VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2019

FURTHER MATHEMATICS

Written examination 1

Wednesday 29 May 2019

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.45 pm (1 hour 30 minutes)

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A – Core	24	24			24
B – Modules	32	16	4	2	16
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question book of 32 pages
- Formula sheet
- Answer sheet for multiple-choice questions
- Working space is provided throughout the book.

Instructions
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book and the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Core

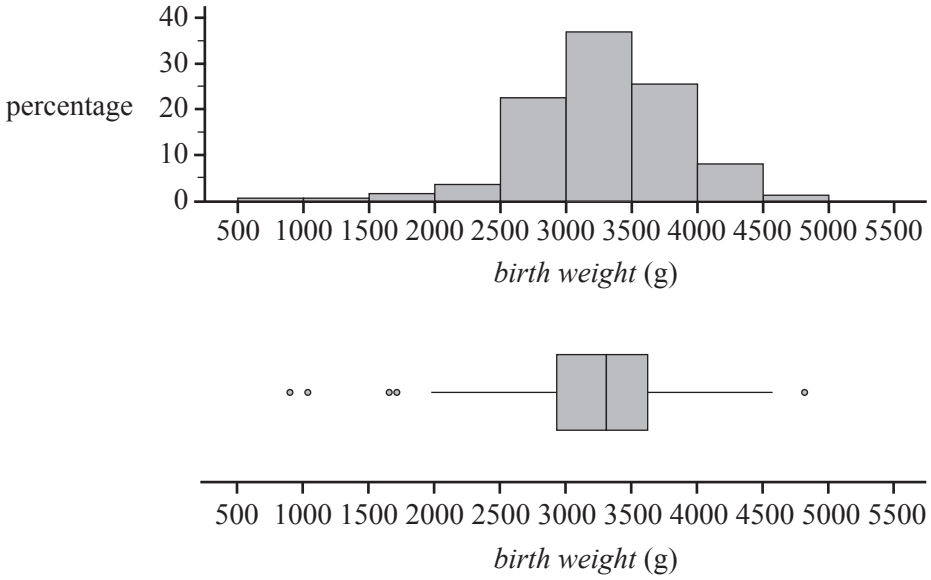
Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Use the following information to answer Questions 1 and 2.

The histogram and boxplot shown below both display the distribution of the *birth weight*, in grams, of 200 babies.



Question 1

The shape of the distribution of the babies' *birth weight* is best described as

A. positively skewed with no outliers.
B. negatively skewed with no outliers.
C. approximately symmetric with no outliers.
D. positively skewed with outliers.
E. approximately symmetric with outliers.

Question 2

The number of babies with a *birth weight* between 3000 g and 3500 g is closest to

A. 30
B. 32
C. 37
D. 74
E. 80

Question 3

The total birth weight of a sample of 12 babies is 39.0 kg.

The mean birth weight of these babies, in kilograms, is

A. 2.50

B. 2.75

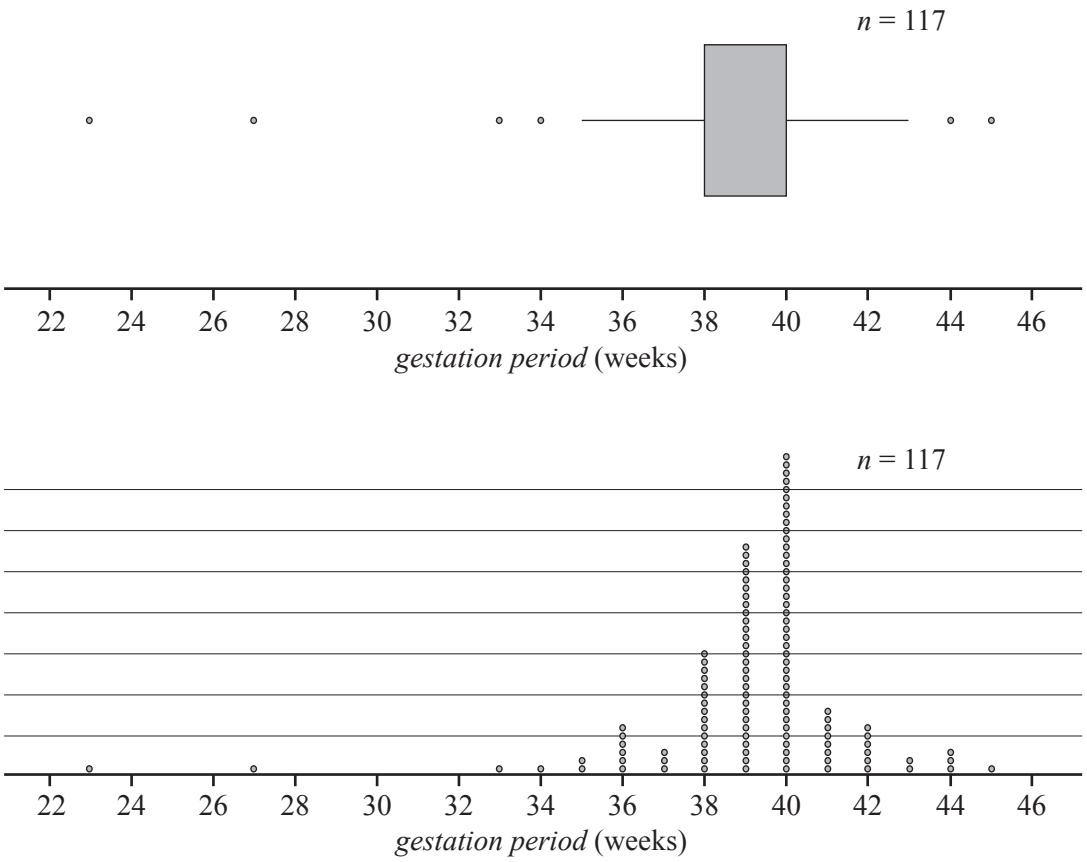
C. 3.00

D. 3.25

E. 3.50

Question 4

The boxplot and dot plot shown below both display the distribution of the *gestation period*, in weeks, for 117 baby girls.



The median *gestation period*, in weeks, of these baby girls is

A. 38

B. 38.5

C. 39

D. 39.5

E. 40

Use the following information to answer Questions 5 to 7.

The birth weights of a large population of babies are approximately normally distributed with a mean of 3300 g and a standard deviation of 550 g.

Question 5

A baby selected at random from this population has a standardised weight of $z = -0.75$

Which one of the following calculations will result in the *actual birth weight* of this baby?

A. *actual birth weight* $= 550 - 0.75 \times 3300$

B. *actual birth weight* $= 550 + 0.75 \times 3300$

C. *actual birth weight* $= 3300 - 0.75 \times 550$

D. *actual birth weight* $= 3300 + \dfrac{0.75}{550}$

E. *actual birth weight* $= 3300 - \dfrac{0.75}{550}$

Question 6

Using the 68–95–99.7% rule, the percentage of babies with a birth weight of less than 1650 g is closest to

A. 0.14%

B. 0.15%

C. 0.17%

D. 0.3%

E. 2.5%

Question 7

A sample of 600 babies was drawn at random from this population.

