2021
VCE
Further Mathematics

Trial Examination 2

Kilbaha Education

Quality educational content

Kilbaha Education **PO Box 2227** **Kew Vic 3101** **Australia**	**Tel: (03) 9018 5376** **kilbaha@gmail.com** **https://kilbaha.com.au**

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Letter

STUDENT NUMBER									

VICTORIAN CERTIFICATE OF EDUCATION 2021

FURTHER MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section A - Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	7	7	36
Section B - Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 36 pages.
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

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Section A – Core

Instructions for Section A

Answer all questions in the spaces provided. Write using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

Question 1 (7 marks)

The Warburn Trail Festival is an annual event involving a weekend of outdoor running races.
One of the races is the River Run.
The table below relates to the 25 competitors in the River Run.

Competitor	*Section*	*Status*	*Time* (minutes)
1	junior	amateur	41
2	senior	amateur	45
3	junior	amateur	62
4	senior	professional	25
5	senior	amateur	34
6	senior	professional	21
7	junior	amateur	41
8	senior	professional	21
9	adult	amateur	40
10	senior	professional	25
11	senior	amateur	47
12	junior	amateur	39
13	junior	amateur	38
14	senior	professional	27
15	adult	amateur	19
16	adult	amateur	22
17	junior	amateur	37
18	adult	professional	17
19	adult	amateur	35
20	junior	amateur	35
21	adult	amateur	32
22	adult	amateur	23
23	senior	amateur	37
24	adult	professional	19
25	senior	amateur	38

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Data Analysis

Question 1 (continued)

The four variables in this data set are:
Competitor – entrant identification number
Section – junior (12-18 years), adult (19-55 years), senior (older than 55 years)
Status – amateur, professional
Time – number of minutes taken to complete the race

a. Which one of the four variables is ordinal? 1 mark

_____

b. How many competitors in the adult section were also professional? 1 mark

_____

c. Use the data table to complete the following two-way frequency table below. 2 marks

	Status	
Section	**Amateur**	**Professional**
junior		
adult	6	
senior		
Total	18	

d. What percentage of amateurs are adults? 1 mark
Give your answer correct to one decimal place.

_____

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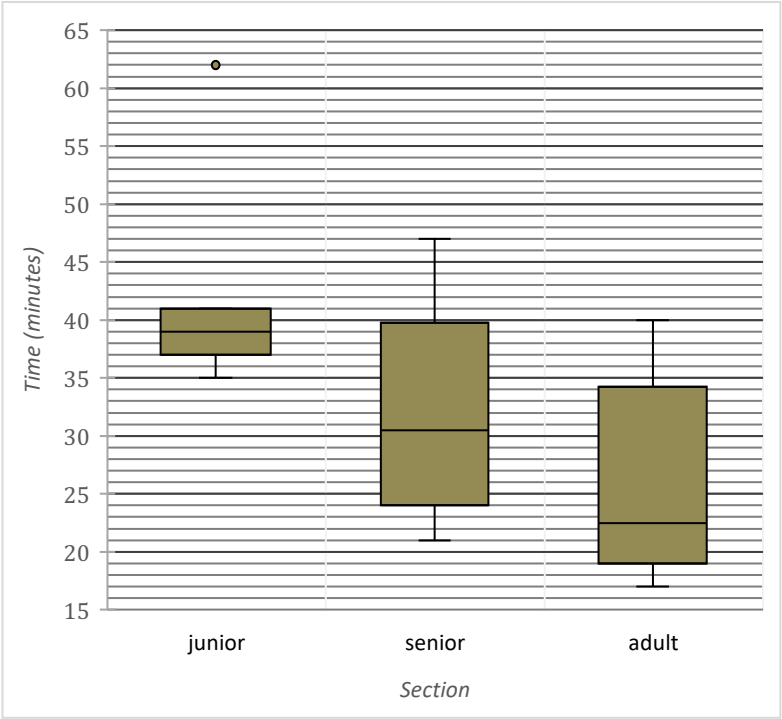
Data Analysis

Question 1 (continued)

The boxplots below show the distribution of *time* for each *section* (junior, senior, adult).



e. Explain why the *time*, in minutes, taken to complete the race is associated with the *section* of the runner. Refer to the values of an appropriate statistic in your answer.

1 mark

f. The data value of 62 for the junior section is an outlier as it is above the upper fence. Determine the value of the upper fence for the junior section. 1 mark

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Data Analysis

Question 2 (3 marks)

In a particular year, a total of 600 competitors participated in the Warburn Trail Festival.
The age of the competitors was approximately normally distributed with
a mean of 33 years and a standard deviation of 7 years.

a. Determine the percentage of competitors expected to be younger than 19 years. 1 mark

_____

_____

b. Determine the number of competitors expected to be aged between 40 and 47 years. 1 mark

_____

_____

c. One particular competitor has a standardised age given by z = -1.6 1 mark
Determine the actual age, in years, of this competitor.
Give your answer correct to one decimal place.

