Neap

Trial Examination 2020

VCE Further Mathematics Units 3&4

Written Examination 2

Suggested Solutions

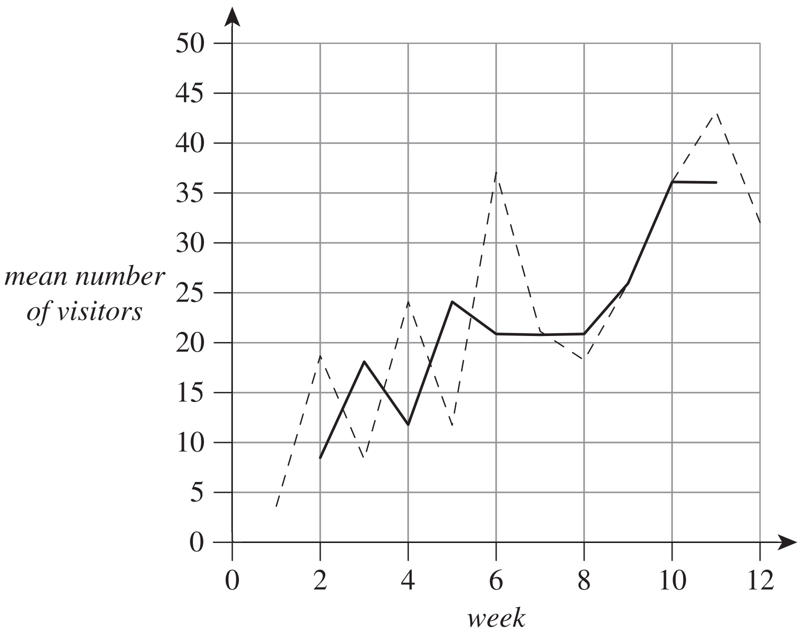
SECTION A – CORE

Data analysis

Question 1 (12 marks)

a. time series graph A1

b.



graph points M1
graph line M1

c. **i.** $\frac{42}{78} \times 100 = 53.8\%$ A1

ii. $\frac{8}{18} \times 100 = 44.4\%$ A1

d. **i.** seasonal variation A1

ii. Monday A1

iii. Although the mean is 57, this is not the number of visitors per day, which increases during the week and peaks on Saturday. On some days there would be excess staff and on weekends not enough. A1

e. **i.** $\dfrac{\left(\frac{600}{450} + \frac{720}{500} + \frac{750}{550}\right)}{3} = 1.4$ A1

ii. The deseasonalised figures for autumn and winter are $\frac{865}{0.9} = 961.1$ and $\frac{743}{0.7} = 1061.4$ respectively. M1

The winter promotion was the more successful. A1

f. $\frac{450 + 380 + 620}{3} = 483.3$ A1

Question 2 (6 marks)

a.

First time visitors	*Stem*	*Repeat visitors*
	0	8
	1	5 5
5 5 0	2	0 4 5 6 8 8
6 5 5	3	2
5 5 5 2	4	
5 0 0	5	
	6	
	7	5

A1

b. The five-number summary is the lowest value, Q_1, the median, Q_3 and the highest value.
Therefore, the five-number summary here is 20, 30, 42, 47.5, 55. A1

c. The mean for repeat visitors is 26.9. A1

d. We are showing that 75 is an outlier, so calculate the upper fence:

$Q_1 + 1.5 \times IQR$

$IQR = Q_3 - Q_1$

$= 28 - 15$

$= 13$

upper fence $= Q_3 + 1.5 \times IQR$

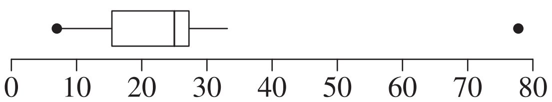
$= 28 + 1.5 \times 13$

$= 47.5$

75 is higher than 47.5 and so must be an outlier. A1

Note: Only award marks for correct answer with calculations.

e.



