Neap:

Trial Examination 2019

VCE Further Mathematics Units 3&4

Written Examination 1

Suggested Solutions

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SECTION A – CORE

Data analysis

Question 1 C

With 23 pieces of data, the median is the twelfth value which falls within the 20–<30 group. Since it can be equal to 20 but not 30, it must be 20.

Question 2 D

The median is 35, so 50% of the data is greater than that.

Question 3 C

A pattern that repeats every twelve months indicates seasonality. Since the horizontal axis is time, the graph is a time-series.

Question 4 B

Since the data repeats every twelve months, the number of points in the moving mean should be a factor of twelve.

Question 5 A

The data must be first placed in ascending order. Then find the median, Q_1 and Q_3. The maximum and minimum vales are 5 and 34.

Question 6 A

The y-intercept is 70 and the gradient $-\frac{70}{630} = -\frac{1}{9}$. The equation is therefore:

fuel remaining $= -\frac{1}{9} \times$ distance travelled + 70

Question 7 B

$r = -0.9$, so $r^2 = 0.81$. The fuel remaining is the response variable, responding to the distance travelled.

Question 8 E

The vertical scale is a log scale. So the value that is read from the graph, 6, means $10^6 = 1\,000\,000$.

Question 9 B

The range of the under 25s and the longest phone call are both 45.

Question 10 A

The median and mode are 10 and 0 respectively so the answer must be either **A** or **D**. Calculating the mean using $\frac{6 \times 0 + 2 + 5 + 8 + 10 + 2 \times 12 + 14 + 15 + 17 + 20 + 23 + 25 + 35}{19} = 10.4$.

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Question 11 E

The total of the seasonal indices for the other eleven months is $12 - 3.2 = 8.8$. The average therefore is $\frac{8.8}{11} = 0.8$.

Question 12 A

95% means within two standard deviations either side of the mean. Therefore, 1.02 – 0.98 = 0.04, which is 4 × the standard deviation.

Question 13 D

A, **B** and **E** are distractions; the question is asking which of the given transformations will produce a straight line. The $\frac{1}{x}$ transformation has the effect of decreasing the larger values of x and increasing the smaller ones. This is what is needed to straighten the line.

Question 14 B

Substituting 80 as the temperature gives $0.4 \times 80 + 15 = 47$

actual value – estimated value = residual

$$50 - 47 = 3$$

Question 15 D

Using the values in $y = ax + b$ gives $y = -4.2\log(x) + 32$. Remember the variable is now log(x).

Question 16 E

Using $z = \frac{x - \mu}{\sigma}$, $1.7 = \frac{x - 66}{18}$

$$x = 1.7 \times 18 + 66$$
$$= 96.6 = 97\%$$

Recursion and financial modelling

Question 17 C

To get the next term the previous term is divided by –2 (or multiplied by $-\frac{1}{2}$). We need the first term, t_1, to know where to begin.

Question 18 D

$t_1 = 3(4) - 4 = 8$

$t_2 = 3(8) - 4 = 20$

$t_3 = 3(20) - 4 = 56$

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Question 19 D

An increase of 42% means multiplying by a factor of 1.42.

Hence, $t_{n+1} = 1.42t_n$; $t_0 = 1000$.

Question 20 A

$I = 1 + \frac{r}{100}$

When this is changed to an interest rate per month, it gives $I = 1 + \frac{\frac{2.3}{12}}{100}$ over 36 months, which gives:

$$T_3 = 20000 \times \left(1 + \frac{\frac{2.3}{12}}{100}\right)^{3 \times 12}$$

Question 21 C

$38\,500 \times 0.8$ is the principal after one year. After the first year 10% is lost (0.9 of the beginning principal each year). This is applied for $n - 1$ years. The first year there was a 20% reduction.

Question 22 C

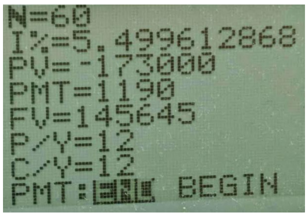
At the end of each year interest of 3% is added and a sum of $5000 is withdrawn.

Question 23 B

The interest to be paid is $\frac{35\,000 \times 0.75 \times 4 \times 8}{100}$ = $8400, which when added to the cost of the boat is $43 400.

Question 24 D

Enter the information into your calculator and find the unknown, $I\%$. The screenshot from a TI-84 is shown.



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SECTION B – MODULES

Module 1 – Matrices

Question 1 **A**

None of the other alternate answers correctly multiply the two matrices together.

Question 2 **C**

No solution when determinant of coefficient matrix = zero.

$$ad - bc = 0$$
$$6 \times -3 - 9 \times -2 = 0$$

Question 3 **D**

The transpose of $A \times B$ is only equal to **D**.

