The Mathematical Association of Victoria

Trial Examination 2019

FURTHER MATHEMATICS

Written Examination 1

STUDENT NAME: _____

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

MULTIPLE-CHOICE QUESTION BOOK

Structure of Book

Section	Number of questions	Number of questions to be answered	Number of Modules	Number of modules to be answered	Number of marks
A – Core	24	24			24
B - Modules	32	16	4	2	16 Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question book of 34 pages
- Formula sheet
- Answer sheet for multiple-choice questions
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Write your **name** on the multiple-choice answer sheet.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Core

Data Analysis

Use the following information to answer Questions 1 and 2.

The age, in months, of 48 students starting school at one primary school is displayed in the dot plot below

Question 1

The percentage of students who are less than 60 months of age when they start school is closest to

A. 3%
B. 4 %
C. 6%
D. 8%
E. 15%

Question 2

The mean ($\bar{x}$) and sample standard deviation (S_x) for this data are closest to

A.   mean = 64.0   standard deviation = 3.84
B.   mean = 64.0   standard deviation = 3.88
C.   mean = 64.5   standard deviation = 3.84
D.   mean = 64.5   standard deviation = 3.88
E.   mean = 65.0   standard deviation = 4.00

Question 3

A school wants to study the amount of time their 240 Year 12 students spend doing homework each day. A random number generator is used to select a sample of 30 students from the Year 12 student enrolment database to survey.

Which one of the following statements is **not** true?

A.   This is an example of random sampling.
B.   The mean number of hours the 30 selected students spend doing homework is called a statistic.
C.   The mean number of hours the population of 240 students spend doing homework is called a parameter.
D.   The mean number of hours the 30 selected students spend doing homework will be the same as the mean number of hours the 240 students spend doing homework.
E.   Every member of the population has an equal chance of being selected as one of the 30 students in the sample.

Question 4

The boxplots below show the immunisation rates for measles for 20 countries in 1990 and 2010:
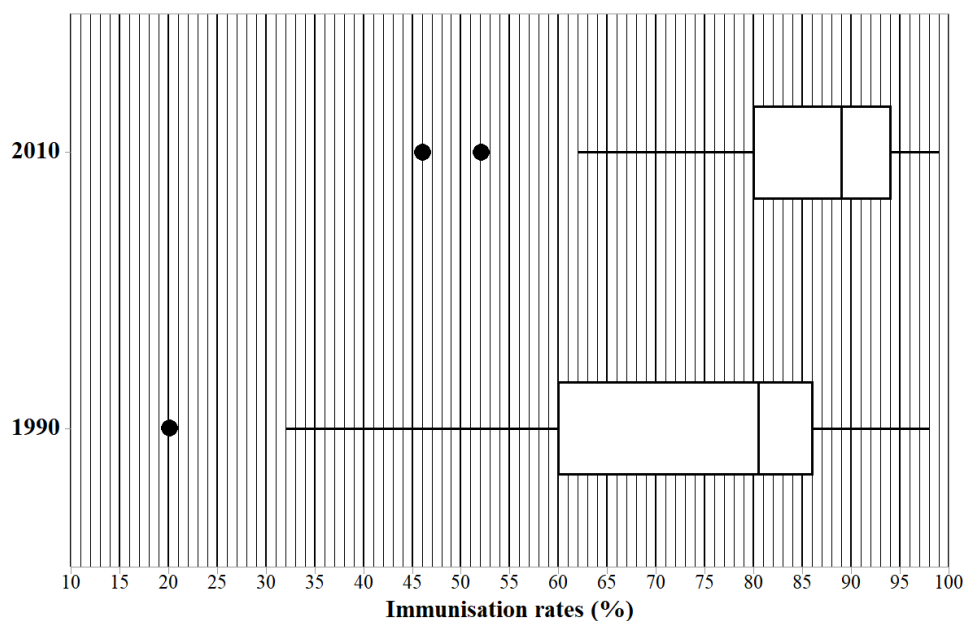
Based on the information contained in the boxplots, which of the following statements is true?

A. Both the 2010 and 1990 distributions are positively skewed with outliers.

B. More than 25% of the countries in 1990 have lower immunisation rates than the lowest of the 2010 immunisation rates.

C. The immunisation rates in 2010 are higher with more variation than the 1990 immunisation rates.

D. The interquartile range for 1990 immunisation rates is larger than the interquartile range for 2010 immunisation rates.

E. Immunisation rates are not associated with the year.

Question 5

The Gross Domestic Profit (GDP), in $US millions, of 236 countries is displayed in the histogram below.
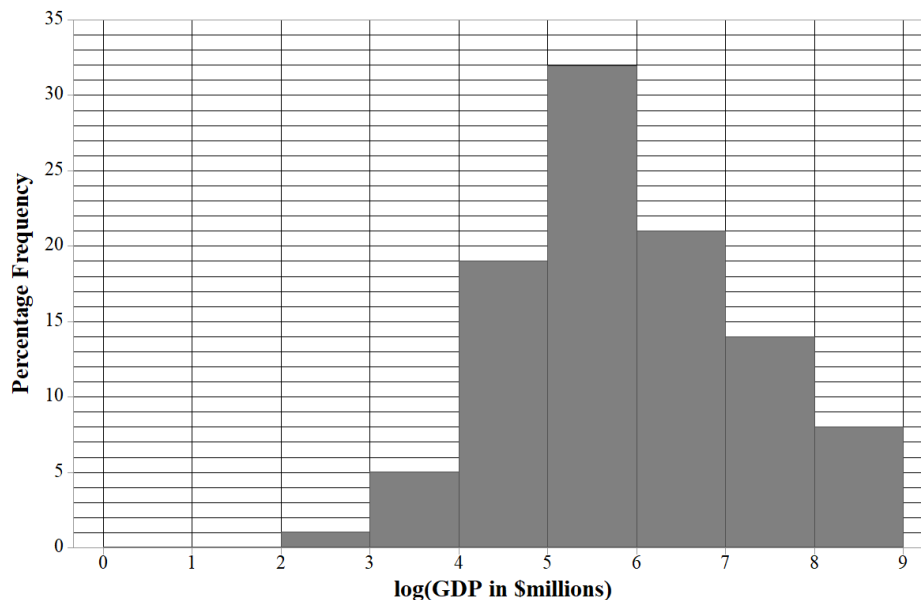
The percentage of countries with a GDP greater than $US 10 000 million is

    A. 6%
    B. 19%
    C. 40%
    D. 75%
    E. 94%

Question 6

The cholesterol level for adult males is approximately normally distributed with a mean of 4.8 mmol/L and a standard deviation of 0.8 mmol/L.