A1

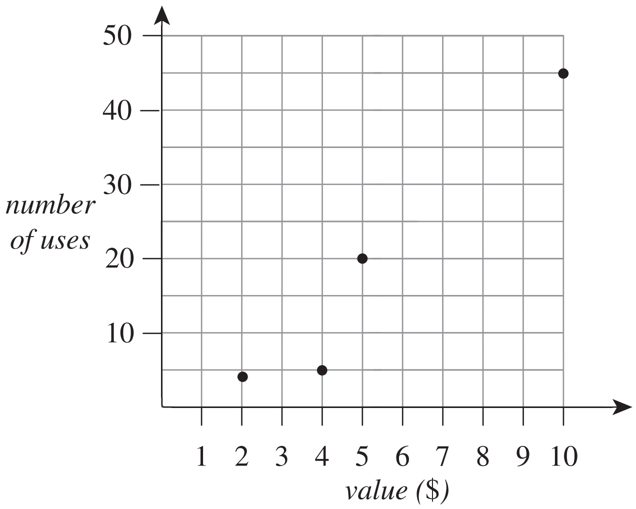
f. range and mean A1

Note: Both answers are required to be awarded the mark.

Question 3 (4 marks)

a. The higher the value of the discount coupon, the more they are used. Therefore, the number of uses is the response variable. A1

b.

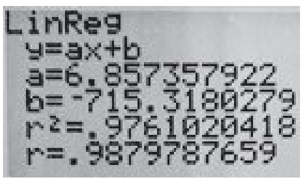


graph points M1
axes and scale M1

c. The gradient is positive, so increasing the value increases the number of times it is used. A1

Question 4 (8 marks)

a. Enter the bivariate figures into your technology and find the least squares regression line.



$I = 6.9v - 715.3$ A1

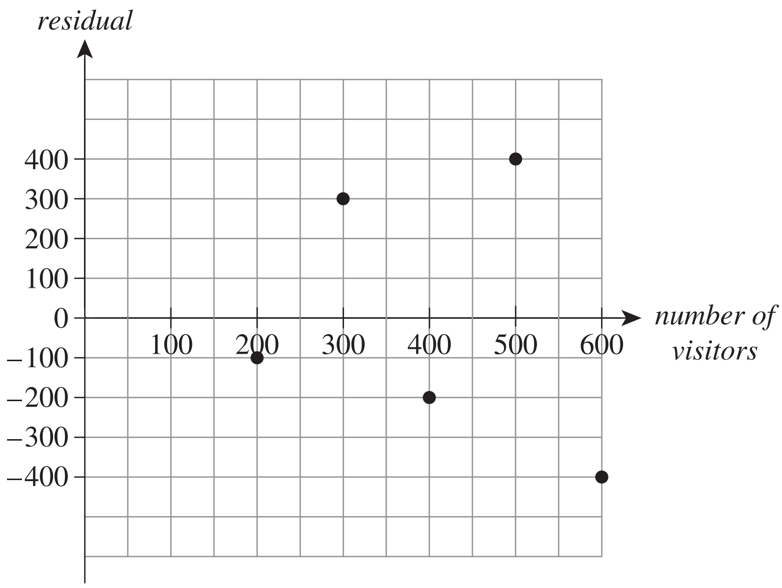
b.

Weekly visitors	200	300	400	500	600
Income (actual)	900	2100	2400	3600	3800
Income (predicted)	1000	1800	2600	3200	**4200**
Residual	–100	300	–200	400	**–400**

A2
Income (predicted) A1
Residual A1

 FMU34EX2_SS_2020.FM

c.

