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YEAR 12 *Trial Exam Paper*

2023

GENERAL MATHEMATICS

Written examination 1

Worked solutions

This book presents:

- worked solutions
- mark allocations
- tips.

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Question	Answer
1	*D*
2	*B*
3	*C*
4	*C*
5	*C*
6	*D*
7	*D*
8	*E*
9	*B*
10	*E*
11	*D*
12	*A*
13	*A*
14	*B*
15	*D*
16	*C*
17	*D*
18	*E*
19	*B*
20	*B*

Question	Answer
21	*D*
22	*C*
23	*A*
24	*A*
25	*C*
26	*C*
27	*E*
28	*D*
29	*B*
30	*E*
31	*B*
32	*D*
33	*E*
34	*C*
35	*D*
36	*B*
37	*C*
38	*C*
39	*C*
40	*E*

Data analysis

Question 1

Answer: D

Explanatory notes

Range = 84 – 17 = 67

Question 2

Answer: B

Explanatory notes

The median is between the ninth and tenth score, which is 39.

The mean = 38.6; IQR = 45 – 26 =19; standard deviation = 16.1; maximum score = 84.

Question 3

Answer: C

Explanatory notes

The data represent a numerical variable (*amount of money spent*) and a categorical variable (*shift*).

A segmented bar chart, scatterplot and dot plot are not suitable: a segmented bar chart requires two categorical variables; a scatterplot requires two numerical variables; and a dot plot uses only one numerical variable.

The back-to-back stem plot and parallel boxplots are both suitable for one numerical and one categorical variable; however, splitting the data over the three shifts will not fit on the stem plot, which would accommodate only two shifts instead of three.

Question 4

Answer: C

Explanatory notes

For positively skewed data, the median is lower than the mean, as the mean has been dragged upwards by the higher extreme scores.

Options A, D and E are incorrect, as the mean is significantly lower than the median.

Option B is incorrect, as the mean and median are the same, indicating that the data are symmetrical instead.

Question 5

Answer: C

Explanatory notes

$\log_{10}(10) = 1$ and $\log_{10}(100) = 2$, so between these values is a frequency of 6.

$$\frac{6}{24} \times 100 = 25\%$$

Question 6

Answer: D

Explanatory notes

As the histogram does not go beyond a log value of 4, which is equivalent to 10 000 cars, there must be no more than 10 000 cars in the car park. This means 6200 is the most appropriate estimate.

Question 7

Answer: D

Explanatory notes

Being placed in the bottom 2.5% is equivalent to two standard deviations below the mean, so the difference of 8 between Anna's mark and the mean is equal to two standard deviations. This means one standard deviation is 4.

This is also shown using the *z*-score formula:

$$\text{solve}\left(-2=\frac{54-62}{x}, x\right) \qquad x=4.$$

Question 8

Answer: E

Explanatory notes

Option A is incorrect because 25% of 21–40-year-olds visit zero to two times and 25% of 21–40-year-olds visit two to three times, making it the same quantity.

Option B is incorrect, as the range is smaller for 21–40-year-olds (i.e. seven visits compared to nine), so is less variable.

Option C is incorrect because although one-quarter of 21–40-year-olds visit the GP five times or more, and half of 41–60-year-olds visit five times or more, this isn't equivalent to a total of three-quarters. 50% of one quantity and 25% of another quantity doesn't equate to a total of 75%.

Option D is incorrect, as the median is two more (not fewer).

Option E is correct because 25% of 21–40-year-olds visit between three and five times, as do 25% of 41–60-year-olds.

Question 9

Answer: B

Explanatory notes

The slope is referring to the value of 1.8, which indicates the change in weight for every 1 unit (i.e. month) increase in age.