A sample of 50 males was selected at random from this population.

The number of these males with a cholesterol level of more than 4.0 mmol/L would be expected to be

- **A.** 32
- **B.** 42
- **C.** 50
- **D.** 64
- **E.** 84

Question 7

The back to back stem and leaf plot shows the maximum daily temperatures (°C) for Ballarat for each of the 31 days in December 2008 and December 2018.

```
          December 2008                              December 2018
                                    4 | 1
            9 9 9 9 8 8 8 7 6 6 | 1 | 6  6  9
    4 3 3 2 1 0 0 0 0 0 0 0 | 2 | 0  0  0  1  1  1  1  2  4  4  4  4
            9 8 7 6 6 6 5 | 2 | 5  6  6  7  8  9
                        2 | 3 | 0  1  1  1  2  2  4
                            | 3 | 6  7  7          Key: 24ºC is 2|4
```

Which one of the following statements is **not** true?

A. The median maximum daily temperature and the modal maximum temperatures for 2008 are the same.

B. There is more variation in the 2008 maximum daily temperatures than the 2018 maximum daily temperatures.

C. There are no outliers for the 2008 maximum daily temperatures.

D. The median maximum daily temperature for 2018 is 5°C higher than median maximum daily temperature for 2008.

E. The interquartile range for the maximum daily temperatures for 2018 is 10°C.
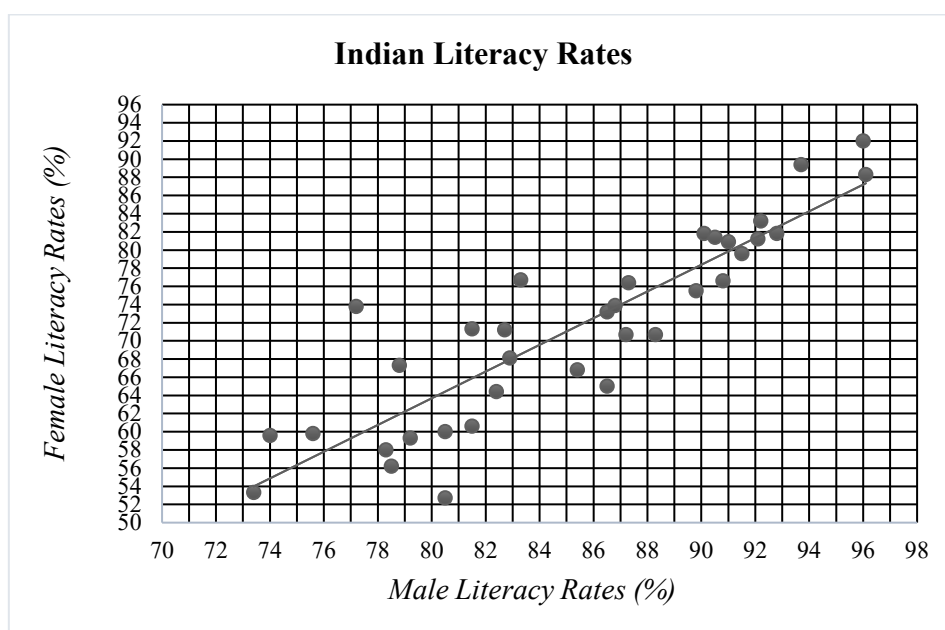
Question 8

Segmented bar charts would be an appropriate graphical tool to investigate the association between the variable "*support for tougher gun control*" (1= yes, 2 = not sure, 3 = no) and the variable

    A. *"number of guns registered"*

    B. *"gender"* (male = 1, female = 2, not stated = 3)

    C. *"age"* in years

    D. *"price of a gun"* in dollars

    E. *"cost of a gun licence"* in dollars

Use the following information to answer Questions 9 and 10

The scatterplot shows the *female literacy rates (%)* plotted against *male literacy rates (%)* for the Indian states and territories from the 2011 census. A least squares line has been fitted to the data

Source: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4649870/

Question 9

There is a positive correlation ($r = 0.881$) between *female literacy rates (%)* and *male literacy rates (%)* for this data.

Which of the following is the most appropriate interpretation of this correlation coefficient?

A. As *male literacy rates* increase the *female literacy rates* tend to increase.
B. On average, the *female literacy rate* for a country is 88% of the *male literacy rates* for the country.
C. There is a moderate, positive correlation between *female literacy rates* and *male literacy rates.*
D. Increasing *male literacy rates* will cause higher *female literacy rates.*
E. For every increase of 1% in the *male literacy rates*, there will be an increase of 0.88% in the *female literacy rates*.

Question 10

The equation of this least squares line is closest to

A. $female\ literacy\ rates = 1.5 \times male\ literacy\ rates - 54$
B. $female\ literacy\ rates = 1.5 \times male\ literacy\ rates + 25$
C. $female\ literacy\ rates = 1.5 \times male\ literacy\ rates + 48$
D. $female\ literacy\ rates = 0.7 \times male\ literacy\ rate - 36$
**E.** $female\ literacy\ rates = 0.7 \times male\ literacy\ rates + 48$

Use the following information to answer Questions 11 and 12

The table and graph below show the mean *age at first marriage* and the *Human Development Index (HDI)* for twenty countries. The HDI is used to rank countries by level of "human development". It contains three dimensions: health level, educational level and living standard.

Age at First Marriage	Human Development Index (HDI)
17.6	0.27
17.8	0.34
20.0	0.42
20.4	0.57
21.4	0.47
22.1	0.57
22.7	0.74
23.0	0.67
23.1	0.69
23.3	0.77
23.9	0.79
24.2	0.75
25.2	0.81
26.5	0.84
26.9	0.86
29.2	0.86
29.8	0.85
30.5	0.89
31.0	0.87
32.4	0.90