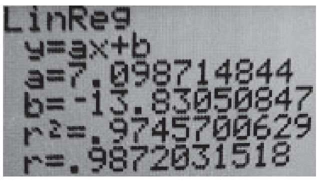


A2
correct labelling A1
correct scatterplot information A1

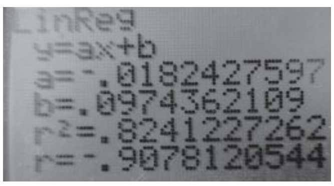
d. There is no pattern, so linearity is confirmed. A1

e. Enter the data into your technology and transform the x values into x^2 and $\frac{1}{y}$, then calculate the value of r for each. M1

x^2:



$\frac{1}{y}$:



Since r is 0.99 for x^2 and –0.91 for $\frac{1}{y}$, the x^2 transformation better linearises the data. A1

Recursion and financial modelling

Question 5 (3 marks)

a. Each successive term is found by multiplying the previous term by 6 and then subtracting 1.

$t_{n+1} = 6t_n - 1;\ t_1 = 1$ A1

b. $t_{n+1} = -0.5t_n + 4;\ t_1 = 400$

$t_1 = 400$

$t_2 = -0.5(400) + 4 = -196$

$t_3 = -0.5(-196) + 4 = 102$ A1

c. $t_{n+1} = 3t_n - 2;\ t_1 = 1$

$t_1 = 1$

$t_2 = 3(1) - 2 = 1$

$t_3 = 3(1) - 2 = 1$

Every term is 1. A1

Question 6 (6 marks)

a. **i.** Reading from the equation, $P = \$125\,000$. A1

ii. $R = 1 + \frac{r}{100}$

$\frac{r}{100} = 1.007 - 1$

$r = 0.7\%$ per month

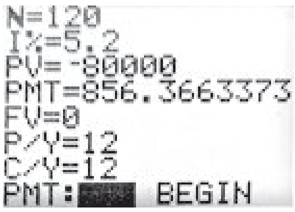
$0.7 \times 12 = 8.4\%$ per year A1

b. The loan is adding compound interest (as the interest is calculated and added monthly, changing the principal). A1

c. $A = 125\,000 \times 1.007^{60}$

$= \$189\,967.04$ A1

d. Enter the data into the financial application of your technology (10 years is 120 compounding periods).

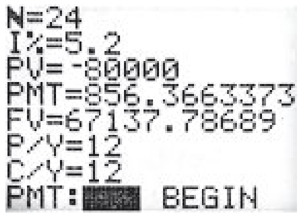


The monthly repayments are $856.37. A1

Note: In this case the PV has been entered as negative, giving a positive payment. It is also correct to enter the PV as positive, which will give the payment as negative.

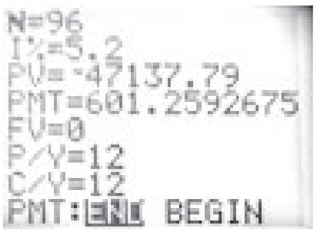
 FMU34EX2_SS_2020.FM

e. Firstly, calculate the amount owed after two years using your technology.



The amount owed after two years is \$67 137.79.

Making a payment of \$20 000 gives a principal of \$47 137.79. Use this as the principal for the remaining eight years to find the new payment.



The payment for the next eight years is \$601.26. A1

Note: In this case the PV has been entered as negative, giving a positive payment. It is also correct to enter the PV as positive, which will give the payment as negative.

Question 7 (3 marks)

a. **i.**

	V_0 = \$25 000
V_{n+1} = 25 000 – 3200 = 21 800	V_1 = \$21 800
V_{n+1} = 21 800 – 3200 = 18 600	V_2 = \$18 600
V_{n+1} = 18 600 – 3200 = 15 400	V_3 = \$15 400

A1

ii. V_4 = \$15 400 – 3200 = 12 200

V_5 = \$12 200 – 3200 = 9000

At the end of the fifth year, the value has reached \$9000. A1

b.