Question 10

Answer: E

Explanatory notes

The age 1.5 years needs to be converted to 18 months, then substituted into the equation as shown:

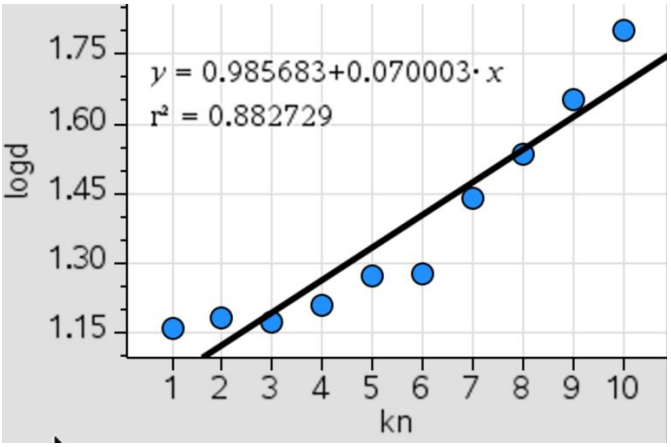
$30.5+1.8 \cdot 18$ $\quad 62.9$

Question 11

Answer: D

Explanatory notes

Transform the data, generate the transformed scatterplot and add the regression equation.



Finding the regression equation of the transformed data results in the values shown below.

a	0.98568...
b	0.07000...

Rounding these to two significant figures results in 0.99 and 0.070.

Tip

- *When rounding significant figures, start counting the significant figures at the first non-zero digit, as any zeros before that are required to stay as placeholders.*

Question 12

Answer: A

Explanatory notes

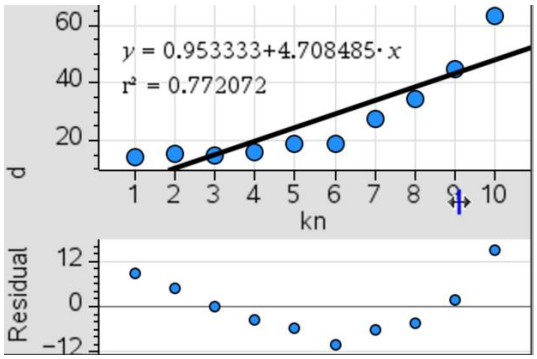
The statement in Option A is incorrect, as the residual plot still clearly shows a pattern.

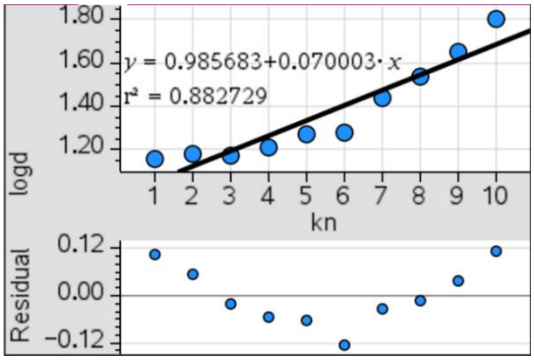
The statement in Option B is correct, as seen by the increase in the r value.

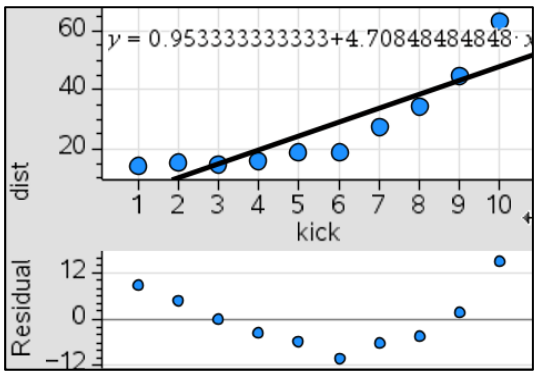
The statement in Option C is again referring to r^2, which is accurate.

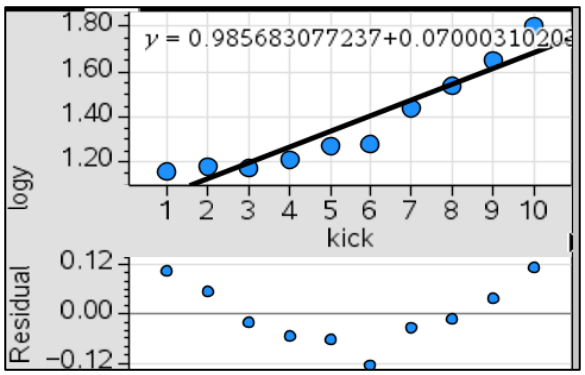
The statement in Option D is referring to $(1 - r^2)$ as a percentage, representing the amount *not* attributed to the variation in kick and distance.

