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YEAR 12 *Trial Exam Paper*

2023

GENERAL MATHEMATICS

Written examination 2

STUDENT NAME:

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTIONAND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
18	18	60
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 23 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

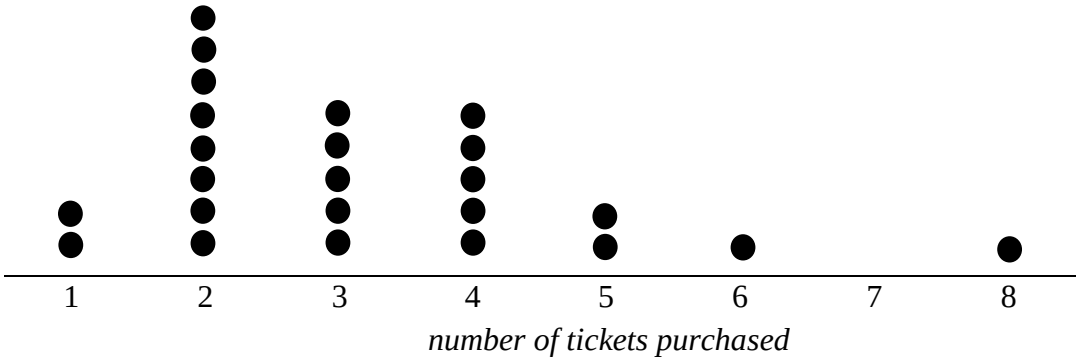
In all questions where a numerical answer is required, you should only round your answer when instructed to do so.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (5 marks)

A musical production is showing at a local theatre. The number of tickets purchased in each of the first 25 online transactions was recorded, as shown by the partially completed dot plot below.



a. Identify the type of data represented by the dot plot.

1 mark

_____

b. Only 24 transactions are represented in the dot plot above. There was a total of 80 tickets purchased in the 25 transactions.

Draw a dot on the graph above to show the missing transaction.

1 mark

c. Determine the mean number of tickets sold in these 25 transactions.

1 mark

_____

d. Show that the transaction in which 8 tickets were purchased is an outlier.

2 marks

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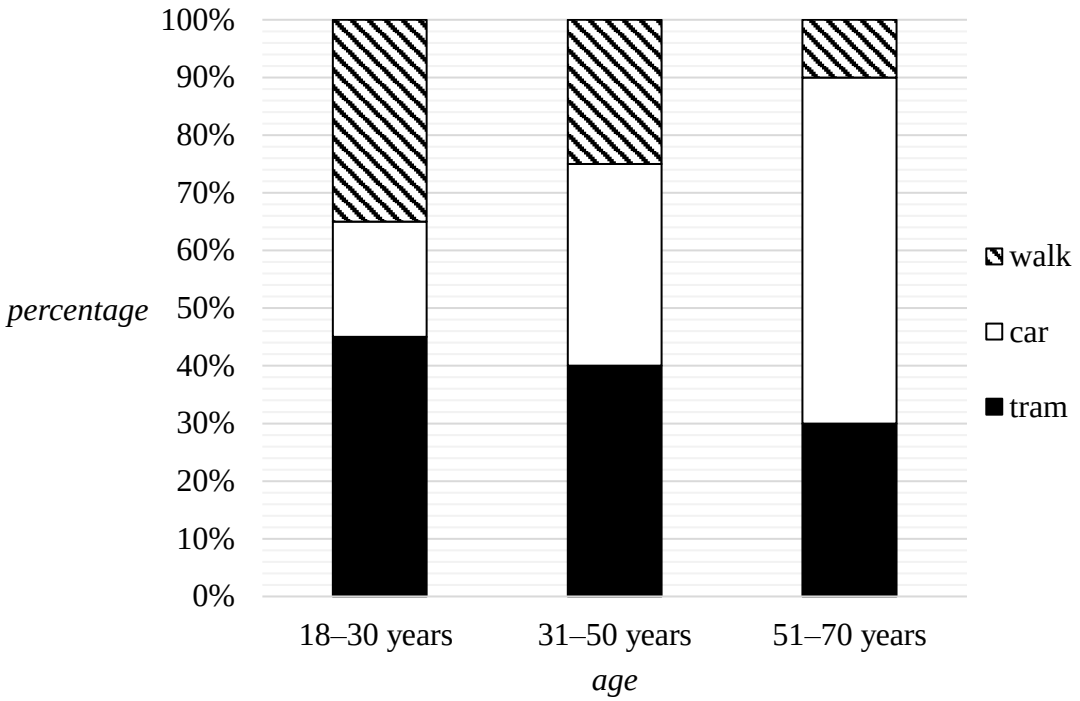
Question 2 (5 marks)