_____

_____

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Data Analysis

Question 3 (11 marks)

The Warburn Tourist Park owner wants to predict the number of campsite bookings needed for the Festival volunteers.
The table below shows the *number of campsite bookings* and the *number of volunteers* for eight past festivals.

number of volunteers	24	29	38	43	49	56	59	66
number of campsite bookings	18	20	22	25	29	33	37	42

a. Determine the equation of the least squares line that can be used to predict the *number of campsite bookings* from the *number of volunteers*.
Write the values of the intercept and slope of this line in the appropriate boxes below.
Give your answers to three significant figures.

2 marks

number of campsite bookings = ☐ + ☐ × *number of volunteers*

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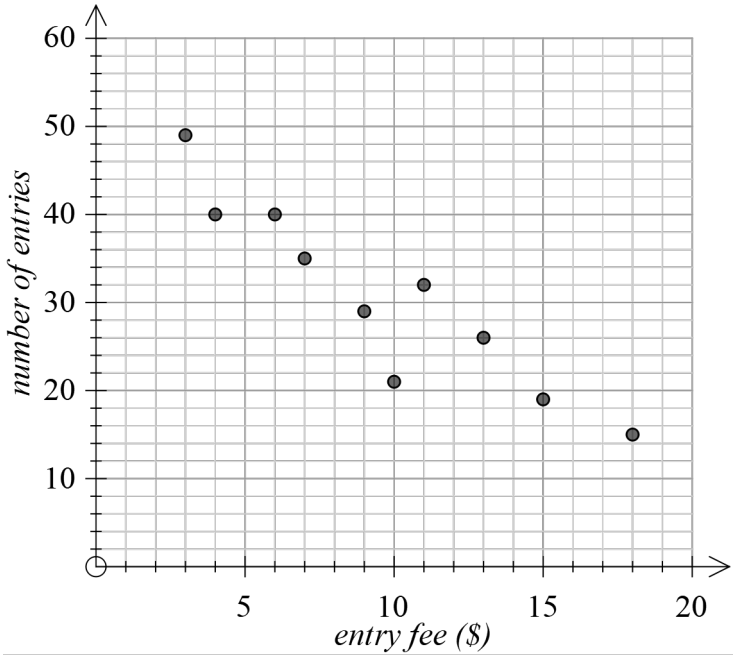
Data Analysis

Question 3 (continued)

b. The festival organisers wish to know if there is an association between the number of entries received for each event, and the entry fee charged.

The *number of entries* for each event and the *entry fee*, in dollars, charged for the event is examined.
A scatterplot of the data is shown below.



The equation of the least-squares line is

$$number\ of\ entries = 50.4 - 2.1 \times entry\ fee$$

i. Name the explanatory variable. 1 mark

ii. Draw the graph of this least squares line **on the scatterplot**. 1 mark

(answer on scatterplot)

iii. Interpret the slope of the regression line in terms of the variables *number of entries* and *entry fee*.

1 mark

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Data Analysis

Question 3b. (continued)

iv. The correlation coefficient is r = -0.927

Determine the percentage of the variation in *number of entries* that can be explained by the variation in *entry fee*.
Round your answer to one decimal place. 1 mark

v. How many dollars is the median entry fee? 1mark

vi. An extra event is to be added to the program. The entry fee charged for this event will be $14.
Use the least-squares regression line equation to predict the number of entries for this event.

1 mark

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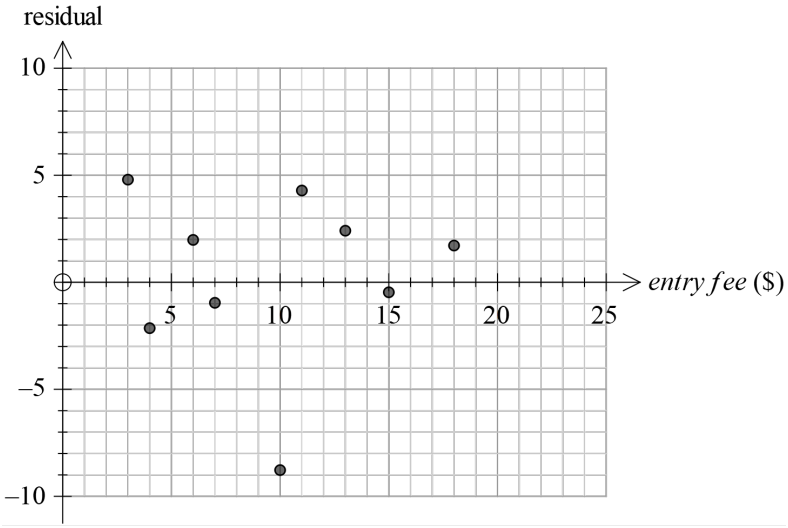
Data Analysis

Question 3 (continued)

c. The residual plot associated with the least squares line is shown below with one point missing.