The statement in Option E is shown below as true because the y-intercept has increased from 0.95 to 0.99 and the slope has decreased from 4.7 to 0.07.









r^2	0.77207...	r^2	0.88272...
r	0.87867...	r	0.93953...

Question 13

Answer: A

Explanatory notes

Option B: The smoothed value on Day 3 is incorrect.

Option C: Day 10 is incorrect.

Option D: Day 6 is incorrect.

Option E: Day 4 is incorrect.

Question 14

Answer: B

Explanatory notes

$a = 4 - (0.867 + 1.157 + 1.044) = 0.93$ or $\dfrac{\text{actual}}{\text{deseasonalised}} = 0.93$

b = actual = deseasonalised × SI = 72 000

62069· 1.157	71813.833

c = deseasonalised = $\dfrac{\text{actual}}{\text{SI}}$

$\dfrac{54000}{0.867}$	62283.7370242

Question 15

Answer: D

Explanatory notes

The quarter number for quarter 2 2018 is 14:

25.4+3.06· 14	68.24

actual = deseasonalised × SI

68.24· 0.98	66.8752

Tip

- *When asked for the actual value, be careful that you don't just stop at finding the deseasonalised value from the equation. That deseasonalised value then needs to be converted to the actual value.*

Question 16

Answer: C

Explanatory notes

The calculation requires finding the average of the four figures around December 2010 (i.e. Oct., Nov., Dec., Jan. and Nov., Dec., Jan. and Feb.), then finding the average of those two values.

$\dfrac{\dfrac{61+63+64+66}{4}+\dfrac{63+64+66+57}{4}}{2}$	63.

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Recursion and financial modelling

Question 17

Answer: D

Explanatory notes

The value decreases from $5000 to $2000 over 10 years, resulting in a total depreciation of $3000.

Question 18

Answer: E

Explanatory notes

Flat rate depreciation occurs when an asset decreases in value by the same amount for each time period, as shown in the graph.

Options A and C: An annuity and reducing balance depreciation both result in a geometric pattern instead of a linear one, as the value decreases by a different amount each time period. (An annuity that is giving regular payments would be decreasing in value over time, but not in a straight line or linear pattern.)

Option B: The value remains the same for a perpetuity, so the graph would consist of a sequence of dots still at $5000.

Option D: Unit cost depreciation involves depreciation based on usage, rather than time.

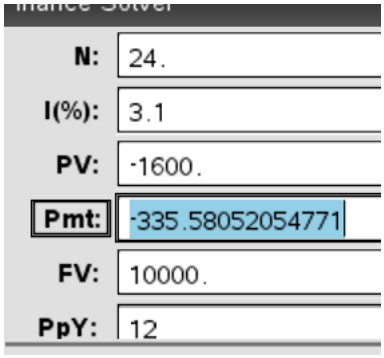
Tip

- *Identifying whether a graph is linear can help you to narrow down which financial model is being used. If it is linear, it is most likely flat rate interest, unit cost depreciation or flat rate depreciation.*

Question 19

Answer: B

Explanatory notes



Question 20

Answer: B

Explanatory notes

Two options round to 5.02%: the first two.

eff(4.9,26)	5.01719207914
eff(4.9,52)	5.01961202877
eff(5.1,12)	5.22091760368
eff(5.1,365)	5.23191441982
eff(5.1,4)	5.19836921141

Question 21

Answer: D

Explanatory notes

Option A is incorrect, as it would grow to only $4250 after 5 years.

Option B is incorrect, as it would result in a reduced value each year instead of growing, due to the decimal of 0.94 being less than 1.

Option C is incorrect, as it would grow to only $4015 after 5 years.

Option D is correct, as shown by the recursion relation below.

3000.· 1.04+250	3370.
3370.· 1.04+250	3754.8
3754.8· 1.04+250	4154.992
4154.992· 1.04+250	4571.19168
4571.19168· 1.04+250	5004.0393472

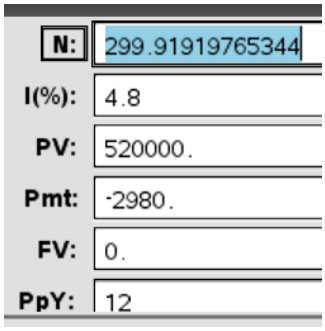
Option E is incorrect, as it would grow to only $2708; a decimal of 0.91 also indicates that it is not an investment that is increasing.

