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YEAR 12 *Trial Exam Paper*

2023

GENERAL MATHEMATICS

Written examination 2

Worked solutions

This book presents:

- worked solutions
- mark allocations
- tips.

Data analysis

Question 1a.

Worked solution

Numerical discrete

Mark allocation: 1 mark

- 1 mark for the correct answer: numerical discrete

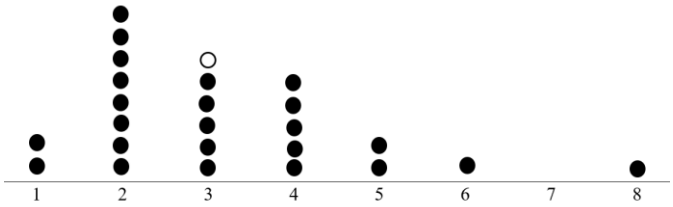
Tip

- *Discrete variables have distinct, countable values. Each discrete variable has a distinct previous and distinct next value on a number line. By contrast, continuous variables can take on any value and so can be placed anywhere on a number line. For continuous variables, there is no set previous or next value.*

Question 1b.

Worked solution

The number of tickets purchased with the partially completed dot plot of 24 transactions is 77 (1 × 2 + 2 × 8 + 3 × 5 + 4 × 5 + 5 × 2 + 6 + 8), leaving a transaction of 3 tickets missing.



Mark allocation: 1 mark

- 1 mark for the correct placement of the additional dot

Question 1c.

Worked solution

80/25 = 3.2

Mark allocation: 1 mark

- 1 mark for the correct answer: 3.2

Tip

- *If a terminating decimal is calculated, don't round it unless it is requested in the question.*

Question 1d.

Worked solution

$Q_1 = 2$, $Q_3 = 4$, so IQR = 2

Upper fence = $Q_3 + (1.5 \times \text{IQR}) = 4 + (1.5 \times 2) = 7$

Since $8 > 7$ it is considered to be an outlier.

Mark allocation: 2 marks

- 1 mark for calculating the upper fence: 7
- 1 mark for confirming that 8 is an outlier as it is greater than the upper fence

Question 2a.

Worked solution

65 – 45 = 20%

Mark allocation: 1 mark

- 1 mark for the correct answer: 20%

Question 2b.

Worked solution

Number of customers in each group:

- 18–30 years: 40% × 150 = 60
- 31–50 years: 40% × 150 = 60
- 51–70 years: 20% × 150 = 30

Number of tram users in each group:

- 18–30 years: 45% × 60 = 27
- 31–50 years: 40% × 60 = 24
- 51–70 years: 30% × 30 = 9

Total number of tram users = 27+ 24 + 9 = 60

Mark allocation: 1 mark

- 1 mark for the correct answer: 60

Question 2c.

Worked solution

Yes, there is an association, as the percentage of use for each mode of transport differs between the age groups:

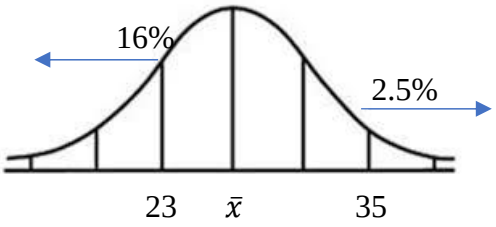
- tram use decreases from 45% (18–30) to 40% (31–50) to 30% (51–70)
- car use increases from 20% (18–30), to 35% (31–50) to 60% (51–70)
- a preference for walking decreases from 35% (18–30) to 25% (31–50) to 10% (51–70).

Mark allocation: 2 marks

- 1 mark for stating that there is an association, as evidenced by a difference in travel mode across the three age groups
- 1 mark for comparing the percentages of at least one mode of transport as evidence

Question 2d.

Worked solution



There are 3 standard deviations between these two ages, which is (35 – 23)/3 = 4 years as one standard deviation.

The mean age is 31 – 4 = 27 years old.

Mark allocation: 1 mark

- 1 mark for the correct answer: 27

Question 3a.

Worked solution

Approximately 95% of the variation in *sales* can be explained by the variation in *number of customers*.

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3b.