Using the 68–95–99.7% rule, the number of these babies with a birth weight between 2200 g and 3850 g is closest to

A. 111

B. 113

C. 185

D. 408

E. 489

Question 8

The variables *recovery time after exercise* (in minutes) and *fitness level* (below average, average, above average) are

A. both numerical.

B. both categorical.

C. an ordinal variable and a nominal variable respectively.

D. a numerical variable and a nominal variable respectively.

E. a numerical variable and an ordinal variable respectively.

Question 9

A least squares line of the form $y = a + bx$ is fitted to a set of bivariate data for the variables x and y.

For this set of bivariate data, $\overline{x} = 5.50$, $\overline{y} = 5.60$, $s_x = 3.03$, $s_y = 1.78$ and $a = 3.1$

The slope of the least squares line, b, is closest to

A. 0.44

B. 0.45

C. 0.58

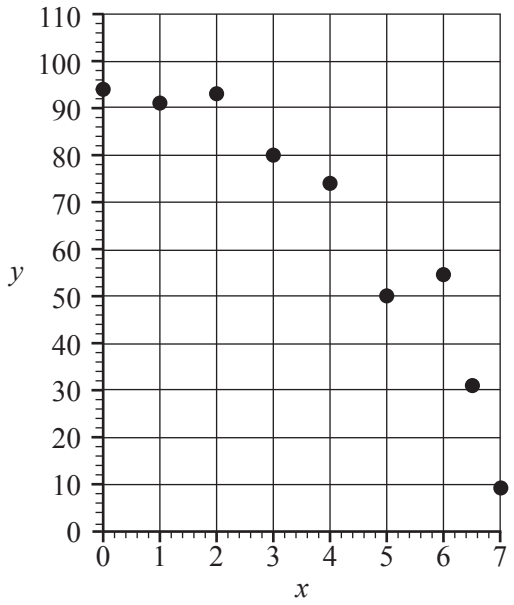
D. 0.59

E. 0.76

Question 10

A student uses the data in the table below to construct the scatterplot shown below.

x	*y*
0	94
1	91
2	93
3	80
4	74
5	50
6	55
6.5	31
7	9



A squared transformation is applied to the variable x to linearise the data.

A least squares line is fitted to this linearised data with x^2 as the explanatory variable.

The equation of this least squares line is closest to

A. $y = 94.1 - 12.3x^2$

B. $y = 95.8 - 1.57x^2$

C. $y = 8.76 - 0.0768x^2$

D. $y = 107 - 111x^2$

E. $y = 107 - 0.0768x^2$

Question 11

The *Human Development Index* (*HDI*) and the mean number of *children* per woman for 13 countries are related.

This relationship is non-linear.

To linearise the data, a $\log_{10}$ transformation is applied to the response variable *children*.

A least squares line is then fitted to the linearised data.

The equation of this least squares line is

$$\log_{10}(children) = 1.06 - 0.00674 \times HDI$$

Using this equation, the mean number of *children* per woman for a country with a *HDI* of 95 is predicted to be closest to

A. 0.4

B. 1.5

C. 2.6

D. 2.9

E. 3.1

Question 12

Which one of the following statements could be true when written as part of the results of a statistical investigation?

A. The correlation coefficient between *height* (in centimetres) and *foot length* (in centimetres) was found to be $r = 1.24$

B. The correlation coefficient between *height* (below average, average, above average) and *arm span* (in centimetres) was found to be $r = 0.64$

C. The correlation coefficient between *blood pressure* (low, normal, high) and *age* (under 25, 25–49, over 50) was found to be $r = 0.74$

D. The correlation coefficient between the *height* of students of the same age (in centimetres) and the *money* they spent on snack food (in dollars) was found to be $r = 0.22$

E. The correlation coefficient between *height of wheat* (in centimetres) and *grain yield* (in tonnes) was found to be $r = -0.40$ and the coefficient of determination was found to be $r^2 = -0.16$

Question 13

The association between *amount of protein consumed* (in grams/day) and *family income* (in dollars) is best displayed using

A. a scatterplot.

B. a time series plot.

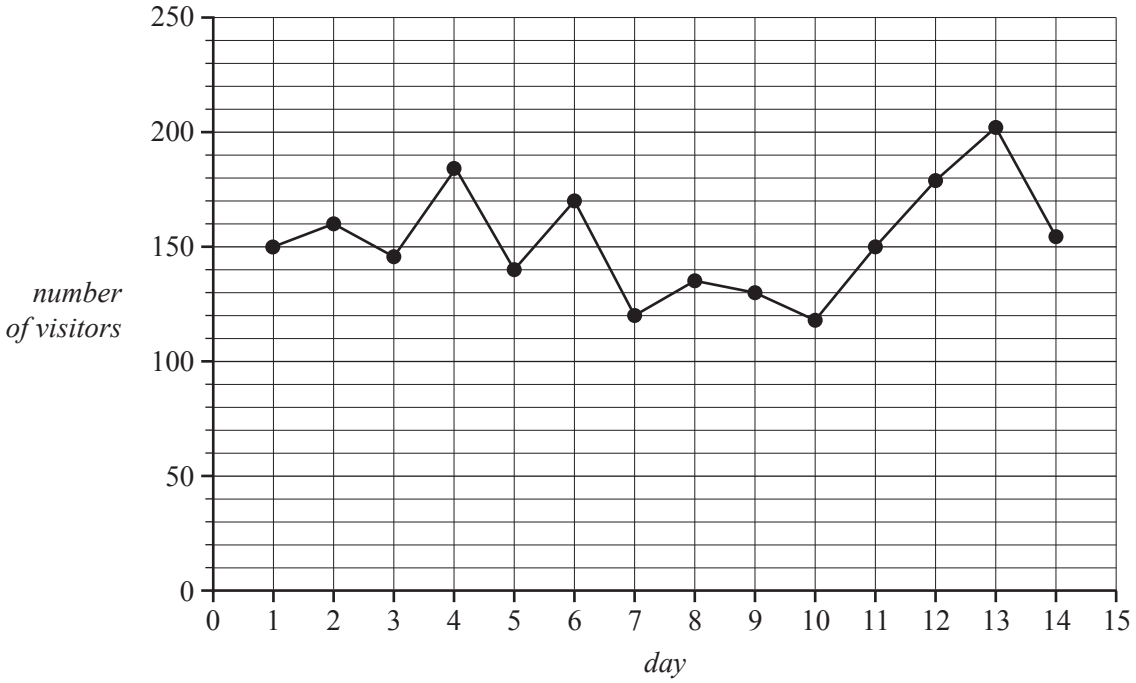
C. parallel boxplots.

D. back-to-back stem plots.

E. a two-way frequency table.

Question 14

The time series plot below shows the daily number of visitors to a historical site over a two-week period.