Question 22

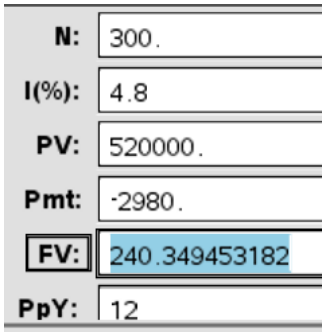
Answer: C

Explanatory notes

The annual rate is 0.004 × 1200 = 4.8%. The number of repayments are then found:



Finding the balance after 300 payments shows an overpayment of $240.35.



So the smaller final repayment is 2980 – 240.35 = $2739.65.

Tip

- *When finding the FV, if you see that it is the same sign as the PV, then more of the loan has been paid back than required.*

Question 23

Answer: A

Explanatory notes

Interest earned in the third quarter = total interest earned over three quarters – total interest earned over two quarters

FV after three quarters = 1257.83; FV after two quarters = 2517.65

Interest earned over two quarters: $2 \times 1285 - (5000 - 2517.65) = 87.65$

Interest earned over three quarters: $3 \times 1285 - (5000 - 1257.83) = 112.83$

Interest = 112.83 – 87.65 = 25.18

Note: An alternative method is:

FV(2) – FV(3) = 1259.82

$1285 of this difference is accounted for by the repayment, so the rest is interest:

1285 – 1259.82 = 25.18

Question 24

Answer: A

Explanatory notes

Use Solve to find the annual rate of interest. Then use this value to find the depreciated value of the car.

$$\text{solve}\left(4000\cdot\left(1+\frac{r}{1200}\right)^{96}=5684, r\right)$$

$r=-2404.40005783$ or $r=4.4000578273$

$$45000\cdot\left(1-\frac{4.4}{100}\right)^{8} \qquad 31396.0962622$$

Matrices

Question 25

Answer: C

Explanatory notes

There were 43 fiction books by an Australian author sold.

Question 26

Answer: C

Explanatory notes

Option A is $(2 \times 3) \times (3 \times 3) = 2 \times 3$.

Option B is $(2 \times 2) \times (2 \times 3) = 2 \times 3$.

Option C is $(3 \times 2) \times (2 \times 3) = 3 \times 3$.

Option D is $(3 \times 3) \times (3 \times 2) = 3 \times 2$.

Option E is $(3 \times 2) \times (2 \times 2) = 3 \times 2$.

Question 27

Answer: E

Explanatory notes

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Option E is correct because the matrix $\begin{bmatrix} S \\ C \\ A \\ R \\ E \end{bmatrix}$ requires the *fifth* letter of ACRES, S, to be first.

Hence, the '1' is in the *fifth* column of the first row; then the second letter of ACRES, C, is shown by the '1' in the second column of the second row; then the first letter, A, is shown by the '1' in the first column of the third row, and so on.

Tip

- *You could also use trial and error for this, using the calculator.*

Question 28

Answer: D

Explanatory notes

The row number (i) and column number (j) are shown for the 2 × 3 matrix:

$$\begin{bmatrix} 1,1 & 1,2 & 1,3 \\ 2,1 & 2,2 & 2,3 \end{bmatrix}$$

The rule $i + 3j$ is correct, as illustrated below:

$1 + 3 \times 1 = 4$ $\quad$ $1 + 3 \times 2 = 7$ $\quad$ $1 + 3 \times 3 = 10$

$2 + 3 \times 1 = 5$ $\quad$ $2 + 3 \times 2 = 8$ $\quad$ $2 + 3 \times 3 = 11$

Question 29

Answer: B

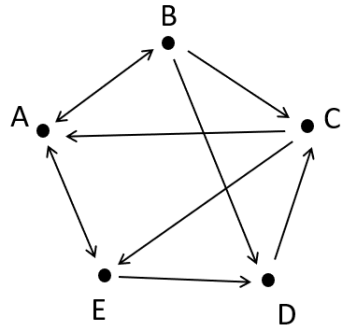
Explanatory notes

Finding the combined one-step and two-step communication matrix shows that there are zero communication links between Danni and Benny (and back to herself).