Worked solution

$r = \sqrt{0.948} = 0.973$

Mark allocation: 1 mark

- 1 mark for the correct answer: 0.97

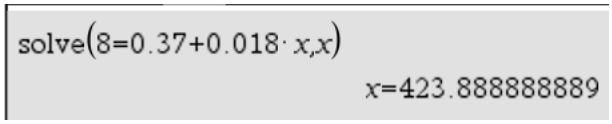
Tip

- *Don't forget to check if r is negative or positive after finding the square root of r^2. This can be determined from the direction of the slope of the scatterplot.*

Question 3c.

Worked solution

8000 needs to be substituted with 8 in the equation for the least squares line, as the units are in thousands.



This value must be rounded to 424, as a fraction of a ticket cannot be purchased.

Mark allocation: 1 mark

- 1 mark for the correct answer: 424

Question 3d.

Worked solution

Interpolation involves estimating values within a range of data, which in this case is from 80 to 320 customers. Hence, $80 \leq$ *number of customers* ≤ 320.

Mark allocation: 1 mark

- 1 mark for giving the values of 80 and 320 (in that order)

Question 3e.

Worked solution

When actual sales are lower than predicted, the residual value is negative: –170 is –0.17.

The predicted sales can be calculated to be 3.97 ($3970) with 200 customers, and then *solve* can be used to determine what the unknown actual sales would be.

$0.37+0.018 \cdot 200$	3.97
$\text{solve}(x-3970=-170,x)$	$x=3800.$

Mark allocation: 1 mark

- 1 mark for the correct answer: $3800

Question 4a.

Worked solution

The scatterplot has a negative direction and is non-linear in form.

Mark allocation: 1 mark

- 1 mark for the answer of negative and non-linear (in any order)

Question 4b.

Worked solution

a	25.8025...
b	-0.0372...
r^2	0.80610...
r	-0.8978...

The only options for this graph are x^2 and y^2. When applying a squared transformation on the explanatory variable, *time spent rehearsing*, the equation would be:

time spent studying = 26 – 0.037 × (*time spent rehearsing*)^2

Mark allocation: 2 marks

- 1 mark for giving the correct form of the equation:
  time spent studying = $a + b \times$ (*time spent rehearsing*)^2
  or
  time spent studying = $b \times$ (*time spent rehearsing*)$^2 + a$
- 1 mark for obtaining the correct values: 26 and –0.037

Question 5a.

Worked solution

Monday, Tuesday, Wednesday and Thursday all have sales values that fall below the weekly sales average, which indicates they have a seasonal index below 1 (where 1 is the average sales).

Although Thursday has one value equal to its weekly average (190), the seasonal index is still less than 1. This is because the other value for Thursday (180) is sufficiently below its weekly average. The calculation is shown below.

$$\frac{\frac{180}{195}+\frac{190}{190}}{2} \qquad 0.961538461538$$

Mark allocation: 2 marks

- 1 mark for answering Monday, Tuesday and Wednesday
- 1 mark for answering Thursday

Question 5b.

Worked solution

$$\frac{\frac{150}{195}+\frac{160}{190}}{2} \qquad 0.805668016194$$

Mark allocation: 1 mark

- 1 mark for the correct answer: 0.81

Question 5c.

