Neap

Trial Examination 2021

VCE Further Mathematics Units 3&4

Written Examination 2

Suggested Solutions

SECTION A – CORE

Data analysis

Question 1 (6 marks)

a. The data is positively skewed. A1

b. $$\frac{740+2\times 210+3\times 10+4\times 10+5\times 5+6\times 5+7\times 20}{1000}=\frac{1425}{1000}$$
$$=1.4$$ A1

c. 1, 1, 1, 2, 7 A1

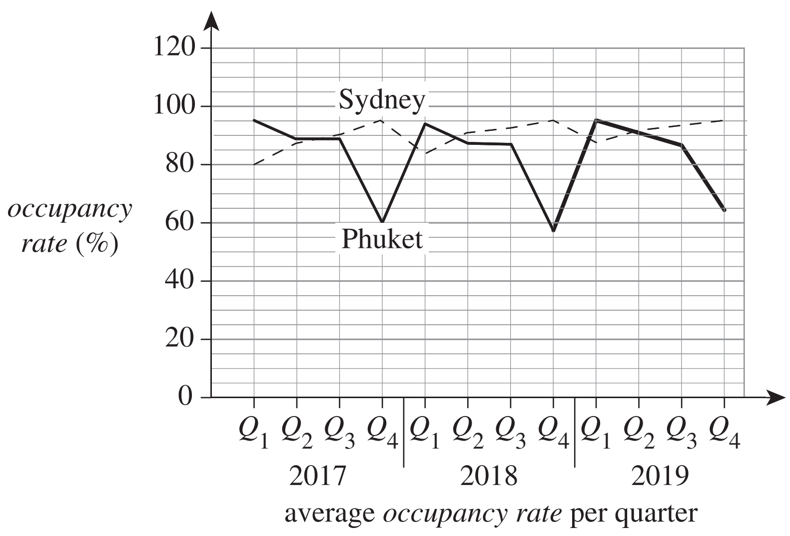
d. $IQR=2-1$
$=1$
$Q_3=2$
The upper fence is $2+1.5\times 1=3.5$.
Therefore 4, 5, 6 and 7 are all outliers. A1

e. 10 (*read from table*) A1

f. $\frac{700}{850}\times 100=82.35\%$
$\frac{120}{150}\times 100=80\%$
The guests who stayed one night were more impressed with the bed. 82.35% thought it was excellent compared to only 80% of the guests who stayed more than one night.
The guests who stayed one night had a higher percentage.
correct calculations and final answer A1

Question 2 (7 marks)

a.



graph line connected to four missing points A1

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b. The occupancy rate in Phuket is seasonal with a drop in the fourth quarter each year. A1

c. $\dfrac{\left(\dfrac{60}{83.75}+\dfrac{58}{81.75}+\dfrac{64}{83.5}\right)}{3}=0.73$ M1, A1

d. The occupancy rate for both hotels dropped in April and then remained at the lower level for the remainder of the year. The hotel occupancy in Phuket dropped much more than Sydney. A1

e.

Sydney		Phuket
	0	
	1	0 0 1 2 4
	2	0 5
	3	5
8 5	4	0
2 1 0 0	5	
4 0 0	6	
	7	0
	8	
2 2 0	9	0 5

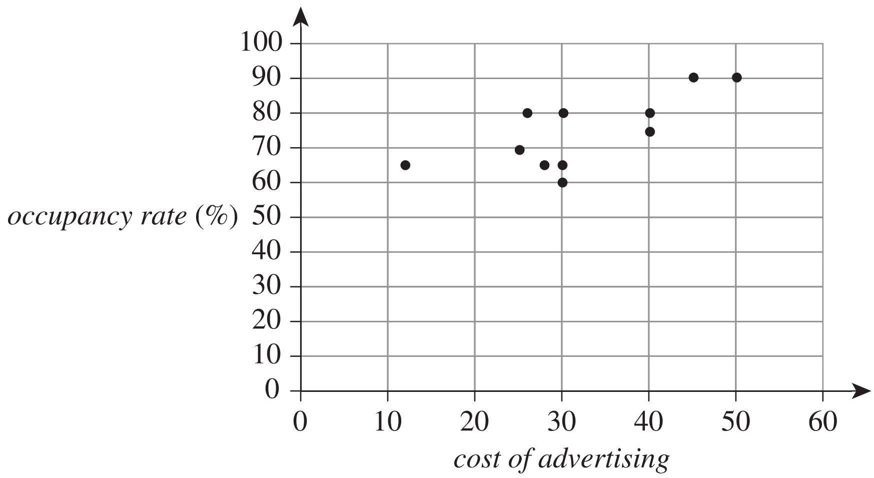
key: 4 | 5 = 45

correct layout and key A1
correct data A1

Question 3 (6 marks)