The Pearson's product moment correlation coefficient for this data is 0.892.
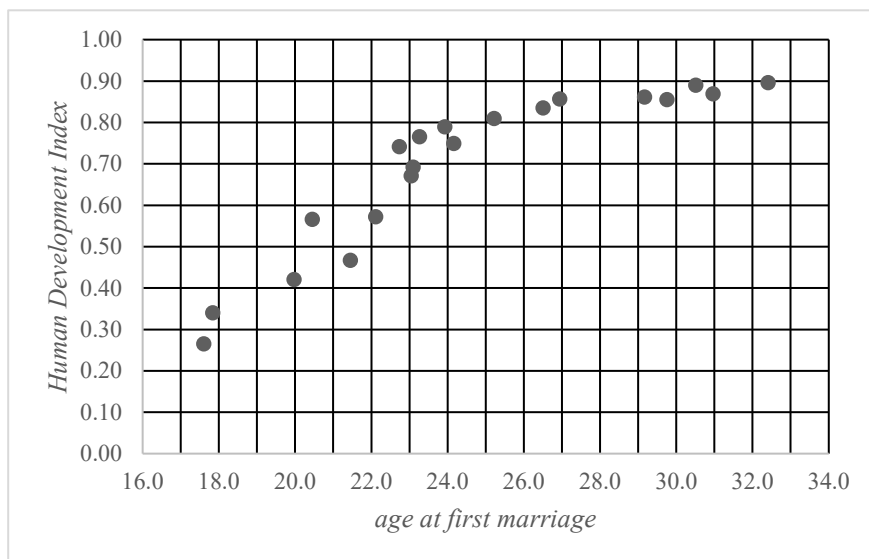
Question 11

Which one of the following statements is true?

A. 89.2% of the variation in *HDI* can be explained by the variation in the *age at first marriage*

B. 89.2% of the variation in *age at first marriage* can be explained by the variation in the *HDI*

C. 79.6% of the variation in *HDI* can be explained by the variation in the *age at first marriage*

D. 79.6% of the variation in *age at first marriage* can be explained by the variation in the *HDI*

E. 94.4% of the variation in *age at first marriage* can be explained by the variation in the *HDI*

Question 12

The relationship between *HDI* and *age at first marriage* is not linear. A reciprocal transformation is applied to the *age at first marriage* variable and the resulting least squares regression line is found.
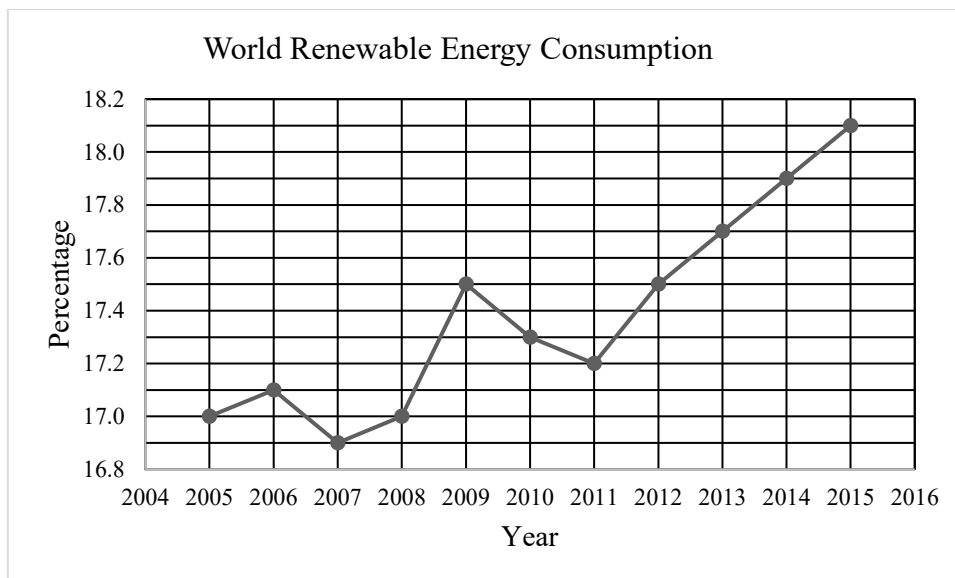
The gradient of the resulting least squares regression line, correct to two significant figures, is

A. -24

B. -24.44

C. -0.036

D. 0.040

E. 0.067

SECTION A – continued
TURN OVER

Question 13

The time series graph shows the world's renewable energy consumption as a percentage of total final energy consumption for a period of 10 years from 2005 to 2015.

Three-median smoothing could be used to smooth the time series plot above.

The three-median smoothed *percentage* value for 2009 is closest to

A. 17.0
B. 17.1
C. 17.2
D. 17.3
E. 17.4

Use the following information to answer Questions 14 and 15

The table below gives the percentage of unemployed 15-24 year old youths for 2017 and 2018

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Percentage Unemployed 2017	13.5	14.0	14.2	12.6	12.1	12.4	12.3	12.5	12.1	11.4	11.4	12.4
Percentage Unemployed 2018	13.4	14.2	13.4	12.5	11.0	10.7	10.7	11.6	11.1	10.6	10.8	11.3
Seasonal Index	1.10	1.15	1.12	1.03	0.95	0.94	0.93	0.97	0.95	1.00	0.91	0.95

If $t = 1$ is January, 2017, $t = 2$ is February, 2017 and so on, the time series graph for this data is

Percentage Unemployed 2017-2018
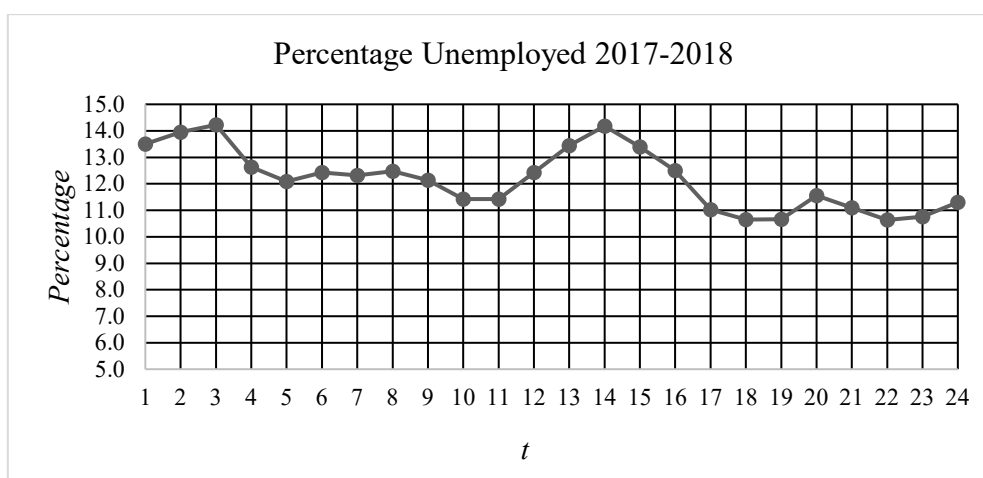
Question 14

The equation of the least squares regression line that can be used to find the *percentage unemployed* is closest to

A. *percentage unemployed* $= 62.66 - 4.12t$

B. *percentage unemployed* $= 14.28 - 0.16t$

C. *percentage unemployed* $= 13.71 - 0.24t$

D. *percentage unemployed* $= 13.71 + 0.24t$

E. *percentage unemployed* $= 13.53 - 0.11t$

Question 15

From the seasonal index for November we know that the unemployment figures for November are generally

A. 9% lower than the yearly average
B. 9% of the yearly average
C. 9% higher than the yearly average
D. 91% lower than the yearly average
E. 91% of the November average

Question 16

The table and graphs gives the *global annual mean temperature* (°C) for five *year* intervals from 1950 to 2010.