This time series plot is to be smoothed using seven-median smoothing.

The smoothed number of visitors on day 4 is closest to

A. 120

B. 140

C. 145

D. 150

E. 160

Question 15

The table below shows the long-term monthly average heating cost, in dollars, for a small office.

Month	Jan.	Feb.	Mar.	Apr.	May	June	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Long-term average heating cost	78	83	73	109	156	130	146	168	159	140	136	122

The seasonal index for April is closest to

A. 0.80

B. 0.87

C. 0.96

D. 1.25

E. 1.56

Question 16

A small cafe is open every day of the week except Tuesday.

The table below shows the daily seasonal indices for labour costs at this cafe.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Seasonal index	1.47	closed	1.24	1.01	0.77	0.76	0.75

Excluding the day that the cafe is closed, the long-term average weekly labour cost is $9786.

The expected daily labour cost on a Wednesday is closest to

A. $1127

B. $1315

C. $1631

D. $1733

E. $2022

Recursion and financial modelling

Question 17

A sequence of numbers is generated by the recurrence relation shown below.

$$P_0 = 2, \qquad P_{n+1} = 3P_n - 1$$

What is the value of P_3?

A. 2
B. 5
C. 11
D. 41
E. 122

Question 18

A truck was purchased for \$134 000.

Using the reducing balance method, the value of the truck is depreciated by 8.5% each year.

Which one of the following recurrence relations could be used to determine the value of the truck after n years, V_n?

A. $V_0 = 134\,000, \qquad V_{n+1} = 0.915 \times V_n$
B. $V_0 = 134\,000, \qquad V_{n+1} = 1.085 \times V_n$
C. $V_0 = 134\,000, \qquad V_{n+1} = V_n - 11\,390$
D. $V_0 = 134\,000, \qquad V_{n+1} = 0.915 \times V_n - 8576$
E. $V_0 = 134\,000, \qquad V_{n+1} = 1.085 \times V_n - 8576$

Question 19

Consider the recurrence relation shown below.

$$V_0 = 125\,000, \qquad V_{n+1} = 1.013\,V_n - 2000$$

This recurrence relation could be used to determine the value of

A. a perpetuity with a payment of \$2000 per quarter.
B. an annuity with withdrawals of \$2000 per quarter.
C. an annuity investment with additional payments of \$2000 per quarter.
D. an item depreciating at a flat rate of 1.3% of the purchase price per quarter.
E. a compound interest investment earning interest at the rate of 1.3% per annum.

Question 20

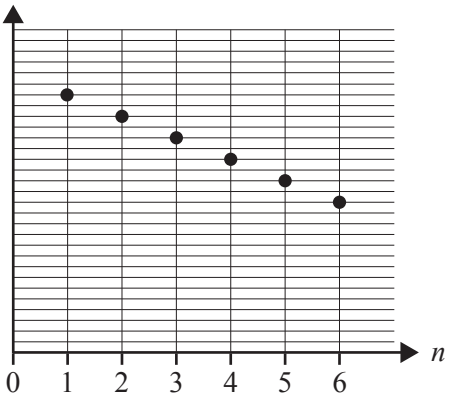
Marty has been depreciating the value of his car each year using flat rate depreciation.

After three years of ownership, the value of the car was halved due to an accident.

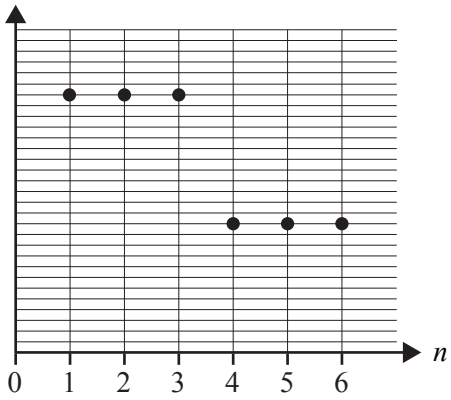
Marty continued to depreciate the value of his car by the same amount each year after the accident.

Which one of the following graphs could show the value of Marty's car after n years, C_n?

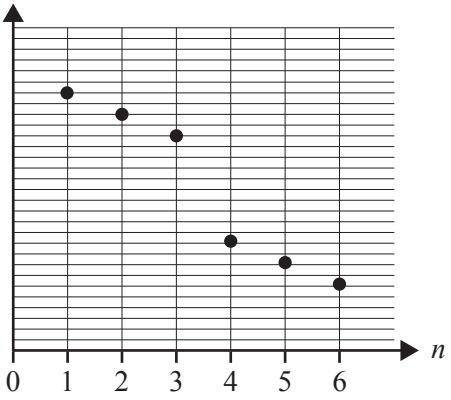
A.



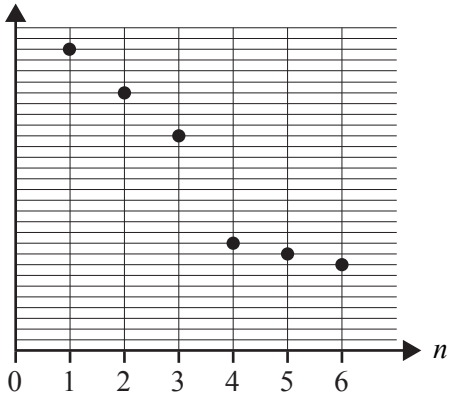
B.



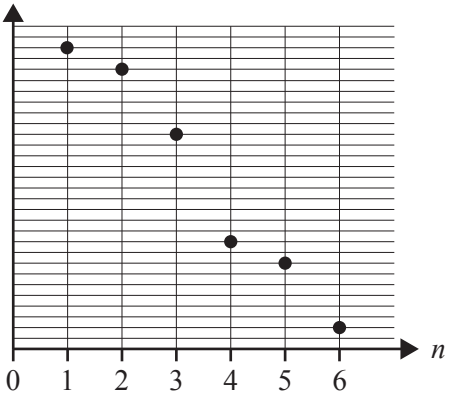
C.



D.



E.



Question 21

Yazhen has a reducing balance loan.

Six lines of the amortisation table for Yazhen's loan are shown below.

Repayment number	Repayment	Interest	Principal reduction	Balance of loan
15	1000.00	349.50	650.50	34 299.50
16	1000.00	343.00	657.00	33 642.50
17	1000.00	403.71	596.29	33 046.21
18	1000.00	396.55	603.45	32 442.76
19	1000.00	389.31	610.69	31 832.07
20	1000.00	381.98	618.02	31 214.05

The interest rate for Yazhen's loan increased after one of these six repayments had been made.

The first repayment made at the higher interest rate was repayment number

A. 15

B. 16

C. 17

D. 18

E. 19

Use the following information to answer Questions 22 and 23.

Armand borrowed $12 000 to pay for a holiday.

He will be charged interest at the rate of 6.12% per annum, compounding monthly.

This loan will be repaid with monthly repayments of $500.