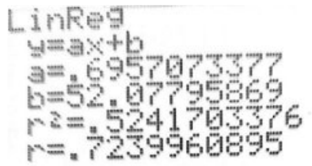
a. Spending on advertising will affect the occupancy rate. Therefore, the occupancy rate is the response variable. A1

b.



correctly plots the data A1
uses correct axes and a suitable scale A1

c. **i.** Enter the data into the spreadsheet on an approved technology, then use the calculate function to find the LinReg.



$r = 0.72$ A1

ii. $y = 0.70x + 52.08$

occupancy rate = 0.70 × *cost of advertising* + 52.08 A1

iii. Spending $1 000 000 is outside the limits of the collected data and is an extrapolation. In this case, substituting 1000 (as x is the cost in 1000s) into the regression equation gives an occupancy rate over 700%, which is impossible. A1

Question 4 (5 marks)

a. $1.3 \times 40 - 2.3 = 49.7$

$35 - 49.7 = -14.7$

The residual is –14.7. A1

b. There is a clear pattern in the residual plot, which does not support the idea of linearity. A1

c. **89.7** % of the change in the value of **the occupancy rate** is due to a change in the value of $\boldsymbol{\dfrac{1}{advertising}}$.

correct first and second boxes A1

correct third box A2

d. A residual plot needs to be done for the $\dfrac{1}{x}$ transformation to find out whether there is a pattern. A1

Recursion and financial modelling

Question 5 (2 marks)

a.

$t_1 = -6$

$t_2 \Rightarrow 2(-6) + 9 = -3$

$t_3 \Rightarrow 2(-3) + 9 = 3$

$t_4 \Rightarrow 2(3) + 9 = 15$

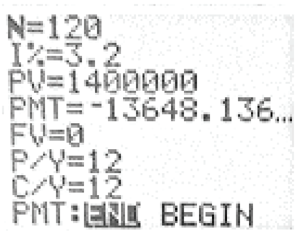
$(-6) + (-3) + 3 + 15 = 9$ A1

b. $t_{n+1} = 4t_n - 5, t_1 = 2$ A1

Note: The numbers increase quite quickly in the pattern, so multiplication by a factor is likely to be involved. By attempting simple factors such as 2, 3, 4 and 5, the factor of 4 gives 8, 12, 28, 92. Each of these is a constant 5 more than the next term, giving the relationship $t_{n+1} = 4t_n - 5, t_1 = 2.$

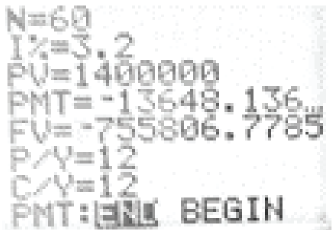
Question 6 (3 marks)

a. Insert values as shown into an approved technology.



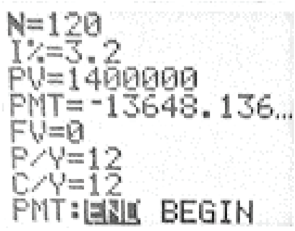
The monthly payment is $13 648. A1

b. Insert values as shown into an approved technology.

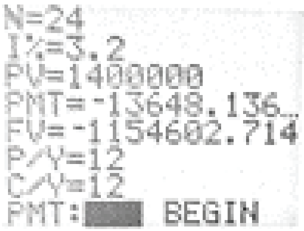


The balance still owing is $755 807. A1

c. Find the monthly repayments for a payout after 10 years. Insert values as shown into an approved technology.



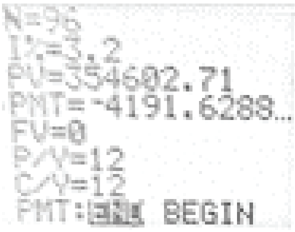
Change N to 24 (months) and calculate the value after 2 years.



The outstanding balance is \$1 154 602.71.

\$1 154 602.71 – \$800 000 = \$354 602.71

Use this figure as the initial balance and calculate the monthly repayments over 8 years.