When the entry fee = \$9, the actual number of entries is 29.
Calculate the residual for when *entry fee* = \$9 and mark it on the residual plot with a cross (X). 1 mark

d. The residual plot in part **b** can be used to test one of the assumptions about the nature of the association between *number of entries* and *entry fee*.

i. What is this assumption? 1 mark

ii. The residual plot supports this assumption.
Explain why. 1 mark

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Data Analysis

Question 4 (3 marks)

Runners visit Warburn throughout each year to train for the Trail Festival.
The table below shows the number of runners visiting to train each season over two years.

	Summer	Autumn	Winter	Spring
2018	436	540	340	604
2019	386	500	298	576

a. The seasonal index for Spring is shown in the table below.
Find the seasonal indices for the other three seasons and write them in the table below.
Round your answers to 2 decimal places. 2 marks

Summer	Autumn	Winter	Spring
			1.28

b. The total number of runners visiting to train each season in 2020 is shown in the table below.

	Summer	Autumn	Winter	Spring
2020	280	306	195	321

Use the appropriate seasonal index from part **a** to deseasonalise the number of runners visiting in Spring 2020.
Round your answer to the nearest whole number 1 mark

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Recursion and financial modelling

Question 5 (7 marks)

Jeff runs a business in which he does the final stitching on quilts with a quilting machine.
Jeff chooses the reducing balance method to calculate the depreciation on his new quilting machine.
The value of the quilting machine, in dollars, after n years, Q_n can be modelled by the recurrence relation

$$Q_0 = 27000, \qquad Q_{n+1} = 0.91Q_n$$

a. What amount, in dollars, did Jeff pay for the new machine? 1 mark

_____

b. Show, with recursion that the value of the machine after 2 years is $22358.70 by filling in the boxes below with the appropriate values.

2 marks

$Q_0 =$ 27000

$Q_1 =$ [ ] x 27000 = [ ]

$Q_2 =$ [ ] x [ ] = [ ]

c. What is the annual percentage rate of depreciation used by Jeff? 1 mark

_____

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Recursion and financial modelling

Question 5 (continued)

d. After how many years will the value of Jeff's quilting machine first fall below $12000?

1 mark

A rule of the form $Q_n = a \times b^n$ can be used to determine the value, in dollars, of the quilting machine, Q_n, after n years.

e. Write down this rule for Q_n 1 mark

Jeff claims the machine's depreciation as a tax deduction each year.

f. By how much does the machine depreciate during the fourth year? Give your answer to the nearest whole dollar. 1 mark

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Recursion and financial modelling

Question 6 (3 marks)

Jeff buys a second quilting machine. The initial value of this machine is $36000.
He decides to depreciate the machine using the unit cost method.
The machine quilts 340 pieces each year.
After five years, the value of the machine is $20700.

a. **Show** that the machine depreciates by $9 for every piece it quilts. 1 mark

b. Let V_n be the value of the machine after n years.

Write down a recurrence relation, in terms of V_0, V_{n+1} and V_n that could be used to model the value of the machine using this unit cost depreciation method.

1 mark

c. The value of the machine continues to depreciate by $9 for every piece quilted. The machine has a scrap value of $1962.
After how many pieces quilted will the machine reach its scrap value? 1 mark

Question 7 (2 marks)

Jeff plans to retire in nine years. He has $247000 in a superannuation account. He plans to deposit $270 every month into this account for the next nine years.
The account earns interest at a rate of 6.6% per annum compounding monthly.

a. What is the balance of this account when he retires in nine year's time?
Give your answer to the nearest cent.

1 mark

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Recursion and financial modelling

Question 7 (continued)

b. Three years before retirement, the interest rate of the account falls to 6.4% per annum.
Calculate the monthly payment Jeff will need to make over the next three years so that the account will have a balance of $490000 on his retirement.
Give your answer to the nearest whole dollar. 1 mark

_____

_____

_____

END OF SECTION A

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Section B – Modules

Instructions for Section B

Select two modules and answer all questions within the selected modules. Write using blue or black pen. You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
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Module 1 - Matrices

Question 1 (3 marks)

Jeff's quilting company stitches three different quilt sizes, Small (S), Medium (M), and Large (L). His charge, in dollars, to stitch each size of quilt is shown in matrix C below.

$$C = \begin{bmatrix} 50 \\ 170 \\ 320 \end{bmatrix} \begin{matrix} \text{S} \\ \text{M} \\ \text{L} \end{matrix}$$

a. What does Jeff charge to stitch a medium sized quilt? 1 mark

_____

b. Write down the order of matrix C

1 mark

_____

c. A client wishes to have three medium and 2 small quilts stitched.
Jeff's total charge to this client can be found by the matrix product $Q \times C$.
Write down the matrix Q. 1 mark

$Q =$

_____

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Question 2 (2 marks)

Lola runs a quilting company in competition with Jeff's.
The table below shows the number of each size of quilt her company stitches in three different weeks, and the total amount charged each week.