	$V_0 = \$25\ 000$
$V_{n+1} = 0.8(25\ 000) = 20\ 000$	$V_1 = \$20\ 000$
$V_{n+1} = 0.8(20\ 000) = 16\ 000$	$V_2 = \$16\ 000$
$V_{n+1} = 0.8(16\ 000) = 12\ 800$	$V_3 = \$12\ 800$
$V_{n+1} = 0.8(12\ 800) = 10\ 240$	$V_4 = \$10\ 240$
$V_{n+1} = 0.8(10\ 240) = 8192$	$V_5 = \$8192$

At the end of the fifth year, the value has reached \$8192. A1

Question 8 (3 marks)

a. $A = PR^n$

$= 600\ 000(1.028)^3$

$= \$651\ 824.37$ A1

b. $600\ 000 - 65\ 000 = 535\ 000$

$V_0 = 535\ 000$, $V_n = V_0 R^n$

$$= 535\ 000\left(1 + \frac{\frac{4.5}{12}}{100}\right)^{36}$$

$= \$612\ 172.59$ A1

c. $V = 65\ 000 \times 0.82 \times 0.88^2$

$= \$41\ 275.52$ A1

SECTION B – MODULES

Module 1 – Matrices

Question 1 (5 marks)

a. 1×3 A1

b. $52 \times M$ A1

c. $M \times N$ A1

d. $52 \times \begin{bmatrix} 45 & 30 & 20 \end{bmatrix} \times \begin{bmatrix} 1300 \\ 900 \\ 1400 \end{bmatrix} = 5\,902\,000$ A1

e. $1.05 \times 1.05 \times \begin{bmatrix} 1300 \\ 900 \\ 1400 \end{bmatrix} = \begin{bmatrix} 1433.25 \\ 992.25 \\ 1543.50 \end{bmatrix}$

1544 Concession members A1

Question 2 (8 marks)

a. 25% of Concession members will become Peak members in the next year. A1

b. $\mathbf{0.1} \times 1300 + \mathbf{0.9} \times 900 + \mathbf{0.15} \times 1400 = 1150$ A1

c.

$$t^{100} \cdot \begin{bmatrix} 1300 \\ 900 \\ 1400 \end{bmatrix} \qquad \begin{bmatrix} 917.647058835 \\ 1976.47058822 \\ 705.882352948 \end{bmatrix}$$

918 Peak members A1

d.

$$\text{solve}\left(t \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 130 \\ 70 \\ -20 \end{bmatrix} = \begin{bmatrix} 1300 \\ 900 \\ 1400 \end{bmatrix}, x, y, z \right)$$

$x=914.961832061$ and $y=483.664122137$ an▸

915 Peak members A1

e. The gym will meet its membership target.

Peak members for 2020 = 1435 A1

The 5% increase would result in 1365 members. A1

f.

$$t \cdot \begin{bmatrix} 1300 \\ 900 \\ 1400 \end{bmatrix} + \begin{bmatrix} 130 \\ 70 \\ -20 \end{bmatrix} \quad \begin{bmatrix} 1435. \\ 1220. \\ 1125. \end{bmatrix}$$

$$t \cdot \begin{bmatrix} 1435. \\ 1220. \\ 1125. \end{bmatrix} + \begin{bmatrix} 130 \\ 70 \\ -20 \end{bmatrix} \quad \begin{bmatrix} 1476.75 \\ 1480.25 \\ 1003. \end{bmatrix}$$

$$t \cdot \begin{bmatrix} 1476.75 \\ 1480.25 \\ 1003. \end{bmatrix} + \begin{bmatrix} 130 \\ 70 \\ -20 \end{bmatrix} \quad \begin{bmatrix} 1488.4875 \\ 1700.35 \\ 951.1625 \end{bmatrix}$$

M1

Note: A valid attempt at using the formula must be shown for full marks.

1700 Off-peak members A1

Question 3 (2 marks)

a. By totalling column 1 from matrix P (8 + 1 + 4), the total number of treadmills bought is 13. A1

b.

$$\begin{bmatrix} 8 & 2 & 0 \\ 1 & 6 & 0 \\ 4 & 1 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 10490 \\ 9591 \\ 10342 \end{bmatrix} \quad \begin{bmatrix} 951.26086957 \\ 1439.9565217 \\ 1699. \end{bmatrix}$$

A rowing machine costs $1439.96. A1

Module 2 – Networks and decision mathematics

Question 1 (7 marks)

a. St Thomas–North Howell–Pieters–Allan Town (7 hours) A1

b. **i.** Hamiltonian cycle A1

ii. 2 + 3 + 4 + 1 + 2 + 8 + 14 + 3 = 37 hours A1

c. **i.** Eulerian circuit A1

ii. The network shows that the trip has three odd-numbered vertices. For an Eulerian circuit to be viable, all vertices must be even. A1

d. The company would need to add three routes in order to make the six odd-numbered vertices even. A1

e. Matthams–Fowl–Pieters–Allan Town A1

Question 2 (6 marks)

a.

Task	Immediate predecessors
A	–
B	***A***
C	***B***
D	***A***
E	***A***
F	***E***
G	***D, F***
H	***C***

A1

b. Task *G* can start on day 11. (Task *A* takes 4 days and task *D* takes 6 days, which is a total of 10 days.) A1

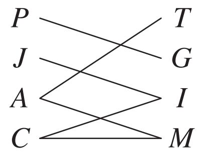
c. The critical path is 15 days (*A*, *D*, *G*). A1

15 days after 24 April is 8 May, which is when work completes. Therefore, Ernest can go away on 9 May. A1

d. Reducing by 1 day would cost $10 and reduce the critical path to 14 days. A1

Any additional changes will not change the overall time, and so this is the cheapest option. A1

Question 3 (2 marks)



P-G = 60
J-I = 40
A-T = 80
C-M = 70

a. Priya should complete the guide book task. A1

b. Priya on guide book (60), Jayden on itinerary (40), Amit on tickets (80) and Charlotte on meals (70) gives a total of 250 hours of work. A1

Module 3 – Geometry and measurement

Question 1 (7 marks)

a. $2 \times (35 \times 7 + 27 \times 7 + 27 \times 35) = 2758 \text{ cm}^2$ A1

b. $6615 - 6615 \times (0.8)^3 = 3228.12 \text{ cm}^3$ A1

c. $\dfrac{1570}{10^2 \times \pi} = 5.00001169218$ A1

d. $\dfrac{1570}{6615} \times 100 = 23.746031746$ A1

Therefore, the box is 76% empty. A1

e. $10^2 \times \pi \times 2 + 5 \times 10 \times 14 + 20 \times \pi \times 5 \times 2 = 1956.63706144$ A1

1957 cm^2 A1

Question 2 (2 marks)

a. $\sqrt{30^2 + (18.2)^2} = 35.089029625$

35 km A1

b.

$\tan^{-1}\left(\dfrac{30}{18.2}\right)$ 58.756207675

$360-58.756207674754$ 301.24379233

301°T A1

Question 3 (6 marks)

a. Washington A1

b.

$\text{solve}\left(\sin(78)=\dfrac{x}{6400},x\right)$ $x=6260.1446447$

6 km A1

c.

$\dfrac{\pi \cdot 51 \cdot 6400}{180}$ 5696.75467851

5697 m A1

d. **i.** The meeting must take place at GMT 5 am, which is 9 pm in Vancouver. A1

ii. The maximum time that the meeting can run for is 1 hour. A1

e. 8.20 am 1 July is 16.20 GMT 1 July.

16.40 3 July is 8.40 GMT 3 July.

Therefore:

- 16.20 GMT 1 July to 16.20 2 July (24 hours);
- 4.20 3 July (36 hours);
- 8.20 3 July (40 hours), and;
- 8.40 3 July 40 hours and 20 minutes.