The monthly repayment is \$4192. A1

Question 7 (3 marks)

a. $50\,000 \times 1.08 = \$54\,000$ A1

b. $t_n = 50\,000 \times 1.08^{(n-1)}$ A1

Note: Accept a similar correct representation.

c. Using recursion:

Year	Calculation
6	$t_6 = 50\,000 \times 1.08^{(6-1)}$ $= 73\,466.40$
8	$t_8 = 50\,000 \times 1.08^{(8-1)}$ $= 85\,691.21$
9	$t_9 = 50\,000 \times 1.08^{(9-1)}$ $= 92\,546.51$

The profit will exceed \$90 000 during the ninth year. A1

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Question 8 (4 marks)

a. Option 1:

$\text{value} = 180\,000 - (180\,000 \times 0.15)n$ A1

OR

$\text{value} = 180\,000 - 27\,000n$ A1

Option 2:

$\text{value} = 180\,000 \times 0.75^n,\ t_0 = 180\,000$ A1

b. $\text{value} = 180\,000 \times 0.75^n$

$= 180\,000 \times 0.75^2$

$= \$101\,250$ A1

c. The scrap value after 6 years using option 1:

$\text{value} = 180\,000 - 27\,000n$

$= 180\,000 - 27\,000 \times 6$

$= 18\,000$ A1

SECTION B – MODULES

Module 1 – Matrices

Question 1 (4 marks)

a. $30 (*read from matrix*) A1

b. $\begin{bmatrix} 4 \\ 6 \\ 1 \end{bmatrix}$ A1

c. $\begin{bmatrix} 15 & 30 & 50 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \\ 1 \end{bmatrix}$ A1

d. $0.95\begin{bmatrix} 15 & 30 & 50 \end{bmatrix}$ A1

Question 2 (3 marks)

a.

$$\begin{array}{c} \text{from} \\ \begin{array}{ccc} B & M & L \end{array} \end{array}$$

$$T = \begin{bmatrix} 0.5 & \mathbf{0.2} & \mathbf{0.15} \\ 0.4 & 0.45 & 0.15 \\ 0.1 & 0.35 & 0.7 \end{bmatrix} \begin{matrix} B \\ M \\ L \end{matrix} \text{ to}$$

A1

b. Element T_{21} represents that 40% (0.4) of the books at Boxville move to Merakton every day. A1

c. On Tuesday, there were 15 000 books in Merakton. 35% of these books will move to Little Furness (5250 books).

On Wednesday there will be 18 900 books in Little Furness. 5250 is 27.78% of 18 900.

28% (to the nearest percentage) of the books expected to be at Little Furness on Wednesday were in Merakton on Tuesday. A1

Question 3 (3 marks)

a.

$t \cdot s - b$ $\quad \begin{bmatrix} 11725. \\ 12900. \\ 17950. \end{bmatrix}$

$t \cdot \begin{bmatrix} 11725. \\ 12900. \\ 17950. \end{bmatrix} - b$ $\quad \begin{bmatrix} 9370. \\ 11532.5 \\ 19247.5 \end{bmatrix}$

9370 books were in Boxville on Wednesday. A1

b.

$$T\begin{bmatrix}11725\\12900\\17950\end{bmatrix}-B=\begin{bmatrix}12750\\13500\\18750\end{bmatrix}$$

$$T^{-1}\begin{bmatrix}12750\\13500\\18750\end{bmatrix}=\begin{bmatrix}15000\\15000\\15000\end{bmatrix}$$

A1

c.

$$t\cdot\begin{bmatrix}6358.70625\\9642.83125\\19298.4625\end{bmatrix}-b \qquad \begin{bmatrix}5355.300625\\8870.0678125\\18649.6315625\end{bmatrix}$$

$$t\cdot\begin{bmatrix}5355.300625\\8870.0678125\\18649.6315625\end{bmatrix}-b \qquad \begin{bmatrix}4543.19879688\\8147.04215625\\17759.7590469\end{bmatrix}$$

15000−8147 6853

6853 books need to be delivered to Merakton. A1

Question 4 (2 marks)

20% of the books at Ababooks move to Extrabooks on any day. If the number at both bookshops remains the same, then 50 books move from Extrabooks to Ababooks and so 50 books was 5% of the number at Extrabooks.