	Week 1	**Week 2**	**Week 3**
Small	23	18	32
Medium	16	22	10
Large	2	1	2
Total charge	\$4615	\$5190	\$3910

Lola charges

\$$a$ for a small quilt
\$$b$ for a medium quilt
\$$c$ for a large quilt

a. Use the information in the table above to complete the following matrix equation by inserting the missing values.

1 mark

$$\begin{bmatrix} 23 & 16 & \_\_ \\ \_\_ & 22 & 1 \\ 32 & \_\_ & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4615 \\ 5190 \\ 3910 \end{bmatrix}$$

b. What is Lola's charge for stitching a medium sized quilt? 1 mark

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Question 3 (5 marks)

Lola runs patchwork classes at three different locations, A, B and C each week.
She has a total of 240 clients participating in the classes.
Let P_n be the state matrix that shows the number of clients at each location in week n.
The initial state matrix, P_1, below shows the number of clients at each location in week one of the course.

$$P_1 = \begin{bmatrix} 60 \\ 100 \\ 80 \end{bmatrix} \begin{matrix} \text{A} \\ \text{B} \\ \text{C} \end{matrix}$$

The clients change their location for the classes from week to week according to the transition matrix, T, shown below.

$$T = \begin{matrix} & \textit{this week} & & \\ & \text{A} \quad \text{B} \quad \text{C} & & \\ \end{matrix} \begin{bmatrix} 0.5 & 0.7 & 0.3 \\ 0.2 & 0.2 & 0.1 \\ 0.3 & 0.1 & 0.6 \end{bmatrix} \begin{matrix} \text{A} \\ \text{B} \\ \text{C} \end{matrix} \quad \textit{next week}$$

a. How many clients are expected to be at location B in week two?

1 mark

_____

_____

b. Of the clients attending location A in week one, how many are expected at location C in week two?

1 mark

_____

_____

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Question 3 (continued)

c. **Show** that the number of clients expected at location A in week two is 124 by filling in the boxes below.

1 mark

$$\square \times 60 + \square \times 100 + \square \times 80 = 124$$

d. In week three, 113 clients are expected at location A.
What percentage of these clients were expected to have been at location A in week two?
Round your answer to one decimal place.

1 mark

_____

_____

_____

e. In the long term, what percentage of all clients are expected to be at location B?
Round your answer to one decimal place.

1 mark

_____

_____

_____

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Question 4 (2 marks)

Lola stocks a total of 1800 spools of different types of thread; Aurifil (A), Glide (G), Polycot (P), and Rasant (R).
Let S_n be the state matrix that gives the number of spools of each type of thread she stocks in the nth year after 2021.
The number of spools stocked in 2021 is shown in matrix S_0 below.
For the 1800 spools of thread, the matrix recurrence relation below can be used to find the number of spools of each type Lola will stock each year.

$$S_{n+1} = TS_n + K$$

where

$$S_0 = \begin{bmatrix} 600 \\ 500 \\ 300 \\ 400 \end{bmatrix} \begin{matrix} \text{A} \\ \text{G} \\ \text{P} \\ \text{R} \end{matrix}$$

$$T = \begin{matrix} & \text{A} & \text{G} & \text{P} & \text{R} \\ \text{A} \\ \text{G} \\ \text{P} \\ \text{R} \end{matrix} \begin{matrix} \textit{This year} \\ \\ \\ \\ \textit{next year} \end{matrix} \begin{bmatrix} 0.1 & 0.1 & 0.0 & 0.2 \\ 0.2 & 0.1 & 0.0 & 0.2 \\ 0.5 & 0.7 & 0.8 & 0.5 \\ 0.2 & 0.1 & 0.2 & 0.1 \end{bmatrix}$$

$$K = \begin{bmatrix} 300 \\ m \\ -500 \\ 100 \end{bmatrix}$$

a. Write down the value of m in matrix K.

1 mark

_____

b. How many reels of Polycot is Lola expected to stock in 2023?

1 mark

_____

_____

_____

End of Module 1: Matrices

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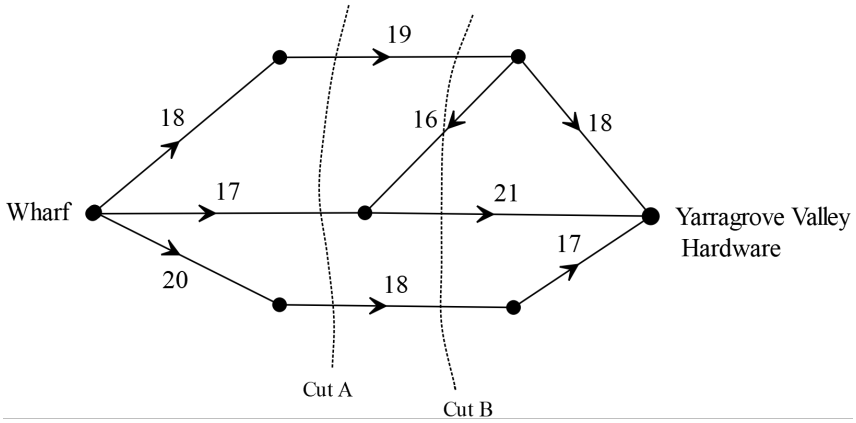
Module 2: Networks and decision mathematics

Question 1 (3 marks)

Miles runs a home maintenance business in the Yarragrove Valley. He relies on equipment and materials being delivered from the wharf.
The graph below shows the number of delivery trucks that can move between the wharf and Yarragrove Valley Hardware each day.
The unmarked vertices represent other hardware stores in the area.