40 hours and 20 minutes A1

Module 4 – Graphs and relations

Question 1 (4 marks)

a. For five hours, the solid dot indicates the cost is \$8 per hour. (The open circle applies to stays of length > 5 hours.)

$$\text{cost} = 5 \times 8$$
$$= \$40$$ A1

b. The maximum time of parking for \$30 is $\frac{30}{8} = 3.75$ hours. A1

c. $\text{cost} = 48 + 8n$ A1

Note: An equivalent expression is also acceptable.

d. The additional cost of an Adventurer pass is 80 – 48 = \$32. Since an individual ride ticket costs \$8 and $\frac{32}{8} = 4$, buying 4 individual ride tickets would result in the same cost as buying an Explorer pass and paying for 4 individual rides. To make the Adventurer pass cheaper, the student must take a minimum of 5 rides. A1

Question 2 (4 marks)

a. The maximum value of x is 15 (half of the 30 m width). Substitute into the equation.

$$h = \frac{2}{15}(15)^2$$
$$= 30 \text{ m}$$ A1

b.
$$h = \frac{2}{15}x^2$$
$$3 = \frac{2}{15}x^2$$
$$\frac{3 \times 15}{2} = x^2$$
$$x = \sqrt{22.5}$$
$$x = 4.7 \text{ m}$$ A1

c. The gradient is the line joining the points (0, 0) and (15, 30). Using $\frac{\text{rise}}{\text{run}}$, the gradient is $\frac{30}{15} = 2$. A1

d. $h = kx^2$

Substitute the point (15, 45).

$$45 = k(15)^2$$

$$k = \frac{45}{15^2}$$

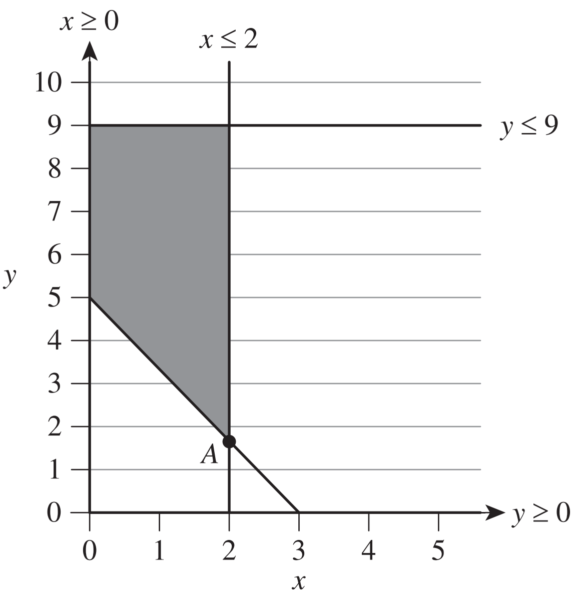
$$= 0.2$$ A1

Question 3 (7 marks)

a. $5x + 3y \geq 15$ A1

b. $Z = 500x + 330y$ A1

c.



A1

d. Point A is at the intersection of the lines $x = 2$ and $5x + 3y \geq 15$. Substitute $x = 2$ into the second inequation.

$$5(2) + 3y \geq 15$$

$$10 + 3y \geq 15$$

$$3y \geq 5$$

$$y \geq \frac{5}{3}$$

The point is $\left(2, \frac{5}{3}\right)$. A1

*Note: Consequential on answer to **Question 3a.***

e. There must be a whole number of helicopters, so only whole-number solutions are feasible. A1

f. To minimise cost, only whole-number solutions within the feasible area need testing. A minimum cost will be found at the intersections of $5x + 3y \geq 15$ with $x = 0$, $x = 1$ and $x = 2$. The three valid points are then (0, 5), (1, 4) and (2, 2). M1

Point	**Number of passengers**	**Cost**
(0, 5)	$0 \times 5 + 5 \times 3 = 15$	$0 \times 500 + 5 \times 330 = \$1650$
(1, 4)	$1 \times 5 + 4 \times 3 = 17$	$1 \times 500 + 4 \times 330 = \$1820$
(2, 2)	$2 \times 5 + 2 \times 3 = 16$	$2 \times 500 + 2 \times 330 = \$1660$

The cheapest option is to hire 5 three-seater helicopters. A1