If 20% is 50, there were 250 books at Ababooks. If 5% is 50, there were 1000 books at Extrabooks. M1

Therefore, in total there are 1250 books at the two stores. A1

Module 2 – Networks and decision mathematics

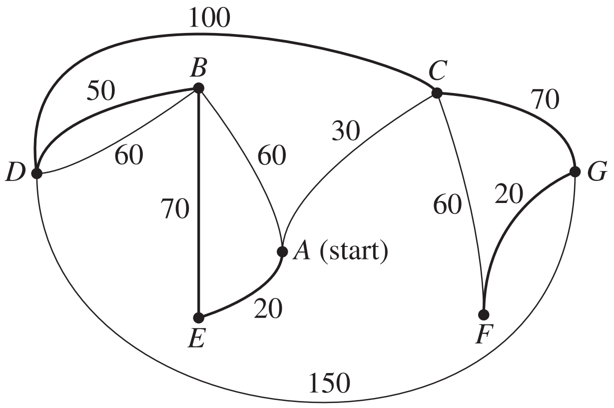
Question 1 (5 marks)

a. $D–C–F = 100 + 60$

$= 160$ m A1

b. **i.** Hamiltonian path A1

ii. 330 m A1



Note: Drawing the path on the diagram is not required.

c. **i.** Activity 2 is possible, as there are exactly 2 odd vertices on the course. A1

ii. *G* (*the odd vertex that is not the starting point*) A1

Question 2 (3 marks)

a. There are 19 possible routes. A1

b. 17 + 12 + 7 = 36 A1

c. No. The minimum cut is 29, which is the maximum flow of the obstacle course. This means that the obstacle course cannot accommodate all 30 students at once. A1

Question 3 (4 marks)

a. 2 *(tasks B and C)* A1

b. 44 minutes *(critical path is A, C, G, I, K)* A1

c. The float time on task *J* is 1 minute. *(It can start at 32 minutes, but must be completed by 37 minutes and takes 4 minutes to complete.)* A1

d. task *G (It is on the critical path and so will reduce the time taken to complete the whole project.)* A1

Module 3 – Geometry and measurement

Question 1 (4 marks)

a. Bangkok (*It has the smallest degree of latitude; the equator is at 0°.)* A1

b. $6400\cos(64) = 2805.57533945$

≈ 2806 km A1

c. $\frac{42+33}{360} \times \pi \times 6400 \times 2 = 8377.58040957$

≈ 8378 km A1

d. 1535 + 9 hours and 47 minutes = 0122 (1:22 am), and Istanbul is 4 hours behind Bangkok.

The flight arrives in Istanbul at 9:22 pm or 21:22. A1

Question 2 (4 marks)

a. $20 \times 25 - 10 \times 3 - 15 \times 8 - 10 \times 6 - \pi \times 3^2 = 261.725666118$ A1

$\approx 262 \text{ cm}^2$

b. $20 \times 4 \times 2 + 25 \times 4 \times 2 + 25 \times 20 = 860 \text{ cm}^2$ A1

c. $15 \times 8 \times 6 = 720 \text{ cm}^3$ A1

d.

solve$(\pi \cdot 3^2 \cdot h=275,h)$	$h=9.72613541117$
$(h=9.7261354111715)-4$	
	$h-4=5.72613541117$

6 cm A1

Question 3 (4 marks)

a.

$400 \cdot \sin(35)$	229.43057454

229 km A1

b. $180 + 35 = 215°$ A1

c. $400\cos(35)+1000\cos(55)=901.237254067$

≈ 901 km A1

d.

$$400\cos(35)+1000\cos(55)=901.237254067$$
$$400\sin(35)+1000\sin(55)=1048.58261883$$
$$\tan^{-1}\left(\frac{1048.58261883}{901.237254067}\right)=49.3215541434$$
$$180+49.3215541434=229.321554143$$
$$\approx 229°$$

A1

Module 4 – Graphs and relations

Question 1 (6 marks)