The mode of transport used to get to a theatre was recorded for the 150 customers attending one night. The data is represented in the following percentage-segmented bar chart.



a. What percentage of customers aged between 18 and 30 used a car to get to the theatre?

1 mark

_____

b. Of the 150 customers surveyed, 40% were in the 18–30 age group, 40% were in the 31–50 age group and the remainder were in the 51–70 age group.

How many people in total used the tram?

1 mark

_____

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c. Does the percentage-segmented bar chart support the contention that mode of transport is associated with age? Refer to the values of an appropriate statistic to support your answer.

2 marks

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_____

d. On another performance night, the age of those who caught the tram was found to be normally distributed.

The youngest 16% of the group were 23 years or younger, and the oldest 2.5% of the group were at least 35 years old.

Using the 68–95–99.7% rule, what is the mean age of those who caught the tram?

1 mark

_____

_____

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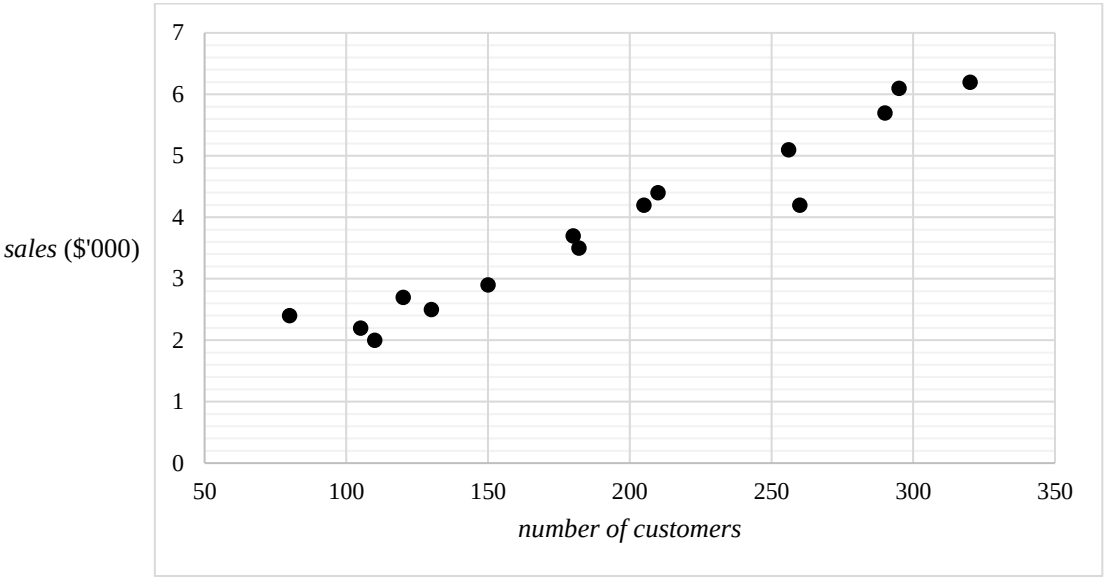
Question 3 (5 marks)

The scatterplot below shows the *number of customers* plotted against *sales* (in thousands of dollars) of merchandise over 15 nights of the performance.