Worked solution

$$\frac{330}{1.69} \qquad 195.266272189$$

Mark allocation: 1 mark

- 1 mark for the correct answer: 195

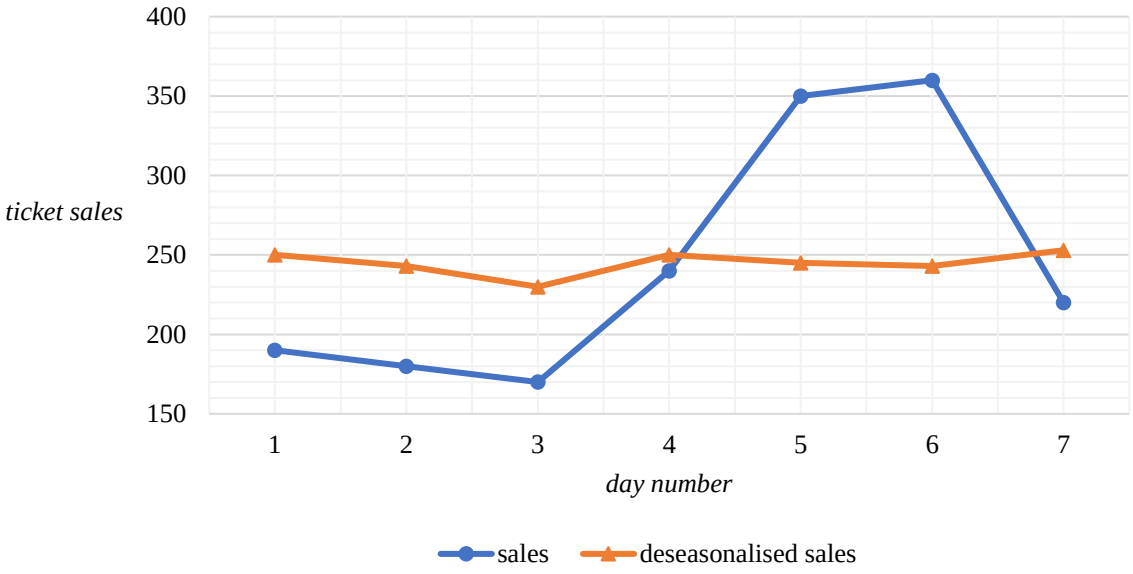
Question 5d.

Worked solution

actual sales = deseasonalised sales × seasonal index

Thursday: 250 × 0.96 = 240 Friday: 245 × 1.43 = 350

Saturday: 243 × 1.48 = 360 Sunday: 253 × 0.87 = 220



Mark allocation: 2 marks

- 2 marks for drawing a graph that is accurate, with no errors in values or lines

Recursion and financial modelling

Question 6a.

Worked solution

$0.93 \times 800 = \$744$

Mark allocation: 1 mark

- 1 mark for the correct answer: $744

Question 6b.

Worked solution

$V_n = 800 \times 0.93^n$

Mark allocation: 1 mark

- 1 mark for writing 800 and 0.93 (in that order)

Question 6c.

Worked solution

$(0.93)^4 \cdot 800$	598.441608
800−598.441608	201.558392

Mark allocation: 1 mark

- 1 mark for the correct answer: $202

Tip

- *Read carefully to determine if the question is asking for the value after depreciation or the amount of depreciation over a period of time.*

Question 7a.

Worked solution

After fortnight 1: $360 \times 1.0016 + 210 = 570.58$

After fortnight 2: $570.58 \times 1.0016 + 210 = 781.49$

After fortnight 3: $781.49 \times 1.0016 + 210 = 992.74$

After fortnight 4: $992.74 \times 1.0016 + 210 = 1204.33$

Mark allocation: 1 mark

- 1 mark for the correct answer: 4

Question 7b.

Worked solution

0.0016 × 26 × 100 = 4.16%

Mark allocation: 1 mark

- 1 mark for the correct answer: 4.16%

Question 7c.

Worked solution

Balance after 3 fortnights = 992.74

Amount invested = 360 + (210 × 3) = 990

Interest = 992.74 – 990 = 2.74

Mark allocation: 1 mark

- 1 mark for the correct answer: $2.74

Question 8a.

Worked solution

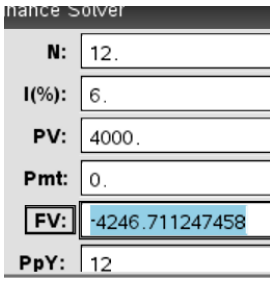
0.06/12 × 4000 = $20 per month = $240 for the year

Mark allocation: 1 mark

- 1 mark for the correct answer: $240

Question 8b.

Worked solution



$4247 – $4000 = $247 interest

Mark allocation: 1 mark

- 1 mark for the correct answer: $247

Question 8c.

Worked solution

4000 – 102 = 3898 (or 3795.49 + 102.51 = 3898)

Mark allocation: 1 mark

- 1 mark for the correct answer: $3898

Question 8d.