Year	Mean Temperature
1950	13.738
1955	13.765
1960	13.920
1965	13.822
1970	13.937
1975	13.903
1980	14.164
1985	14.034
1990	14.333
1995	14.358
2000	14.326
2005	14.559
2010	14.601
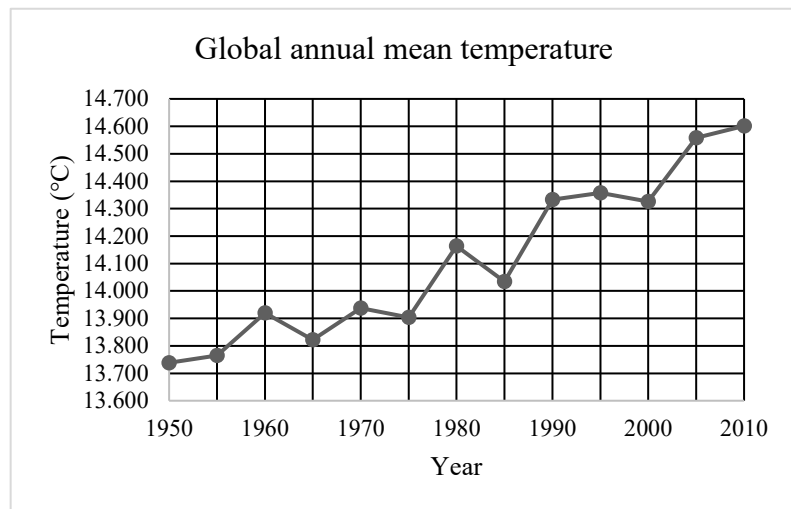
If a four-mean smoothing with centring is used to smooth the time series plot above, correct to three decimal places, the four-mean smoothed *global annual mean temperature* centred on 1970 would be

A. 13.896
B. 13.957
C. 13.878
D. 13.896
E. 13.926

Recursion and Financial Modelling

Question 17

The yearly balance of a simple interest investment is given by the recurrence relation

$$D_0 = 1200 \quad D_{n+1} = D_n + 50.4$$

The interest rate per annum for this investment is

A. 3.1%
B. 4.2%
C. 5.04%
D. 5.4%
E. 23.8%

Question 18

Thomas takes out an **interest only loan** on his car.

He has borrowed an amount of $20 000 at 7.2% per annum compounding monthly.

The amount that Thomas will owe on his loan after two years would be

A. $20 000
B. $23 088
C. $22 984
D. $12 849
E. $22 880

Question 19

The balance of a loan of P is given by the following recurrence relation

$$L_0 = P, \quad L_{n+1} = 1.045 \times L_n - 540$$

Which of the following statements is **not** true?

A. The loan is charged 4.5% interest each time period
B. If P is $12 000 the loan would be an interest only loan
C. If P is $15 000 the loan would never be paid off under this arrangement
D. If P is $8000 the loan would be paid off in 24 time periods
E. The loan repayment is $540 each time period

Question 20

The graph below shows the balances of two different investments on the 1st of January each year over a ten year period.

Investment A started with an initial investment of $3000 and has a balance of $4800 after 4 years.

Investment B started with an initial investment of $2000 and has a balance of $2300 after 1 year.

Both investments earned interest, added annually, but no other additional payments were made into the accounts.
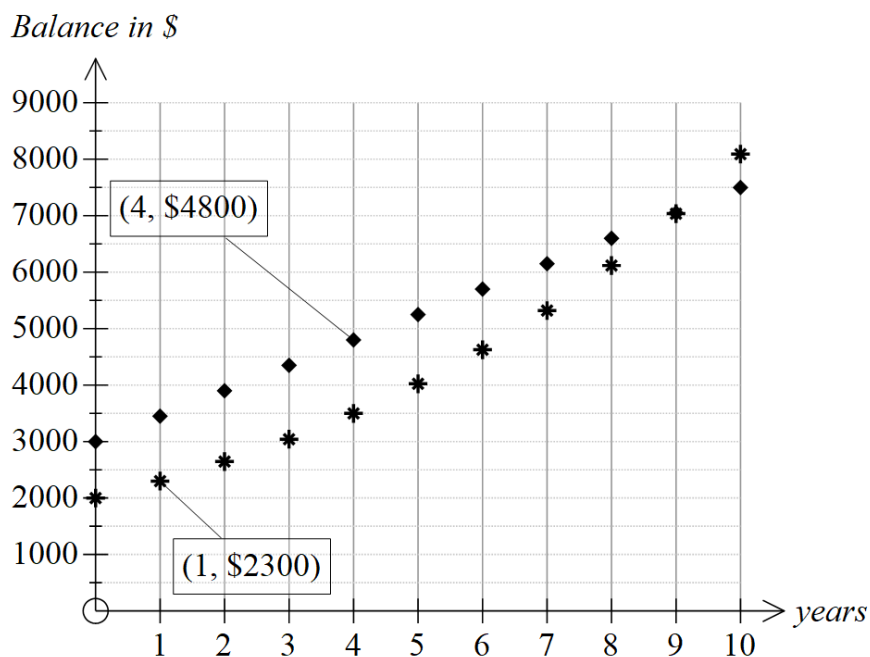
Which of the following statements is true?

A. Investment A was earning compound interest
B. Investment B was earning simple interest
C. The per annum interest rate is different for both Investment A and Investment B
D. The balance of Investment B first exceeds the balance of Investment A after 8 years
E. The biggest difference in balance during the 10 year period between Investment A and Investment B is less than $1500

Question 21

Stefanos deposits $5000 into an investment account at 4.7% per annum compounding quarterly. A second bank offers Stefanos a rate of 5.3% compounding monthly instead.

If Stefanos changes to the second bank, the increase in effective rate per annum for his investment would be closest to

A. 0.60%
B. 0.08%
C. 0.13%
D. 0.73%
E. 0.65%

Question 22

Robbie buys a large industrial printing press for his business that costs $140 000.