Question 22

After four months, the total interest that Armand will have paid is closest to

A. $231

B. $245

C. $255

D. $734

E. $1796

Question 23

After eight repayments, Armand decided to increase the value of his monthly repayments.

He will make a number of monthly repayments of $850 and then one final repayment that will have a smaller value.

This final repayment has a value closest to

A. $168

B. $169

C. $180

D. $586

E. $681

Question 24

Robyn has a current balance of $347 283.45 in her superannuation account.

Robyn's employer deposits $350 into this account every fortnight.

This account earns interest at the rate of 2.5% per annum, compounding fortnightly.

Robyn will stop work after 15 years and will no longer receive deposits from her employer.

The balance of her superannuation account at this time will be invested in an annuity that will pay interest at the rate of 3.6% per annum, compounding monthly.

After 234 monthly payments there will be no money left in Robyn's annuity.

The value of Robyn's monthly payment will be closest to

A. $3993

B. $5088

C. $6664

D. $8051

E. $9045

END OF SECTION A
TURN OVER

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

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Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

Module 1 – Matrices

Before answering these questions, you must **shade** the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The number of individual points scored by Rhianna (R), Suzy (S), Tina (T), Ursula (U) and Vicki (V) in five basketball matches (F, G, H, I, J) is shown in matrix P below.

$$P = \begin{array}{c} \\ R \\ S \\ T \\ U \\ V \end{array} \begin{array}{c} \begin{array}{ccccc} \multicolumn{5}{c}{match} \\ F & G & H & I & J \end{array} \\ \begin{bmatrix} 2 & 0 & 3 & 1 & 8 \\ 4 & 7 & 2 & 5 & 3 \\ 6 & 4 & 0 & 0 & 5 \\ 1 & 6 & 1 & 4 & 5 \\ 0 & 5 & 3 & 2 & 0 \end{bmatrix} \end{array} \quad player$$

Who scored the highest number of points and in which match?

A. Suzy in match I

B. Tina in match H

C. Vicki in match F

D. Ursula in match G

E. Rhianna in match J

Question 2

Four teams, blue (B), green (G), orange (O) and pink (P), played each other once in a competition.

There were no draws in this competition.

The results of the competition are shown in the matrix below.

$$\begin{array}{c} winner \\ \\ B \\ G \\ O \\ P \end{array} \begin{array}{c} \begin{array}{cccc} & loser & & \\ B & G & O & P \end{array} \\ \begin{bmatrix} - & 1 & v & 1 \\ 0 & - & 1 & 1 \\ 0 & w & - & 0 \\ 0 & 0 & x & - \end{bmatrix} \end{array}$$

The letters v, w and x each have a value of '0' or '1'.

A '1' in the matrix shows that the team named in that row defeated the team named in that column.

A '0' in the matrix shows that the team named in that row was defeated by the team named in that column.

A dash (–) in the matrix shows that no game was played.

The values of v, w and x are

A. $v = 0, w = 1, x = 0$

B. $v = 0, w = 1, x = 1$

C. $v = 1, w = 0, x = 1$

D. $v = 1, w = 1, x = 0$

E. $v = 1, w = 1, x = 1$

Question 3

In matrix A, the element $a_{21} = 7$.

The order of matrix A is 3×2.

Matrix B is the transpose of matrix A.

Matrix B could be

A. $\begin{bmatrix} 3 & 9 & 4 \\ 2 & 7 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 7 & 2 \\ 1 & 0 & 5 \end{bmatrix}$

C. $\begin{bmatrix} 4 & 1 & 3 \\ 7 & 8 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 5 & 6 \\ 7 & 1 \\ 4 & 2 \end{bmatrix}$

E. $\begin{bmatrix} 9 & 7 \\ 3 & 4 \\ 0 & 2 \end{bmatrix}$

Question 4

In a game between two teams, Hillside and Rovers, each team can score points in two ways.

The team may hit a Full or the team may hit a Bit.

More points are scored for hitting a Full than for hitting a Bit.

A team's total point score is the sum of the points scored from hitting Fulls and Bits.

The table below shows the scores at the end of the game.

	Full	**Bit**	**Total**
Hillside	4	8	52
Rovers	5	2	49

Let f be the number of points scored by hitting one Full.

Let b be the number of points scored by hitting one Bit.

Which one of the following matrix products can be evaluated to find the matrix $\begin{bmatrix} f \\ b \end{bmatrix}$?

A. $\begin{bmatrix} 4 & 8 \\ 5 & 2 \end{bmatrix}^{-1} \times \begin{bmatrix} 52 \\ 49 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 8 \\ 5 & 2 \end{bmatrix} \times \begin{bmatrix} 52 \\ 49 \end{bmatrix}$

C. $\begin{bmatrix} 52 & 49 \end{bmatrix} \begin{bmatrix} 4 & 8 \\ 5 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 52 & 49 \end{bmatrix} \begin{bmatrix} 4 & 8 \\ 5 & 2 \end{bmatrix}^{-1}$

E. $\begin{bmatrix} 4 & 8 & 52 \\ 5 & 2 & 49 \end{bmatrix}^{-1}$

Question 5

A population of birds feeds at two different locations, A and B, on an island.

The change in the percentage of the birds at each location from year to year can be determined from the transition matrix T shown below.

$$T = \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} this\ year & \\ A & B \end{array} \\ \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} \end{array} \begin{array}{l} \\ next\ year \end{array}$$

In 2018, 55% of the birds fed at location B.

In 2019, the percentage of the birds that are expected to feed at location A is

A. 32%

B. 42%

C. 48%

D. 58%

E. 62%

Question 6

Matrix W is a 3×2 matrix.

Matrix Q is a matrix such that $Q \times W = W$.

Matrix Q could be

A. [1]

B. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

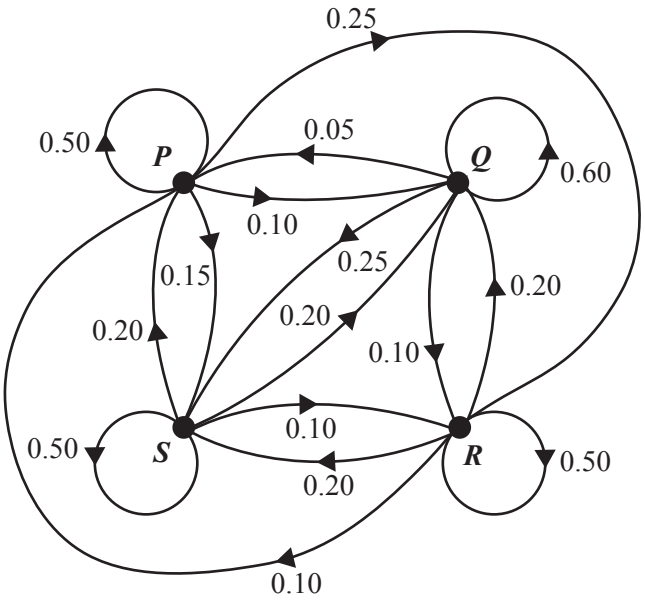
Use the following information to answer Questions 7 and 8.

