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General Mathematics

Written Examination 2

2024 Insight Year 12 Trial Exam Paper

Worked Solutions

This book presents:

- worked solutions
- mark allocations
- tips.

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Data analysis

Question 1a.

Worked solution

Despite being in numerical form, *site number* is categorical as the site number is assigned, like the name of a group of items.

Similarly, *distance travelled* has been presented in various groups, making it a categorical variable.

Repeat booking is also categorical, indicated by the *yes* or *no* response provided.

Mark allocation: 1 mark

- 1 mark for the correct answer: *site number, repeat booking* and *distance travelled*

Tip

- *Some variables that are in numerical form are still considered categorical, as they are an assigned grouping or ranking.*

Question 1b.

Worked solution

When the values are placed in order, the middle value is found to be 7. This can be found using the statistics function on the calculator.

Mark allocation: 1 mark

- 1 mark for the correct answer: 7

Question 1c.

Worked solution

The mean, i.e. the average of the scores, is 3.95, which is 4 rounded to the nearest whole number.

Mark allocation: 1 mark

- 1 mark for the correct answer: 4

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Question 1d.

Worked solution

There are 7 sites out of 20 that had campers staying between 1 and 4 nights: $\frac{7}{20} \times 100 = 35\%$

Mark allocation: 1 mark

- 1 mark for the correct answer: 35

Question 1e.

Worked solution

Length of stay (nights)	Repeat booking	
	Yes	No
1–3	4	1
4–7	4	7
8+	1	3
Total	9	11

Mark allocation: 1 mark

- 1 mark if all values are correct

Question 1f.

Worked solution

$\text{IQR} = 41 - 21 = 20$

Upper fence $= 41 + 1.5 \times 20 = 71$

76 is above the upper fence of 71, so it is an outlier.

Mark allocation: 2 marks

- 1 mark for correctly calculating the upper fence: 71
- 1 mark for concluding that since $76 > 71$, it follows that 76 is an outlier

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Question 2a.i

Worked solution

$0.475 \times 400 = 190$

Mark allocation: 1 mark

- 1 mark for the answer of 190

Question 2a.ii

Worked solution

16% were at the river less than 55 minutes, so 84% were there longer than 55 minutes.

$0.84 \times 400 = 336$

Mark allocation: 1 mark

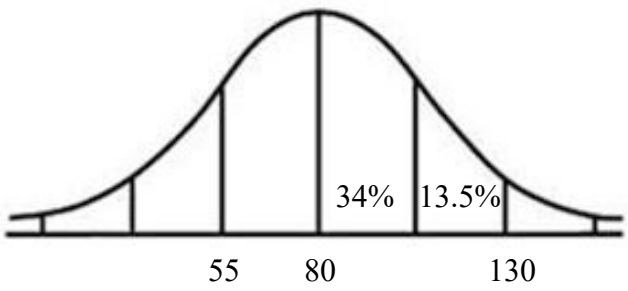
- 1 mark for the correct answer: 336

Question 2b.

Worked solution

$47.5\% = 34\% + 13.5\%$, which means that two standard deviations are between the times of 80 and 130 minutes. This implies that one standard deviation is 25 minutes.

This is also consistent with 55 minutes being one standard deviation below the mean of 80 minutes.



Mark allocation: 2 marks

- 1 mark for the correct mean: 80 minutes
- 1 mark for the correct standard deviation: 25 minutes