Worked solution

Interest = 0.06/12 × 3795.49 = 18.98

Principal reduction = 122 – 18.98 = $103.02

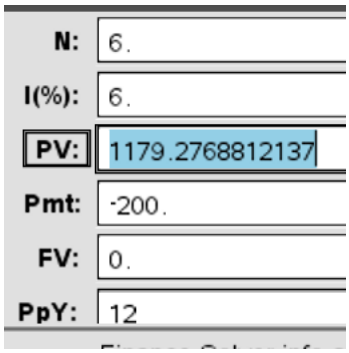
Mark allocation: 1 mark

- 1 mark for the correct answer: $103.02

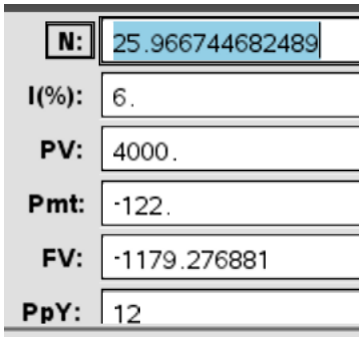
Question 8e.

Worked solution

Begin by finding the balance when the final 6 months of higher repayments begins:



Then use that balance (FV) to determine how many months of $122 payments were required:



26 + 6 = 32 months, which is 4 months less than the original 36 months

Mark allocation: 2 marks

- 2 marks for the correct answer: 4 months

  Note: Award 1 mark if the answer is incorrect but there is evidence that $1179 was obtained in calculations (or $1175.43, the amount owing for the last 6 months)

Matrices

Question 9a.

Worked solution

The order of a matrix is the number of rows by the number of columns: in this case 3 × 1.

Mark allocation: 1 mark

- 1 mark for the correct answer: 3 × 1

Question 9b.

Worked solution

$[1 \quad 1 \quad 1] \times E = [297]$

Mark allocation: 1 mark

- 1 mark for the correct answer: $[1 \quad 1 \quad 1] \times E = [297]$

Question 10a.

Worked solution

The number of columns of A is equal to the number of rows of B.

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 10b.

Worked solution

Element M_{12} is the total cost of grain eaten each week: \$70.

$$[8 \quad 4 \quad 7] \cdot \begin{bmatrix} 5 & 3 \\ 2 & 1 \\ 6 & 6 \end{bmatrix} \qquad [90 \quad 70]$$

Mark allocation: 1 mark

- 1 mark for the correct answer: the total cost of grain eaten per week (\$70)

Question 11

Worked solution

$\text{Det} = ad - bc = 2x - 8$

$$\frac{1}{2x-8}\begin{bmatrix} x & -2 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -1.5 & 1 \\ 2 & y \end{bmatrix}$$

Taking one part of the calculation:

$$\frac{1}{2x-8} \times x = -1.5$$

$$\text{solve}\left(\frac{1}{2 \cdot x - 8} \cdot x = -1.5, x\right) \qquad x = 3.$$

Substituting x into the initial equation and solving gives:

$$\begin{bmatrix} 2 & 2 \\ 4 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} -1.5 & 1 \\ 2 & -1 \end{bmatrix}$$

Thus the values are $x = 3$ and $y = -1$.

Mark allocation: 2 marks

- 1 mark for the correct answer for x: 3
- 1 mark for the correct answer for y: –1

Question 12a.

Worked solution

Using just the transition matrix with a high power for the long term shows the overall percentage of cattle expected at each location. For paddock *A*, it is 39.6% (rounded to 40%).

$$\begin{bmatrix} 0.5 & 0.4 & 0.15 \\ 0.3 & 0.6 & 0.3 \\ 0.2 & 0 & 0.55 \end{bmatrix}^{50}$$

$$\begin{bmatrix} 0.395604395604 & 0.395604395604 & 0.395 \\ 0.428571428571 & 0.428571428571 & 0.428 \\ 0.175824175824 & 0.175824175824 & 0.175 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 0.4 & 0.15 \\ 0.3 & 0.6 & 0.3 \\ 0.2 & 0 & 0.55 \end{bmatrix}^{50}$$

$$\begin{bmatrix} 5604 & 0.395604395604 & 0.395604395604 \\ 8571 & 0.428571428571 & 0.428571428571 \\ 5824 & 0.175824175824 & 0.175824175824 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for the correct answer: 40%

Question 12b.

Worked solution

The value of x can be used for the initial values in both A and C as they are the same.

$\text{solve}(0.2 \cdot x + 0.55 \cdot x = 30, x) \qquad x = 40.$

If the number of cattle in both A and C is 40, then B must have 170 – (40 + 40) = 90 cattle.

Mark allocation: 1 mark

- 1 mark for the correct answer: $S_o = \begin{bmatrix} 40 \\ 90 \\ 40 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$

Question 13a.

