

2021 VCE

Further Mathematics Trial Examination 1 Suggested Solutions



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Quality educational content

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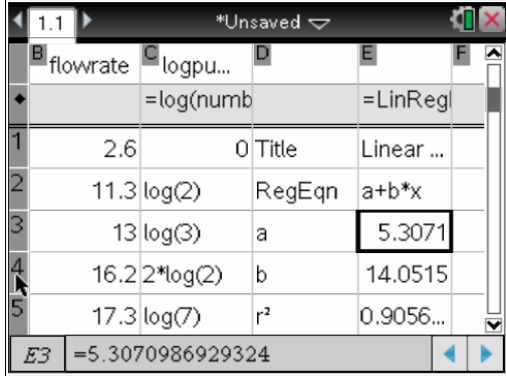
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Section A Core – Data Analysis

<p>Question 1 D</p> <p>More than 17 goals is 18 or above. There are four dots for 18 or above. So 4.</p>	<p>Question 2 B</p> <p>The median is between the 24th and 25th value. This occurs between two values of 13.</p>
<p>Question 3 D</p> $IQR = Q_3 - Q_1 = 15 - 12 = 3$	<p>Question 4 A</p> <p>Adding frequencies shows there are 25 drill bits. Median value will be the 13th. This is in the first column between 10^{-1} and 10^0 ie. between one tenth of a mm and 1 mm. These values are between 0 and 1.</p>
<p>Question 5 C</p> <p>22.2 is 2 standard deviations below the mean. 2.5% of the sample are expected to be less than this.</p> $2.5\% \times 1600 = 40$	<p>Question 6 D</p> $-1.4 = \frac{x - 25.6}{1.7}$ $x = 23.22$ ≈ 23.2
<p>Question 7 A</p> <p><i>Age group</i> is a categorical variable that can be put in order, so ordinal. <i>Opinion</i> is a categorical variable that can be put in order, so also ordinal.</p>	<p>Question 8 D</p> <p>From the graph, we see about 33% of this age group somewhat approve.</p> $33\% \times 160 = 52.8$

Section A Core – Data Analysis

<p>Question 9 C</p> <p>We need to compare at least two age groups to see if there is an association, so B, D and E can be eliminated.</p> <p>Option A does not compare the same thing across age groups.</p> <p>Only option C compares the same opinion across two age groups.</p>	<p>Question 10 B</p> $y = a + bx$ $b = -0.62 \frac{15.0}{1.3} = -7.15384 \dots$ $a = 78.0 - (-7.15384) \dots \times 5.6 = 118.061 \dots$ <p>Closest equation is $y = 120 - 7.2x$</p>
<p>Question 11 E</p> <p>The coefficient of -1.82 gives the expected rate of change of <i>temperature</i> with <i>time</i>. Being negative, we expect as <i>time</i> increases, <i>temperature</i> decreases.</p> <p>(NOTE: Response B is incorrect, as in this case, r must be negative. Watch out for this trap)</p>	<p>Question 12 C</p> <p>Predicted temperature $= 95.8 - 1.82 \times 22 = 55.76$</p> <p>Residual = actual – predicted $= 54.6 - 55.76 = -1.16$</p>
<p>Question 13 A</p> $\frac{1}{\text{time}} = -0.013 + 0.041 \times 8 = 0.315$ $\text{time} = \frac{1}{0.315} \approx 3.2$	<p>Question 14 E</p> <p>Use technology to apply a \log_{10} transformation to <i>number of pumps</i>, then carry out linear regression with $\log(\text{number of pumps})$ as the explanatory variable.</p>  <p>This gives</p> <p><i>flow rate</i> = $5.3071 +$ $14.0515 \times \log_{10}(\text{number of pumps})$</p>

Section A Core – Data Analysis

<p>Question 15 D</p> <p>The values for August, September October November, December are 8.2, 6.4, 5.8, 8.2, 6.2 respectively. Placing these in ascending order, the median is 6.4</p>	<p>Question 16 C</p> <p>Daily average = $513 \div 7 = 73.285 \dots$ Thursday's figure is 53% above this value. Expected number of golf balls sold on a Thursday = $1.53 \times 73.285 \dots \approx 112$</p>
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Section A Core – Recursion and financial modelling

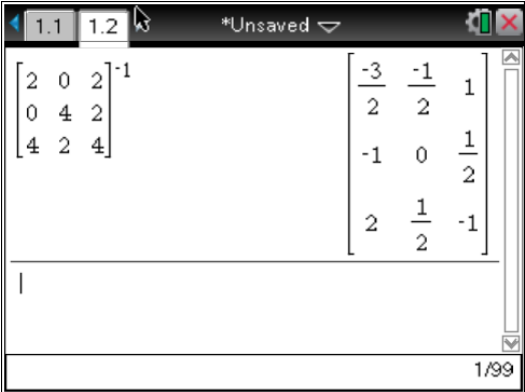
<p>Question 17 C</p> <p>The constant deduction is 950.</p>	<p>Question 18 B</p> <p>Calculation (i) would give the interest earned minus \$950. Calculation (iv) would give the interest earned during the <i>fifth</i> month.</p> <p>Interest earned during the fourth month will be 0.45% of the value at the end of the third month, or $0.0045 \times V_3$, so calculation (ii) will work.</p> <p>Also, $V_4 = V_3 + \text{interest} - 950$ or, $\text{interest} = 950 - V_3 + V_4$ so calculation (iii) will work.</p> <p>Calculation (v) has the signs for V_4 and V_3 the wrong way round.</p> <p>Calculations (ii) and (iii) are the only ones that will calculate the interest earned during the fourth month.</p>
<p>Question 19 B</p> $\text{effective rate} = \left[\left(1 + \frac{4.65}{400} \right)^4 - 1 \right] \times 100$ ≈ 4.73	<p>Question 20 D</p> <p>The principal of a perpetuity remains constant, so the withdrawal must equal the interest earned. Option D is the only one in which this occurs.</p> $1.076 \times 57000 - 4332 = 57000$
<p>Question 21 A</p> <p>Payment number 7: Interest rate $= \frac{1271.87}{100147.63} \times 100 \approx 1.27\%$ per quarter</p> <p>Payment number 8: Interest rate $= \frac{1178.58}{95819.50} \times 100 \approx 1.23\%$ per quarter</p> <p>This is a decrease of 0.04% per quarter which is equivalent to $0.04 \times 4 = 0.16\%$ per annum</p>	<p>Question 22 E</p> <p>Depreciation over 5 years $= 17000 - 2000 = 15000$</p> <p>Depreciation per unit $= \frac{15000}{25000 \times 5} = 0.12$</p> <p>Value starts at 17000 and decreases 0.12 for each brochure printed, ie $V_n = 17000 - 0.12n$</p>

Section A Core – Recursion and financial modelling

<p>Question 23 C Using finance solver</p> $N = 20 \times 12 = 240$ $I = 4.25$ $PV = 250000$ $PMT =$ $FV = 0$ $P/Y = 12$ $C/Y = 12$ <p>This gives $PMT = -1548.086\dots$</p> <p>Aimee's repayment is \$1548 each month</p>	<p>Question 24 B Find how long it will take to pay off the loan after the lump sum payment is made.</p> $N =$ $I = 4.25$ $PV = 130000$ $PMT = -1650$ $FV = 0$ $P/Y = 12$ $C/Y = 12$ <p>This gives $N = 92.5415\dots$</p> <p>Adding to the first 10 years, gives</p> $92.54 + 120 \approx 213$
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End of solutions Section A Core

Module 1 – Matrices

<p>Question 1 D</p> <p>A binary matrix has elements of only 0 and 1.</p>	<p>Question 2 D</p> <p>P must be a 5 x 5 permutation matrix consisting of a 1 and four zeros in each row. This eliminates A, C and E.</p> <p>Option B will produce $\begin{bmatrix} N \\ A \\ E \\ L \\ G \end{bmatrix}$ so is incorrect.</p>
<p>Question 3 E</p> <p>Let the rule be $a_{ij} = di + ej + f$ Pick 3 elements, not all from the same column, for example: $a_{11} = 3$ $a_{21} = 6$ and $a_{23} = 8$ This gives us 3 simultaneous equations</p> $\begin{aligned} d + e + f &= 3 \\ 2d + e + f &= 6 \\ 2d + 3e + f &= 8 \end{aligned}$ <p>Solving this gives $d = 3$, $e = 1$, $f = -1$</p>	<p>Question 4 B</p> <p>For an inconsistent system, there are no solutions. The graphs of the lines are parallel. For no solution, determinant = 0</p> <p>So, $(3 \times -3) - 9a = 0$ $a = -1$</p> <p>Now if $b = 2$, the lines will be exactly the same (dependent system). So, $b = -4$ will give an inconsistent system</p>
<p>Question 5 E</p> <p>Use technology to find the inverse of the matrix of coefficients.</p> 	<p>Question 6 A</p> <p>There is only one given option where the proportion of Apple eaters choosing an Apple the following week is greater than the proportion of Banana eaters choosing a Banana the following week ie A to A (0.7) is greater than B to B (0.5)</p>

Module 1 – Matrices (continued)**Question 7 D**

The proportion of each changing their choice is
0.7 of B, 0.6 of C, 0.8 of D

$$0.7 \times 35 + 0.6 \times 52 + 0.8 \times 63 = 106.1$$

$$\frac{106.1}{150} \times 100 \approx 71$$

Question 8 A

Find the steady state matrix by raising T to a large
power e.g. 99.

In the long term:

$$\begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.6 & 0.4 & 0.3 \\ 0.1 & 0.4 & 0.2 \end{bmatrix}^{99} \begin{bmatrix} 35 \\ 52 \\ 63 \end{bmatrix} = \begin{bmatrix} 46.1538 \\ 65.3846 \\ 38.4615 \end{bmatrix}$$

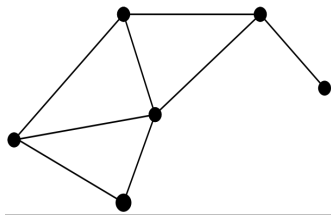
Now, 0.4 of those choosing Carrot cake in one
week are expected to choose Date slice in the
next.

$$0.4 \times 65.3846 = 26.15 \dots \approx 26$$

End of Module 1 Solutions

Module 2 – Networks and decision mathematics

Question 1 D



Degrees of odd vertices are 3, 3, 3 and 1
Sum = 10

Question 2 E

An Eulerian trail can only be made if there are two and only two odd vertices.
The network has four odd vertices; B, E, F and H.
Joining B and F makes them both even, leaving only two odd vertices.

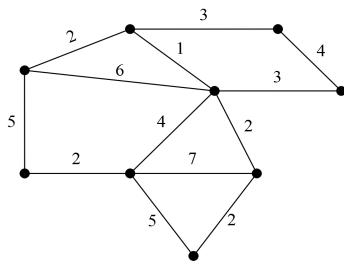
Question 3 E

The graph has 6 vertices, 12 edges and 8 faces
(remember that the outside area counts as a face)
so, $e = 12, v = 6, f = 8$

Question 4 C

There are two edges between A and C, so elements in row A column C and row C column A should both be 2. This eliminates options A, B, D and E.
Also, because the connections work in both directions, the adjacency matrix should be symmetric.
Other methods of elimination could be used, such as row and column E should only contain one 1 and the rest zeros.

Question 5 D



The minimal spanning tree has a weight of
 $2 + 3 + 2 + 1 + 3 + 4 + 2 + 2 = 19$

Module 2 – Networks and decision mathematics (continued)

Question 6 B

6	5	7	8
8	5	6	7
5	4	5	4
7	9	11	8

performing row reduction gives



1	0	2	3
3	0	1	2
1	0	1	0
0	2	4	1

Need only 3 lines to cover zeros. Cannot make an allocation.