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}^2$$

$$\begin{bmatrix} 2. & 1. & 1. & 2. & 1. \\ 2. & 1. & 2. & 1. & 2. \\ 2. & 1. & 0. & 1. & 2. \\ 1. & 0. & 1. & 0. & 1. \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

This can also be found using a diagram to represent the communications:



The diagram shows Danni can only communicate directly with Carly, who can communicate with Alfie and Ellie (but not Benny).

Question 30

Answer: E

Explanatory notes

Option A has the survival rates for *J* and *A* under *A* and *S* instead.

Option B has the birth rate for *A* under *J* instead.

Option C has the birth rate for *A* and *S* under *J* and *A* instead.

Option D has the survival rate for *A* under *S* instead.

Question 31

Answer: B

Explanatory notes

30% of the visitors at *C* on Monday then visited *E* on Tuesday, and then 20% of them visited *F* on Wednesday.

$(0.3 \times 50) \times 0.2 = 3$

Question 32

Answer: D

Explanatory notes

Using the initial state matrix, where the only unknown for Monday is for Franklin Park:

$$S_{\text{Mon}} = \begin{bmatrix} 50 \\ 40 \\ \text{x} \end{bmatrix}$$

There were 38 people at Colton Reserve on Tuesday, so:

$0.6 \times 50 + 0.1 \times 40 + 0.1 \times x = 38$

$x = 40$

So to determine Wednesday:

$$\begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.3 & 0.7 & 0.15 \\ 0.1 & 0.2 & 0.75 \end{bmatrix}^2 \times \begin{bmatrix} 50 \\ 40 \\ 40 \end{bmatrix} = \begin{bmatrix} 32 \\ 52.15 \\ 45.85 \end{bmatrix}$$

There were 46 visitors at Franklin Reserve.

Networks and decision mathematics

Question 33

Answer: E

Explanatory notes

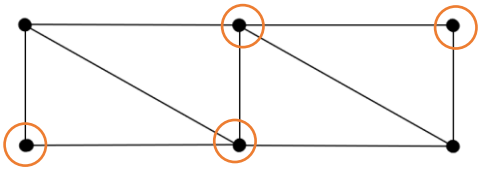
A Hamilton cycle involves visiting every vertex/landmark once only, beginning and ending at the same point.

Question 34

Answer: C

Explanatory notes

The circled vertices have even degrees of 2 or 4. The other two vertices are odd (degree of 3):



Question 35

Answer: D

Explanatory notes

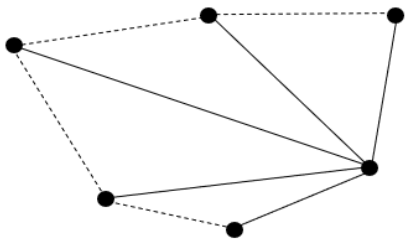
Wilma can only do Task A, and only Yasmin can do Task C; this means Task D must be allocated to Xavier and Task B must be allocated to Zoe.

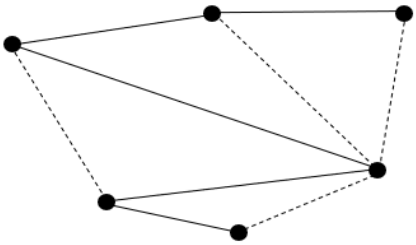
Question 36

Answer: B

Explanatory notes

There are many combinations that create a tree, but each of them requires four edges to be removed in order to remove any possible cycles. Two examples are shown.





Tip

- *A tree cannot contain any loops or cycles, but must include every vertex.*

Question 37

Answer: C

Explanatory notes

Option A is incorrect because there are three paths (not two) between locations *C* and *D*.

Option B is incorrect because there are three paths (not two) between locations *C* and *D* and a loop at *D* (one not zero).

Option D is incorrect because there are only two paths between locations *A* and *D* (not three).

Option E is incorrect because there is a loop at *D*, paths between *C* and *D* and paths between *A* and *D*.

Question 38

Answer: C

Explanatory notes

Option A would produce a shortest path of 10 km along *A–B–C–E–H–I*.

Option B is incorrect as the shortest path is 11 km along *A–B–C–E–H–I*.

Option C is correct as the shortest path is 12 km along *A–B–C–E–H–I*.

Option D would result in a shortest path of 13 km along numerous paths.