When a least squares line is fitted to the scatterplot, its equation is:

$$sales = 0.37 + 0.018 \times number\ of\ customers$$

The coefficient of determination of the data is 0.948.

a. Interpret the coefficient of determination in terms of the variables *number of customers* and *sales*, writing the value as a percentage to the nearest whole number.

1 mark

b. Calculate the value of the correlation coefficient, *r*.

Round your answer to 2 decimal places.

1 mark

c. The theatre's management wants to predict how many customers would be needed to obtain sales of $8000. Based on the scatterplot above, how many customers would be needed to achieve this?

1 mark

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d. The prediction in **part c.** is an example of extrapolation.

Complete the following to show the values for the *number of customers* that would result from a prediction using interpolation.

1 mark

$\square \leq$ *number of customers* $\leq \square$

e. On a night when there were 200 customers, the sales were $170 less than predicted.

What were the sales on that night?

1 mark

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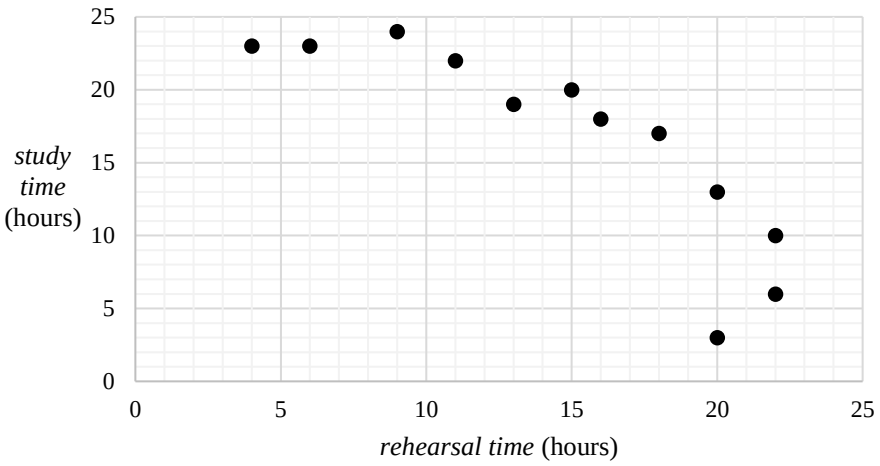
Question 4 (3 marks)

Billy is one of the performers at the theatre. He is also studying at university. The number of hours he rehearses and the number of hours he spends studying per week are recorded.



***Time spent rehearsing* (hours)**	***Time spent studying* (hours)**
6	23
9	24
13	19
18	17
11	22
20	3
20	13
16	18
15	20
22	10
22	6
4	23

a. Identify the direction and form of the scatterplot shown.

1 mark

_____

b. In order to linearise the data, a transformation is performed on the explanatory variable, *time spent rehearsing*. This results in the correlation coefficient, *r*, increasing from –0.852 to –0.898.

Apply an appropriate transformation to the variable *time spent rehearsing.* Write the equation of the regression line fitted to the transformed data. Round the *y*-intercept and slope to 2 significant figures.

2 marks

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_____

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Question 5 (6 marks)

The daily ticket sales for 12 days of performances by the theatre are shown below.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**Average**
Week 1	120	110	150	180	290	320	195
Week 2	100	90	160	190	270	330	190

a. The seasonal index for Saturday is 1.69.

Which days are expected to have a seasonal index below 1?

2 marks

_____

b. Calculate the seasonal index for Wednesday, rounded to 2 decimal places.

1 mark

_____

c. Calculate the deseasonalised ticket sales for Saturday in Week 2, rounded to the nearest whole number.

1 mark

_____

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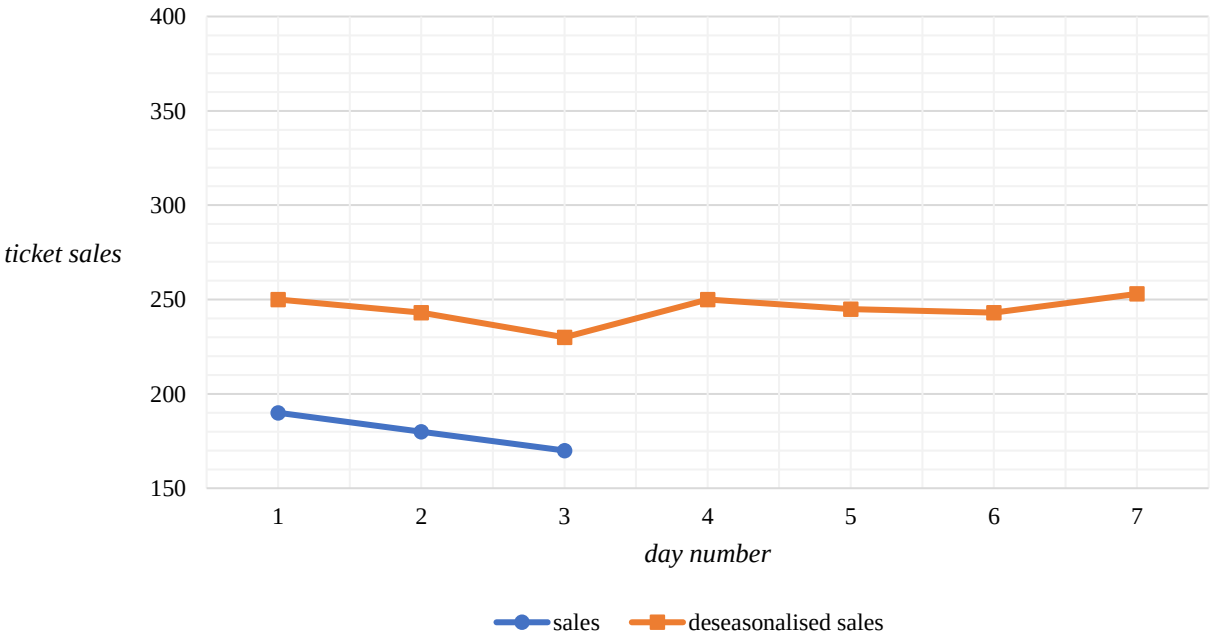
d. After a promotion, an extra performance was added on Sunday and an increase in sales was observed across the week. New seasonal indices as well as the deseasonalised sales for a particular week are shown in the table below.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**Sunday**
Seasonal index	0.78	0.74	0.74	0.96	1.43	1.48	0.87
Deseasonalised sales	250	243	230	250	245	243	253

Complete the graph below by plotting the remainder of the actual sales for the weeks.

2 marks



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Recursion and financial modelling

Question 6 (3 marks)

Seth purchases an electric scooter for $800. The scooter depreciates by 7% per annum on a reducing balance basis.

a. What is the value of the scooter after the first year?

1 mark

_____

b. Complete the rule that gives the value of the scooter, V_n, in dollars, after n years in the boxes below.

1 mark

$$V_n = \square \times \square^{n}$$

c. How much has the scooter depreciated by after 4 years?

Round your answer to the nearest dollar.

1 mark

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Question 7 (3 marks)

In order to save for an overseas trip, Seth invests money in an account that earns interest compounding fortnightly, as shown by the recurrence relation below.

$$V_0 = 360 \qquad V_{n+1} = 1.0016V_n + 210$$

a. After how many fortnights will the balance of Seth's account first exceed $1200?

1 mark

b. What is the annual interest rate of this account?

1 mark

c. How much interest was earned over 3 fortnights?

1 mark

Question 8 (6 marks)

Seth did not have sufficient funds for his overseas travels, so he borrowed $4000 from his sister Isabel.

They agreed she would charge him 6% per annum, compounding monthly, with an interest-only period for the first 12 months.

a. How much in total would Seth need to pay Isabel for the 12-month interest-only period?

1 mark

b. If instead of paying Isabel, Seth had left the balance to accrue with compound interest during that 12-month period, how much interest would he owe at the end of that period?

Round your answer to the nearest dollar.

1 mark

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c. Once the interest-only period finishes, Seth begins making regular monthly repayments for both the principal and interest. He expects it will take him three years of repayments at $122 per month to pay Isabel back.

The first four lines of the amortisation table of the loan schedule for the remaining three years are shown below.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	4000.00
1	122.00	20.00	102.00	
2	122.00	19.49	102.51	3795.49
3	122.00			

Calculate the balance after the first payment.

1 mark

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_____

d. Calculate the principal reduction for the third month.

1 mark

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_____

e. After some time, Seth increases his repayments, which results in a shorter loan term.

If his last 6 repayments are $200 per month, by how many months is the term of the 3-year loan reduced?

2 marks

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Matrices

Question 9 (2 marks)

Mac owns a farm. The number of sheep (*S*), cattle (*C*) and goats (*G*) in the east part of the farm is represented by matrix *E* below.

$$E = \begin{matrix} S \\ C \\ G \end{matrix}\begin{bmatrix} 120 \\ 135 \\ 42 \end{bmatrix}$$

a. What is the order of matrix *E*?

1 mark

_____

b. Write a matrix product with a summing matrix to calculate the total number of sheep, cattle and goats in the east part of the farm.

1 mark

_____

Question 10 (2 marks)

Mac purchases three types of grain to feed his lambs: Superior (*S*), Extralife (*E*) and Pure (*P*).

Matrix *A* shows the cost in dollars per kilogram of each type of grain.

$$\begin{matrix} & S & E & P \end{matrix}$$
$$A = \begin{bmatrix} 8 & 4 & 7 \end{bmatrix}$$

Matrix *B* shows the number of kilograms (*K*) of each grain delivered per week, and the number of kilograms (*L*) of each grain that is eaten each week.

$$\begin{matrix} & K & L \end{matrix}$$
$$B = \begin{matrix} \\ \\ \\ \end{matrix}\begin{bmatrix} 5 & 3 \\ 2 & 1 \\ 6 & 6 \end{bmatrix}\begin{matrix} S \\ E \\ P \end{matrix}$$

a. Explain why the matrix product *AB* is defined.

1 mark

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b. The matrix product *AB* is $M = [90 \quad 70]$. Interpret the element M_{12}.

1 mark

_____

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Question 11 (2 marks)

Mac wants to determine the cost of two types of shed that neighbouring farmers have purchased. He uses simultaneous equations to determine the unknown costs. Part of the process involves finding the relevant inverse matrix, as shown below.

$$\begin{bmatrix} 2 & 2 \\ 4 & x \end{bmatrix}^{-1} = \begin{bmatrix} -1.5 & 1 \\ 2 & y \end{bmatrix}$$

Determine the values of x and y.

Question 12 (2 marks)

Each week the cattle are rotated between three paddocks, *A*, *B* and *C*, as shown in the matrix *T* below.

$$T = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \text{This week} \\ \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.15 \\ 0.3 & 0.6 & 0.3 \\ 0.2 & 0 & 0.55 \end{bmatrix} \end{array} \begin{array}{l} \\ A \\ B \\ C \end{array} \text{ Next week}$$

a. In the long term, what percentage of the cattle is in paddock *A*? Round your answer to the nearest whole number.

1 mark

b. There are 170 cattle in total. Before the first rotation, there was an equal number of cattle in paddocks *A* and *C*. After the first week, there were 30 cattle in paddock *C*.

Complete the initial state matrix below.

1 mark

$$S_0 = \begin{bmatrix} \quad \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

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Question 13 (4 marks)

The survival and birth rate of a rodent population that lives amongst Mac's crops is shown below.

The life span of the rodents can be divided into four age groups: *A* (0−3 months), *B* (3−6 months), *C* (6−9 months) and *D* (9−12 months).

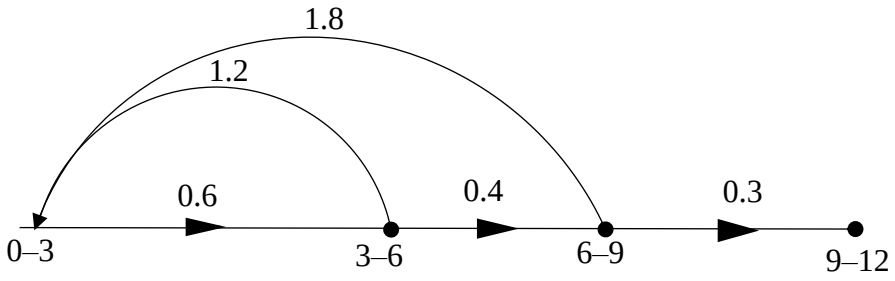
The Leslie matrix, *L*, that models the breeding patterns of this rodent population is as follows.

$$L = \begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 0 & 1.2 & 1.8 & 0 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{bmatrix} \end{array} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$$

The initial state matrix for the rodent population, S_0, is shown below.

$$S_0 = \begin{bmatrix} 80 \\ 90 \\ 50 \\ 30 \end{bmatrix}$$

The same information is presented in the transition diagram below.



a. Use the Leslie matrix, initial state matrix and diagram to complete the table below.

2 marks

Age	Current population	Birth rate	Survival rate
A: 0–3 months	80		
B: 3–6 months	90	1.2	
C: 6–9 months	50		0.3
D: 9–12 months		0.0	0.0

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b. Using the equation $S_{n+1} = L \times S_n$, determine the number of rodents aged between 9 and 12 months after one 3-month period.

1 mark

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_____

c. Determine the percentage, to the nearest whole number, by which the total rodent population increased in that 3-month period.

1 mark

_____

_____

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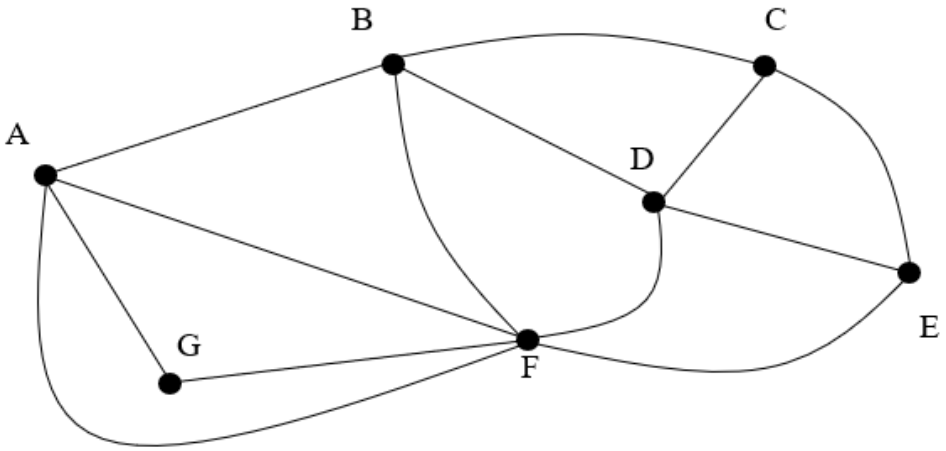
Networks and decision mathematics

Question 14 (2 marks)

Maraika works at the Botanical Gardens.

There are 7 greenhouses connected by walking paths and each is surrounded by lawns that can be used for picnics and playing. The graph below represents the paths between each greenhouse.



a. Complete Euler's formula in the boxes below to show the number of vertices, faces and edges of the graph.

1 mark

$\square + \square - \square = 2$

b. Given that Euler's formula holds for this graph, what type of graph is it?