Two cuts are shown on the graph, Cut A and Cut B.

a. Write down the capacity of Cut A. 1 mark

_____

b. Write down the capacity of Cut B. 1 mark

_____

c. Determine the maximum number of delivery trucks that can go from the wharf to Yarragrove Valley Hardware each day.

1 mark

_____

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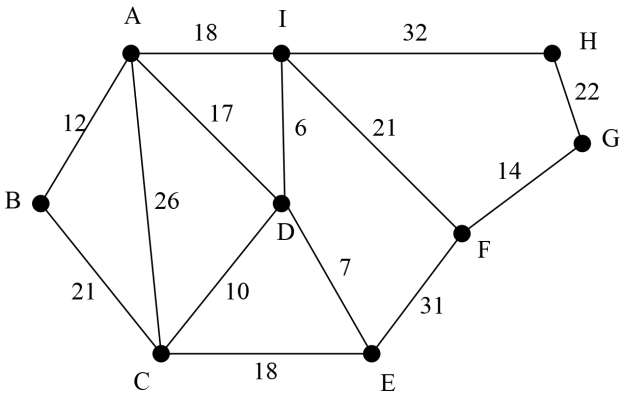
Question 2 (5 marks)

Miles has jobs at nine different homes in Yarragrove Valley.
The graph below shows the homes labelled A to I.
The distance, in kilometres, between homes is shown on the edges.



a. How many vertices have degree 2?

1 mark

_____

b. What is the length, in kilometres, of the shortest path between home B and home G?

1 mark

_____

c. Miles wishes to begin at home E and follow an Eulerian trail.
At which home does he finish? 1 mark

_____

d. On another day, Miles will begin at home D. He will visit every home once only, returning to home D.
If C is the second home he visits, write down the path he takes.

1 mark

_____

e. What mathematical term is used to describe the route taken in part **d**?

1 mark

_____

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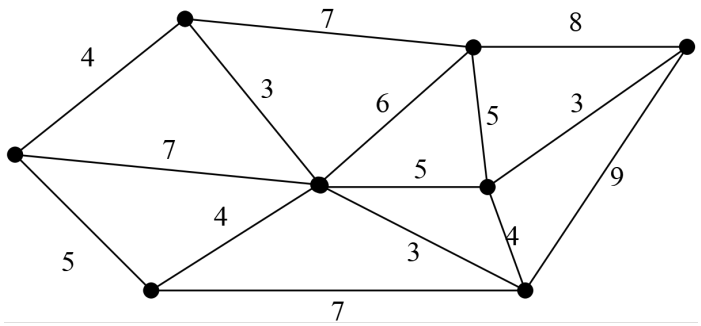
Question 3 (2 marks)

Miles is landscaping a garden at one of the homes.
He plants eight ornamental trees and wishes to connect them with pathways.
The diagram below shows the trees represented as vertices, and the edges showing the distance, in metres, between them.



To reduce costs, Miles will construct the minimum length of path possible whilst ensuring the trees remain connected.

a. On the graph above, draw the minimum spanning tree.

1 mark

(answer on graph above)

b. What is the minimum length of path, in metres, that Miles will need to construct?

1 mark

_____

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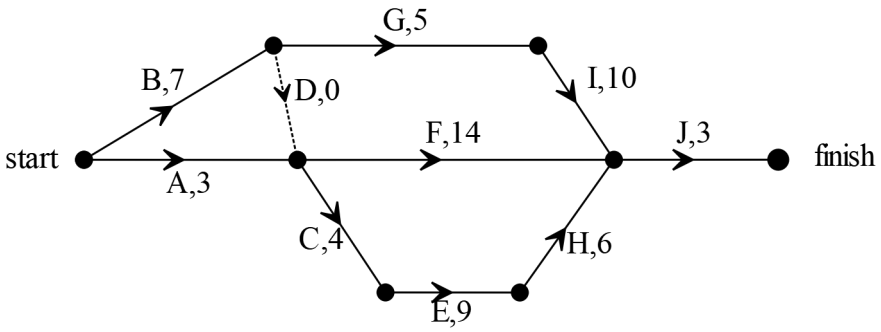
Question 4 (2 marks)

Miles is cleaning and repairing a roof of one of the homes. This requires a number of activities. The directed graph below shows the activities involved and the duration time, in hours, for each.



a. Write down the critical path.

1 mark

_____

b. Miles finds that only one of the activities can have its duration time reduced
What is the maximum possible reduction in completion time of the job? 1 mark

_____

_____

_____

End of Module 2: Networks and decision mathematics

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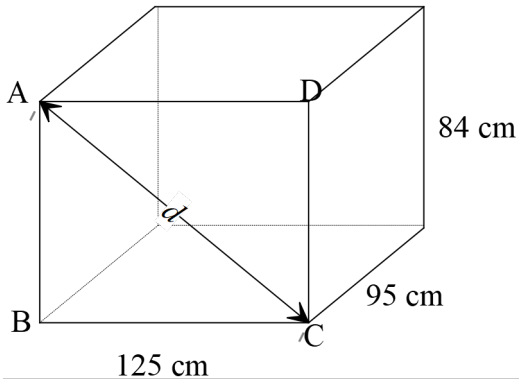
Module 3: Geometry and measurement

Question 1 (4 marks)

Granite blocks used to build the Shaky Rock lighthouse are brought in as rectangular prisms with dimensions as shown in the diagram below.



a. What is the length, in centimetres, of the diagonal, d, from A to C?
Give your answer correct to one decimal place.

1 mark

b. What is the total surface area, in cm^2, of the block?

1 mark

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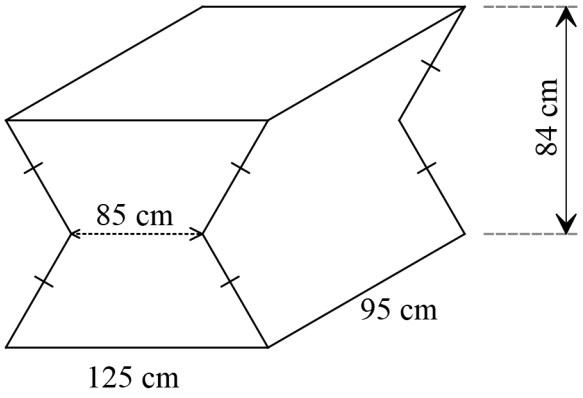
Question 1 (continued)

To give strength to the walls of the lighthouse, the blocks must be made to interlock. This is done by removing a triangular prism from each side as shown to make an interlocking block.



c. **Show** that the volume of one of the triangular prisms that needs to be removed is 79800 cm^3.

1 mark

d. Calculate the volume, in cm^3, of the interlocking block. 1 mark

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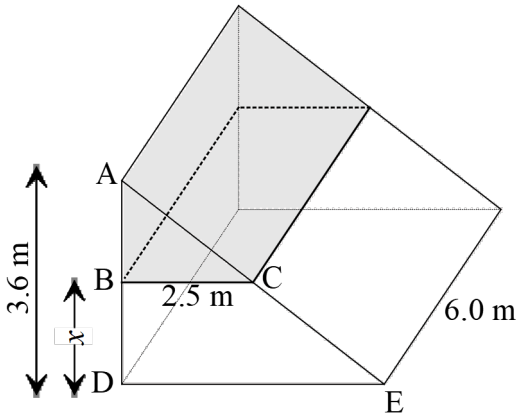
Question 2 (3 marks)

A storage shed next to the lighthouse is in the shape of a right triangular prism as shown in the diagram below.
It is divided into two sections, upper and lower. The base of the upper section is parallel to the base of the lower.
The volume of the upper (shaded) section is one quarter of the volume of the whole shed.



a. **Show** that the length x, the distance of B to D, is 1.8 metres. 1 mark

b. What is the length, in metres, of D to E? 1 mark

c. Calculate the volume, in m^3, of the whole shed. 1 mark

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Question 3 (5 marks)

Shaky Rock lighthouse is located at latitude 54° N and longitude 6° W.

Windy Squalls lighthouse is located at latitude 48° N and longitude 6° W.

a. Referring to the latitude and longitude references given, explain how we know that Windy Squalls lighthouse is closer to the equator.

1 mark

b. Assume that the radius of Earth is 6400 km.
Calculate the shortest great circle distance between Shaky Rock lighthouse and Windy Squalls lighthouse.
Round your answer to the nearest kilometre. 1 mark

Lonely Shoal lighthouse is located at latitude 54° N and longitude 51° W
Assume that the radius of Earth is 6400 km.

c. **i** Write a calculation that shows that the radius of the small circle of Earth at latitude 54° N is 3762 km rounded to the nearest kilometre. 1 mark

ii Find the shortest small circle distance between Shaky Rock lighthouse **(**54° N, 6° W) and Lonely Shoal lighthouse.
Round your answer to the nearest kilometre 1 mark

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Question 3 (continued)

Margite, a lighthouse keeper, is flown from Shaky Rock lighthouse (54° N, 6° W) to Lonely Shoal lighthouse (54° N, 51° W).
She departs Shaky Rock lighthouse at 11:15 pm on Monday 12th of January 2021.
She arrives at Lonely Shoal lighthouse at 7:52 am on Tuesday 13th of January 2021.
The time difference between Shaky Rock and Lonely Shoal is three hours.

d. How long, in hours and minutes, did Margite take to fly from Shaky Rock lighthouse to Lonely Shoal lighthouse?