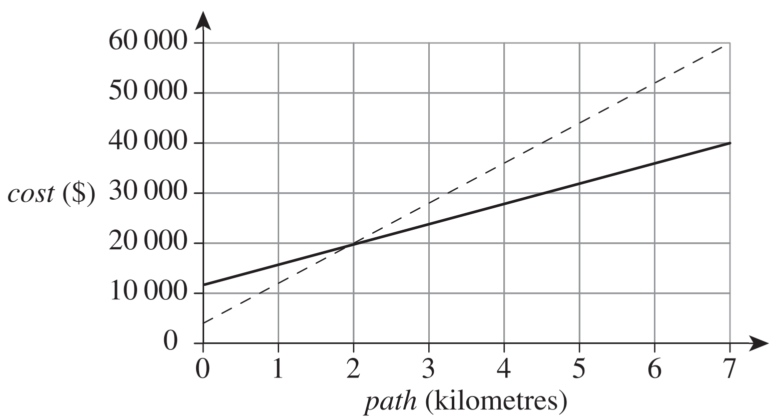
a. The delivery cost is **$4000**. A1

The cost per km is **$8000**. A1

b. $C = 12000 + 4000n$ A1

Note: Accept any equivalent equation using the variables C and n.

c.



Note: The dashed line represents the existing graph line.

correct y-intercept A1

correct gradient A1

d. For a length of path that is less than **two** kilometres, it is cheaper to use the **crushed rock** option. A1

Question 2 (2 marks)

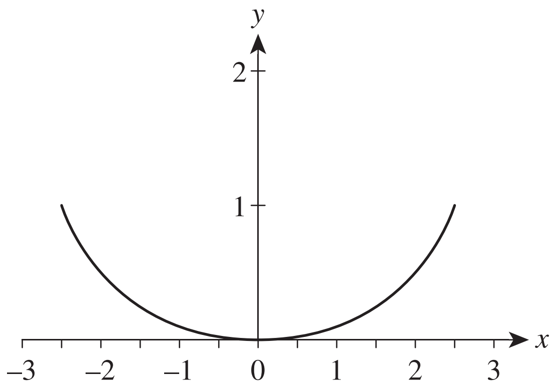
a. Substitute the point (2.5, 1) into $y = kx^2$.

$$1 = k\,2.5^2$$

$$k = \frac{1}{6.25}$$

$$= 0.16$$

A1



Note: Graphs are not required as part of the response, but may be used to help visualise the problem.

b. The end of the curved section of the halfpipe is at (x, 1.4).

Note: The height of the halfpipe is 0.2 metres higher than the equation predicts due to 20 cm gap between the base of the ramp structure and the curve of the halfpipe.

Solve $y = 0.16x^2$ using the point (x, 1.4).

$$1.4 = 0.16x^2$$

$$\frac{1.4}{0.16} = x^2$$

$$x = \sqrt{\frac{1.4}{0.16}}$$

$$x = 2.96$$

The length that the base needs to be is therefore 2 × (2.96 + 0.5) = 6.92 m. A1

Question 3 (4 marks)

a. $20x + 30y \le 1800$ A1

b. $\text{income} = 25x + 8y$ A1

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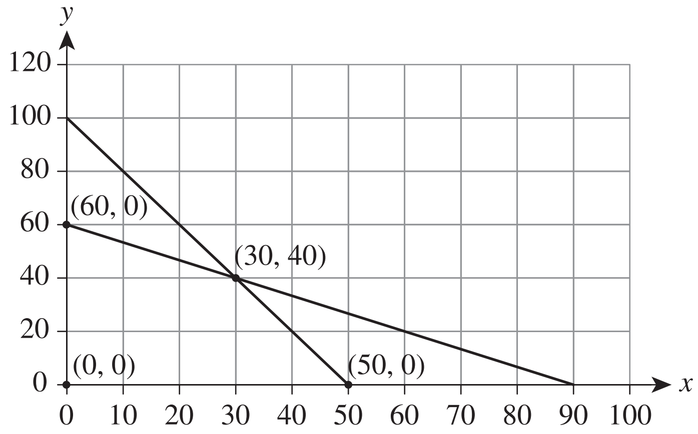
c. The maximum income occurs when the shop sells 50 T-shirts and 0 tea towels. A1

The coordinates of the corners of the feasibility area are shown in the following graph and table.



Point	income $= 25x + 8y$
(0, 0)	$\text{income} = 25 \times 0 + 8 \times 0$ $= 0$
(0, 60)	$\text{income} = 25 \times 0 + 8 \times 60$ $= 480$
(30, 40)	$\text{income} = 25 \times 30 + 8 \times 40$ $= 1070$
(50, 0)	$\text{income} = 25 \times 50 + 8 \times 0$ $= 1250$

each point checked to find the maximum income M1