Worked solution

Age	Current population	Birth rate	Survival rate
A: 0–3 months	80	0.0	0.6
B: 3–6 months	90	1.2	0.4
C: 6–9 months	50	1.8	0.3
D: 9–12 months	30	0.0	0.0

Mark allocation: 2 marks

- 2 marks for all five values calculated correctly
- 1 mark if there is no more than one error

Question 13b.

Worked solution

The age distribution after 1 period of 3 months is:

$$\begin{bmatrix} 0 & 1.2 & 1.8 & 0 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 80 \\ 90 \\ 50 \\ 30 \end{bmatrix} \qquad \begin{bmatrix} 198. \\ 48. \\ 36. \\ 15. \end{bmatrix}$$

Thus there are 15 rodents aged 9–12 months.

Mark allocation: 1 mark

- 1 mark for the correct answer: 15

Question 13c.

Worked solution

The total population increased from 250 to 297 (that is, 198 + 48 + 36 + 15). This increase is 47/250 × 100 = 18.8%, or 19% to the nearest whole number.

Mark allocation: 1 mark

- 1 mark for the correct answer: 19%

Networks and decision mathematics

Question 14a.

Worked solution

$V + F - E = 2$

$7 + 8 - 13 = 2$

Mark allocation: 1 mark

- 1 mark for the correct answer: either 7, 8, 13 or 8, 7, 13

Question 14b.

Worked solution

Euler's formula holds for planar graphs.

Mark allocation: 1 mark

- 1 mark for the correct answer: planar

Tip

- *Be careful not to confuse Euler's formula with Euler paths or circuits.*

Question 15a.

Worked solution

C-D-B-A-G-F-E or *C-B-A-G-F-D-E*

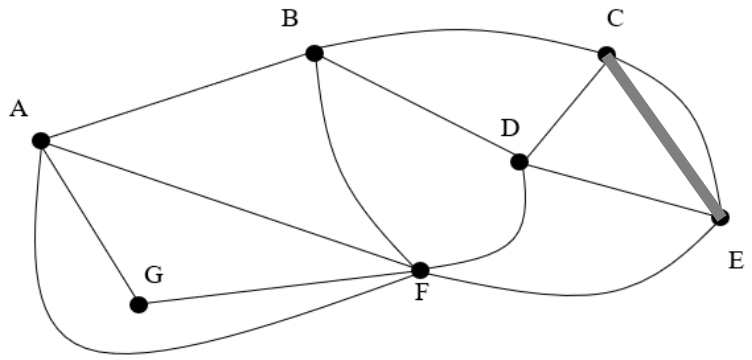
Mark allocation: 1 mark

- 1 mark for identifying one of the paths listed

Question 15b.

Worked solution

All vertices are even except *C* and *E*, so in order to create an Euler circuit where Maraika can follow every edge and return to where she started, an extra edge is required between *C* and *E*.



Mark allocation: 1 mark

- 1 mark for drawing a line between *C* and *E*

Question 16a.

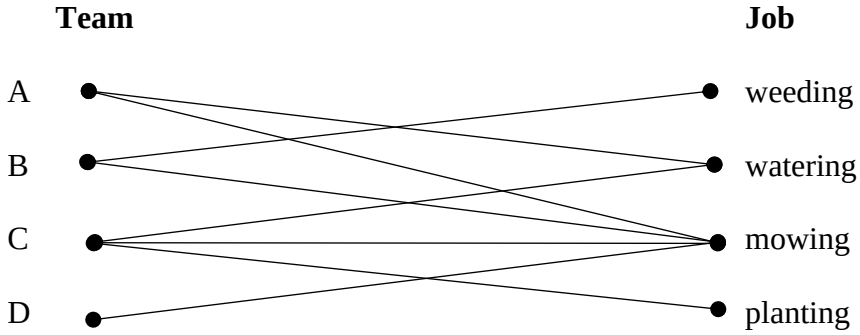
Worked solution

The Hungarian Algorithm used to identify the allocation with the minimum time is:

$$\begin{bmatrix} 17 & 6 & 5 & 14 \\ 15 & 7 & 5 & 12 \\ 21 & 7 & 6 & 12 \\ 15 & 8 & 4 & 19 \end{bmatrix} \begin{matrix} -5 \\ -5 \\ -6 \\ -4 \end{matrix} = \begin{bmatrix} 12 & 1 & 0 & 9 \\ 10 & 2 & 0 & 7 \\ 15 & 1 & 0 & 6 \\ 11 & 4 & 0 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 1 & 0 & 9 \\ 10 & 2 & 0 & 7 \\ 15 & 1 & 0 & 6 \\ 11 & 4 & 0 & 15 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 5 & 0 & 0 & 0 \\ 1 & 3 & 0 & 9 \end{bmatrix}$$

$$-10 \quad -1 \quad 0 \quad -6$$



So the allocation that results in the least time is:

Team	Job
A	watering
B	weeding
C	planting
D	mowing

Mark allocation: 2 marks

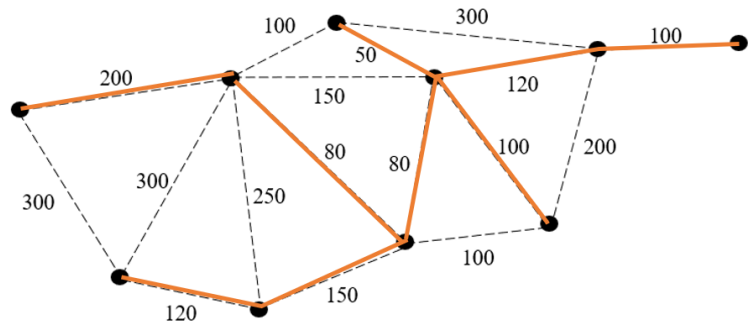
- 2 marks for allocating teams as shown in the table above

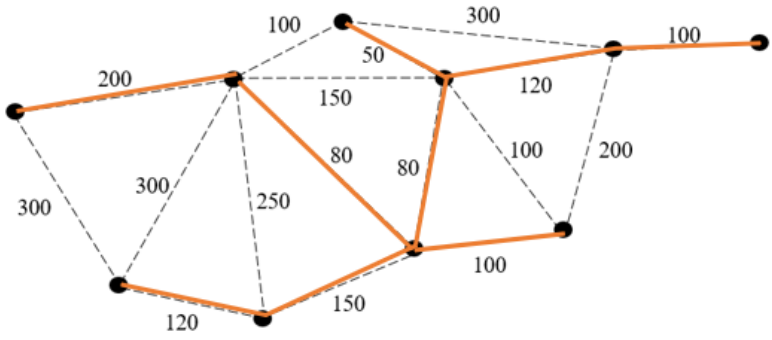
Note: Award 1 mark if two or three allocations are correct

Question 16b.

Worked solution

There are two solutions:





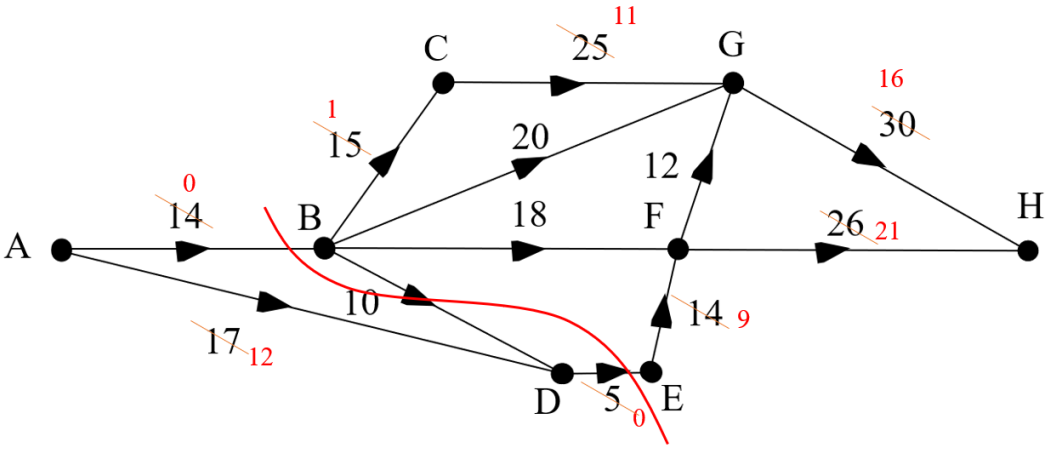
Mark allocation: 1 mark

- 1 mark for either of the spanning trees drawn above

Question 16c.

Worked solution

The minimum cut is 14 + 5 = 19 litres per hour. The value of 10 (*B* to *D*) is running from the sink side of the cut to source side of the cut (rather than source to sink) and so is not included.



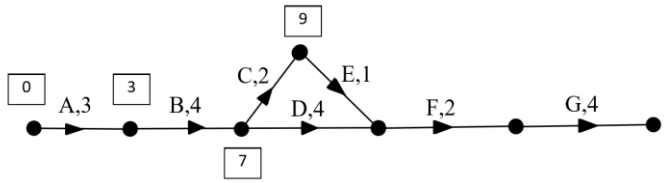
Mark allocation: 1 mark

- 1 mark for the correct answer: 19 litres per hour

Question 17a.

Worked solution

Activity *E* cannot begin until *A*, *B* and *C* are completed, and together *A*, *B* and *C* requires 9 hours.

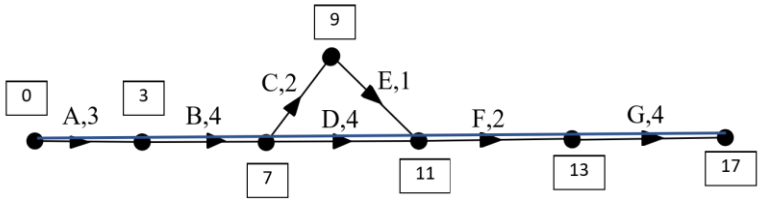


Mark allocation: 1 mark

- 1 mark for the correct answer: E

Question 17b.

Worked solution



The critical path is *A*-*B*-*D*-*F*-*G*, which leaves *C* and *E* not on the critical path.

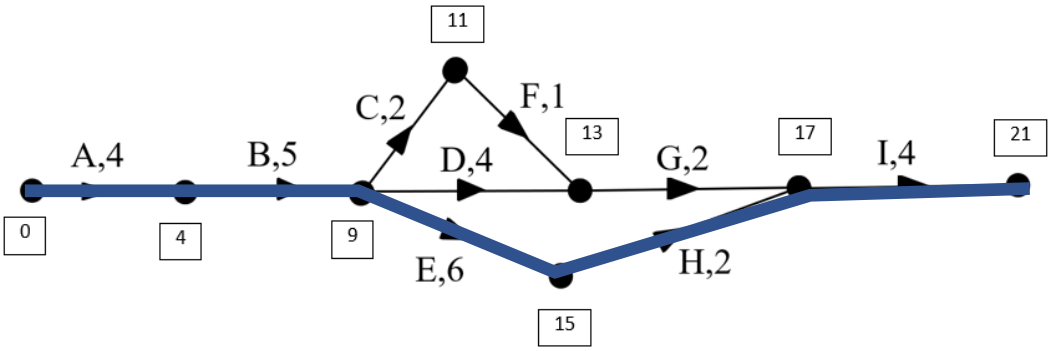
Mark allocation: 1 mark

- 1 mark for the correct answer: *C* and *E*

Question 18a.

Worked solution

A-B-E-H-I, 21 hours



Mark allocation: 1 mark

- 1 mark for the correct answer: *A-B-E-H-I*, 21 hours

Question 18b.

Worked solution

There are three paths in the network, which means all three will be critical paths.

A-B-C-F-G-I already takes 18 hours, which means no reduction is required.

A-B-D-G-I takes 19 hours, which means it needs to be reduced by 1 hour. Without impacting *A-B-C-F-G-I*, the only option is to reduce *D* by 1.

A-B-E-H-I takes 21 hours, which means it needs to be reduced by 3 hours. Only *E* and *H* can be reduced by 3 hours on that path without further impacting the other two paths. *H* is only 2 hours, so it must be *E* that is reduced by 3.

Activity	Reduction in hours
E	3
D	1

Mark allocation: 1 mark

- 1 mark for stating that E must be reduced by 3 hours and D must be reduced by 1 hour

Tip

- *Reducing one edge may have a different impact on the minimum completion of the project, as the critical path may shift. Always check every path to see what has changed.*

END OF WORKED SOLUTIONS