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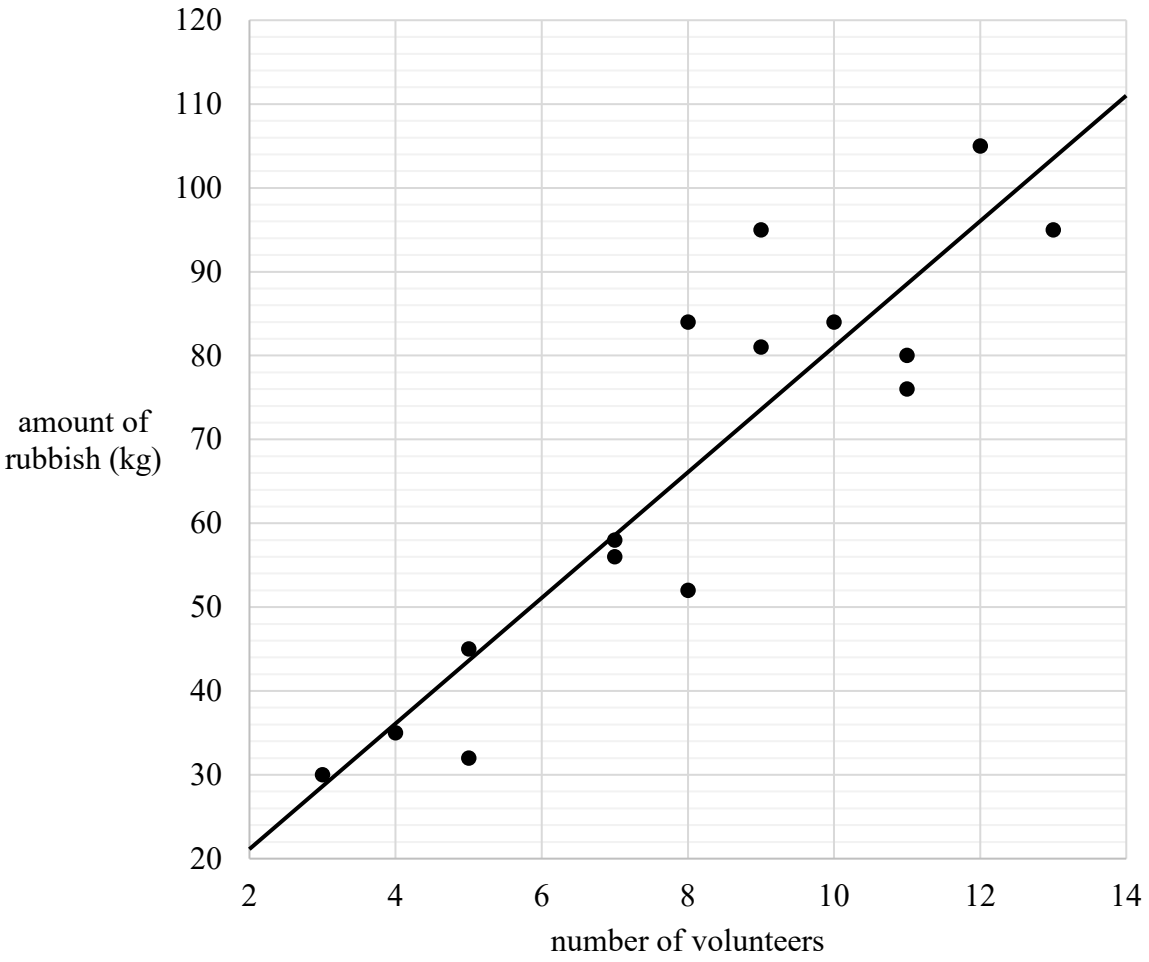
Question 3a.

Worked solution

Substituting the end points of the explanatory axis (2 and 14) into the equation will identify where the regression line should begin and end on the graph.

$x = 2$ $\quad 6.55 + 7.46 \times 2 = 21.47$ $\quad$ start point = (2, 21.5)

$x = 14$ $\quad 6.55 + 7.46 \times 14 = 110.99$ $\quad$ end point = (14, 111)



Mark allocation: 1 mark

- 1 mark for placing the start point of the regression line between 20 and 22 kg and the end point between 110 and 112 kg

Note: Award no marks if the line falls outside either of these ranges.

Tip

- *When drawing a least squares regression line, you should always extend to the ends of the x-axis.*

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Question 3b.

Worked solution

For an increase of one volunteer, the amount of rubbish collected is expected to increase by 7.46 kg.

Mark allocation: 1 mark

- 1 mark for a sentence referring to an increase of one volunteer resulting in an increase of 7.46 kg of rubbish collected

Note: Award no marks if the sentence refers to 'for each volunteer' or similar, instead of referring to an *increase in one volunteer.*

Tip

- *The interpretation of the slope requires careful wording. It should be expressed in a way that makes clear that an increase of one unit in the explanatory variable results in the increase or decrease of the response variable.*

Question 3c.

Worked solution

Using the data in Table 2, $r = 0.9035\ldots = 0.904$

Mark allocation: 1 mark

- 1 mark for the correct answer: 0.904

Question 3d.

Worked solution

Inputting the data from the table into a CAS, r^2 is calculated to be 0.8163.

$1 - 0.8163 = 0.1837$, so 18.4% of the variation in the response cannot be explained by the variation in the explanatory variable.

Mark allocation: 1 mark

- 1 mark for the correct answer: 18.4

Question 3e.

Worked solution

$6.55 + 7.46 \times 30 = 230.35$

Rounding to the nearest whole number, the answer is 230.

Mark allocation: 1 mark

- 1 mark for the correct answer: 230

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Question 3f.

Worked solution

Predicted amount of rubbish $= 6.55 + 7.46 \times 10 = 81.15$

Residual = actual amount – predicted amount

$= 89 - 81.15$

$= 7.85$

Mark allocation: 1 mark

- 1 mark for the correct answer: 7.85

Tip

- *If rounding has not been specified, the terminating decimal should be used in full, unless referring to people or objects that should be referred to by whole numbers.*

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Question 4a.

Worked solution

Linearity, that is, the assumption that the data is linear.

Mark allocation: 1 mark

- 1 mark for the correct answer: linearity

Question 4b.

Worked solution

No, since the residual plot shows a clear pattern.

Mark allocation: 1 mark

- 1 mark for the correct answer: the pattern in the residual plot indicates that the data is not linear

Question 4c.

Worked solution

$\log_{10}(number\ of\ sightings) = 2.0 - 0.061 \times day$

The values need to be rounded correctly and in the correct order.

Mark allocation: 1 mark

- 1 mark for the correct rounded values placed in the correct boxes

Tip

- *Ensure that the slope includes a negative in examples such as these, as the plus sign in the equation may cause it to be overlooked.*

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Question 5a.

Worked solution

Average for 2023: 49.25

Average for 2024: 42.25

Q2:

$$\frac{\frac{45}{49.25}+\frac{39}{42.25}}{2} \qquad 0.918391253417$$

Q4:

$$\frac{\frac{46}{49.25}+\frac{35}{42.25}}{2} \qquad 0.881206259574$$

Mark allocation: 2 marks

- 2 marks for 0.92 and 0.88 (in that order)

  Note: Award 1 mark for one correct answer, even if it is rounded incorrectly.

Question 5b.

Worked solution

Deseasonalised value $= \dfrac{\text{actual value}}{\text{seasonal index}} = \dfrac{63}{1.44} = 43.75 = 44\%$ rounded to the nearest whole number.

Mark allocation: 1 mark

- 1 mark for the correct answer: 44

Question 5c.

Worked solution

Q3 in 2025 has the quarter number of 11.
Deseasonalised $= 52.3 - 1.45 \times 11 = 36.35$
Actual = deseasonalised $\times$ seasonal index
$= 36.35 \times 1.44$
$= 52.344$, or 52% when rounded to the nearest whole number

Mark allocation: 1 mark

- 1 mark for the correct answer: 52

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Recursion and financial modelling

Question 6a.

Worked solution

B_0 is the principal amount invested: $\$1800$.

Mark allocation: 1 mark

- 1 mark for the correct answer: $\$1800$

Question 6b.

Worked solution

$1800 \times 1.0015 = 1802.70$

$1802.70 \times 1.0015 = 1805.40$

Mark allocation: 1 mark

- 1 mark for the two calculations shown, with the correct rounding of both.

Tip

- *Recursion requires the value of the last term to be used in the calculation of the value of the next term.*

Question 6c.

Worked solution

$(1.0015 - 1) \times 26 \times 100 = 3.9$ or $(1.0015 - 1) \times 2600 = 3.9$

Mark allocation: 1 mark

- 1 mark for the calculation $(1.0015 - 1) \times 26 \times 100 = 3.9$ or similar

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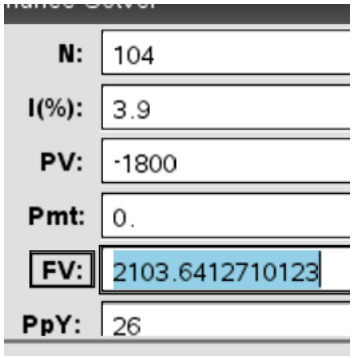
Question 6d.

Worked solution

The rule $B_n = 1.0015^n \times 1800$ can be used, with $4 \times 26 = 104$ fortnights. Alternatively, the financial solver on a CAS could be used.



Mark allocation: 1 mark

- 1 mark for the correct answer: $\$2103.64$

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Question 7a.

Worked solution

$0.08 \times 2800 = 224$

$\text{solve}(2800+224 \cdot n=5600, n) \quad n=12.5$

12.5 years is rounded up to 13 years.

Mark allocation: 1 mark

- 1 mark for the correct answer: 13 years

Question 7b.

Worked solution

8% depreciation results in 92% of the value remaining = 0.92.

$\text{solve}(2800 \cdot (0.92)^n=1400, n) \quad n=8.31295041412$

This value must also be rounded up to 9 years.

Mark allocation: 1 mark

- 1 mark for the correct answer: 9 years

Question 7c.

Worked solution

Total depreciation = $1700 - 712 = \$988$

Per use depreciation = $\dfrac{988}{(4 \times 52 \times 5 \text{ years})} = \$0.95$

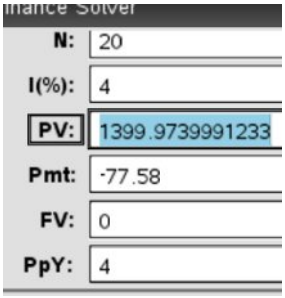
Mark allocation: 1 mark

- 1 mark for the correct answer: $\$0.95$

Question 7d.

Worked solution

Using the financial solver on a CAS:



Mark allocation: 1 mark

- 1 mark for the correct answer: $\$1400$

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Question 8a.

Worked solution

An investment which earns compound interest and has withdrawals that are greater than the interest earned is called an annuity.

Mark allocation: 1 mark

- 1 mark for the correct answer: an annuity

Note: Award no marks for 'annuity investment' or other variations on the word.

Question 8b.

Worked solution

$P \times \frac{4.8}{1200} \times 24 \text{ months} = 2400 \quad P = 25\,000$

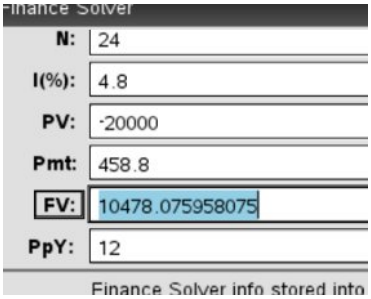
$\text{solve}\left(\frac{4.8}{1200} \cdot 24 \cdot x = 2400, x\right) \quad x = 25000.$

Mark allocation: 1 mark

- 1 mark for the correct answer: $\$25\,000$

Question 8c.

Worked solution



Balance after two years at 4.8%: $\$10\,478.06$

Interest = withdrawals – principal reduction

$= (458.80 \times 24) - (20\,000 - 10\,478.08)$

$= 11011.20 - 9521.92$

$= 1489.28$

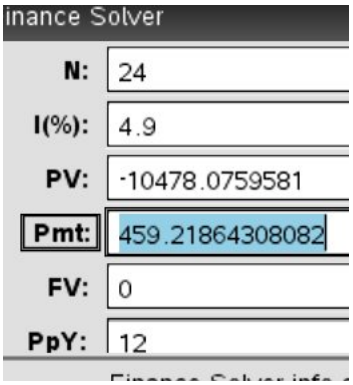
New withdrawal amount after two years (with new rate): $\$459.22$

Interest = withdrawals – principal reduction

$= (459.22 \times 24) - 10\,478.08$

$= 11021.28 - 10\,478.08$

$= 543.20$



Total interest $= 1489.28 + 543.20$

$= 2032.48$

$= \$2032$ (rounded)

Mark allocation: 2 marks

- 2 marks for the correct answer: $\$2032$

Note: Award 1 mark if the answer is incorrect but there is evidence that the values $\$1489$ or $\$543$ involved relevant calculation.