Question 4 **D**

Row elements increase by 1, so i; column elements increase by 2, so $2j$.

$i + 2j - 1$ is consistent with all elements.

Question 5 **C**

Find the sum of one-step and two-step matrices.

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Leading diagonals represent redundant communication links.

Therefore the total is 6.

Question 6 **B**

The columns should total to 1.

Question 7 **B**

Find the long-term steady state by multiplying the transition matrix to a large power by a column matrix that sums to 60 000.

$$\begin{bmatrix} 0.6 & 0.3 & 0.2 \\ 0.3 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix}^{100} \begin{bmatrix} 20\,000 \\ 20\,000 \\ 20\,000 \end{bmatrix} = \begin{bmatrix} 23\,414/6 \\ 20\,487.8 \\ 16\,097.6 \end{bmatrix}$$

Question 8 **E**

The number of players in each squad at the end of the season is equal to $T \times S$.

$$\begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0.3 & 0.5 & 0.3 & 0 \\ 0.1 & 0.3 & 0.5 & 0 \\ 0.1 & 0.2 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 25 \\ 25 \\ 0 \end{bmatrix} = \begin{bmatrix} 15 \\ 29 \\ 23 \\ 13 \end{bmatrix}$$

The club needs to recruit 17 players and release 4.

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Module 2 – Networks and decision mathematics

Question 1 D

A simple graph has no loops or multiple edges. **D** has multiple edges.

Question 2 C

Planar graphs can be redrawn with no overlapping edges, as in **C**.

Question 3 C

element 0 – no edge between vertices

element 1 – one edge between vertices

element 2 – two edges between vertices

Question 4 C

In a complete graph, every vertex is connected to every other vertex.

8 vertices are all connected to the other 7.

$8 \times 7 = 56$

To avoid counting edges twice, we must divide by 2.

$\frac{56}{2} = 28$

Question 5 A

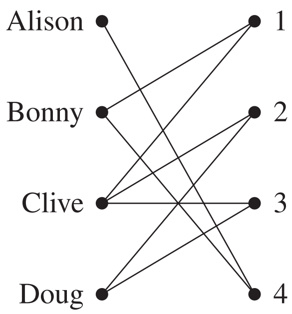
$$\begin{aligned}\text{minimum cut} &= 9 + 8 + 10 \\ &= 27\end{aligned}$$

Question 6 D

For a Eulerian trail, two vertices of an odd degree are needed.

Removing *EG* and adding *AD* creates a Eulerian circuit.

Question 7 D



Task 2 could be allocated to either Clive or Doug.

Question 8 C

Two critical paths are present: *C-D-H-I-J-K* and *C-E-I-J-K*. Therefore **C** is not true.

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Module 3 – Geometry and measurement

Question 1 **C**

Using Pythagoras' theorem:

$\sqrt{140^2 - 60^2} = 126.491106407$

OR

$140^2 - 60^2 = 16\ 000$

$\sqrt{16\ 000} = 126.491106407$

OR

solve $(x^2 + 60^2 = 140^2, x)$

$x = -126.491106407$ or $x = 126.491106407$

Question 2 **B**

Find the angle between 140 m and 60 m, then subtract this from 90.

$$\cos^{-1}\left(\frac{60}{140}\right) = 64.6230664748$$

$90 - 64.6230664748 = 25.3769335252$

Question 3 **E**

The time difference is 6 hours behind Madrid.

$6 \times 15 = 90°$

As time is behind, the location from Madrid is 90°W further.

$3 + 90 = 93°$W

Question 4 **A**

As we have three sides and a missing angle we can use the cosine rule.

The correct use of the cosine rule to find BAC is $\cos^{-1}\left(\frac{9^2 + 10^2 - 6^2}{2 \times 9 \times 10}\right)$.

Question 5 **C**

$$\begin{aligned}\text{shaded area} &= \text{area of sector} - \text{area of triangle}\\ &= \frac{57}{360} \times \pi \times 8^2 - \frac{1}{2} \times 8 \times 8 \times \sin(57)\\ &= 4.99734738212\\ &\approx 5\end{aligned}$$

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Question 6 B

Equate both volumes using appropriate formula and solve for r.

$$\text{solve}\left(\frac{4}{3} \times \pi \times r^2 = \frac{1}{3} \times 8 \times 8 \times 20, r\right)$$

$r = 4.67017729976$

The answer is 4.7 cm.