He will depreciate the printing press using the unit cost method.

The printing press will depreciate by $4.60 for every 5 000 pages printed.

The number of pages that have been printed when the printing press has a book value of $71 000 is

A. 3
B. 15 000
C. 75 000
D. 15 000 000
E. 75 000 000

Question 23

Petra invests $30 000 in an account earning 3.4% per annum interest compounding monthly.

At the end of every month Petra deposits an additional $900 to the account.

The total interest Petra has earned at the end of three years is closest to

A. $67 276
B. $37 276
C. $4876
D. $4279
E. $2963

Question 24

Luka takes out a loan of $10 000 to buy a car. He is charged 8.4% per annum interest compounding monthly and he will pay the loan off entirely over a 24 month period with equal monthly payments.

Which of the following statements about Luka's loan is **not** true?

A. Over the 24 month period Luka will pay a total of $898 interest correct to the nearest dollar
B. After 12 months, Luka still owes $5209 correct to the nearest dollar on his loan
C. After 6 months Luka has paid $2345 correct to the nearest dollar off the balance of the loan
D. The principal reduction during the third month is $392 correct to the nearest dollar
E. The balance of the loan at the end of 23 months would be $451 correct to the nearest dollar

SECTION B – Modules

<table>
<tr><td>Instructions for Section B</td></tr>
</table>

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple – choice questions.
Show the modules you are answering by shading the matching boxes on your multiple - choice answer sheet **and** writing the name of the module in the box provided.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale

Contents   **Page**

Module 1: Matrices

Before answering these questions, you must **shade** the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Consider the matrix equation below:

$$2\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} - 3\begin{bmatrix} -5 \\ x \\ 4 \end{bmatrix} = \begin{bmatrix} 17 \\ 8 \\ -6 \end{bmatrix}$$

The matrix equation would be correct if x was equal to

A. 4
B. -4
C. 2
D. -2
E. 1

Question 2

$$\text{Matrix } F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Matrix F cannot be described as

A. A binary matrix
B. A square matrix
C. A lower triangular matrix
D. A regular matrix
E. A symmetric matrix

Question 3

The matrix $\begin{bmatrix} 6 & a \\ b & 12 \end{bmatrix}$ would have an inverse if

A. $a = 8$ and $b = 9$
B. $a = -6$ and $b = -12$
C. $a = -8$ and $b = 9$
D. $a = -8$ and $b = -9$
E. $a = -36$ and $b = -2$

<div align="right">

SECTION B – Module 1 continued
TURN OVER

</div>

Question 4

Five people, George (G), Harry (H), Ivan (I), James (J) and Kevin (K), play a round robin tennis tournament. Each person plays each other person exactly once and a winner is declared for each game.

A matrix showing the winner and loser of each game is shown below. In this matrix a "1" represents a win. For example in row 1, the "1" shows that George defeated Harry.

Some entries in the matrix have not been completed.

$$
\begin{array}{c} \\ \\ winner \end{array}
\begin{array}{c} \\ G \\ H \\ I \\ J \\ K \end{array}
\begin{array}{c} loser \\ \begin{array}{ccccc} G & H & I & J & K \end{array} \\
\left[\begin{array}{ccccc}
0 & 1 & 0 & 0 & 0 \\
- & 0 & 0 & 1 & 0 \\
- & - & 0 & 0 & 1 \\
- & - & - & 0 & 0 \\
- & - & - & - & 0
\end{array}\right]
\end{array}
$$

The completed matrix would be:

A.

$$
\begin{array}{c} \\ \\ winner \end{array}
\begin{array}{c} \\ G \\ H \\ I \\ J \\ K \end{array}
\begin{array}{c} loser \\ \begin{array}{ccccc} G & H & I & J & K \end{array} \\
\left[\begin{array}{ccccc}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{array}\right]
\end{array}
$$

B.

$$
\begin{array}{c} \\ \\ winner \end{array}
\begin{array}{c} \\ G \\ H \\ I \\ J \\ K \end{array}
\begin{array}{c} loser \\ \begin{array}{ccccc} G & H & I & J & K \end{array} \\
\left[\begin{array}{ccccc}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 0
\end{array}\right]
\end{array}
$$

C.

$$
\begin{array}{c} \\ \\ winner \end{array}
\begin{array}{c} \\ G \\ H \\ I \\ J \\ K \end{array}
\begin{array}{c} loser \\ \begin{array}{ccccc} G & H & I & J & K \end{array} \\
\left[\begin{array}{ccccc}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0
\end{array}\right]
\end{array}
$$

D.

$$
\begin{array}{c} \\ \\ winner \end{array}
\begin{array}{c} \\ G \\ H \\ I \\ J \\ K \end{array}
\begin{array}{c} loser \\ \begin{array}{ccccc} G & H & I & J & K \end{array} \\
\left[\begin{array}{ccccc}
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{array}\right]
\end{array}
$$

E.

$$
\begin{array}{c} \\ \\ winner \end{array}
\begin{array}{c} \\ G \\ H \\ I \\ J \\ K \end{array}
\begin{array}{c} loser \\ \begin{array}{ccccc} G & H & I & J & K \end{array} \\
\left[\begin{array}{ccccc}
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 0
\end{array}\right]
\end{array}
$$

Question 5

Members of a weekly gym club can choose to do Aerobics (A), Ballet (B) or Cycling (C). The transition diagram below shows the movement between the activities from week to week:
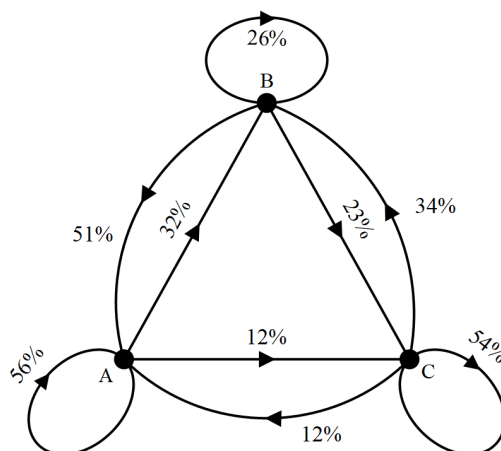
A transition matrix that could also represent this situation is

A.

this week

$$\begin{array}{ccc} A & B & C \end{array}$$
$$\begin{bmatrix} 0.56 & 0.34 & 0.12 \\ 0.51 & 0.26 & 0.23 \\ 0.12 & 0.34 & 0.54 \end{bmatrix} \begin{array}{l} A \\ B \quad \textit{next week} \\ C \end{array}$$

B.

this week

$$\begin{array}{ccc} A & B & C \end{array}$$
$$\begin{bmatrix} 0.56 & 0.51 & 0.12 \\ 0.32 & 0.26 & 0.34 \\ 0.12 & 0.23 & 0.54 \end{bmatrix} \begin{array}{l} A \\ B \quad \textit{next week} \\ C \end{array}$$

C.

this week

$$\begin{array}{ccc} A & B & C \end{array}$$
$$\begin{bmatrix} 0.51 & 0.56 & 0.12 \\ 0.26 & 0.32 & 0.34 \\ 0.23 & 0.12 & 0.54 \end{bmatrix} \begin{array}{l} A \\ B \quad \textit{next week} \\ C \end{array}$$

D.

this week

$$\begin{array}{ccc} A & B & C \end{array}$$
$$\begin{bmatrix} 0.56 & 0.23 & 0.54 \\ 0.12 & 0.26 & 0.12 \\ 0.32 & 0.51 & 0.54 \end{bmatrix} \begin{array}{l} A \\ B \quad \textit{next week} \\ C \end{array}$$

E.

this week

$$\begin{array}{ccc} A & B & C \end{array}$$
$$\begin{bmatrix} 0.56 & 0.26 & 0.34 \\ 0.32 & 0.51 & 0.12 \\ 0.12 & 0.23 & 0.54 \end{bmatrix} \begin{array}{l} A \\ B \quad \textit{next week} \\ C \end{array}$$

Question 6

In a shopping centre there are three coffee shops, Edge (E), Flavours (F) and Good Life (G). A number of customers are surveyed and it is found the customers change between the coffee shops according to the transition matrix below:

$$\begin{array}{c} \textit{this time} \\ \begin{array}{ccc} E & F & G \end{array} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.3 & 0.1 & 0.4 \\ 0.2 & 0.6 & 0.4 \end{bmatrix} \begin{array}{l} E \\ F \quad \textit{next time} \\ G \end{array} \end{array}$$

Which of the following statements is true?

A.   The same number of people will change each time from Edge to Flavours as will change from Flavours to Edge
B.   In the long run Edge will have the most customers
C.   Each time half of Flavours' customers will change to Good Life
D.   Half of the customers who go to either Flavours or Good Life this time will go to Edge next time
E.   Half of the customers who go to Edge this time will go to either Flavours or Good Life next time

Question 7

Consider the matrix relation below:

$$A_{n+1} = H \times A_n + B \text{ where } H = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.1 & 0.5 & 0.4 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}, \ B = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix} \text{ and } A_3 = \begin{bmatrix} 1407 \\ 1651 \\ 1742 \end{bmatrix}$$

The matrix A_1 in this relationship would be

A. $\begin{bmatrix} 1780 \\ 2070 \\ 2151 \end{bmatrix}$

B. $\begin{bmatrix} 1240 \\ 1430 \\ 1530 \end{bmatrix}$

C. $\begin{bmatrix} 1100 \\ 1200 \\ 1300 \end{bmatrix}$

D. $\begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix}$

E. $\begin{bmatrix} 1589 \\ 1863 \\ 1948 \end{bmatrix}$

Question 8

A dealer sells SUVs and utes.

He buys vehicles at a wholesale price and then adds 25% to the SUV wholesale price and 15% to the ute wholesale price to determine the selling price.

The wholesale price for SUVs is $20 000 each and for utes is $30 000 each.

He then adds $500 in registration and fees to every vehicle to determine the drive-away price.

In January, the dealer sells 12 SUVs and 9 utes.

In February, the dealer sells 17 SUVs and 11 utes.

In March, the dealer sells 14 SUVs and 10 utes.

A matrix equation that would give the total money collected by the dealer from the drive-away prices of SUVs and utes sold during these three months would be

A.
$$\begin{bmatrix} 12 & 9 \\ 17 & 11 \\ 14 & 10 \end{bmatrix} \left(\begin{bmatrix} 1.25 & 0 \\ 0 & 1.15 \end{bmatrix} \begin{bmatrix} 20\,000 \\ 30\,000 \end{bmatrix} + \begin{bmatrix} 500 \\ 500 \end{bmatrix} \right) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

B.
$$\begin{bmatrix} 12 & 9 \\ 17 & 11 \\ 14 & 10 \end{bmatrix} \left(\begin{bmatrix} 0.25 & 0 \\ 0 & 0.15 \end{bmatrix} \begin{bmatrix} 20\,000 \\ 30\,000 \end{bmatrix} + \begin{bmatrix} 500 \\ 500 \end{bmatrix} \right) \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

C.
$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 12 & 9 \\ 17 & 11 \\ 14 & 10 \end{bmatrix} \left(\begin{bmatrix} 1.25 & 0 \\ 0 & 1.15 \end{bmatrix} \begin{bmatrix} 20\,000 \\ 30\,000 \end{bmatrix} + \begin{bmatrix} 500 \\ 500 \end{bmatrix} \right)$$

D.
$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 12 & 9 \\ 17 & 11 \\ 14 & 10 \end{bmatrix} \left(\begin{bmatrix} 0.25 & 0 \\ 0 & 0.15 \end{bmatrix} \begin{bmatrix} 20\,000 \\ 30\,000 \end{bmatrix} + \begin{bmatrix} 500 \\ 500 \end{bmatrix} \right)$$

E.
$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 12 & 9 \\ 17 & 11 \\ 14 & 10 \end{bmatrix} \begin{bmatrix} 1.25 & 0 \\ 0 & 1.15 \end{bmatrix} \begin{bmatrix} 20\,000 \\ 30\,000 \end{bmatrix} + \begin{bmatrix} 500 \\ 500 \end{bmatrix}$$

Module 2: Networks and decision mathematics

Before answering these questions, you must **shade** the 'Networks and decision mathematics' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The number of edges in a complete graph with 6 vertices is

A. 5
B. 6
C. 10
D. 15
E. 21

Question 2

The map below shows the road connections between three towns A, B and C.
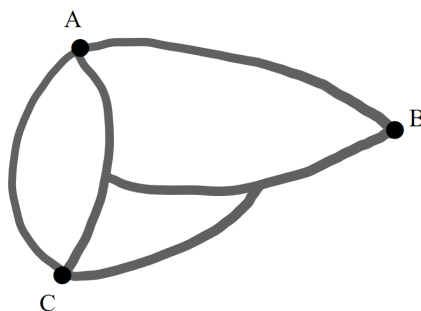
The graph that could be used to model the connections is

A.

B.

C.

D.

E.

Question 3

A planar graph has 6 faces and 8 vertices, the number of edges is

A. 4
B. 6
C. 8
D. 12
E. 14

Question 4

The digraph below shows the flow capacity for pipes in a watering system.
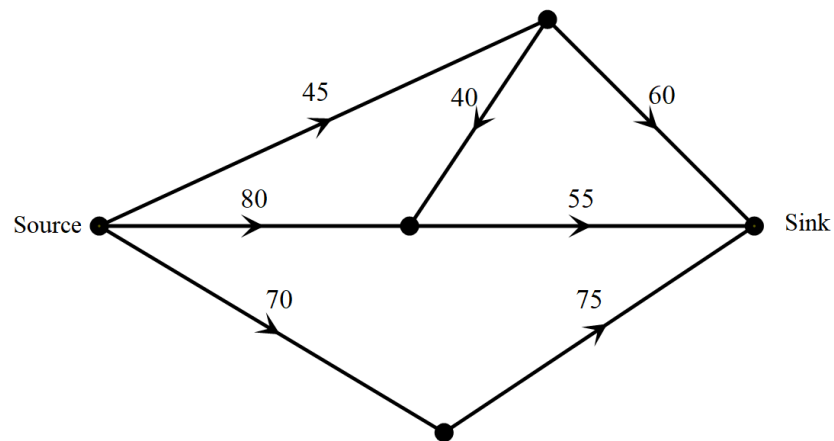
The maximum flow from the source to the sink is

A. 145
B. 170
C. 175
D. 185
E. 195

Question 5

A manufacturing procedure has four processes. Four employees, Alice, Bjorn, Charlotte and David, are able to perform different parts of the process apart from Charlotte who cannot do process 3.
The table below shows the time it takes each person to complete each part of the process, in minutes.

	Process 1	Process 2	Process 3	Process 4
Alice	23	18	12	18
Bjorn	28	24	17	26
Charlotte	17	20		23
David	15	23	19	27

If each person is allocated one process, the minimum time total for the manufacturing process is

A. 48 minutes
B. 60 minutes
C. 67 minutes
D. 70 minutes
E. 73 minutes

Use the following information to answer Questions 6 and 7.

The activity network below gives the time, in hours, for activities in a project.
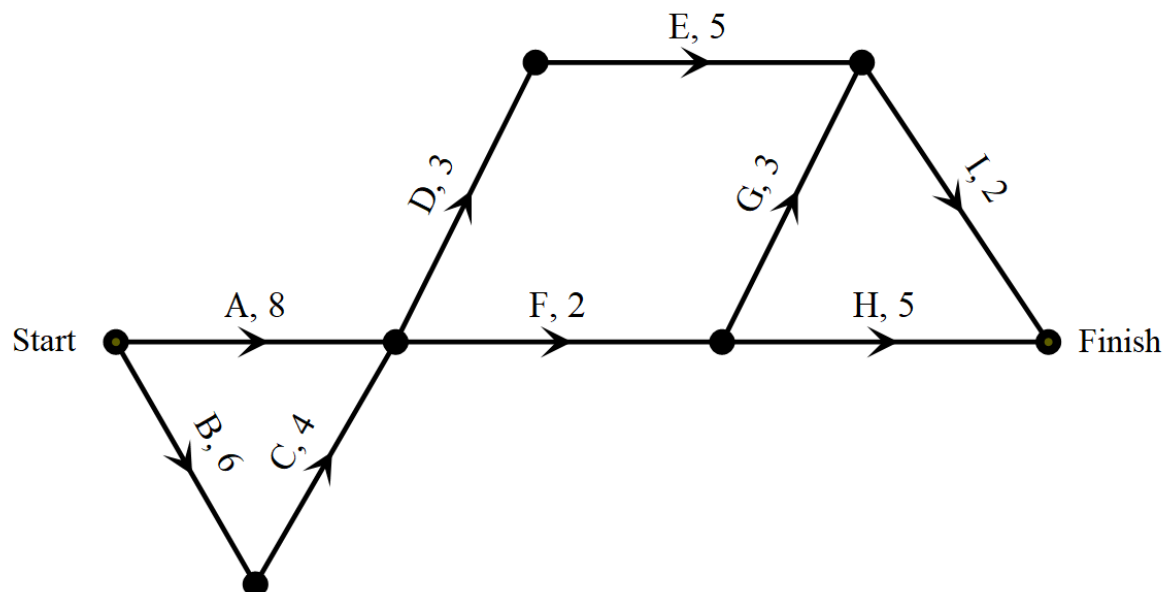
Question 6

The latest starting time for activity F is

A. 8 hours
B. 10 hours
C. 12 hours
D. 13 hours
E. 14 hours

Question 7

Which of the following would create a new critical path?

A. Reducing A by three hours
B. Increasing I by one hour
C. Reducing C by one hour
D. Increasing B by three hours
E. Reducing E by three hours

Question 8

The following precedence table lists the activities involved in a project

Activity	Immediate predecessor
A	-
B	A
C	A
D	B
E	C
F	D, E
G	C

Which of the following directed graphs represents the activity network?

A.
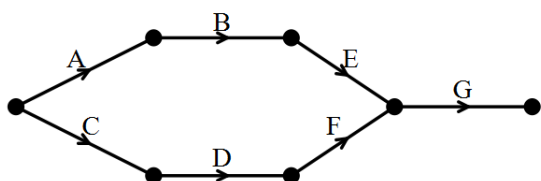
B.
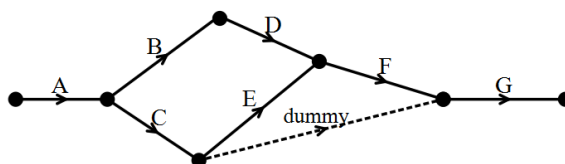
C.
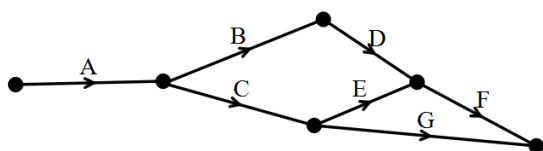
D.
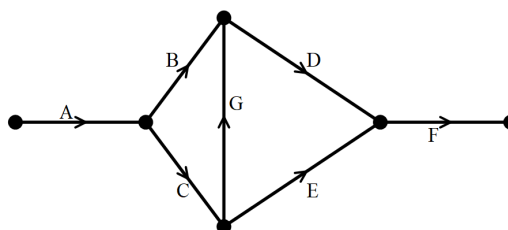
E.
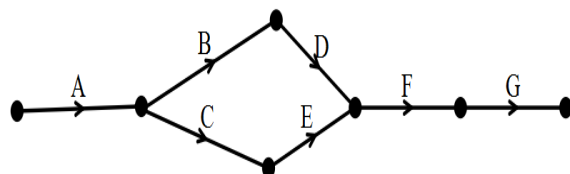
SECTION B – Module 2 continued

