark**