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GENERAL MATHS UNITS 3&4
TRIAL EXAMINATION 2
SOLUTIONS
2024

Data analysis

Question 1 (8 marks)

a. There are two numerical variables in Table 1. **(1 mark)**
Note that they are *weight* and *wingspan*. The variable *tag number* identifies each bird, it doesn't count or measure anything, so it is a categorical variable (and a nominal variable).

b. Enter all the data for the variable *weight* into your CAS and using One-Variable stats calculate the mean, $\bar{x} = 38$. **(1 mark)**

c. There are seven wild birds in the sample. Their weights, in order, are 32, 34, 35, 36, 36, 38, 39. The middle weight, i.e. the median is 36 g. **(1 mark)**

d.

	Type	
Colour	Captive	Wild
blue	4	0
green	4	6
yellow	2	1
Total	10	7

(1 mark) for four correct entries **(1 mark)** for remaining four entries

e. $\dfrac{(82+205)}{(180+240)} \times 100\% = 68.3333\ldots\%$

$= 68\%$ (to nearest whole number) **(1 mark)**

f. Yes the data **does** support the contention that the colour of a bird is associated with the type of bird because for blue birds, there is a difference in the percentage that are captive compared to wild. **(1 mark)**

The percentage of captive birds that are blue is $\dfrac{55}{180} \times 100\% \approx 31\%$ which is higher than the percentage of wild birds that are blue which is $\dfrac{8}{240} \times 100\% \approx 3\%$ **(1 mark)**

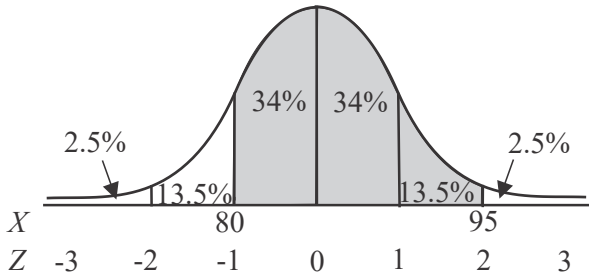
This answer is sufficient. As an alternative however, you could quote statistics for green birds **or** for yellow birds.

That is, $\dfrac{82}{180} \times 100\% \approx 46\%$ of captive birds are green whereas a much higher $\dfrac{205}{240} \times 100\% \approx 85\%$ of wild birds are green.

Alternatively, $\dfrac{43}{180} \times 100\% \approx 24\%$ of captive birds are yellow whereas only $\dfrac{27}{240} \times 100\% \approx 11\%$ of wild birds are yellow.

Question 2 (5 marks)

a. The value 95 is two standard deviations above the mean.
The value 80 is one standard deviation below the mean.



Therefore, over 15 cm, there are three standard deviations so one standard deviation is 5 cm. The mean therefore is equal to $95 - 2 \times 5 = 85$.
(Alternatively, $80 + 1 \times 5 = 85$.)
The mean = 85 and the standard deviation = 5.

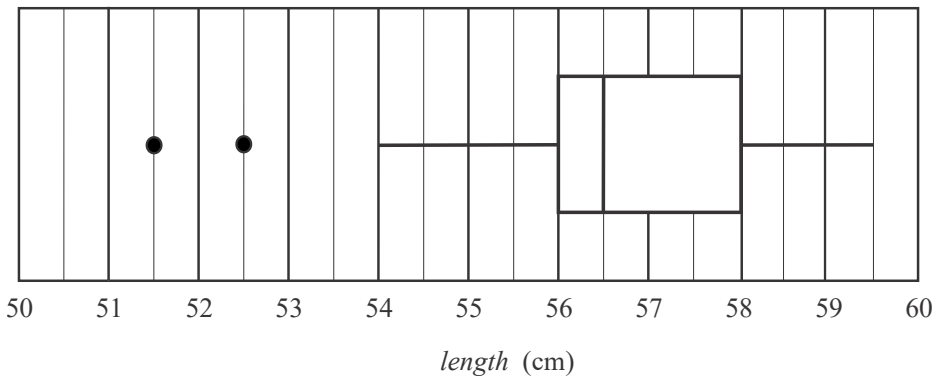
(1 mark) for the mean
(1 mark) for the standard deviation

b. lower fence $= Q_1 - 1.5 \times \text{IQR}$
$= 56 - 1.5 \times (58 - 56)$
$= 53$

upper fence $= Q_3 + 1.5 \times \text{IQR}$
$= 58 + 1.5 \times (58 - 56)$
$= 61$

(1 mark)

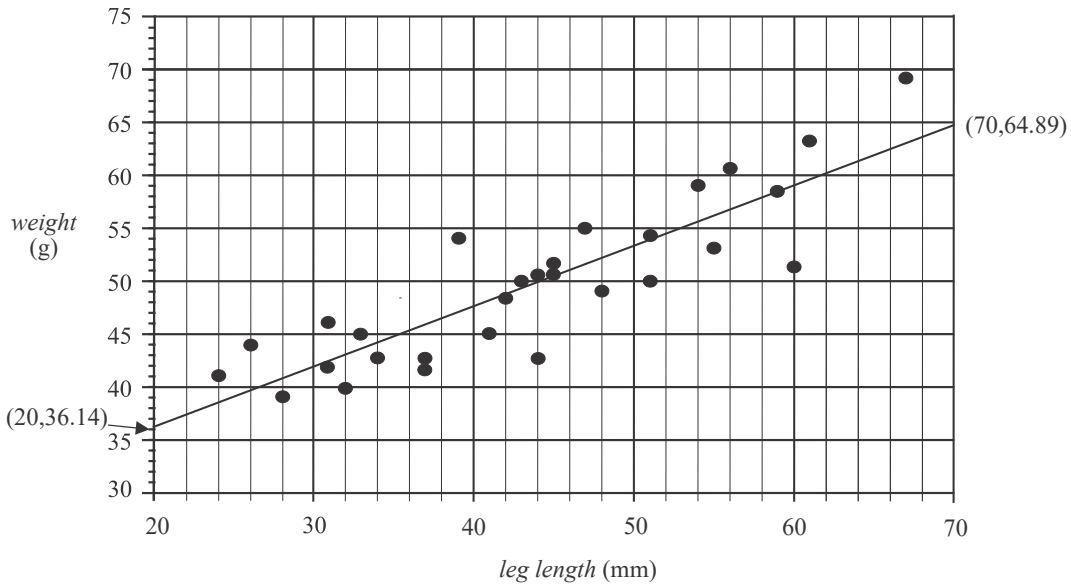
c. The boxplot is shown below. Note that there are two outliers i.e. 51.5 and 52.5.
These values are outliers because they are both less than the lower fence of 53.
The next lowest data value (from the dot plot) is 54. Since it is greater than the lower fence of 53, it is the value at the end of the lower whisker.



(1 mark) correct box and median
(1 mark) correct whiskers and outliers

Question 3 (5 marks)

a.



Whilst not necessary to gain the mark, it is very useful to show the first and last points on the line i.e. (20, 36.14) and (70, 64.89), and then just use a ruler to connect them.

(1 mark)

b. $weight = 24.64 + 0.575 \times leg\ length$

$= 24.64 + 0.575 \times 22$

$= 37.29$

(1 mark)

c. It is an example of extrapolation. **(1 mark)**

(This is because the value 22 lies outside the range of values of *leg length* shown on the scatterplot, i.e. the smallest value of *leg length* indicated on the scatterplot was 24.)

d. The slope of the least squares line is 0.575. This means that on average, for every increase of 1 mm in the *leg length* of a bird, it is predicted that there will be an increase of 0.575 g in the *weight* of a bird. **(1 mark)**

e. The residual plot shows no clear pattern i.e. the residuals are randomly scattered. **(1 mark)**

Question 4 (2 marks)

When the distribution of data on the scatterplot has the shape shown, we could use a log y transformation or a $\frac{1}{y}$ transformation or an x^2 transformation.
Because we have been specifically asked for a transformation to the variable *age*, which lies on the horizontal or x-axis, we will use the x^2 transformation i.e. the $(age)^2$ transformation.
Using our CAS, enter the data for the variables *age* and *length* and then create a third column for the variable $(age)^2$.
Now calculate the least squares regression line using $(age)^2$ as the x variable and *length* as the y variable.
The equation is $length = 4.28641\ldots + 0.40315\ldots \times (age)^2$

Rounding the intercept and slope to four significant figures gives:
$length = 4.286 + 0.4032 \times (age)^2$

(1 mark) for 4.286
(1 mark) for 0.4032

Question 5 (4 marks)

a. $\frac{49.8 + 51.9 + 58.4 + 65.2}{4} = 56.325$

$\frac{51.9 + 58.4 + 65.2 + 67.5}{4} = 60.75$

To centre these two values $\frac{56.325 + 60.75}{2} = 58.5375$

The answer is 58.5% (to one decimal place). **(1 mark)**

b. Because four-mean smoothing is used, the first month that can be smoothed is March (i.e. Jan, Feb, Mar, Apr) and the last is October (i.e. Sep, Oct, Nov, Dec).
So only $12 - 2 - 2 = 8$ months can be smoothed.
There would be 8 points on the smoothed plot. **(1 mark)**

c. average humidity $= \frac{42.1 + 45.7 + 49.8 + \ldots + 54.4 + 57.8 + 49.3}{12} = 55.05$
(1 mark)

Seasonal index for September $\frac{58.2}{55.05} = 1.0572\ldots$
$= 1.06$ (rounded to two decimal places)
1 mark)

Recursion and financial modelling

Question 6 (3 marks)

a. It represents Dean's monthly repayment of the loan. **(1 mark)**

b. $V_0 = 750\,000$

$V_1 = 1.0055 \times 750\,000 - 5636.04$

$= 748\,488.96$

$V_2 = 1.0055 \times 748\,488.96 - 5636.04$

$= 746\,969.6092\ldots$

The balance of the loan after two months is \$746 969.61. **(1 mark)**

c. 0.55% is the monthly interest rate so $0.55\% \times 12 = 6.6\%$ is the annual compound interest rate for the loan. **(1 mark)**

Question 7 (5 marks)

a. $D_2 = 0.62 \times D_1$

$= 0.62 \times 0.62 \times 10\,500$

$= 4036.2$

After two years the value of the office equipment is \$4036.20 **(1 mark)**
(Don't forget that we are talking about the "value" of the office equipment so we must express the answer in dollars and cents, hence the need to add the zero to 4036.2)

b. Annual rate of depreciation is $(1 - 0.62) \times 100\% = 38\%$ **(1 mark)**

c. Generate the sequence on your CAS.

\$10 500, \$6510, \$4036.20, \$2502.44, \$1551.52, \$961.94, \$596.40, \$369.77, …
After seven years the value of the office equipment first drops below \$500.
(1 mark)

Note that the value after one year is \$6510 not \$10 500.

d. $D_n = 10\,500 \times 0.62^n$ **(1 mark)**

e. It is the flat rate method of depreciation. **(1 mark)**

Question 8 (4 marks)

a. The monthly interest rate will be $3.6\% \div 12 = 0.3\%$, so the recurrence relation will be

$$I_0 = 180\,000, \qquad I_{n+1} = 1.003I_n + 1500$$

(1 mark)

b. Using finance solver:

N: ?
I(%): 3.6
PV: − 180 000
Pmt: − 1500
FV: 500 000
Ppy: 12
Cpy: 12

N: 128.74…

After 129 months the balance of the account will first exceed \$500 000.

(1 mark)

c. Using finance solver, after 3 years:

N: 36
I(%): 3.6
PV: − 180 000
Pmt: − 1500
FV: ?
Ppy: 12
Cpy: 12

FV: 257 429.9981…

(1 mark)

Using finance solver, after a further 4 years:

N: 48
I(%): 3.6
PV: − 257 429.9981…
Pmt: ?
FV: 350 000
Ppy: 12
Cpy: 12

Pmt: − 1023.6144…

Dean's reduced monthly payment is \$1023.61. **(1 mark)**

Matrices

Question 9 (3 marks)

a. $i = 1$ and $j = 2$ **(1 mark)**

b. $A = \begin{bmatrix} 0 & 2 \end{bmatrix}$ **(1 mark)**

c. The required scalar is 0.3. Note that a 70% discount means that owners are only paying 30% of the original fee. **(1 mark)**

Question 10 (2 marks)

a. The order is *D, E, B, C, A*. **(1 mark)**

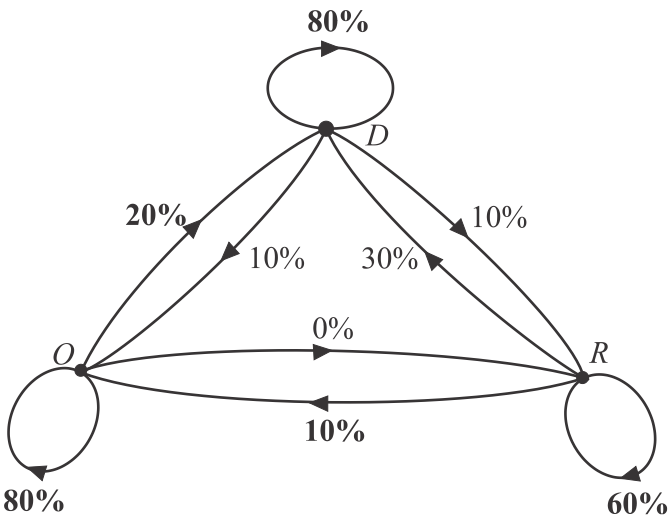
b. *P* is the square matrix below.

$$\begin{bmatrix} B & E & D & C & A \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} E & D & C & A & B \end{bmatrix}$$

(1 mark)

Question 11 (3 marks)

a.



(1 mark)

b. Method 1

Using the transition matrix *T*, 0.1 of 20 = 2 went from working at the depot to working on the trucks, 0.6 of 15 = 9 stayed working on the trucks and none of the 5 working at the office went to working on the trucks.
In total, there were 2 + 9 = 11 workers working on the trucks at the start of 2023.
(1 mark)

Method 2

$$S_{2023} = T S_{2022}$$

$$= \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 20 \\ 15 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 21.5 \\ 11 \\ 7.5 \end{bmatrix}$$

There were 11 workers working on the trucks at the start of 2023. **(1 mark)**

c.

$$S_1 = T S_0 + B$$
$$= \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 22 \\ 9 \\ 9 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$
$$= \begin{bmatrix} 24.1 \\ 12.6 \\ 6.3 \end{bmatrix}$$

$$S_2 = T S_1 + B$$
$$= \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 24.1 \\ 12.6 \\ 6.3 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$
$$= \begin{bmatrix} 26.32 \\ 14.97 \\ 4.71 \end{bmatrix}$$

There are predicted to be 15 workers working on the trucks at the start of 2026.
(1 mark)

Question 12 (4 marks)

a. The element in the third row and second column of the Leslie matrix gives us the proportion of female insects in the second year of their life that are expected to survive into the third year of their life. The answer is 0.5. **(1 mark)**

b. Only female insects in their second year and in their third year can produce offspring.
At the start of the study there are 55 female insects in their second year and they are expected to each produce, on average, 6.8 offspring. $55 \times 6.8 = 374$.
At the start of the study there are 40 female insects in their third year and they are expected to each produce, on average, 9.2 offspring. $40 \times 9.2 = 368$.
In total there are $374 + 368 = 742$ offspring expected to be produced in the coming year. **(1 mark)**

c.

$$S_1 = L S_0$$
$$= \begin{bmatrix} 742 \\ 64 \\ 27.5 \\ 12 \end{bmatrix}$$

After one year of the study there are expected to be 12 female insects in the fourth year of their life. **(1 mark)**

d.

$$S_2 = L^2 S_0$$
$$= \begin{bmatrix} 688.2 \\ 593.6 \\ 32 \\ 8.25 \end{bmatrix}$$

Female insects in their first year and fourth year will not produce offspring.

$$\frac{688.2 + 8.25}{688.2 + 593.6 + 32 + 8.25} = 0.526795\ldots$$

The percentage required is 53% (to the nearest whole number). **(1 mark)**

Networks and decision mathematics

Question 13 (3 marks)

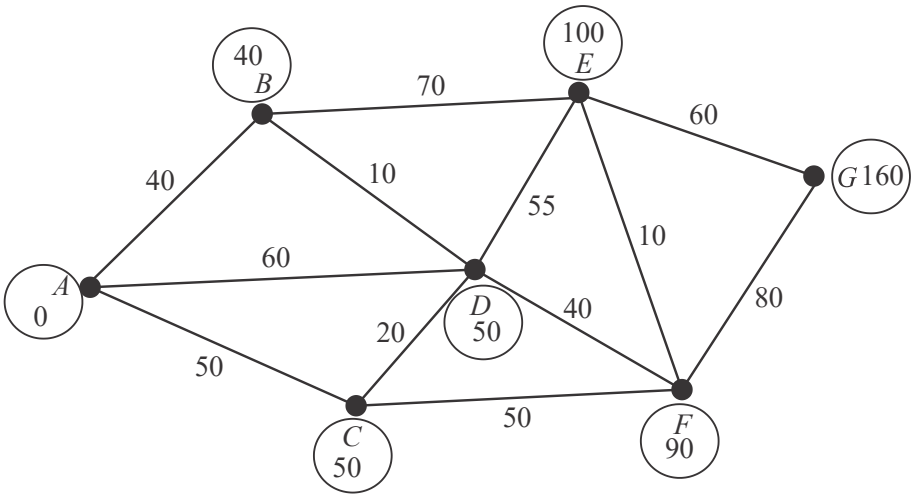
a. The meerkats enclosure can be reached directly from every other enclosure. **(1 mark)**

b. The route is a Hamiltonian cycle. **(1 mark)**

c. An Eulerian trail follows every edge of a graph just once. The graph has two odd vertices, one at the vertex *M* and the other at the vertex *T*.
So the zoo keeper must finish at the tiger enclosure. **(1 mark)**

Question 14 (2 marks)

a. Use Dijkstra's algorithm to find the shortest distance.



The shortest distance between *A* and *G* is 160 metres. **(1 mark)**

b. The shortest path is achieved using the route *A* – *B* – *D* – *F* – *E* – *G*.
Therefore feeding location *C* will be missed. **(1 mark)**

Question 15 (3 marks)

a. We use the Hungarian algorithm.

	C	*F*	*H*	*D*
A	100	50	35	55
L	60	45	70	55
R	40	70	50	30
Z	70	50	70	70

Subtract the minimum value in each row from each other element in that row.

	C	*F*	*H*	*D*
A	65	15	0	20
L	15	0	25	10
R	10	40	20	0
Z	20	0	20	20

The first column contains no zeros so subtract the minimum element in that column, i.e. 10, from each of the other elements in the column.
And then try and cover all the zeros with a minimum number of straight lines.

	C	*F*	*H*	*D*
A	55	15	0	20
L	5	0	25	10
R	0	40	20	0
Z	10	0	20	20

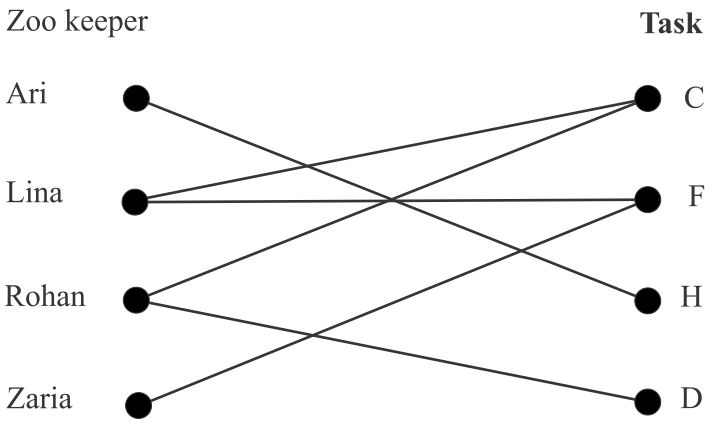
The zeros can be covered with 3 lines. In order to be able to make an allocation, this needs to be 4.
The smallest of the uncovered numbers is 5.
Add this to the numbers that are covered by two lines and subtract it from all the uncovered values.

	C	*F*	*H*	*D*
A	50	15	0	15
L	0	0	25	5
R	0	45	25	0
Z	5	0	20	15

The minimum number of lines required to cover all the zeros is now 4.

	C	*F*	*H*	*D*
~~*A*~~	~~50~~	~~15~~	~~0~~	~~15~~
~~*L*~~	~~0~~	~~0~~	~~25~~	~~5~~
~~*R*~~	~~0~~	~~45~~	~~25~~	~~0~~
~~*Z*~~	~~5~~	~~0~~	~~20~~	~~15~~

Now we can draw a bipartite graph.



(1 mark)

b. Use the bipartite graph to allocate.

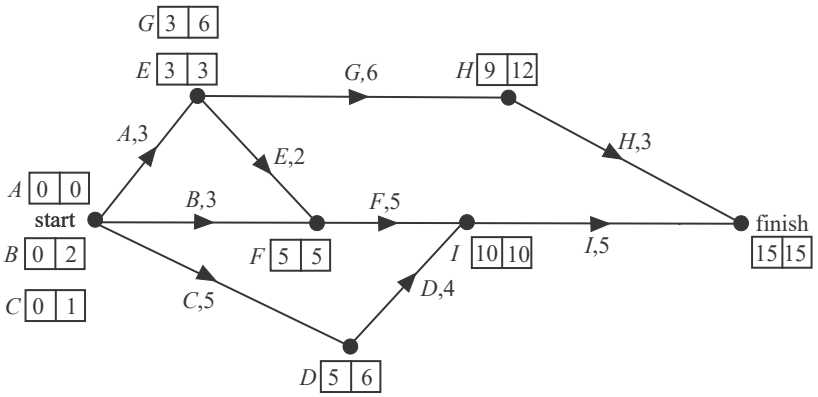
Zoo keeper	Task
Ari	Health check
Lina	Cleaning
Rohan	Data collection
Zaria	Food prep.

(1 mark) correct allocation

c. The minimum total time, in minutes, for the four tasks to be completed is
$35 + 60 + 30 + 50 = 175$ **(1 mark)**

Question 16 (4 marks)

a. Do a forward and backward scan. Note that this will be required in order to answer other parts in the question.



The critical path is A – E – F – I. There are 4 activities on the critical path.
(1 mark)

b. Activity B has a float time of 2 weeks. **(1 mark)**

c. The four possible paths and their times without reduction are

A G H	12 weeks
A E F I	15 weeks
B F I	13 weeks
C D I	14 weeks

If we apply all the possible reductions to these four paths we obtain

A G H	10 weeks	(reduce *A* by 2 weeks)
A E F I	9 weeks	(reduce *A* by 2 weeks, *F* by 2 weeks, *I* by 2 weeks)
B F I	7 weeks	(reduce *B* by 2 weeks, *F* by 2 weeks, *I* by 2 weeks)
C D I	10 weeks	(reduce *D* by 2 weeks, *I* by 2 weeks)

Since path *A G H* cannot be reduced any further than 10 weeks, then this is the minimum completion time. So activity *A* must be reduced by 2 weeks.
Similarly path *C D I* cannot be reduced any further than 10 weeks. To achieve this, activities *D* and *I* must each be reduced by 2 weeks.
There is no point in paths *A E F I* nor *B F I* being reduced below 10 weeks because *A G H* and *C D I* will take that long to complete. To reduce *A E F I* or *B F I* below 10 weeks would be wasting money.
To get *A G H* and *C D I* down to 10 weeks, we had to reduce activities *A, D* and *I* each by 2 weeks.
Path *A E F I* will therefore be reduced to 15 - 4 = 11 weeks.
Path *B F I* will therefore be reduced to 13 – 2 = 11 weeks.

We need to reduce each of these paths by 1 week. Since they both contain activity *F*, we can reduce activity *F* by 1 week which will reduce both of these paths to 10 weeks.
In summary, we need to reduce *A, D* and *I* each by 2 weeks and *F* by 1 week.

i. The minimum number of weeks in which the restructure can now be completed is 10 weeks. **(1 mark)**

ii. The cost of reducing *A, D* and *I* each by 2 weeks and *F* by 1 week is
$2\times \$\,2000+2\times \$\,500+2\times \$\,1500+1\times \$\,2000=\$\,10\,000$ **(1 mark)**