1 mark

_____

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Question 15 (2 marks)

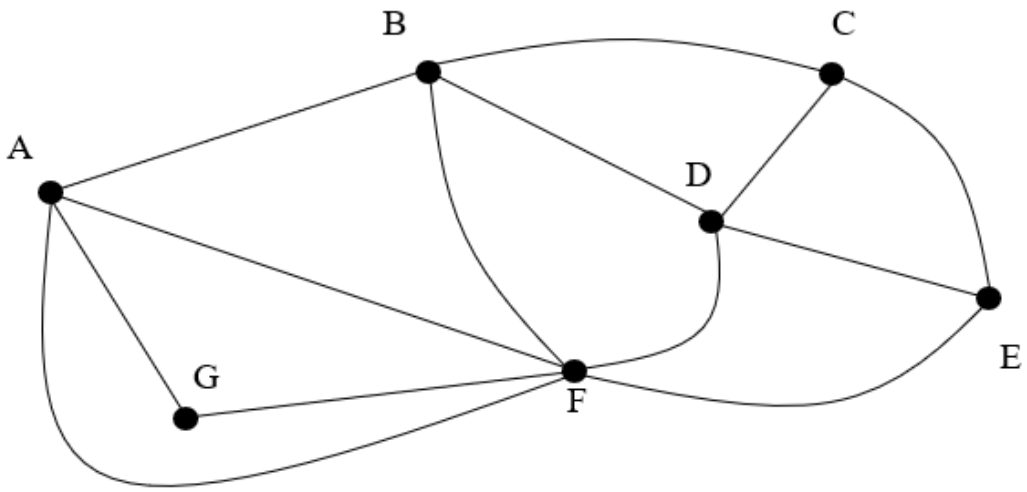
a. Maraika wants to use a Hamilton path to visit every greenhouse, starting at *C* and ending at *E*. Write down a route she can take.

1 mark

_____

b. Maraika wants to inspect each walking path, starting at and returning to the same greenhouse. On the graph below, draw the missing edge required for her to do this.

1 mark



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Question 16 (4 marks)

a. Four teams of staff, A, B, C and D, were allocated to four jobs in the northern part of the Gardens. The time each team took to complete each job, in hours, is shown in the table below.

	Hours spent on each job			
Team	**Weeding**	**Watering**	**Mowing**	**Planting**
A	17	6	5	14
B	15	7	5	12
C	21	7	6	12
D	15	8	4	19

The northern part of the Gardens needed further attention. This time, each team will be allocated just one job in order to minimise the total time needed to complete all four jobs.

Which job should each team be allocated to so that the job is completed in the minimum amount of time?

Write your answer in the table below.

2 marks

Team	**Job**
A	
B	
C	
D	

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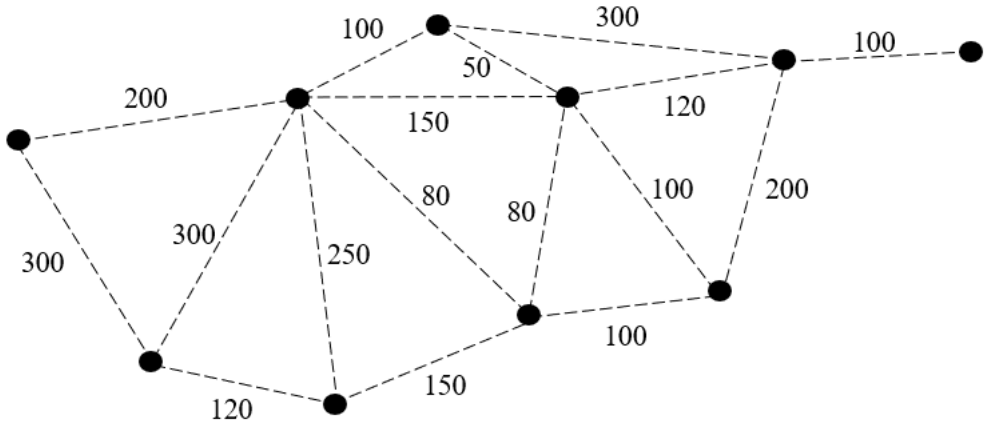
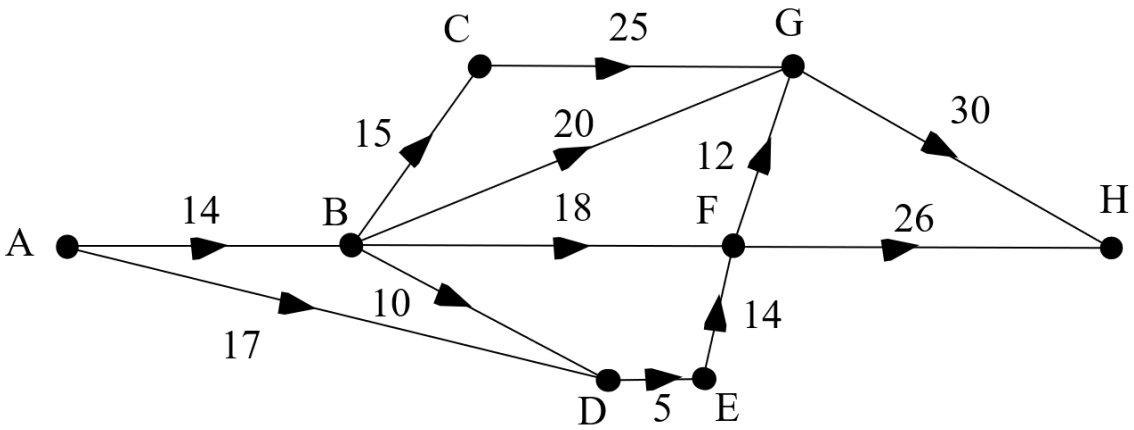
b. A watering system is set up between several garden beds. The distance, in metres, between each garden bed is shown below. In order to minimise the length of pipe running between the beds, a minimum spanning tree is used.

