Neap

Trial Examination 2023

VCE General Mathematics Units 3&4

Written Examination 1

Suggested Solutions

1	A	B	C	**D**	E		15	A	B	C	**D**	E		29	A	B	C	**D**	E
2	A	**B**	C	D	E		16	A	B	C	**D**	E		30	**A**	B	C	D	E
3	A	B	**C**	D	E		17	A	B	C	**D**	E		31	A	B	C	D	**E**
4	A	**B**	C	D	E		18	A	B	C	D	**E**		32	A	B	C	D	**E**
5	A	**B**	C	D	E		19	A	B	C	**D**	E		33	A	B	**C**	D	E
6	**A**	B	C	D	E		20	A	B	C	**D**	E		34	A	B	**C**	D	E
7	A	B	C	**D**	E		21	A	**B**	C	D	E		35	A	B	C	**D**	E
8	A	B	**C**	D	E		22	A	B	C	**D**	E		36	**A**	B	C	D	E
9	A	B	**C**	D	E		23	A	B	**C**	D	E		37	**A**	B	C	D	E
10	A	**B**	C	D	E		24	A	B	C	D	**E**		38	**A**	B	C	D	E
11	A	B	C	D	**E**		25	A	**B**	C	D	E		39	A	B	**C**	D	E
12	A	**B**	C	D	E		26	A	B	C	D	**E**		40	A	**B**	C	D	E
13	A	B	C	**D**	E		27	**A**	B	C	D	E							
14	A	**B**	C	D	E		28	A	B	C	**D**	E							

DATA ANALYSIS

Question 1   D

D is correct; and **B**, **C** and **E** are incorrect. The mean of method 1 is 1.95 g, and the mean of method 2 is 1.9 g. Producing more of the chemical means that the process is more efficient. Therefore, the method that produces a larger mean mass of the final product (method 1) is more efficient.

A is incorrect. This option refers to the mean of the frequency of each method $\left(\dfrac{20}{4} = 5 \right)$, not the mass of the final product.

Question 2   B

A continuous variable is measured and not counted. Height and weight are both continuous numerical variables. Eye colour and favourite TV series are categorical variables, and age in years is a discrete variable.

Question 3   C

C is correct. This graph shows the results of the survey as percentages.

A is incorrect. This graph is a segmented bar chart of the raw data.

B is incorrect. This graph is a segmented bar chart showing the breakdown of each gender.

D is incorrect. This graph is a vertical segmented bar chart of the female percentages per segment.

E is incorrect. This graph is a horizontal segmented bar chart of the female percentages per segment.

Question 4   B

When the value of 20 is changed to 12, the data set becomes 2, 2, 4, 7, 10, 10, 12, 12, 15.

Therefore, the five-number summary becomes 2, 3, 10, 12, 15.

The minimum (2), Q_1 $\left(\dfrac{4+2}{2} = 3 \right)$ and the median (10) remain the same. Therefore, the values in the five-number summary that change are Q_3 and the maximum. As Q_3 changes, the IQR will also change $(Q_3 - Q_1 = 12 - 3 = 9)$, but the IQR is not part of the five-number summary.

Question 5   B

B is correct.

$$b = r\frac{s_y}{s_x}$$

$$= 0.7 \times \frac{15}{10}$$

$$= 1.05$$

$$a = \bar{y} - b\bar{x}$$

$$= 35 - 1.05 \times 20$$

$$= 14$$

Substituting $b = 1.05$ and $a = 14$ into $y = a + bx$ gives:

$C = 14 + 1.05t$

A is incorrect. This option may be reached by miscalculating the values of b and a.

C, **D** and **E** are incorrect. These options may be reached by confusing the explanatory and response variables.

Question 6   A

A is correct and **B** is incorrect. Finding the coefficient of determination gives:

$$r^2 = 0.7^2$$

$$= 0.49$$

$$= 49\%$$

Since C is the response variable, the change in t has caused 49% of the change in C.

C and **D** are incorrect. These options use the correlation coefficient, not the coefficient of determination.

E is incorrect. A positive correlation coefficient still has a coefficient of determination.

Question 7   D

D is correct. There are seven students use video chat out of a class of 24. Therefore, finding the percentage gives:

$$\frac{7}{24} \times 100 = 29.1\%$$

A is incorrect. This option is the number of students who use both message chat and video chat.

B is incorrect. This option is the number of students who do not use video chat.

C is incorrect. This option may be reached by finding the percentage of students who use both message chat and video chat.

E is incorrect. This option may be reached by finding the percentage of students who use both message and video chat from the students who use video chat.

Question 8   C

C is correct. Five students use both video chat and message chat out of the 18 students who use message chat. Therefore, finding the percentage gives:

$$\frac{5}{18} \times 100 = 27.77$$
$$\approx 28\%$$

A is incorrect. This option is the number of students who use both message chat and video chat.

B is incorrect. This option is the number of students who use message chat only.

D is incorrect. This option may be reached by finding the percentage of students who use video chat.

E is incorrect. This option may be reached by finding the percentage of students who use message chat.

Question 9   C

C is correct. The horizontal axis of the histogram uses a $\log_{10}$ scale. Since $1000 = 10^3$, the number of countries where fewer than 1000 cars were sold is the total of the first three columns.

Therefore, finding the percentage gives:

$$\frac{26}{53} \times 100 = 49\%$$

A and **D** are incorrect. These options may be reached by totalling the incorrect columns.

B is incorrect. This option is the total number of countries, not the percentage.

E is incorrect. This option may be reached by misinterpreting the $\log_{10}$ scale and assuming that all the countries sell ≤ 7 cars per month.

Question 10   B

B is correct.

$$\text{three-point moving mean} = \frac{32 + 44 + 50}{3}$$
$$= \frac{126}{3}$$
$$= 42$$

A is incorrect. This option may be reached by calculating the mean of all the data points.

C is incorrect. This option may be reached by calculating the five-point moving mean centred on 44 seconds.

D is incorrect. This option states the centre point of the moving mean.

E is incorrect. This option may be reached by finding the mean of the last three data points.

Question 11　E

E is correct. For equations in the form $y = a + bx$, a is the y-intercept and b is the gradient. Therefore, the equation of the transformed data is $y = 32 - 4.1x$. As a log transformation has been applied to x, the equation of the original data can be found by substituting $\log_{10}(x)$; that is, $y = 32 - 4.1 \times \log_{10}(x)$.

A is incorrect. This option is the equation of the transformed data.

B is incorrect. This option may be reached by using a combination of $\dfrac{1}{x}$ and $\log_{10}(x)$.

C is incorrect. This option may be reached by substituting r^2 as the y-intercept.

D is incorrect. This option may be reached by substituting r as the gradient.

Question 12　B

Entering the data into L_1 and L_2 on a CAS calculator gives:

L_3 represents the $\dfrac{1}{x}$ transformation of L_1, L_4 represents the x^2 transformation of L_1, and L_5 represents the $\log(x)$ transformation of L_1.

The coefficient of determination, r^2, can be used to determine the most appropriate transformation for the data.

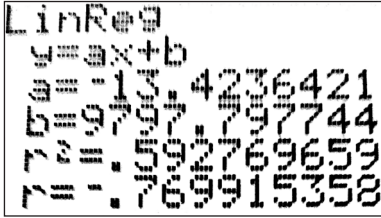Using the linear regression function for L_2 and L_3 gives:

Therefore, the $\dfrac{1}{x}$ transformation gives $r^2 = 0.835$.
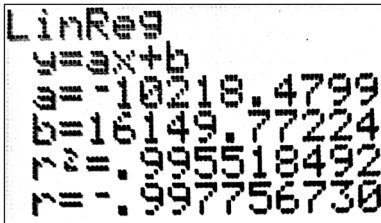
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Using the linear regression function for L_2 and L_4 gives:

Therefore, the x^2 transformation gives $r^2 = 0.593$.

Using the linear regression function for L_2 and L_5 gives:

Therefore, the $\log(x)$ transformation gives $r^2 = 0.996$.

Hence, the $\log(x)$ transformation has the highest value of r^2 and is the most appropriate transformation; this means that equation **B** is correct.

Question 13   D

D is correct.

$$\begin{aligned} \text{residual} &= \text{actual value} - \text{predicted value} \\ &= 290 - (\sim 250) \\ &= 40 \end{aligned}$$

A is incorrect. This option may be reached by subtracting the actual value from the predicted value.

B is incorrect. This option may be reached by misreading the scale on the graph and subtracting the actual value from the predicted value.

C is incorrect. This option may be reached by misreading the scale on the graph.

E is incorrect. This option states the predicted value.

Question 14   B

B is correct. Reading from the graph, the *y*-intercept is approximately 18 and the gradient is positive.

Using two points on the trend line, (30, 100) and (80, 250), to find the gradient gives:

$$\begin{aligned} \text{gradient} &= \frac{250 - 100}{80 - 30} \\ &= \frac{150}{50} \\ &= 3 \end{aligned}$$

Therefore, the equation is closest to *money* = 18 + 3 × *distance*.

A is incorrect. This equation has a negative gradient.

C and **D** are incorrect. These equations use the incorrect gradient.

E is incorrect. This equation uses the incorrect *y*-intercept.

Question 15   D

D is correct. Finding the seasonal index for winter in each year gives:

$$\begin{aligned} \text{winter 2020} &= \frac{\text{actual figure for the season}}{\text{mean for the year}} \\ &= \frac{7400}{4800} \\ &= 1.542 \end{aligned}$$

$$\begin{aligned} \text{winter 2021} &= \frac{9000}{5675} \\ &= 1.586 \end{aligned}$$

$$\begin{aligned} \text{winter 2022} &= \frac{8100}{5300} \\ &= 1.528 \end{aligned}$$

Finding the mean of the seasonal indices gives:

$$\begin{aligned} \text{overall seasonal index} &= \frac{1.542 + 1.586 + 1.528}{3} \\ &= 1.552 \end{aligned}$$

A is incorrect. This option may be reached by using $\dfrac{1}{\text{seasonal index}}$.

B is incorrect. This option is the seasonal index for winter 2022.

C is incorrect. This option is the seasonal index for winter 2020.

E is incorrect. This option is the seasonal index for winter 2021.

Question 16   D

D is correct.

$$\text{mean for summer } 2021 = \frac{42\ 900}{4}$$
$$= 10\ 725$$

$$\text{seasonal index for summer } 2021 = \frac{12\ 400}{10\ 725}$$
$$= 1.156$$

$$\text{mean for summer } 2022 = \frac{46\ 800}{4}$$
$$= 11\ 700$$

$$\text{seasonal index for summer } 2022 = \frac{13\ 000}{11\ 700}$$
$$= 1.111$$

$$\text{seasonal index for summer} = \frac{1.156 + 1.111}{2}$$
$$= 1.134$$

$$\text{deseasonalised figure for summer } 2022 = \frac{13\ 000}{1.134}$$
$$= 11\ 463.84$$

A is incorrect. This option is the seasonal index for summer.

B is incorrect. This option is the mean for summer 2021.

C is incorrect. This option is the deseasonalised figure for summer 2021.

E is incorrect. This option is the mean for summer 2022.

RECURSION AND FINANCIAL MODELLING

Question 17   D

D is correct. Calculating the interest earned over four years gives:

$$I = \frac{23\,000 \times 5 \times 4}{100}$$
$$= 4600$$

Therefore, the total amount to be repaid over 48 equal repayments is $23\,000 + 4600 = \$27\,600$.

Finding the monthly repayment gives:

$$\text{repayment} = \frac{27\,600}{48}$$
$$= 575$$

A is incorrect. This option may be reached by dividing the interest by 48 and not considering the principal.

B is incorrect. This option may be reached by dividing the interest by 12.

C is incorrect. This option may be reached by dividing the principal by 48 and not considering the interest.

E is incorrect. This option is the interest charged on the loan.

Question 18   E

E is correct. $142\,800 \times 0.6^3 \times 0.75^4 = 9759.49$

The value will decrease by 40% for the first three years then decrease by 25% for the next four years.

A is incorrect. This option may be reached by assuming that 40% is a flat rate rather than a decreasing value.

B is incorrect. $142\,800 \times 0.6^3 \times 0.75 = 23\,133.60$

C is incorrect. $142\,800 \times 0.6^3 \times 0.75^2 = 17\,350.20$

D is incorrect. $142\,800 \times 0.6^3 \times 0.75^3 = 13\,012.65$

Question 19   D

D is correct. According to the recurrence relation, 1200 is added each year.

A and **B** are incorrect. While the investment has an interest rate of 6.2% per annum, it compounds annually.

C is incorrect. After five years, the value of the investment will be $74\,336.48.

E is incorrect. After six years, the value of the investment will be $80\,145.34.

Question 20   D

Using the finance solver of a CAS calculator gives:

The balance will fall below $1000 after 22.82 payments. However, as partial payments are not possible, the balance will fall below $1000 after 23 payments.

Question 21   B

B is correct. Each term in the series is given by halving the previous term and adding 4. As the initial term, t_0, is 10, option **B** is the only correct relation.

A is incorrect. This relation does not show the first term of the series.

C is incorrect. This relation describes 10 as t_1, not t_0.

D and **E** are incorrect. These options do not describe the relationship for all terms.

Question 22   D

Finding the monthly interest rate gives:

$$R = 1 + \frac{r}{100}$$
$$1.0015 = 1 + \frac{r}{100}$$
$$r = 0.15$$

Therefore, the annual interest rate is $0.15 \times 12 = 1.8\%$.

Question 23   C

C is correct. The depreciation of 24% as a decimal is $1 - 0.24 = 0.76$. The depreciation of 12% as a decimal is $1 - 0.12 = 0.88$. As the depreciation of 24% occurs in the first year and the depreciation of 12% occurs for the next seven years (for a total of eight years), the value of the electric vehicle is given by $58\,000 \times 0.76 \times 0.88^7$.

A and **B** are incorrect. These expressions find the appreciated value of the vehicle, not the depreciated value.

D is incorrect. This finds the value after nine years.

E is incorrect. This option does not allow for the increased depreciation rate in the first year.

Question 24   E

E is correct. As the video is viewed 10 times on the first day, $t_0 = 10$.

An increase of 140% is given by $1 + 1.4 = 2.4$.

Therefore, the recurrence relation is $t_0 = 10, t_{n+1} = 2.40 t_n$.

A, **B** and **D** are incorrect. These recurrence relations use the incorrect rate of increase.

C is incorrect. This recurrence relation uses the incorrect value for t_1 and does not add 10 each time.

MATRICES

Question 25   B

B is correct.

$$A - B = \begin{bmatrix} 1-2 & 0-2 \\ a-1 & b-0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 \\ a-1 & b \end{bmatrix}$$

A is incorrect. This matrix is the result of AB.

C is incorrect. This matrix is the result of $A + B$.

D is incorrect. This matrix is A^{-1}.

E is incorrect. This matrix is B^{-1}.

Question 26   E

E is correct.

$$\det = ad - bc$$

$$= 5 \times 2 - 4 \times 3$$

$$= -2$$

A is incorrect. This matrix has a determinant of 1.

B is incorrect. This matrix has a determinant of –1.

C is incorrect. This matrix has a determinant of 2.

D is incorrect. This matrix has a determinant of 0.

Question 27   A

A is correct. A CAS calculator can be used to define matrices A and B and then find matrix A^2B.

$$A^2B = \begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}^2 \times \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 & 12 \\ 34 & 36 & 86 \end{bmatrix}$$

B is incorrect. This matrix is incorrectly factorised.

C is incorrect. This matrix is the result of AB.

D is incorrect. This matrix may be reached by swapping the rows and columns of matrix AB.

E is incorrect. If the number of columns in the first matrix matches the number of rows in the second matrix, a multiplication can occur.

Question 28   D

D is correct. Letting the transition matrix, T, be $\begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.5 \end{bmatrix}$ and the initial state matrix, S_0, be $\begin{bmatrix} 60 \\ 60 \end{bmatrix}$ gives:

$$S_1 = TS_0$$
$$= \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.5 \end{bmatrix} \times \begin{bmatrix} 60 \\ 60 \end{bmatrix}$$
$$= \begin{bmatrix} 0.6 \times 60 + 0.5 \times 60 \\ 0.4 \times 60 + 0.5 \times 60 \end{bmatrix}$$
$$= \begin{bmatrix} 66 \\ 54 \end{bmatrix}$$

A and **B** are incorrect. These options represent the transition matrix.

C is incorrect. This option represents the 60 students who initially chose from each sport.

E is incorrect. This option has the volleyball and badminton numbers reversed.

Question 29   D

D is correct. $a_{3,1}$ means the element in the third row and first column. Therefore, $a_{3,1}$ is 4.

A is incorrect. This option is $a_{1,3}$.

B is incorrect. This option is $a_{2,2}$.

C is incorrect. This option is $a_{3,3}$.

E is incorrect. This option is $a_{1,1}$.

Question 30   A

A is correct. The formula for predicting a population using a Leslie matrix, L, is $S_{n+1} = L^n \times S_0$.

For the population of female bears, matrix L is:

$$\begin{bmatrix} 0.3 & 1.25 & 0.45 \\ 0.55 & 0 & 0 \\ 0 & 0.81 & 0 \end{bmatrix}$$

Matrix S_1 is the number of individuals in each age group at the end of the first year;

that is, $\begin{bmatrix} 75 \\ 45 \\ 50 \end{bmatrix}$.

Therefore, the population of female bears in the fourth year, S_4, can be calculated using:

$$\begin{bmatrix} 0.3 & 1.25 & 0.45 \\ 0.55 & 0 & 0 \\ 0 & 0.81 & 0 \end{bmatrix}^3 \times \begin{bmatrix} 75 \\ 45 \\ 50 \end{bmatrix}$$

B is incorrect. This option raises matrix L to the power of 4, which would calculate the population of bears in the fifth year of the study.

C, D and **E** are incorrect. These options use incorrect Leslie matrices.

Question 31   E

E is correct. Using the formula for the inverse of a 2×2 matrix gives:

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} 4 & x \\ y & 5 \end{bmatrix}^{-1} = \frac{1}{4 \times 5 - x \times y}\begin{bmatrix} 5 & -x \\ -y & 4 \end{bmatrix}$$

$$= \frac{1}{20 - xy}\begin{bmatrix} 5 & -x \\ -y & 4 \end{bmatrix}$$

A and **B** are incorrect. These options may be reached by calculating the incorrect determinant.

C is incorrect. This option may be reached by not altering the elements of the original matrix.

D is incorrect. This option may be reached by swapping elements b and c, rather than making them negative.

Question 32   E

E is correct. Matrix P must be the identity matrix, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. This is an example of a permutation matrix, as it is a square matrix where each line (a row or a column) contains one element of 1 and the remaining elements of the line are 0.

A and **B** are incorrect. Multiplying matrix Q by 1 would not result in matrix Q.

C is incorrect. This option is the inverse of matrix Q without the determinant.

D is incorrect. This option is the inverse of matrix Q, which results in $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ when multiplied by matrix Q.

NETWORKS AND DECISION MATHEMATICS

Question 33   C

There are three possibilities for the shortest path: $X–E–H–G–Y$, $X–E–H–I–D–Y$ or $X–E–H–I–G–Y$.
All these paths have a length of 14.

Question 34   C

C is correct. A planar graph should satisfy Euler's formula, $f + v = e + 2$. Substituting 4 faces, 1 vertex and 3 edges gives $4 + 1 = 3 + 2$; thus, option **C** describes a planar graph.

A, **D** and **E** are incorrect. These options do not satisfy Euler's formula.

B is incorrect. Substituting 4 faces, 0 vertices and 2 edges gives $4 + 0 = 2 + 2$; however, it is not possible to draw a planar graph with 4 faces, 0 vertices and 2 edges.

Question 35   D

D is correct. An Eulerian trail is a path that travels each edge in a network exactly once; as an isolated vertex has no edges joining it to other vertices, it cannot have an Eulerian trail.

A is incorrect. Reducing the duration of one of the activities on the critical path does not always reduce the duration of the project.

B is incorrect. An Eulerian trail can exist when a network has two odd vertices.

C is incorrect. The critical path does not have to include the longest activity; it can be formed by shorter activities.

E is incorrect. The number of vertices with an odd degree does not affect whether a graph is planar.

Question 36   A

The maximum flow from *A* to *B* is equal to the minimum cut of flows from left to right. Each possible cut in the graph is shown below.

As the minimum cut is 14, the maximum flow is also 14.

Question 37   A

Y can only be allocated to task 1, which means *V* must be allocated to task 3. *X* can only be allocated to task 4, which means *W* must be allocated to task 2.

Question 38   A

A is correct. This spanning tree connects all the vertices in the graph without a cycle and has a length of 17; thus, it is the minimum spanning tree for the graph.

B is incorrect. This spanning tree has a length of 19.

C is incorrect. This spanning tree has a length of 20.

D is incorrect. This spanning tree has a length of 18.

E is incorrect. This spanning tree has a length of 20.

Question 39   C

C is correct. A row reduction involves subtracting the minimum element of each row from the remaining elements in that row. Therefore:

$$\begin{bmatrix} 8-2 & 6-2 & 2-2 & 2-2 \\ 2-0 & 0-0 & 6-0 & 2-0 \\ 4-2 & 2-2 & 6-2 & 2-2 \end{bmatrix} = \begin{bmatrix} 6 & 4 & 0 & 0 \\ 2 & 0 & 6 & 2 \\ 2 & 0 & 4 & 0 \end{bmatrix}$$

A and **E** are incorrect. These matrices may be reached by factorising the original matrix. Matrix **E** has also omitted the factor outside the matrix.

B is incorrect. This matrix may be reached by subtracting 2 from each non-zero element.

D is incorrect. This option may be reached by dividing the 6 in the third row by 2, instead of subtracting 2.

Question 40   B

B is correct. This matrix contains the correct number of pathways between each vertex.

A is incorrect. This matrix represents the total number of pathways from each vertex.

C and **D** are incorrect. These matrices only count each pathway once.

E is incorrect. This option is not a matrix representation; it is an Eulerian trail.