A farm contains four water sources, *P*, *Q*, *R* and *S*.

Question 7

Cows on the farm are free to move between the four water sources.

The change in the number of cows at each of these water sources from week to week is shown in the transition diagram below.



Let C_n be the state matrix for the location of the cows in week n of 2019.

The state matrix for the location of the cows in week 23 of 2019 is $C_{23} = \begin{bmatrix} 180 \\ 200 \\ 240 \\ 180 \end{bmatrix} \begin{matrix} P \\ Q \\ R \\ S \end{matrix}$

The state matrix for the location of the cows in week 24 of 2019 is $C_{24} = \begin{bmatrix} 160 \\ 222 \\ 203 \\ 215 \end{bmatrix} \begin{matrix} P \\ Q \\ R \\ S \end{matrix}$

Of the cows expected to be at *Q* in week 24 of 2019, the percentage of these cows at *R* in week 23 of 2019 is closest to

A. 8%

B. 9%

C. 20%

D. 22%

E. 25%

Question 8

Sheep on the farm are also free to move between the four water sources.

The change in the number of sheep at each water source from week to week is shown in matrix T below.

$$T = \begin{array}{c} \\ P \\ Q \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cccc} \textit{this week} & & & \\ P & Q & R & S \end{array} \\ \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0.2 & 0.1 & 0.5 & 0.3 \\ 0.1 & 0.3 & 0.1 & 0.2 \\ 0.3 & 0.3 & 0.2 & 0.4 \end{bmatrix} \end{array} \begin{array}{l} P \\ Q \\ R \\ S \end{array} \quad \textit{next week}$$

In the long term, 635 sheep are expected to be at S each week.

In the long term, the number of sheep expected to be at Q each week is closest to

A. 371

B. 493

C. 527

D. 607

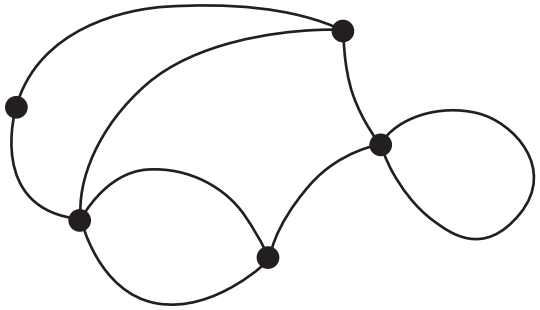
E. 635

Module 2 – Networks and decision mathematics

Before answering these questions, you must **shade** the 'Networks and decision mathematics' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The graph below has five vertices and eight edges.



How many of the vertices in this graph have an even degree?

A. 0

B. 1

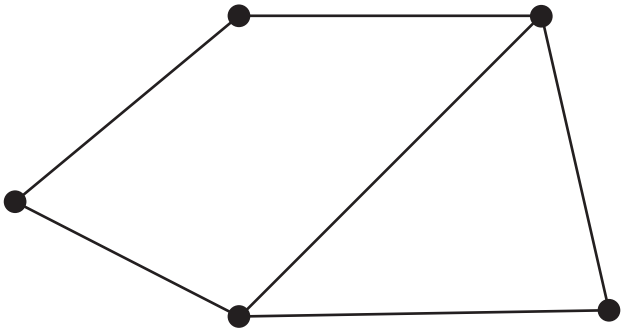
C. 2

D. 3

E. 4

Question 2

Consider the graph below.



Euler's formula will be verified for this graph.

What values of e, v and f will be used in this verification?

A. $e = 5, v = 5, f = 2$

B. $e = 5, v = 5, f = 3$

C. $e = 6, v = 5, f = 2$

D. $e = 6, v = 5, f = 3$

E. $e = 6, v = 6, f = 3$

Question 3

Four students, Alice, Brad, Charli and Dexter, are working together on a school project.

This project has four parts.

Each of the students will complete only one part of the project.

The table below shows the time it would take each student to complete each part of the project, in minutes.

	Part 1	**Part 2**	**Part 3**	**Part 4**
Alice	5	5	3	5
Brad	3	3	6	4
Charli	6	5	4	3
Dexter	4	5	6	5

The parts of this project must be completed one after the other.

Which allocation of student to part must occur for this project to be completed in the minimum time possible?

A.

Part 1	Part 2	Part 3	Part 4
Brad	Dexter	Alice	Charli

B.

Part 1	Part 2	Part 3	Part 4
Brad	Dexter	Charli	Alice

C.

Part 1	Part 2	Part 3	Part 4
Dexter	Alice	Charli	Brad

D.

Part 1	Part 2	Part 3	Part 4
Dexter	Brad	Alice	Charli

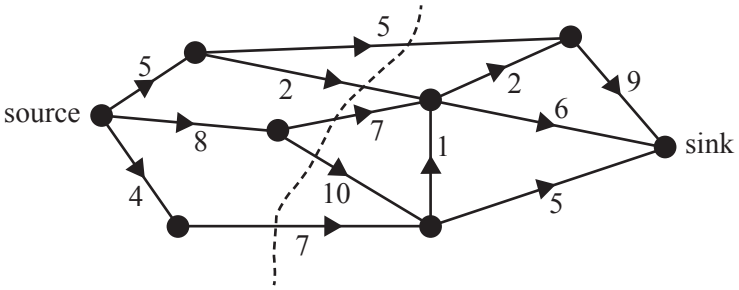
E.

Part 1	Part 2	Part 3	Part 4
Dexter	Brad	Charli	Alice

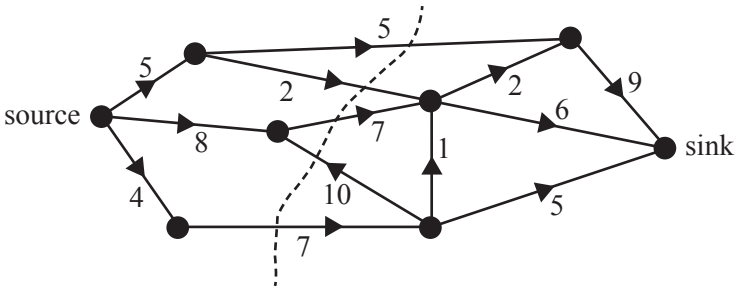
Question 4

Which one of the following flow diagrams shows a cut that has a capacity of 19?

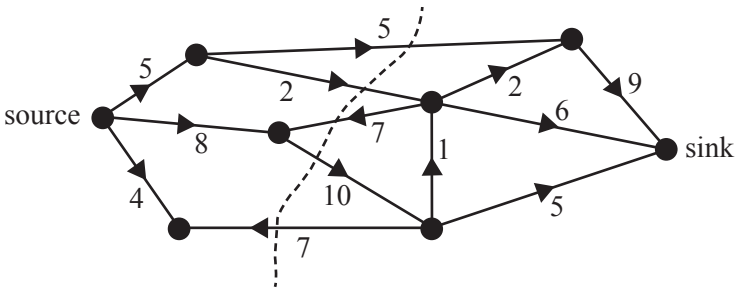
A.