1 mark

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End of Module 3: Geometry and measurement

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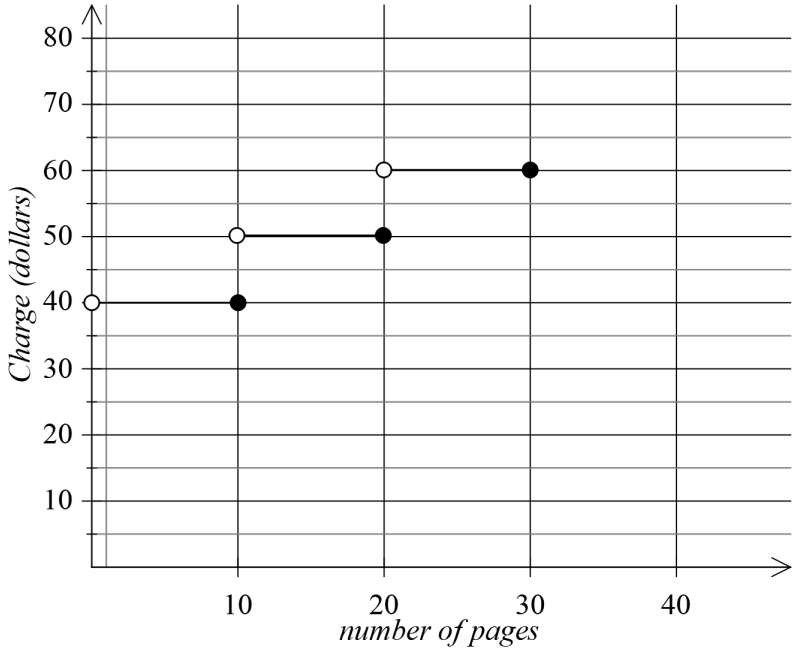
Module 4 : Graphs and relations

Question 1 (5 marks)

Carolyn runs an online craft business and is investigating the cost of getting a website designed. The graph below shows the amount, in dollars, that Wanda's Web Designs charges depending on the number of pages required



a. Wanda's Web Designs charges $70 for a website containing more than 30 pages up to and including 40 pages. Draw this information on the graph above.

(answer on graph above) 1 mark

b. What does Wanda's Web Designs charge for a website with 20 pages? 1 mark

_____

c. Carolyn wants to have three different websites created.
The websites will have 8, 14 and 37 pages.
How much will the total charge be for these websites using Wanda's Web Designs?
1 mark

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Question 1 (continued)

Another company, Magic Web Designs, charges a fixed amount of $4 and a further $2 per page. An equation used to calculate their charge, in dollars, to design a website is

$$\boldsymbol{Charge = 4.00 + 2.00 \times number\ of\ pages}$$

d. Carolyn decides she wants one of her website to have 19 pages.
How much, in dollars, will she save by using Magic Web Designs instead of Wanda's Web Designs for this website?

1 mark

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e. If Carolyn decides to have a website of between 20 and 30 pages, at what number of pages is the charge the same for both Wanda's Web Designs and Magic Web Designs?

1 mark

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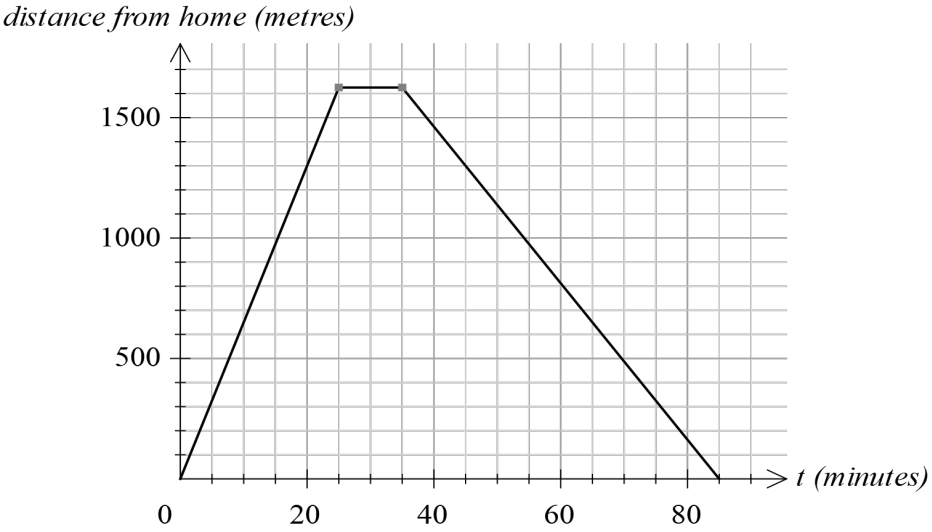
Question 2 (2 marks)

Carolyn is walking to the printers to collect some cards for her business.
The graph below shows her *distance from home*, in metres, t minutes after she started the walk.



The relation that describes this walk is given by

$$D = \begin{cases} 65t & 0 \leq t < 25 \\ 1625 & 25 \leq t < 35 \\ \boldsymbol{k} - 32.5t & 35 \leq t \leq 85 \end{cases}$$

Where D is her *distance from home*, in metres, t minutes after she starts the walk.

a. How far, in metres, is Carolyn from home 17 minutes after she started the walk?

_____ 1 mark

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b. What is the value of $\boldsymbol{k}$ in the above relation? 1 mark

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Question 3 (5 marks)

Carolyn makes and sells blocks of modelling clay for her clients every month.
There are two types of modelling clay; Squishy and Tuff.
Let x be the number of blocks of Squishy made each month.
Let y be the number of blocks of Tuff made each month.

She must make at least 20 blocks each month but can make no more than 60.
She must also make at least two blocks of Tuff for every three blocks of Squishy.
Inequalities 1 to 3 represent these constraints.

Inequality 1	$\boldsymbol{x + y \leq 60}$
Inequality 2	$\boldsymbol{x + y \geq 20}$
Inequality 3	$\boldsymbol{y \geq \frac{2x}{3}}$

Another constraint is given by

Inequality 4	$\boldsymbol{y \geq 12}$

a. Describe inequality 4 in terms of the blocks of modelling clay Carolyn makes each month.

1 mark

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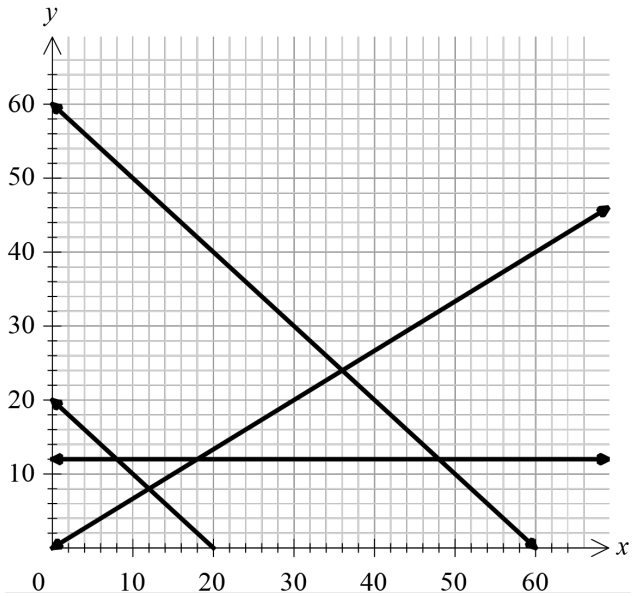
b. The graph below shows the lines representing the boundaries of these constraints.
On the graph, shade the region that satisfies the constraints. 1 mark



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Question 3 (continued)

c. In a particular month, Carolyn makes 21 blocks of Squishy.
What is the minimum number of blocks of Tuff that she needs to make that month?

1 mark

Carolyn can make a profit of $5 on each block of Squishy and a profit of $4 on each block of Tuff. The total profit, in dollars, that she can make each month is given by

$$P = 5x + 4y$$

d. Determine the maximum profit she can make in a month by making and selling the blocks of modelling clay.

1 mark

e. Determine the maximum profit to be made in a month when a total of 45 blocks are made and sold.

1 mark

End of Module 4 : Graphs and relations

End of 2021 VCE Further Mathematics Written Examination 2
Question and Answer Book

Kilbaha Education **PO Box 2227** **Kew Vic 3101** **Australia**	**Tel: (03) 9018 5376** **kilbaha@gmail.com** **https://kilbaha.com.au**

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FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

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Further Mathematics Formulas

Core: Data analysis

standardised score:	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line:	$y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value:	residual value = actual value – predicted value
seasonal index:	seasonal index$= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core: Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1: Matrices

determinant of a 2 × 2 matrix:	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix:	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$
recurrence relation:	$S_0 =$ initial state, $\quad S_{n+1} = TS_n + B$

Module 2: Networks and decision mathematics

Euler's formula:	$v + f = e + 2$

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Module 3: Geometry and measurement

area of a triangle:	$A = \frac{1}{2}bc\sin(\theta^0)$
Heron's formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$
sine rule:	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule:	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle:	$2\pi r$
length of an arc:	$r \times \frac{\pi}{180} \times \theta^0$
area of a circle:	πr^2
area of sector	$\pi r^2 \times \frac{\theta^0}{360}$
volume of a sphere:	$\frac{4}{3}\pi r^3$
surface area of a sphere:	$4\pi r^2$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a prism:	area of base × height
volume of a pyramid:	$\frac{1}{3}$ × area of base × height

Module 4: Graphs and relations

gradient (slope) of a straight line:	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line:	$y = mx + c$

END OF FORMULA SHEET

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