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Student Name……

GENERAL MATHEMATICS UNITS 3 & 4

TRIAL EXAMINATION 2

2024

Reading Time: 15 minutes
Writing time: 1 hour 30 minutes

Instructions to students

This exam consists of 16 questions.
All 16 questions should be answered.
There are a total of 60 marks available for this exam.
The marks allocated to each of the questions are indicated throughout.
Students may bring one bound reference into the exam.
Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.
Where a numerical answer is required, students should only round their answer when instructed to do so.
Unless otherwise stated, the diagrams in this exam are not drawn to scale.
A formula sheet can be found on the last page of this exam.

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Data analysis

Question 1 (8 marks)

Bird watchers collected data in order to study a particular type of bird.

The variables included in the study were:

- *tag number* number used to identify each bird
- *weight* weight of bird in grams (g)
- *colour* main colour of bird: blue, green, yellow
- *wingspan* wingspan of bird in centimetres (cm)
- *type* whether the bird lives as a pet or in the wild: captive, wild

The data shown in Table 1 below was collected for 17 of these birds.

Table 1

Tag number	***Weight (g)***	***Colour***	***Wingspan (cm)***	***Type***
1	37	blue	32	captive
2	39	green	34	captive
3	34	green	30	wild
4	35	blue	28	captive
5	36	yellow	29	wild
6	32	green	32	wild
7	40	blue	30	captive
8	43	green	33	captive
9	39	green	35	captive
10	39	green	31	wild
11	38	green	26	wild
12	45	yellow	29	captive
13	41	green	31	captive
14	35	green	33	wild
15	39	yellow	34	captive
16	36	green	30	wild
17	38	blue	29	captive

a. How many variables in Table 1 are numerical variables? 1 mark

b. Determine the mean *weight*, in grams, of all the birds in the sample. 1 mark

mean = ☐

c. Determine the median *weight*, in grams, of the wild birds in the sample. 1 mark

median = ☐

d. Use the data in Table 1 to complete the following two-way frequency table. 2 marks

Table 2

	Type	
Colour	Captive	Wild
blue		
green		
yellow		
Total		

A larger study is undertaken and a second set of data is collected. A random sample of captive and wild birds is taken and their colour is classified as blue, green or yellow. The number of birds of each colour for this second set of data is shown in the two-way table below.

Table 3

	Type	
Colour	**Captive**	**Wild**
blue	55	8
green	82	205
yellow	43	27
Total	180	240

e. Calculate the percentage of the total number of birds in this second data set that are classified as green.
Round the percentage to the nearest whole number. 1 mark

f. Does the data in Table 3 support the contention that the colour of a bird is associated with the type of bird? Justify your answer by quoting appropriate percentages which have been rounded to the nearest whole number. 2 marks

Question 2 (5 marks)

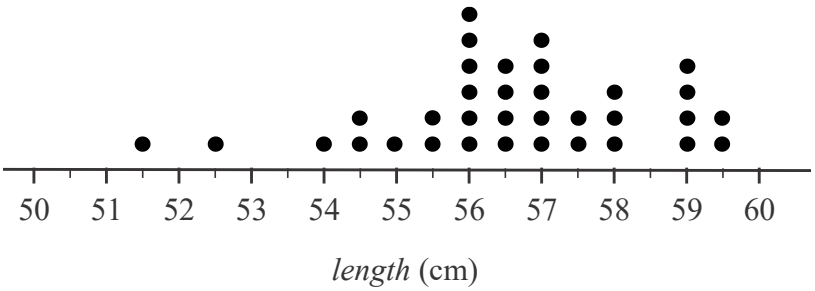
The wingspan, in centimetres, of a population of a different type of bird is normally distributed. Recent studies found that:

- 2.5% of the birds had a wingspan greater than 95 cm
- 16% of the birds had a wingspan less than 80 cm

a. Use the 68-95-99.7% rule to find the mean and standard deviation of this normal distribution. 2 marks

mean = ☐ standard deviation = ☐

The length, in centimetres, of 34 of these birds was recorded and displayed on the dot plot below.