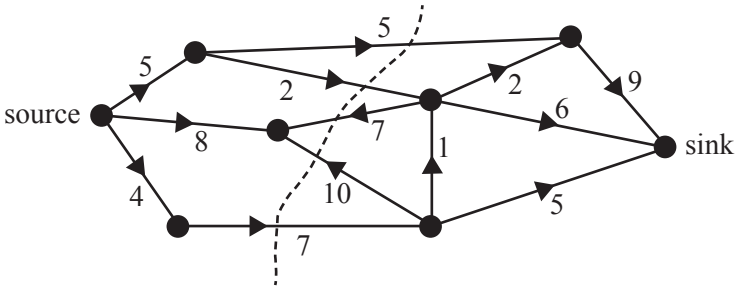
B.



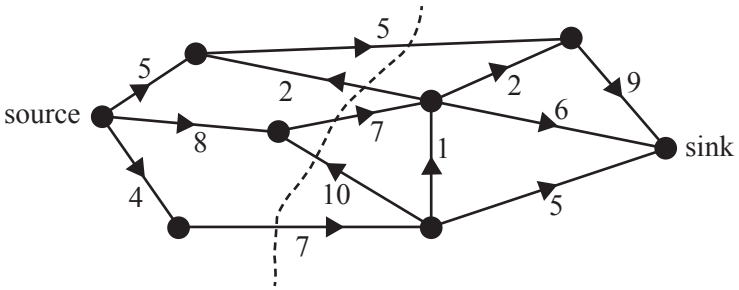
C.



D.



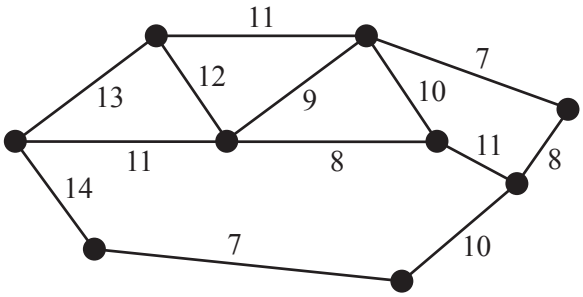
E.



Question 5

In the graph below, the vertices represent electricity transformer substations.

The numbers on the edges of the graph show the length, in kilometres, of cables that connect these substations.



What is the minimum length of cable, in kilometres, that is necessary to make sure that each substation remains connected to the network?

A. 65

B. 71

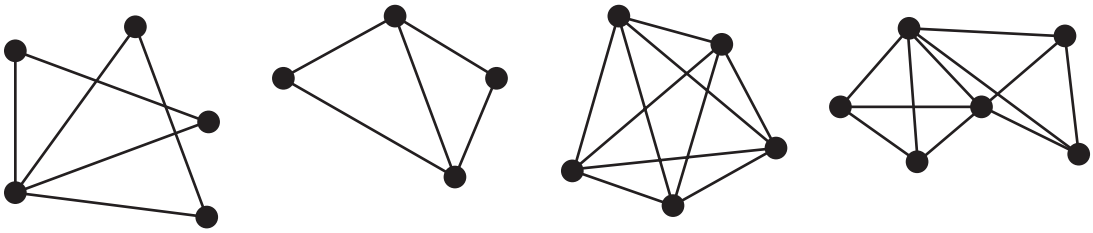
C. 73

D. 74

E. 77

Question 6

Four graphs are shown below.



How many of these graphs are planar?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 7

A graph has five vertices, A, B, C, D and E.

The adjacency matrix for this graph is shown below.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 2 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Which one of the following statements about this graph is **not** true?

A. The graph is connected.

B. The graph contains an Eulerian trail.

C. The graph contains an Eulerian circuit.

D. The graph contains a Hamiltonian cycle.

E. The graph contains a loop and multiple edges.

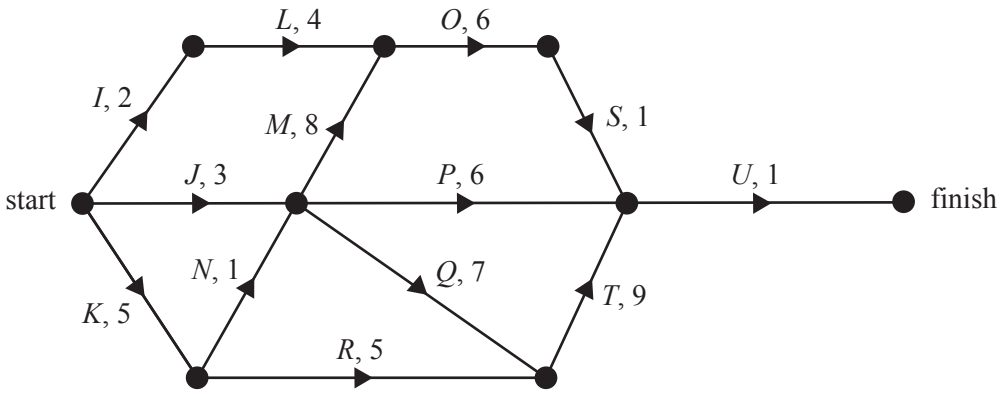
Question 8

A landscape gardener is planning the construction of a garden.

This project requires 13 activities, I to U.

The directed graph below shows each of these activities represented by edges.

The numbers on the edges are the durations of the activities, in days.



The cost of reducing the completion time of any activity in this project is \$1000 per day.

The landscape gardener has a maximum of \$3000 to spend on reducing the minimum completion time of this project.

The total completion time of the project can be reduced by three days by reducing

A. activity K by one day and activity Q by two days.

B. activity K by two days and activity Q by one day.

C. activity K by two days and activity M by one day.

D. activity K by one day and activity M by two days.

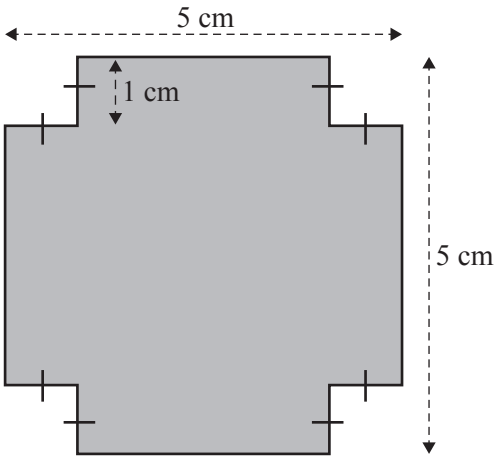
E. activity K by one day, activity M by one day and activity Q by one day.

Module 3 – Geometry and measurement

Before answering these questions, you must **shade** the 'Geometry and measurement' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

A piece of cardboard is shown in the diagram below.



The area of the cardboard, in square centimetres, is

A. 4
B. 5
C. 21
D. 25
E. 29

Question 2

Which one of the following locations is closest to the Greenwich meridian?