Question 7 C

Going from big container to small container:

$$\text{volume scale factor} = \frac{1.5}{2}$$
$$= \frac{3}{4} \text{ or } 0.75$$

length scale factor:

$\sqrt[3]{0.75} = 0.908560296416$

area scale factor:

$(0.908560296416)^2 = 0.825481812224$

surface area:

$1000 \times 0.825481812224 = 825.481812224$

OR

Going from small container to big container:

$$\text{volume scale factor} = \frac{2}{1.5}$$
$$= \frac{4}{3} \text{ or } 1.33333333333333$$

length scale factor:

$\sqrt[3]{1.33333333333} = 1.1006424163$

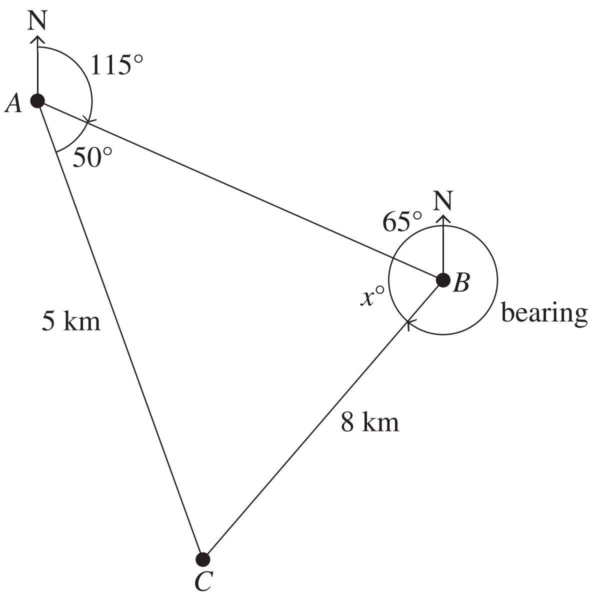
area scale factor:

$(1.1006424163)^2 = 1.21141372855$

surface area:

$$\frac{1000}{1.21141372855} = 825.481812224$$

Question 8 B



Find $\angle ABC$:

$$\sin^{-1}\left(\frac{5\sin(50)}{8}\right) = 28.6056070881$$

To find bearing, calculate the clockwise angle from the north:

$$360 - (65 + 28.6056070881) = 266.394392912$$

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Module 4 – Graphs and relations

Question 1 **D**

The question tells us that the equation is linear, so it is in the form $y = ax + b$. The y-intercept is 4 and the gradient is $-\frac{1}{2}$, so the equation is $y = 4 - \frac{1}{2}x$. Rearranging gives $y + \frac{1}{2}x - 4 = 0$. Multiply by 2 to remove the fraction, giving $2y + x - 8 = 0$.

Question 2 **D**

Since skis are physical objects, neither type of ski can have a negative value and so both $x \geq 0$ and $y \geq 0$. The number of snow skis, x, is at least three times the number of water skis, so $x \geq 3y$.

Question 3 **E**

The section of graph from 5 pm to 6 pm has the steepest gradient and therefore the fastest speed. The negative slope refers to the direction Rumesh is travelling.

Question 4 **D**

The equations of the three lines are $y = x + 2$, $y = 5$ and $y = -\frac{5}{3}x + \frac{50}{3}$. The intervals are $[-2, 1]$, $[1, 4]$ and $(4, 10]$.

Question 5 **A**

The gradient is 2 and the y-intercept is 3. The line is solid and the area shaded is above the line, so we are after greater-than-or-equal-to (≥).

Question 6 **D**

Substitute the coordinates into the objective function $Z = 4x + 5y$, looking for the maximum value.

	A	*B*	*C*	*D*	*E*
Coordinates	(0, 0)	(0, 4)	(4, 5)	(6, 3)	(5, 0)
Substitution	$Z = 4(0) + 3(0)$ $= 0$	$Z = 4(0) + 3(4)$ $= 12$	$Z = 4(4) + 3(5)$ $= 31$	$Z = 4(6) + 3(3)$ $= 33$	$Z = 4(5) + 3(0)$ $= 20$

The maximum value of 33 occurs at point *D*.

Question 7 **E**

Substitute one of the points into the equation to find the value of *k*.

$$-2 = -\frac{(2)^2}{k}$$

$$k = \frac{-4}{-2}$$

$$= 2$$

Question 8 C

Students may solve this by inspection.

Let x be the cost of a meat pie and y be the cost of a sausage roll.

$20x + 15y = 138$ (Equation 1)

$6x + 10y = 59$ (Equation 2)

(Equation 1) $\times 2 = 40x + 30y = 276$ (Equation 3)

(Equation 2) $\times 3 = 18x + 30y = 177$ (Equation 4)

(Equation 3) – (Equation 4) $\rightarrow 22x = 99$

$$x = \frac{99}{22}$$

$$= \$4.50$$

Substitute in (Equation 2).

$6(4.50) + 10y = 59$

$y = \$3.20$

The cost is $8 \times 4.50 + 12 \times 3.20 = \$74.40$.

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