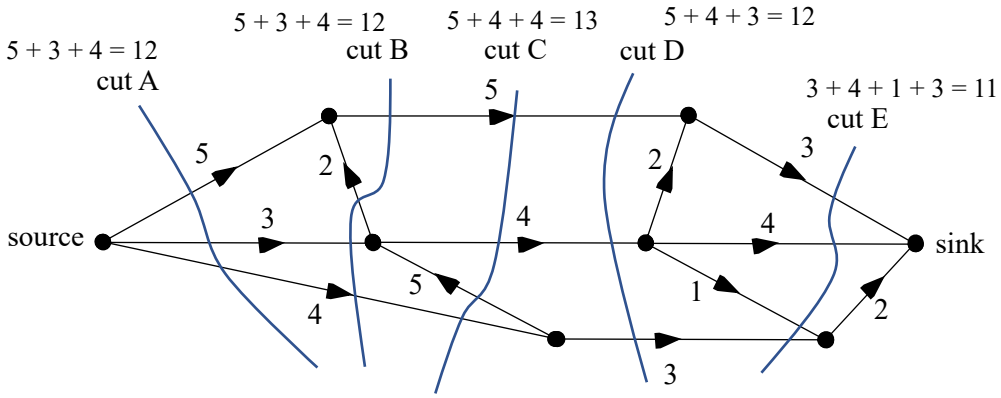
Option E would also result in a shortest path of 13 km along numerous paths.

Question 39

Answer: C

Explanatory notes

Cut B does not include the edge of 2 that it runs through, as it runs from sink to source instead of source to sink. Similarly, cut C does not include the edge of 5 for the same reason.



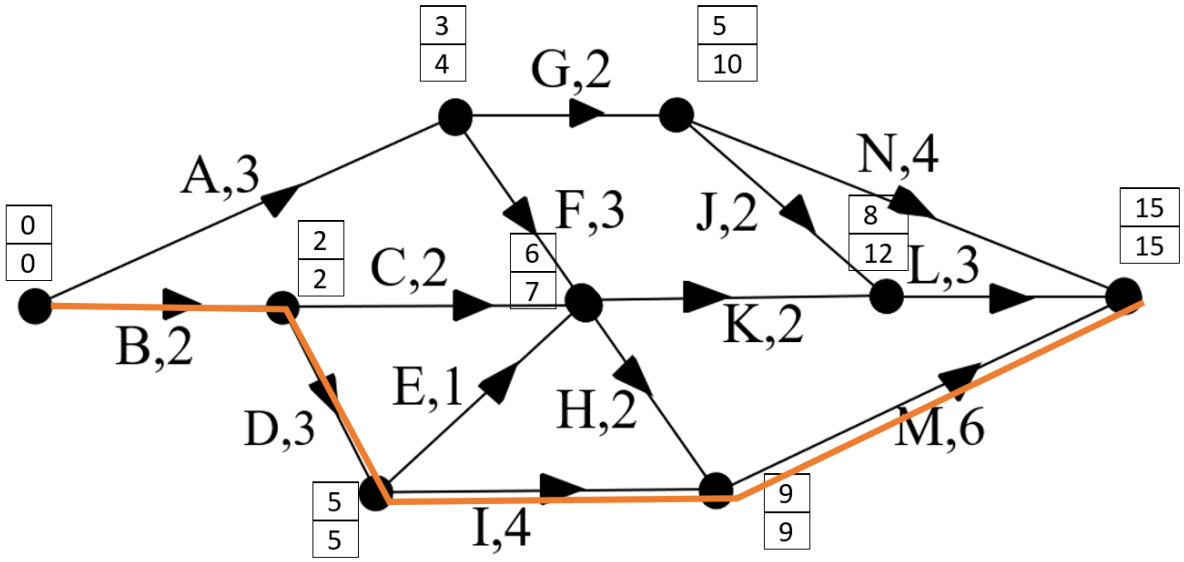
Question 40

Answer: E

Explanatory notes

The graph below shows all statements are correct.

- The critical path is *B*–*D*–*I*–*M*, with a time of 15 hours.
- The earliest start time of *L* is 8 hours.
- The latest finish time of *I* is 9 hours.
- The LST for *K* is 12 – 2 = 10 hours; so the float time is LST – EST = 10 – 6 = 4 hours.



END OF SOLUTIONS BOOK