A. 32° S, 40° E
B. 32° S, 80° E
C. 32° S, 60° W
D. 32° S, 120° E
E. 32° S, 160° W

Question 3

A waterfall in a national park is 4 km east of a camp site.

A lookout tower is 4 km south of the waterfall.

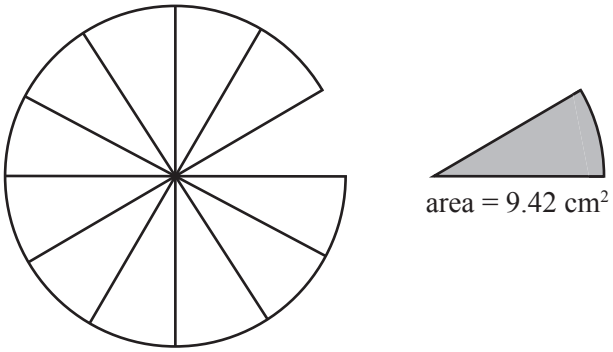
The bearing of the camp site from the lookout tower is

A. 045°
B. 090°
C. 135°
D. 300°
E. 315°

Question 4

A pizza in the shape of a circle has been cut into 12 equal slices.

The area of the top surface of one slice is 9.42 cm^2, as shown shaded in the diagram below.



The perimeter of the top surface of one slice of pizza, in centimetres, is closest to

A. 9.14

B. 15.14

C. 18.00

D. 21.99

E. 40.84

Question 5

The cities of Lima and Washington, DC have the same longitude of 77° W.

The shortest great circle distance between Lima and Washington, DC is 5697 km.

Assume that the radius of Earth is 6400 km.

Lima has a latitude of 12° S and is located due south of Washington, DC.

What is the latitude of Washington, DC?

A. 39° N

B. 51° S

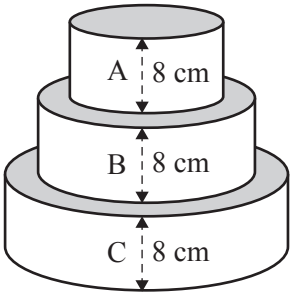
C. 51° N

D. 63° N

E. 65° S

Question 6

A cake in the shape of three cylindrical sections is shown in the diagram below.



Each section of the cake has a height of 8 cm, as shown in the diagram.

The middle section of the cake, B, has twice the volume of the top section of the cake, A.

The bottom section of the cake, C, has twice the volume of the middle section of the cake, B.

The volume of the top section of the cake, A, is 900 cm^3.

The diameter of the bottom section of the cake, C, in centimetres, is closest to

A. 12

B. 18

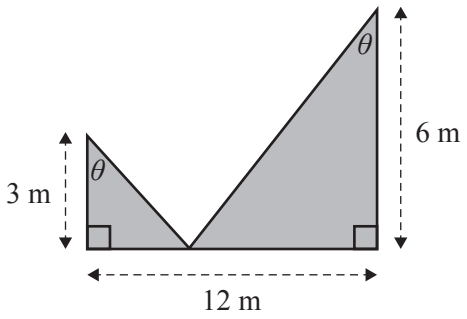
C. 24

D. 36

E. 48

Question 7

Two similar right-angled triangles are shown in the diagram below.



The total perimeter of the two triangles, in metres, is

A. 21

B. 30

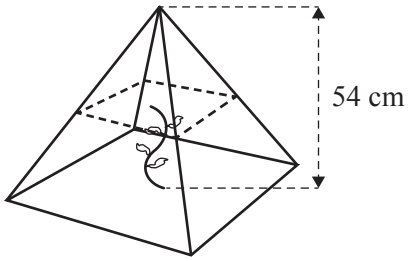
C. 36

D. 42

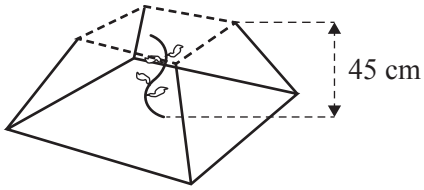
E. 45

Question 8

A young tree is protected by a tree guard in the shape of a square-based pyramid.
The height of the tree guard is 54 cm, as shown in the diagram below.



The top section of the tree guard is removed along the dotted line to allow the tree to grow.
Removing this top section decreases the height of the tree guard to 45 cm, as shown in the diagram below.



The ratio of the volume of the tree guard that is removed to the volume of the tree guard that remains is

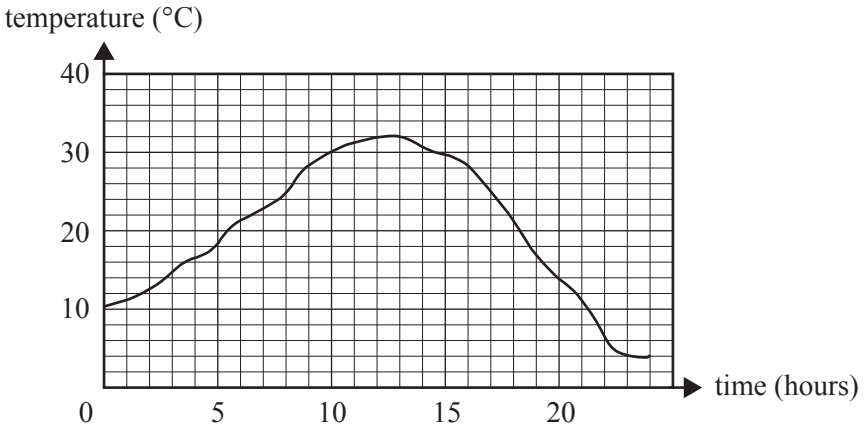
A. 1:6
B. 1:35
C. 1:36
D. 1:215
E. 1:216

Module 4 – Graphs and relations

Before answering these questions, you must **shade** the 'Graphs and relations' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The graph below shows the temperature, in degrees Celsius, over 24 hours.



The difference between the highest and lowest temperatures on this day, in degrees Celsius, is closest to

A. 16
B. 20
C. 24
D. 28
E. 32

Question 2

Two straight lines have the equations $3x - 2y = 3$ and $-2x + 5y = 9$.

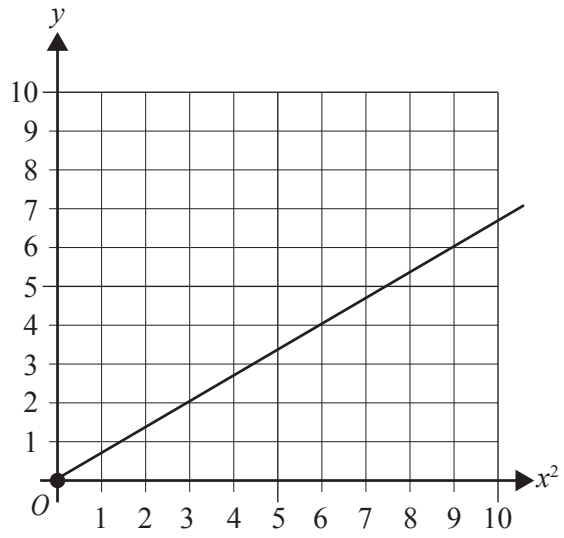
These lines have one point of intersection.