Draw a minimum spanning tree on the graph below.

1 mark



c. A section of the watering system is shown below. The capacity of each pipe in litres per hour as water flows from the source, *A*, to the sink, *H*, is also shown.



Determine the maximum flow from *A* to *H*.

1 mark

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Question 17 (2 marks)

The schedule of activities for the redesign of one of the gardens, as well as their duration, is shown in the table below.

Activity	Duration (hours)	Immediate predecessors
A	3	–
B	4	*A*
C	2	*B*
D	4	*B*
E	1	*C*
F	2	*D*, *E*
G	4	*F*

a. Which activity has an earliest start time of 9 hours?

1 mark

_____

_____

_____

b. The critical path has a minimum completion time of 17 hours.

List the activities that are not on the critical path.

1 mark

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_____

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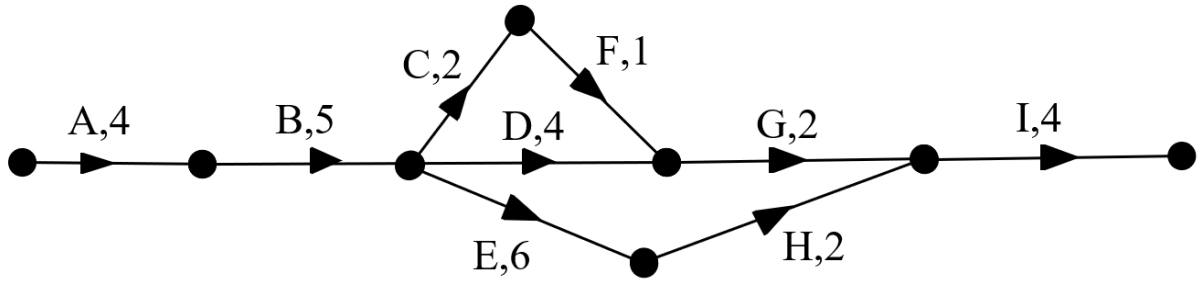
Question 18 (2 marks)

Some changes are made to the project, with the new schedule represented by the directed graph below. The duration of each activity is in hours.



a. What is the critical path and completion time?

1 mark

_____

b. Two activities have their lengths reduced, resulting in three critical paths with a minimum completion time of 18 hours.

Complete the table below to list the two activities that have had their completion times reduced. Show the amount by which they are reduced so that the overall completion time is 18 hours.

1 mark

Activity	Reduction in hours

END OF QUESTION AND ANSWER BOOK

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