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Matrices

Question 9a.

Worked solution

The adult ticket cost of $\$18$ is the element in the second row, first column: t_{21}.
Reject uppercase: T_{21}

Mark allocation: 1 mark

- 1 mark for the correct answer: t_{21}.

Question 9b.

Worked solution

A summing matrix for just for the first two elements (child and adult) is used:

$$\begin{bmatrix}1 & 1 & 0 & 0\end{bmatrix} \cdot \begin{bmatrix}12\\18\\15\\55\end{bmatrix} \qquad \begin{bmatrix}30\end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for the correct answer: $\begin{bmatrix}1 & 1 & 0 & 0\end{bmatrix}$

Question 9c.

Worked solution

The matrix product NT is a 1×1 matrix, and T is a 4×1, which means that matrix N needs to be a row matrix of order 1×4: $(1 \times 4) \times (4 \times 1) = 1 \times 1$.

$$N = \begin{bmatrix}140 & 72 & 12 & 55\end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for the correct answer: $\begin{bmatrix}140 & 72 & 12 & 55\end{bmatrix}$

Question 9d.

Worked solution

Decreasing by 25%: $100 - 25\% = 75\% = 0.75$

Mark allocation: 1 mark

- 1 mark for the correct answer: 0.75

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Question 10a.

Worked solution

The matrix shows the movement by following the 1's from *this exhibit* to the *next exhibit*.

$$\begin{array}{c} \begin{matrix} E & G & L & P & S & T \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} E \\ G \\ L \\ P \\ S \\ T \end{matrix} \end{array}$$

Mark allocation: 1 mark

- 1 mark for the matrix values shown above

Question 10b.

Worked solution

Ana can send messages to Bernie, Diego and Erin; but she can only receive messages from Diego and Erin.

Mark allocation: 1 mark

- 1 mark for the correct answer: Diego and Erin

Question 10c.

Worked solution

The two-step matrix is:

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 2 & 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 1 & 0 & 2 & 2 \end{bmatrix}$$

Column 1 shows the two-step communication *to* Ana. This shows that B and C can send a message indirectly to Ana. The value of row 1, column 1 is excluded as it represents Ana communicating back to herself.

Mark allocation: 1 mark

- 1 mark for the correct answer: Bernie and Celia

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Question 11a.

Worked solution

$0.05 + 0.05 = 0.10 = 10\%$ are going to the aquarium or zoo instead of returning to the museum next month.

Mark allocation: 1 mark

- 1 mark for the correct answer: 10%

Question 11b.

Worked solution

25% of those who visited the zoo this month are expected to visit the aquarium next month.

Mark allocation: 1 mark

- 1 mark for the correct answer: 25%

Question 11c.

Worked solution

$0.15 \times 100 = 15$

Mark allocation: 1 mark

- 1 mark for the correct answer: 15

Question 11d.

Worked solution

July to September is two transitions:

$$\begin{bmatrix} 0.8 & 0.25 & 0.05 \\ 0.15 & 0.6 & 0.05 \\ 0.05 & 0.15 & 0.9 \end{bmatrix}^2 \cdot \begin{bmatrix} 12000 \\ 8000 \\ 7000 \end{bmatrix} \quad \begin{bmatrix} 11702.5 \\ 6367.5 \\ 8930. \end{bmatrix}$$

The number expected at the museum is 8930.

Mark allocation: 1 mark

- 1 mark for the correct answer: 8930

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Question 11e.

Worked solution

$$\begin{bmatrix} 0.8 & 0.25 & 0.05 \\ 0.15 & 0.6 & 0.05 \\ 0.05 & 0.15 & 0.9 \end{bmatrix} \cdot \begin{bmatrix} 12000 \\ 8000 \\ 7000 \end{bmatrix} \qquad \begin{bmatrix} 11950. \\ 6950. \\ 8100. \end{bmatrix}$$

The transition from the first month to the next shows that the zoo needs $9000 - 6950 = 2050$ additional visitors and the museum needs 900 additional visitors.

Mark allocation: 1 mark

- 1 mark for the correct answer: 2050 more visitors to the zoo and 900 more visitors to the museum

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Networks and decision mathematics

Question 12a.

Worked solution

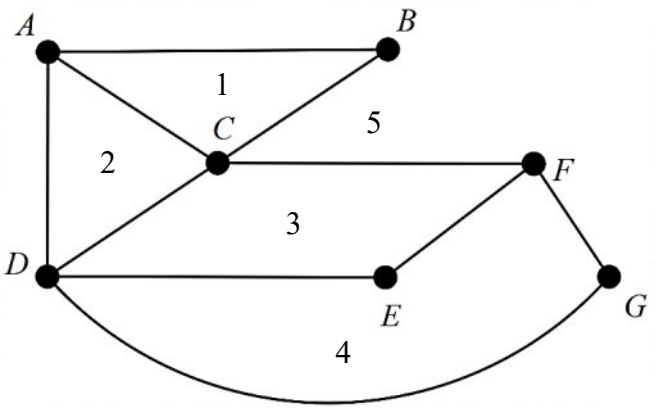
There are five faces, bearing in mind that the outside region is also considered a face.

This can be verified by Euler's rule after noting that there are 10 edges connecting the seven vertices:

$V + F - E = 2$

$7 + F - 10 = 2$

$F = 5$



Mark allocation: 1 mark

- 1 mark for the correct answer: 5

Question 12b.

Worked solution

There are roads from D that directly lead to A, C, E and G.

Mark allocation: 1 mark

- 1 mark for the correct answer: A, C, E and G

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Question 12c.

Worked solution

The Euler path used to travel along every edge will finish at F, as both A and F are the vertices that have an odd degree, so travelling back along F–G again will be required.

Mark allocation: 1 mark

- 1 mark for the correct answer: F–G or G–F

Tip

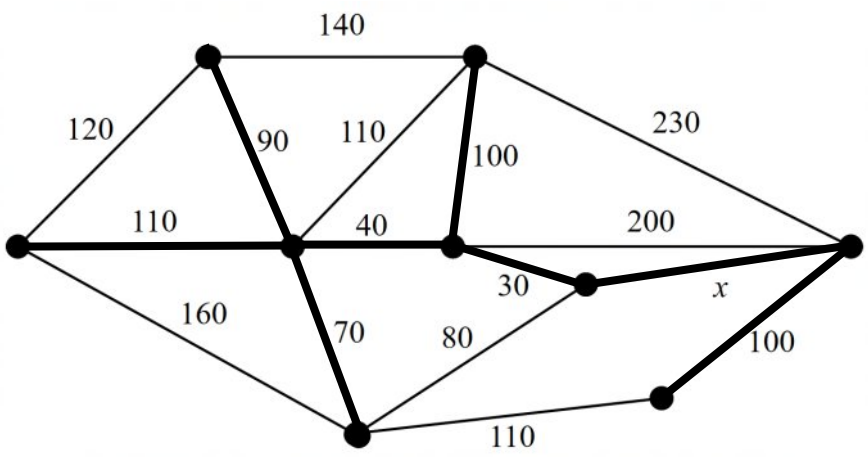
- *Remember that an Euler path (a path that travels along every edge only once) must begin and end at an odd vertex.*

Question 13a.

Worked solution

The minimum spanning tree is shown in bold.

Adding the lengths of the drains in the spanning tree gives 540. $540 + x = 600$, so $x = 60$.



Mark allocation: 1 mark

- 1 mark for the correct answer: 60 metres

Question 13b.

Worked solution

The shortest distance is along A–D–H–I = 370 metres.

Mark allocation: 1 mark

- 1 mark for the correct answer: 370

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Question 13c.

Worked solution

A Hamiltonian path travels to every vertex. The path from A to H that is 870 metres is $A–D–G–F–C–B–E–I–H$.

Mark allocation: 1 mark

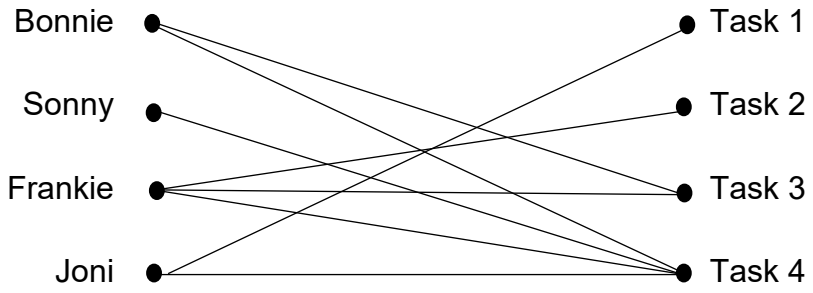
- 1 mark for the correct answer: $A–D–G–F–C–B–E–I–H$

Question 13d.

Worked solution

Row reduction: $\begin{bmatrix} 2 & 4 & 2 & 0 \\ 3 & 2 & 5 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 3 & 4 & 0 \end{bmatrix}$ Column reduction: $\begin{bmatrix} 2 & 3 & 0 & 0 \\ 3 & 1 & 3 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \end{bmatrix}$

The minimum number of vertical or horizontal lines that cover all the zeros is four, which is the same as the number of people, so we can allocate:



Worker	Task
Bonnie	3
Sonny	4
Frankie	2
Joni	1

Mark allocation: 1 mark

- 1 mark for the correct answer: Bonnie does task 3, Sonny does task 4, Frankie does task 2 and Joni does task 1

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Question 14a.

Worked solution

The activities with two immediate predecessors are H, G, J and K.

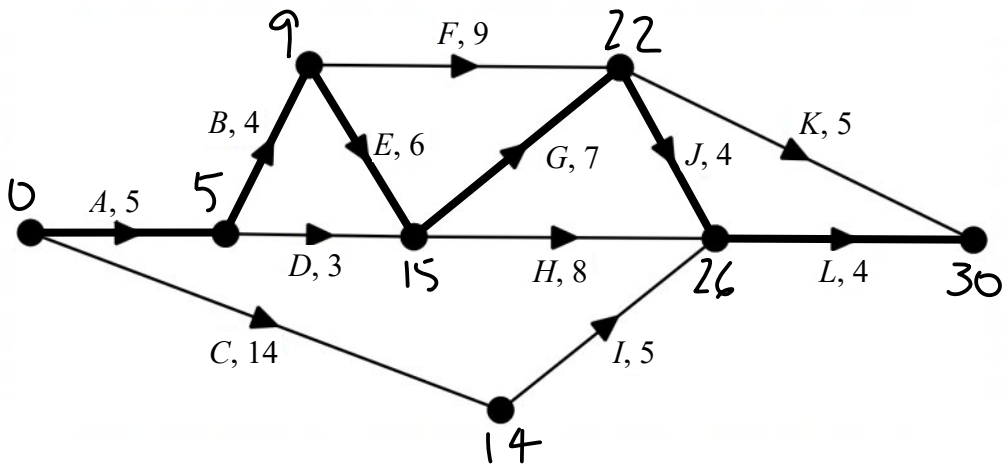
Mark allocation: 1 mark

- 1 mark for the correct answer: H, G, J and K.

Question 14b.

Worked solution

The critical path of 30 days is $A–B–E–G–J–L$.



Mark allocation: 1 mark

- 1 mark for the correct answer: $A–B–E–G–J–L$

Question 14c.

Worked solution

As can be seen from the diagram above, K and J both have an earliest start time of 22 days.

Mark allocation: 1 mark

- 1 mark for the correct answer: K and J

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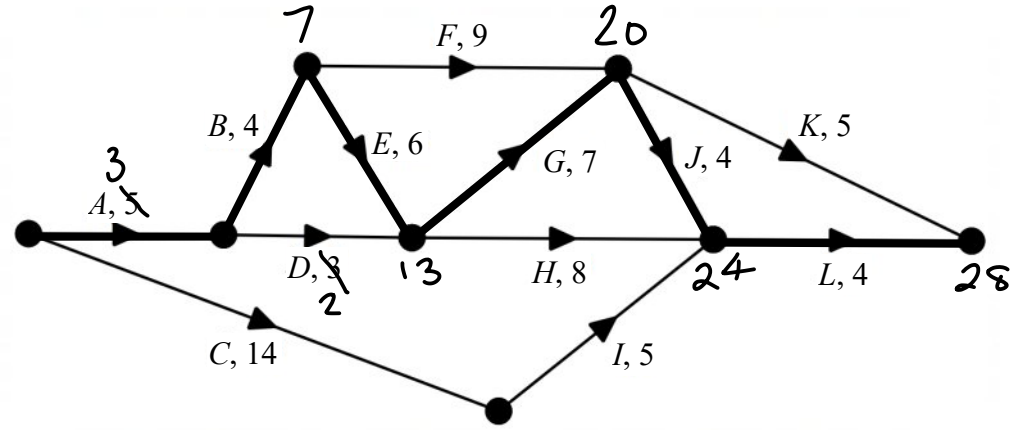
Question 14d.

Worked solution

The reduction in D does not affect the completion time as it is not on the critical path. The reduction of A by two days results in a new completion time of 28 days.



Mark allocation: 1 mark

- 1 mark for the correct answer: 28 days

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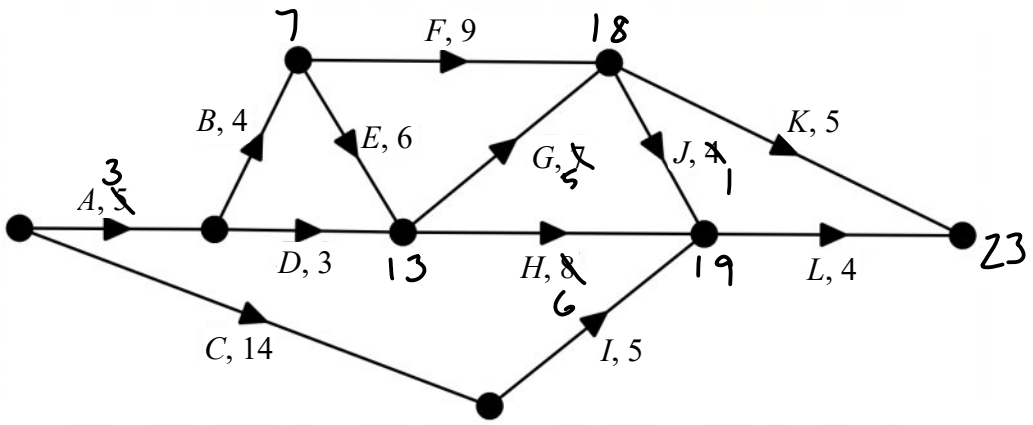
Question 14e.

Worked solution



Reducing all possible activities on the critical path, A–B–E–G–J–L, (i.e. reducing A, G, J and L) results in a new critical time of 21 along that path. However, the critical path has changed: A–B–E–G–K is now 23 and A–B–E–H–L is 25.
In order to address this:
A–B–E–G–K: there are no possible reductions of K to allow that path to also reduce to 21, so instead of reducing L, it should remain with a duration of four, so A–B–E–G–J–L is also 23 days (reducing A, G and J only).
A–B–E–H–L: H can be reduced by two, so the path now has a critical time of 23 also.

Activity	A	C	D	G	H	J	L
Maximum reduction applied (days)	2	0	0	2	2	3	0
Cost of reduction ($)	2000	0	0	4000	3000	3000	0

This is a total of $\$12\,000$.

Mark allocation: 1 mark

- 1 mark for $\$12\,000$

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