Another line that also passes through this point of intersection has the equation

A. $y = -x$
B. $y = x$
C. $y = -2x$
D. $y = 2x$
E. $y = 3x$

Question 3

The graph below shows a relationship between x and y.



The rule that represents this relationship between x and y is

A. $y = \frac{2}{3}x$

B. $y = \frac{3}{2}x$

C. $y = \frac{2}{3}x^2$

D. $y = \frac{3}{2}x^2$

E. $y = \sqrt{\frac{3}{2}}x$

Question 4

A farm has x cows and y sheep.

On this farm there are always at least twice as many sheep as cows.

The relationship between the number of cows and the number of sheep on this farm can be represented by the inequality

A. $x \leq \frac{y}{2}$

B. $y \leq \frac{x}{2}$

C. $2x \geq y$

D. $2y \geq x$

E. $xy \geq 2$

Question 5

The revenue, R, in dollars, that a company receives from selling n caps is given by the equation

$$R = \begin{cases} 25n & n \leq 1000 \\ 20n + c & n > 1000 \end{cases}$$

The graph of this revenue equation consists of two straight lines that intersect at the point where $n = 1000$.

What is the value of c in this revenue equation?

A. 0

B. 1000

C. 2000

D. 4000

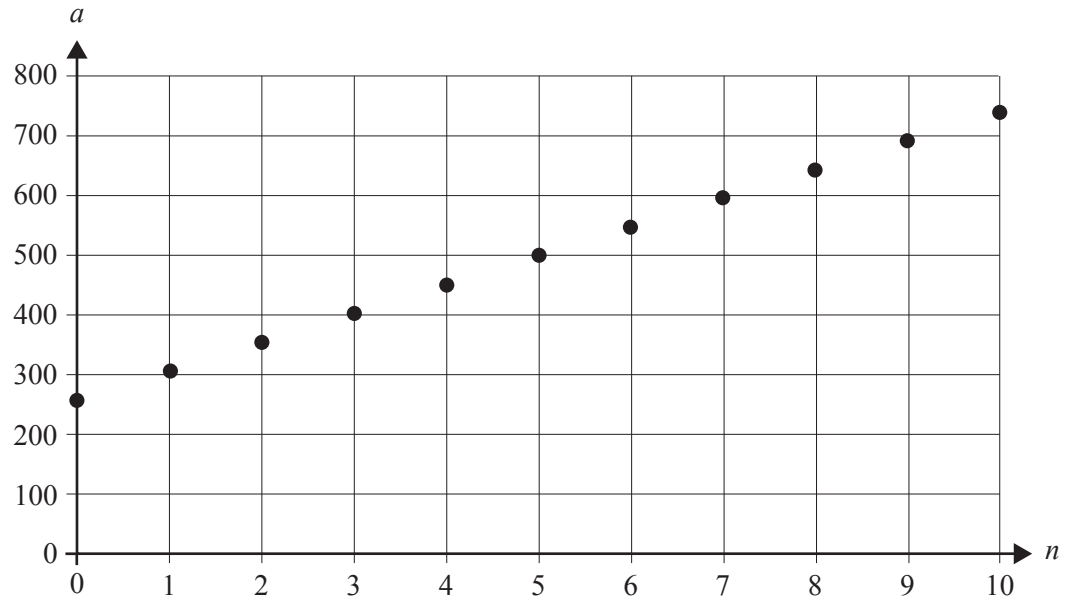
E. 5000

Question 6

In January 2018, an online shop had 260 customer accounts.

In June 2018, the shop had 500 customer accounts.

The graph below shows the number of customer accounts, a, with the online shop n months after January 2018, for a period of 10 months.



The growth in the number of customer accounts that this graph shows is expected to continue beyond these 10 months, following the same trend.

How many customer accounts can the online shop expect to have at the end of December 2019 ($n = 23$)?

A. 1364

B. 1376

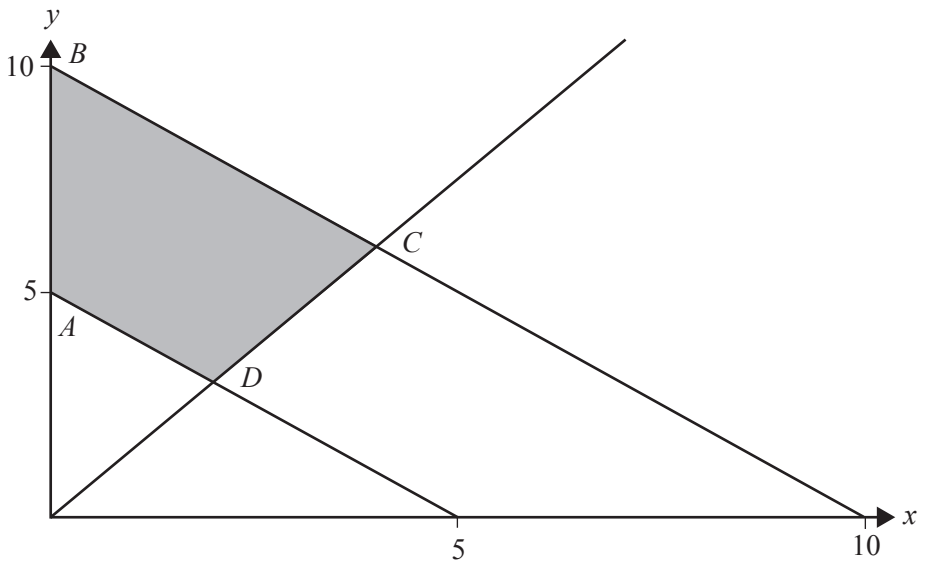
C. 1388

D. 1400

E. 1412

Question 7

The shaded area in the graph below represents the feasible region for a linear programming problem.



The maximum value of the objective function $Z = -2x - 2y$ occurs at

A. point C only.

B. any point along line segment BC.

C. any point along line segment AD.

D. any point along line segment AB.

E. any point along line segment DC.

Question 8

Jamie sold bottles of homemade lemonade to his neighbours on Saturday.

The *revenue*, in dollars, he made from selling n bottles of lemonade is given by

$$revenue = 3.5n$$

The *cost*, in dollars, of making n bottles of lemonade is given by

$$cost = 60 + n$$

The profit made by Jamie on Saturday could have been

A. \$162

B. \$165

C. \$168

D. \$173

E. \$177

END OF MULTIPLE-CHOICE QUESTION BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2019

FURTHER MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A multiple-choice question book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	$\text{seasonal index} = \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a$, $u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 – Matrices

determinant of a 2 × 2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	S_0 = initial state, $S_{n+1} = T S_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base × height
volume of a pyramid	$\frac{1}{3}$ × area of base × height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET