GENERAL MATHEMATICS

Units 3 & 4 – Written examination 2

TSSM™
Creating VCE Success

2023 Trial Examination 2

SOLUTIONS

SECTION A: Core – Data Analysis

Question 1.

a. 10.8% (one-variable stats on CAS)

1 mark

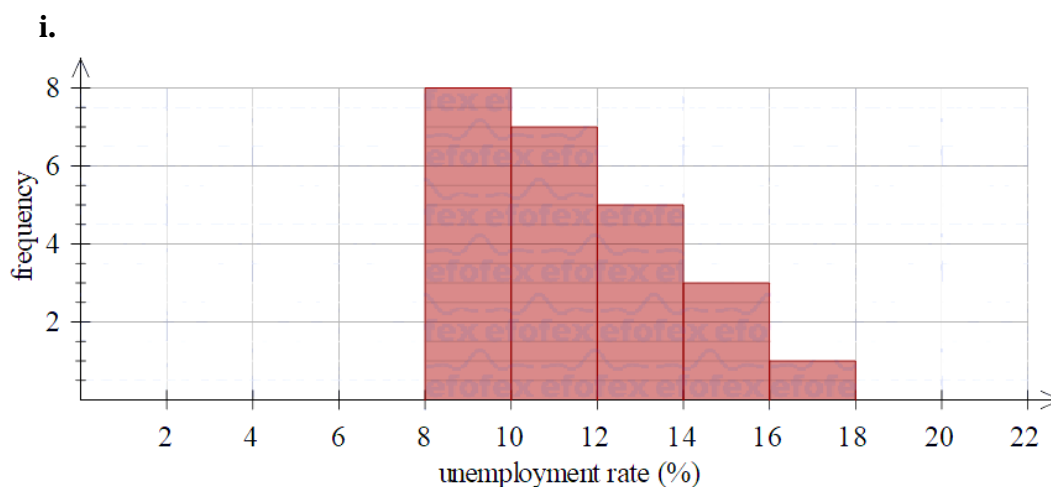**b.** $\frac{15}{24} \times 100 = 62.5\%$

1 mark

c. $Upper\ fence = 12.95 + 1.5 \times (12.95 - 9.8) = 17.525\%$

Since 17.8% is larger than the upper fence value (17.525%), hence it is an outlier.

2 marks

d.
  i.

2 marks

  ii. The distribution is positively skewed.

1 mark

Question 2.

a.

    i. 50% (below the median)

                                                          1 mark

    ii. The *unemployment rate* for males is more variable as compared to that of females. The inter-quartile range for males is 4.6 higher than the inter-quartile range for females (3.6).

                                                          2 marks

b. The median unemployment rate of males (16%) is higher than the median unemployment rate of females (11%) indicating there is an association between unemployment rate and gender.

Students could compare IQRs here too.

                                                          2 marks

Question 3.

a. The plot shows an outlier trend with a downward trend.

                                                           2 marks

b. On average, the unemployment rate decreases by 0.026832% with each additional month.

                                                          1 mark

c. $unemployment\ rate(\%) = \boxed{5.977} - \boxed{0.02683} \times month\ number$

                                                          2 marks

d. $Predicted = 5.9767 - 0.02683 \times 46 = 4.7425\%$
$0.5 = data - 4.7425 \rightarrow data = 5.2\%$

                                                          2 marks

e. Since the residual plot shows a clear pattern, the association between month number and unemployment rate is non-linear.

                                                          1 mark

Question 4.

a. $\dfrac{22+27+35+23+23}{5} = 26°C$

<div align="right">1 mark</div>

b. With five-mean smoothing, we lose 4 data points at the start and 4 at the end.
$14 - 8 = 6$

<div align="right">1 mark</div>

c. $28°C$

<div align="right">1 mark</div>

d. $4.17°C$

<div align="right">1 mark</div>

Core: Recursion and financial modelling

Question 5.

a. $V_1 = 180000 - 10500 = \$169500$

1 mark

b. $V_o = 180000, \ V_{n+1} = V_n - 10500$

1 mark

c. Use recursive technique on CAS
8 years

1 mark

d. $\frac{10500}{180000-10500} \times 100 = 6.19\%$

1 mark

Question 6.

a. $2980.03

1 mark

b. $\frac{3}{1200} \times 357\ 919.97 = \$894.80$

1 mark

c. $R = 1 + \frac{3}{1200} = 1.0025$
$A_0 = 360\ 000, \ A_{n+1} = 1.0025 \times A_n - 2980.03$

1 mark

d. $\frac{3}{1200} \times 360000 = \$900$

1 mark

Question 7.

a. $L_1 = 1.00175 \times 540\,000 - 2305.80 = 538639.20$
$L_2 = 1.00175 \times 538639.20 - 2305.80 = 537276.02$

1 mark

b. $1 + \frac{r}{1200} = 1.00175 \rightarrow r = 2.1\%$

1 mark

c.

N:	301.60927246255
I(%):	2.1
PV:	540000.
Pmt:	-2305.8
FV:	0.
PpY:	12

N:	290.
I(%):	2.1
PV:	540000.
Pmt:	-2375.9584304515
FV:	0.
PpY:	12

$a = \$2375.96$

1 mark

SECTION B – Matrices

Question 1.

a. $[\,5\,] \times [110 \quad 80 \quad 65] = [550 \quad 400 \quad 325]$

$k = 5, \ a = 550, \ b = 400, \ c = 325$

2 marks

b. $[110 \quad 80 \quad 65] \times \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$

$L = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$

1 mark

Question 2.

a.

$$\begin{bmatrix} 40 & 60 \\ 70 & 50 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2160 \\ 2460 \end{bmatrix}$$

1 mark

b.

   i. $\det(A) = 40 \times 50 - 70 \times 60 = 2000 - 4200 = -2200$

      Since det(A) is not zero, hence A^{-1} exists.

1 mark

   ii. $B = \begin{bmatrix} 40 & 60 \\ 70 & 50 \end{bmatrix}^{-1} = \begin{bmatrix} -\dfrac{1}{44} & \dfrac{3}{110} \\ \dfrac{7}{220} & -\dfrac{1}{55} \end{bmatrix}$

1 mark

c. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 & 60 \\ 70 & 50 \end{bmatrix}^{-1} \begin{bmatrix} 2160 \\ 2460 \end{bmatrix} = \begin{bmatrix} 18 \\ 24 \end{bmatrix}$

$2 \times 18 + 3 \times 24 = \$108$

1 mark

Question 3.

a. Employees who travel to Sydney for work one week do not travel to Perth in the following week.

1 mark

b. $0.7 \times 80 + 0.7 \times 50 + 0.8 \times 40 = 123$

1 mark

c. $\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.7 & 0.1 \\ 0 & 0.1 & 0.8 \end{bmatrix}^4 \begin{bmatrix} 80 \\ 50 \\ 40 \end{bmatrix} = \begin{bmatrix} 61.97 \\ 72.416 \\ 35.614 \end{bmatrix}$

62

1 mark

Question 4.

a. The survival rate of the species from age group 1 to age group 2.

1 mark

b. $\begin{bmatrix} 0 & 2.4 & 0.3 \\ 0.6 & 0 & 0 \\ 0 & 0.2 & 0 \end{bmatrix} \begin{bmatrix} 200 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 540 \\ 120 \\ 40 \end{bmatrix}$

There are 120 in age group 2.

1 mark

SECTION B – Networks and decision mathematics

Question 1.

a.

    i. $S - G - E - D - C - B - A - F - S$

1 mark

    ii.   Hamiltonian cycle

1 mark

b.

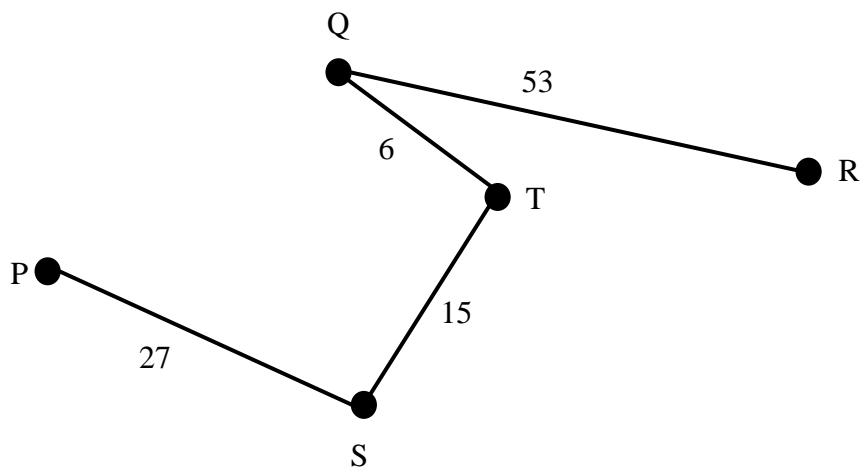   **i.** A (must finish at vertex with odd degree)

1 mark

    ii.   Eulerian trail.

1 mark

Question 2.

a.

    i.

1 mark

    ii. $27 + 15 + 6 + 53 = 101$ km

1 mark

b. To travel along each road exactly once, the degree of each vertex must be even.
The vertex Q is odd degree (3)

1 mark

Question 3.

a. 2 (C and H)

1 mark

b. Critical path is BCE (or BDG).
Minimum completion time = 15 weeks

1 mark

c. Shortening F will have no effect since it is not a critical activity.
Shortening E would only be effective if D or G were reduced as well, as E is only on one critical path.
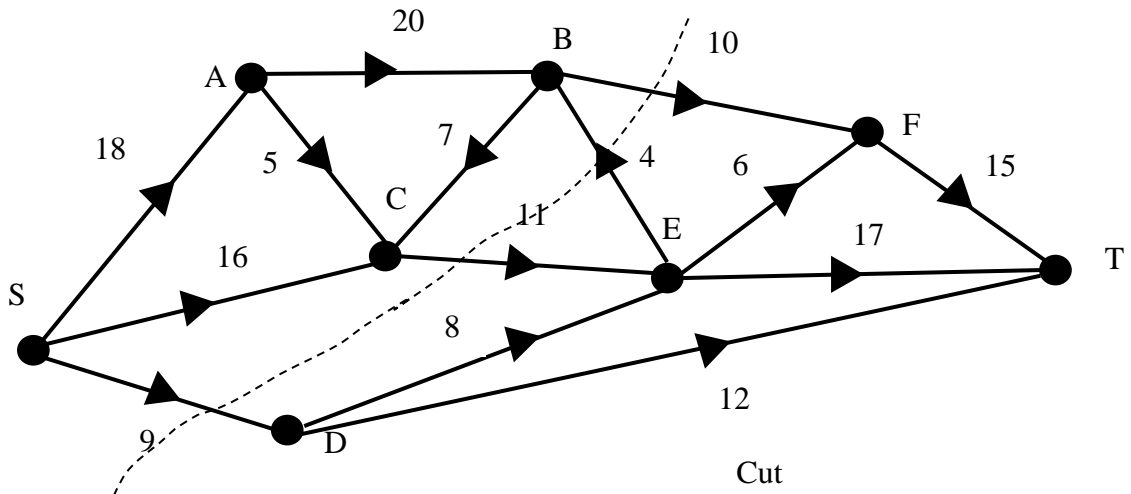
1 mark

Question 4.

a. 10 + 11 + 8 + 12 = 41 kilolitres

1 mark

b. Maximum flow = 10 + 11 + 9 = 30 kilolitres

1 mark