Performing column reduction gives

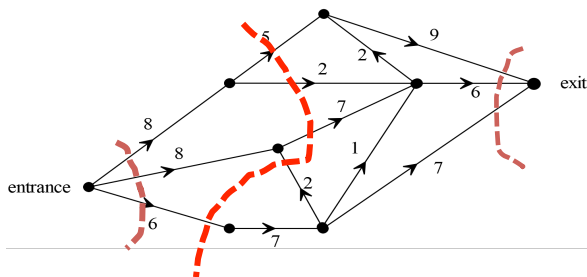
1	0	1	3
3	0	0	2
1	0	0	0
0	2	1	1

Need 4 lines to cover zeros, so can make an allocation.

Hilda must rinse, Guyan must put away, and Emma must wash. This leaves Frank to dry.

Minimum time taken = $7 + 4 + 5 + 6 = 22$ minutes.

Question 7 B



Minimum cut = maximum flow.

Cuts at the entrance and exit both have a value of 22.

We are looking for something lower if possible.

Cut created by thick dotted line above has a value of $5 + 2 + 7 + 6 = 20$

There are no cuts with value less than this.

Question 8 C

B-D-E is the critical path with the earliest completion time of 30 days.

Reducing B and D by one day each creates a new critical path A-C-E taking 29 days.

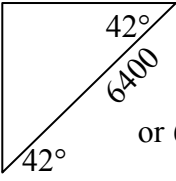
Now reduce A **or** C by one day, making B-D-E and A-C-E both critical paths of 28 days.

(There is no point in reducing both A **and** C as B-D-E will become the critical path again).

Reduce E by one day, bringing the completion time to 27 days.

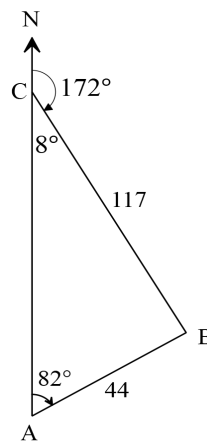
The greatest reduction in time is 3 hours, at a cost of $4 \times 250 = \$1000$

Module 3 – Geometry and measurement

<p>Question 1 B By Pythagoras,</p> $YB = \sqrt{193^2 - 95^2} = 168$	<p>Question 2 C</p> <p>The latitude of a location will give us how many degrees north or south of the equator it is. In this case, we are looking for the location furthest south. Out of the options, 42°S is closest to the South Pole.</p>
<p>Question 3 D</p> $AB = \sqrt{10^2 + 14^2 - 2 \times 10 \times 14 \cos 125^\circ}$ $\approx 21.368 \dots$ <p>Perimeter = 14 + 10 + 21.368 ≈ 45</p>	<p>Question 4 E</p>  <p>R could be found by calculating either $6400 \times \cos 42^\circ$ or $6400 \times \sin 48^\circ$</p> <p>The cosine option is the only one given here.</p>
<p>Question 5 D</p> <p>Area of larger semicircle – area of smaller semicircle</p> $= \frac{\pi \times 4.5^2}{2} - \frac{\pi \times 3.5^2}{2}$ $= \frac{20.25\pi}{2} - \frac{12.25\pi}{2}$ $= \frac{8\pi}{2} = 4\pi$	<p>Question 6 A</p> <p>$V = \frac{1}{3} \times a \times h$, where a is the area of the base.</p> <p>Solving $84.50 = \frac{1}{3} \times a \times 15.60$ gives $a = 16.25$</p> <p>Now $x^2 = 16.25$ Length AB $\sqrt{x^2 + x^2}$ $= \sqrt{16.25 + 16.25} \approx 5.70$</p>

Module 3 – Geometry and measurement (continued)**Question 7 C**

Length ratio of small cone to large cone
 $= 12:42 = 2:7$
 area ratio of small cone to large cone
 $= 2^2:7^2 = 4:49$
 So, area of small cone to area of non-painted
 part of large cone
 $= 4: (49-4)$
 $= 4: 45$

Question 8 B

$$\text{Angle } B = 180 - 82 - 8 = 90^\circ$$

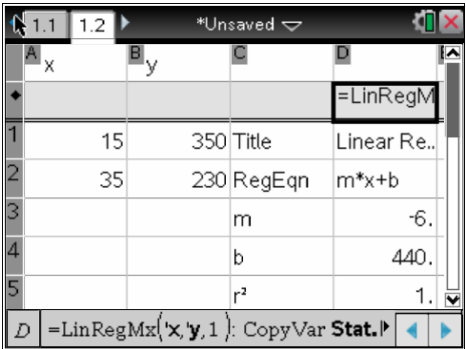
$$AC = \sqrt{117^2 + 44^2} = 125$$

$$CJ = \sqrt{137^2 - 125^2} \approx 56.0714$$

Closest answer is 56 metres.

(Out of interest, a jacquemart is an automaton that strikes a bell to mark the hour. They are usually found on or near the top of clocktowers)

Module 4 – Graphs and relations

<p>Question 1 C</p> <p>At 5:00 pm, volume of water = 28 L. At 7:30 pm, volume of water = 15 L. The difference in volume is 13 L.</p>	<p>Question 2 A</p> <p>The volume of water is 20 litres or more between about 3:40 pm and 7:15 pm. This is 3 hours and 35 minutes = $180 + 35 = 215$ minutes</p>
<p>Question 3 C</p> <p>Gradient = $\frac{0 - -20}{-30 - 0} = \frac{20}{-30} = \frac{-2}{3}$</p> <p>y intercept is at -20, so equation of the line is</p> $y = \frac{-2}{3}x - 20$ <p>When $y = -14$,</p> $x = 6 \times \frac{3}{-2} = -9$	<p>Question 4 B</p> <p>Costs = $560 + 45 \times 5 \times 4 = 1460$ His revenue = $5 \times a$ where a is the hourly charge to his client.</p> <p>Profit = Revenue – Cost $750 = 5a - 1460$ $5a = 2210$ $a = 2210 \div 5 = \\$442$</p>
<p>Question 5 E</p> <p>gradient $\frac{350 - 230}{15 - 35} = -6$</p> <p>Substitute (15, 350) into</p> $V = c - 6t$ $350 = c - 6 \times 15$ $c = 440$ <p>So, $V = 440 - 6t$</p> <p style="text-align: center;">OR</p> <p>Enter the points into Lists and Spreadsheets and perform linear regression.</p> 	<p>Question 6 D</p> <p>First find the rule for graph 2. We know it is of the form $y = \frac{k}{x^2}$ where $k = \frac{108}{8} = \frac{27}{2}$</p> $y = \frac{27}{2x^2}$ <p>When $x = 3$, $y = \frac{27}{2 \times 9} = \frac{3}{2}$</p>

Module 4 – Graphs and relations (continued)

<p>Question 7 A</p> <p>Sarah has a maximum of $40 \times 7 = \\$280$ to spend on cups and saucers. Cost of cups = $\\$2.7x$ Cost of saucers = $\\$1.90y$</p> <p>The total cost must be less than or equal to $\\$280$</p> <p>So, $2.7x + 1.9y \leq 280$</p>	<p>Question 8 D</p> <p>To have a maximum value at point P only, the gradient of the line running from $(0,18)$ to $(n, 0)$ must not be as steep as the gradient of the objective function.</p> <p>Ignoring the negative signs, the slope of the objective function is $\frac{24}{20} = 1.2$</p> <p>The value of $\frac{18}{n}$ must be less than 1.2</p> $\frac{18}{n} < 1.2$ $n > \frac{18}{1.2}$ $n > 15$ <p>So the only option greater than 15 is 16.</p>
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End of Module 4 Solutions

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