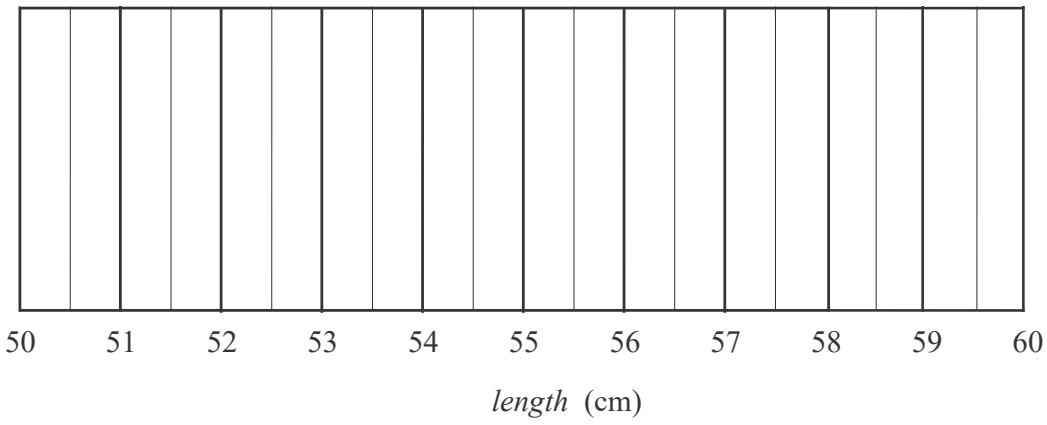
The five-number summary for this sample of lengths, in centimetres, is given below.

Statistic	*Length* (cm)
minimum	51.5
first quartile	56.0
median	56.5
third quartile	58.0
maximum	59.5

A boxplot is to be constructed using this five-number summary and the dot plot.

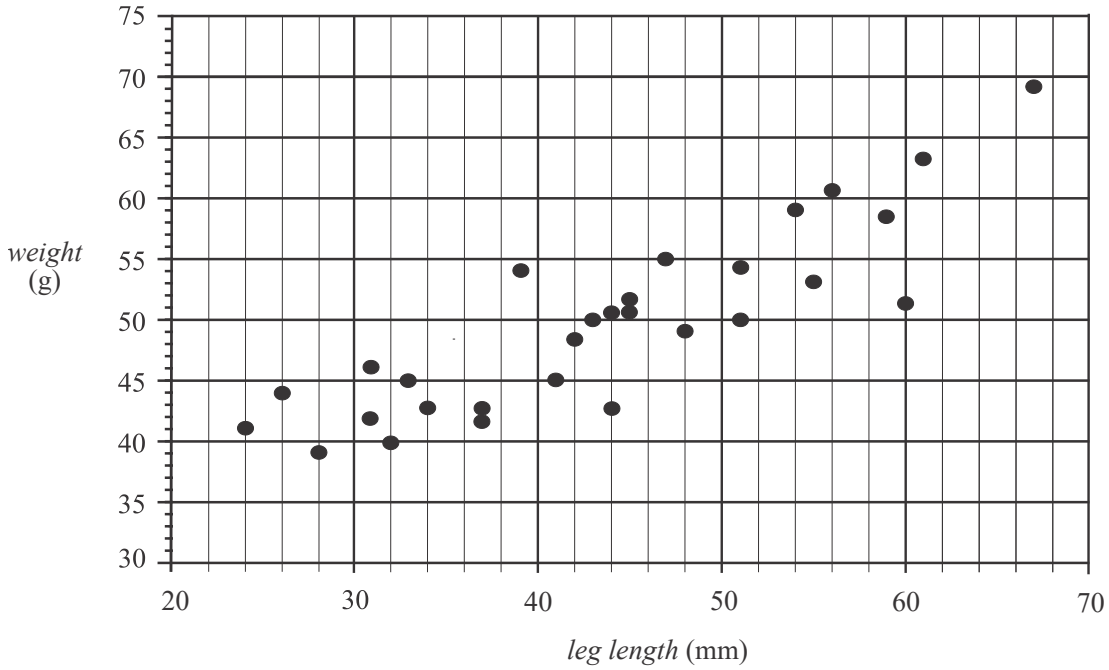
b. Show that the lower and upper fences for the boxplot are 53 cm and 61 cm respectively. 1 mark

c. Construct the boxplot, showing outliers if appropriate, on the grid below. 2 marks



Question 3 (5 marks)

The scatterplot below shows the *leg length*, in millimetres, against *weight*, in grams, for 29 endangered birds.



The equation of the least squares line fitted to this data is

$$weight = 24.64 + 0.575 \times leg\ length$$

a. Draw the graph of the least squares line on the scatterplot above. 1 mark

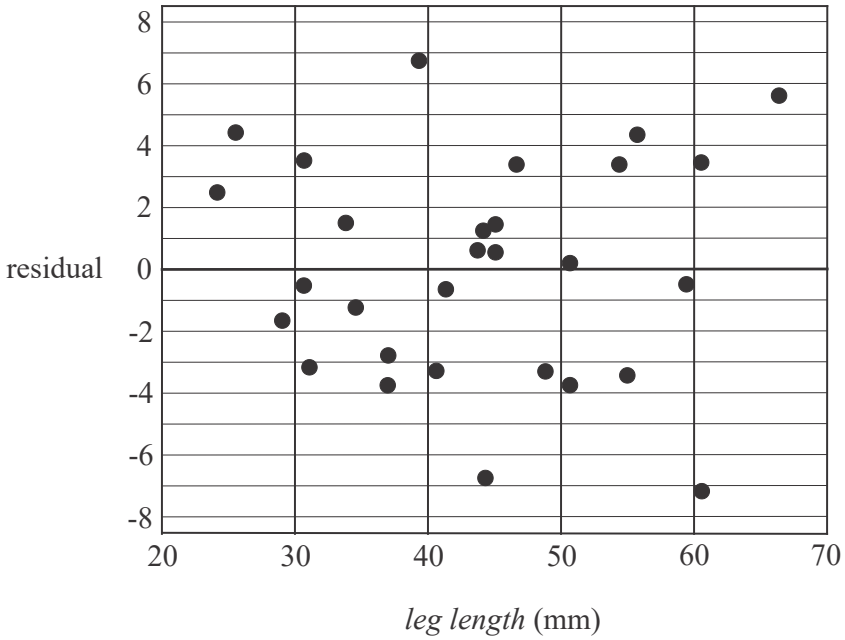
(Answer on the scatterplot above.)

b. Use the equation of the least squares line to predict the *weight* of an endangered bird when its *leg length* is 22 millimetres. 1 mark

c. Write down whether the prediction made in part **b.** is an example of interpolation or extrapolation. 1 mark

d. Interpret the slope of the least squares line in terms of *weight* and *leg length.* 1 mark

e. The residual plot associated with fitting a least squares line to this data is shown below.



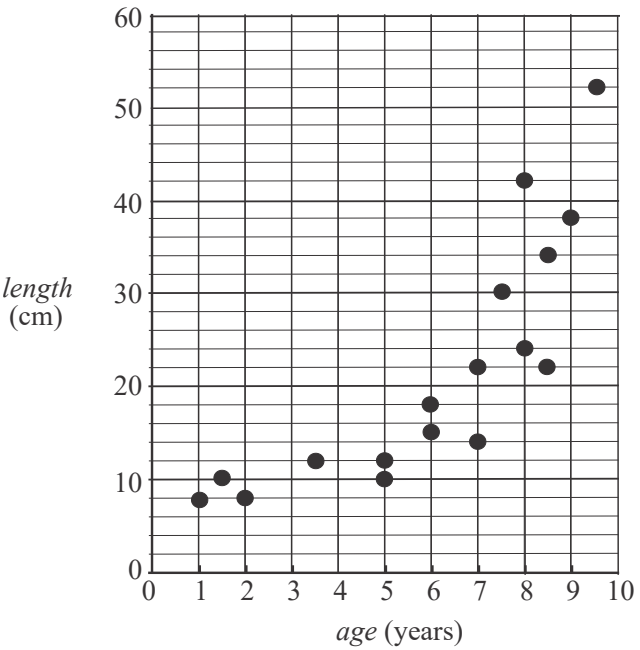
The residual plot supports the assumption that the association between *weight* and *leg length* is linear.
Explain why. 1 mark

Question 4 (2 marks)

Table 4 below shows the *length*, in centimetres, and the *age*, in years, of 17 endangered birds that have been raised in captivity.
A scatterplot for the data is also shown.

Table 4

Age (years)	***Length*** (cm)
1	8
1.5	10
2	8
3.5	12
5	10
5	12
6	15
6	18
7	14
7	22
7.5	30
8	24
8	42
8.5	22
8.5	34
9	38
9.5	52



The relationship between *length* and *age* is clearly non-linear.
Apply an appropriate transformation to the variable *age* in order to linearise the data.
Fit a least squares line to this transformed data and write its equation below.
Round the values of the intercept and slope to four significant figures.

Question 5 (4 marks)

The average monthly humidity, expressed as a percentage, for a city in 2023 is shown in Table 5 below.

Table 5

***Humidity* (%)**												
Year	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
2023	42.1	45.7	49.8	51.9	58.4	65.2	67.5	60.3	58.2	54.4	57.8	49.3

a. Calculate the four-mean smoothed humidity with centring for May.
Round your answer to one decimal place. 1 mark

b. Four-mean smoothing with centring is to be applied to the whole 12 month period.
Determine how many points would appear in this smoothed plot. 1 mark

c. Show that the seasonal index for September 2023 estimated from this data is 1.06 when rounded to two decimal places. 2 marks

Recursion and financial modelling

Question 6 (3 marks)

Dean borrowed $750 000 to purchase a house. Interest will be charged on the loan and will be calculated monthly. He will make monthly repayments over 20 years in order to fully repay the loan.
The balance of the loan, in dollars, after n months, V_n, can be modelled by the recurrence relation

$$V_0 = 750\,000, \qquad V_{n+1} = 1.0055V_n - 5636.04$$

a. Explain what the number 5636.04 represents. 1 mark

b. Showing recursive calculations, determine the balance of the loan after two months. Round your answer to the nearest cent. 1 mark

c. Determine the annual compound interest rate for this loan. 1 mark

Question 7 (5 marks)

Dean purchases office equipment for the home office in his new house. He depreciates this office equipment using the reducing balance method.
The recurrence relation that models the depreciation, D_n, in dollars, of the office equipment after n years is given by

$$D_0 = 10\,500, \qquad D_{n+1} = 0.62 \times D_n$$

a. Determine the value of the office equipment after two years. 1 mark

b. What is the annual rate of depreciation used to depreciate the office equipment? 1 mark

c. After how many years will the value of the office equipment first drop below $500? 1 mark

d. Complete the rule below that gives the value of the office equipment, D_n, in dollars, after n years.
Write your answers in the boxes provided. 1 mark

$$D_n = \square \times \square^{n}$$

e. Dean depreciates the value of the office equipment by a fixed percentage of its current value each year.
An alternative method of depreciation that Dean could have used is to depreciate the value by a fixed percentage of its original value each year.
What name is given to this alternative method of depreciation? 1 mark

Question 8 (4 marks)

After paying off his house, Dean starts planning for his retirement.
He has saved $180 000 which he invests in an account that earns interest at the rate of 3.6% per annum compounding monthly.
Each month, Dean will add a payment of $1500 to this investment account, immediately after the interest is added.

a. Let I_n, in dollars, be the balance of this investment account after n months.
Write a recurrence relation in terms of I_0, I_{n+1}, and I_n that models the balance from month to month. 1 mark

b. After how many months will the balance of this investment account first exceed $500 000?
Round your answer to the nearest whole number. 1 mark

c. After three years of making regular monthly payments of $1500, Dean has to reduce the amount of this monthly payment.
Four years after having paid this reduced monthly payment, the balance of Dean's account is $350 000.
Determine the amount of Dean's reduced monthly payment.
Round your answer to the nearest cent. 2 marks

Matrices

Question 9 (3 marks)

The type of registration fees charged by a local council to dog and cat owners are shown in the table below.

Fee type	Type of animal	
	Dog	Cat
Full fee	\$210	\$170
Concession	\$105	\$85

The matrix F below summarises these fees.

$$F = \begin{bmatrix} 210 & 170 \\ 105 & 85 \end{bmatrix}$$

a. The element f_{ij} in matrix F represents the full fee for the registration of a cat.
What are the values of i and j in this case? 1 mark

$i =$ ☐ $j =$ ☐

b. Joan owns two dogs and two cats and receives a concession on her fees.
The matrix product $A \times F$ produces a matrix that gives Joan the cost of registering her dogs and the cost of registering her cats.
Write down matrix A in the space below. 1 mark

$A =$

c. The local council offers a 70% discount on the registration fees of dogs and cats that have been desexed.
The matrix expression below will give the registration fees for desexed dogs and cats for full fee and concession fee paying owners.

☐ $\times F$

In the box above, write down the missing scalar value. 1 mark

Question 10 (2 marks)

A dog ranger from the council makes one visit each day to each of five local parks, *A, B, C, D* and *E*.

The matrix below shows the movement of the ranger between the five parks during a day.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} \multicolumn{5}{c}{\textit{this visit}} \\ A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \textit{next visit} \\ \\ \\ \end{array}$$

a. The ranger started his visits one day at park *D*.
Write down, in order, the parks visited by the ranger on this day. 1 mark

Because of traffic works, the order in which the ranger visits the five parks changes.
The order in which he will visit the parks in the next couple of days is shown in the table below.

	Order of visit				
	1st	2nd	3rd	4th	5th
Day 1	*B*	*E*	*D*	*C*	*A*
Day 2	*E*	*D*	*C*	*A*	*B*

The order in which the ranger visits the parks on Day 1 can be presented in the matrix

$$M = \begin{bmatrix} B & E & D & C & A \end{bmatrix}.$$

By multiplying matrix *M* by a permutation matrix *P*, the order of visits will change from Day 1 to Day 2.

b. Write down matrix *P* in the space below. 1 mark

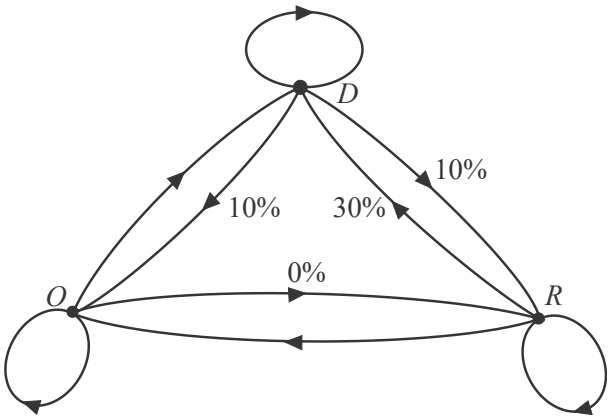
$P =$

Question 11 (3 marks)

Workers in the Waste and Environment team at the council work either at the depot (D), on the trucks (R) or in the office (O).
The proportion of workers who work at each location changes from year to year, as shown in the transition matrix T below.

$$T = \begin{array}{c} \\ D \\ R \\ O \end{array} \begin{array}{c} \begin{array}{ccc} D & R & O \end{array} \\ \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \end{array} \begin{array}{l} \text{this year} \\ \\ \\ \\ \text{next year} \end{array}$$

The transition diagram for matrix T is shown below. This transition diagram is incomplete.



a. Complete the transition diagram **above** by writing the missing values, as percentages, on the edges. 1 mark

(Answer on the transition diagram above.)

At the start of 2022, the Waste and Environment team had 40 workers.
There were 20 working at the depot, 15 working on the trucks and 5 working in the office.

b. How many workers were working on the trucks at the start of 2023? 1 mark

At the start of 2024, the council decides to employ, each year, an additional two workers to work at the depot and an additional five workers to work on the trucks.
They anticipate that four office workers will take redundancies and leave each year.
Let S_n be the state matrix that shows the number of workers at the depot, on the trucks and in the office at the start of the n^{th} year after 2024.
The recurrence relation that is used to predict the number of workers in each location is given by

$$S_{n+1} = T S_n + B,$$

where $S_0 = \begin{bmatrix} 22 \\ 9 \\ 9 \end{bmatrix} \begin{matrix} D \\ R \\ O \end{matrix}$ $\quad T = \begin{matrix} & \text{this year} \\ & D \quad R \quad O \end{matrix} \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \begin{matrix} D \\ R \\ O \end{matrix}$ *next year* and $B = \begin{bmatrix} - \\ - \\ - \end{bmatrix} \begin{matrix} D \\ R \\ O \end{matrix}$

Matrix B is incomplete.

c. How many workers in the Waste and Environment team are expected to be working on the trucks at the start of 2026?
Round your answer to the nearest whole number. 1 mark

Question 12 (4 marks)

The council oversees a regeneration program around a local lake. As part of that program, the female population of a rare insect is being studied.
These female insects have a life span of four years but breed only in their second year and in their third year.
The Leslie matrix, L, below, is used to model the female population distribution of these insects.

$$L = \begin{bmatrix} 0 & 6.8 & 9.2 & 0 \\ 0.8 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{bmatrix}$$

a. What proportion of the female insects in their second year are expected to survive into their third year? 1 mark

The breeding patterns of the female insect population can be modelled by the recurrence relation

$$S_{n+1} = L\,S_n$$

where S_n represents the number of insects at each stage of life after n years of the study.
At the start of the study, the initial state matrix for this female insect population, S_0, is given by

$$S_0 = \begin{bmatrix} 80 \\ 55 \\ 40 \\ 35 \end{bmatrix}$$

b. At the start of the study, what is the expected number of offspring, on average, to be produced by the female population in the coming year? 1 mark

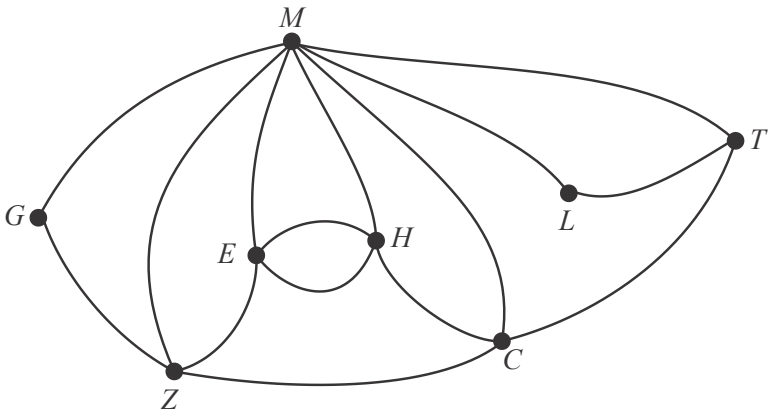
c. After one year of the study, determine how many insects in this female population are expected to be in the fourth year of their life. 1 mark

d. After two years of the study, what percentage of the population of female insects are expected to **not** produce offspring in the coming year?
Round your answer to the nearest whole number. 1 mark

Network and decision mathematics

Question 13 (3 marks)

Outland Zoo has separate enclosures for eight different species of animals: cheetahs (*C*), elephants (*E*), giraffes (*G*), hippos (*H*), lions (*L*), meerkats (*M*), tigers (*T*) and zebras (*Z*).
In the network below the vertices represent these enclosures. The edges of the network represent the pathways between the enclosures.



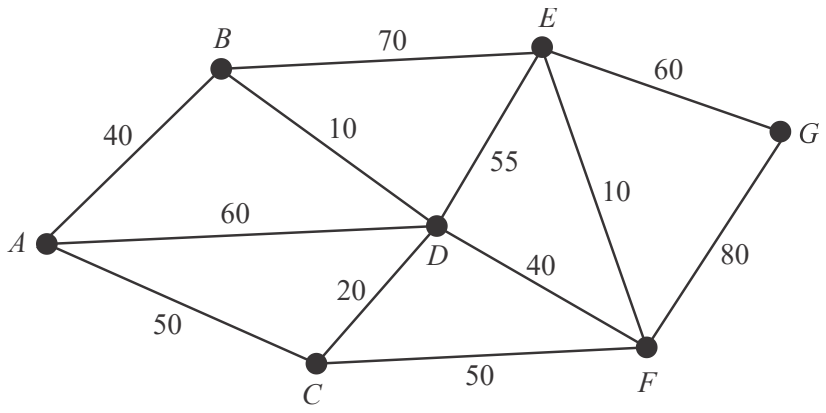
a. Which enclosure at the zoo can be reached directly from all the other enclosures? 1 mark

b. A tour of the zoo begins every hour, starting and finishing at the meerkats enclosure, and visiting each of the other enclosures just once.
What is the mathematical term for this route? 1 mark

c. A zoo keeper starts at the meerkats enclosure and completes an Eulerian trail.
Which enclosure will she finish at? 1 mark

Question 14 (2 marks)

Inside the tiger enclosure there are seven feeding locations.
The vertices *A, B, C, D, E, F* and *G* on the graph below, show these feeding locations.
The edges represent the paths connecting the feeding locations. The number on the edges indicate the distances, in metres, between adjacent feeding locations.
The staff entrance is located at vertex *A* and the exit is located at vertex *G*.



a. What is the shortest distance, in metres, between the entrance at *A* and the exit at *G*? 1 mark

b. If the shortest path between the entrance at *A* and the exit at *G* is taken by a staff member, which feeding location will be missed? 1 mark

Question 15 (3 marks)

A team of zoo keepers, Ari, Lina, Rohan and Zaria work in the giraffe enclosure.
Four of the daily tasks in this enclosure include cleaning, food preparation, health checks and data collection.
The following table shows the usual time taken, in minutes, by each zoo keeper for each daily task.

	Time taken for each daily task (minutes)			
Zoo keeper	**Cleaning**	**Food prep.**	**Health checks**	**Data collection**
Ari	100	50	35	55
Lina	60	45	70	55
Rohan	40	70	50	30
Zaria	70	50	70	70

Management will allocate each of the four zoo keepers to one daily task in order to minimize the time taken to complete all four tasks.

a. Construct a bipartite graph to show all the possible allocations. 1 mark

b. Complete the table below by allocating a task to each of the four zoo keepers so that the four tasks are completed in the minimum time possible. 1 mark

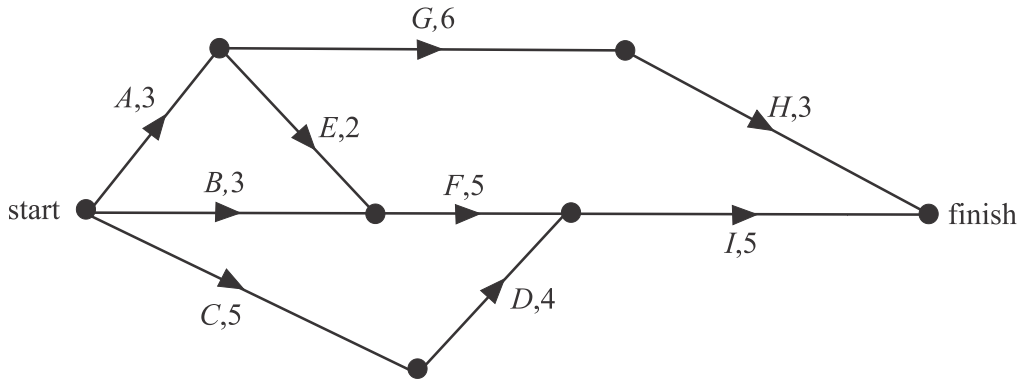
Zoo keeper	Task
Ari	
Lina	
Rohan	
Zaria	

c. What is the minimum total time, in minutes, required to complete all four tasks?

1 mark

Question 16 (4 marks)

The zoo is planning to restructure the elephant enclosure.
The project involves nine activities *A* to *I*.
The directed network below shows these activities and their completion times in weeks.



a. How many activities are on the critical path? 1 mark

_____

b. Which activity has a float time of two weeks? 1 mark

_____

It is possible to reduce the completion time of activities *A, B, D, F* and *I* by employing more workers.
The reduction in completion time for each of these five activities will incur an additional cost for the zoo.
The table below shows the five activities that can have their completion times reduced and the associated weekly reduction cost in dollars.

Activity	Weekly reduction cost ($)
A	2000
B	1000
D	500
F	2000
I	1500

The completion time for each of these five activities can be reduced by a maximum of two weeks.
The zoo requires that the reduction in the completion time be achieved at minimum cost.

c. **i.** What is the minimum number of weeks in which the restructure can now be completed? 1 mark

_____

ii. What is the minimum cost of completing the restructure in this time? 1 mark

_____

END OF EXAMINATION