Module 3: Geometry and Measurement

Before answering these questions, you must **shade** the 'Geometry and measurement' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Consider the diagram below:
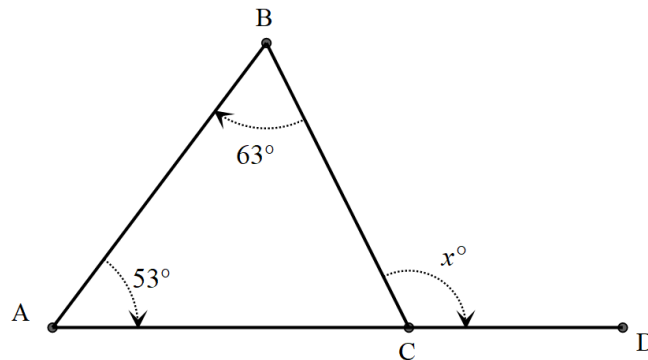
The size of the angle marked $x°$, is

A. 53º
B. 64º
C. 116º
D. 117º
E. 127

Question 2

Which of the following locations would be closest in distance to 75ºN, 23ºE?

A. 75ºN, 33ºE
B. 75ºN, 10ºE
C. 85ºN, 33ºE
D. 85ºN, 23ºE
E. 75ºS, 23ºE

Question 3

Paul flies from Madrid, Spain (40ºN, 4ºW) to Kansas City, USA (40ºN, 94ºW). The plane flies the shortest possible distance.

Paul leaves Madrid at 5 pm, Wednesday, local time, and the flight takes 9 hours.

Assuming that every 15 degrees of longitude is one hour of time difference, the local time when Paul will land in Kansas City would be closest to

A. 8 am Thursday
B. 4 pm Wednesday
C. 6 pm Wednesday
D. 2 am Thursday
E. 8 pm Wednesday

Question 4

Assuming that the radius of the Earth is 6400 km, the shortest distance between Kyushu, Japan (34ºN, 131ºE) and Uluru in the Northern Territory, Australia (25ºS, 131ºE) is closest to

A. 1005 km
B. 2793 km
C. 3798 km
D. 6590 km
E. 7536 km

Question 5

A golfer strikes the ball from the tee (the starting point). If he had hit the ball in a straight line to the hole, the ball would have travelled 110 metres. The golfer mishits the ball and it travels 54 metres in a straight line in a different direction. From where the ball lands, the golfer hits the ball a distance of 65 metres in a straight line to get to the hole.

The angle between the direct line from the tee to the hole and the line where the golfer first hits the ball is closest to

A. 20.3º
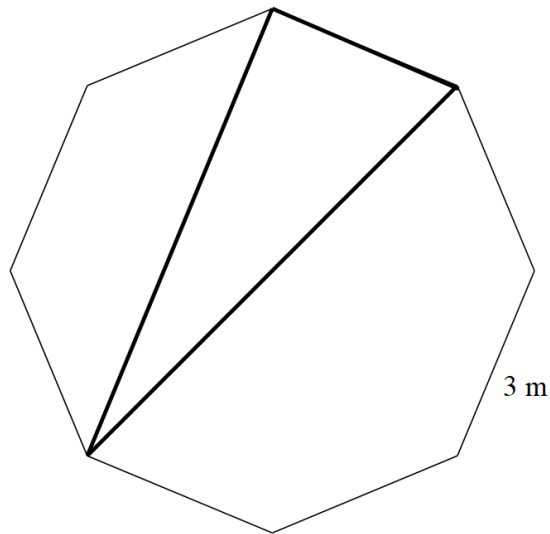B. 24.7º
C. 29.4º
D. 45.1º
E. 134.9º

Question 6

A 30 metre long hose has an outer diameter of 15 mm with 2.5 mm walls. The amount of water in the filled hose, in litres, is closest to

A. 2.36 litres
B. 3.68 litres
C. 4.71 litres
D. 5.30 litres
E. 235.62 litres

SECTION B – Module 3 continued

Question 7

A triangle is inscribed within a regular octagon of side length 3 metres as shown below:

3 m

The perimeter of the inscribed triangle is closest to

A. 7.8 m
B. 7.2 m
C. 15.1 m
D. 18.1 m
E. 21.5 m

Question 8

A child's toy consists of a spherical ball that contains three smaller balls.

The diameter of the largest inner ball is equal to the radius of the outer ball.

The radius of the middle inner ball is half the radius of the largest inner ball.

The radius of smallest inner ball is half the radius of the middle inner ball.

The toy is shown in the diagram below:
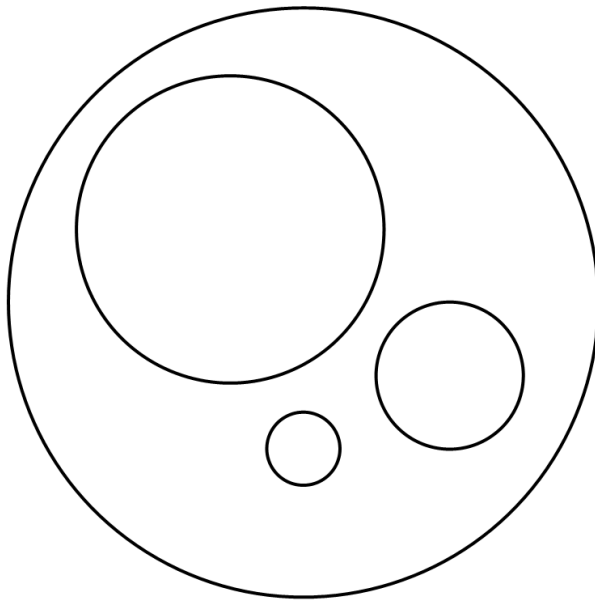
The fraction of the volume of the outer ball **not** taken up by the inner balls is equal to

A. $\dfrac{1}{8}$

B. $\dfrac{21}{64}$

C. $\dfrac{43}{64}$

D. $\dfrac{73}{512}$

E. $\dfrac{439}{512}$

Module 4: Graphs and Relations

Before answering these questions, you must **shade** the 'Graphs and relations' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.
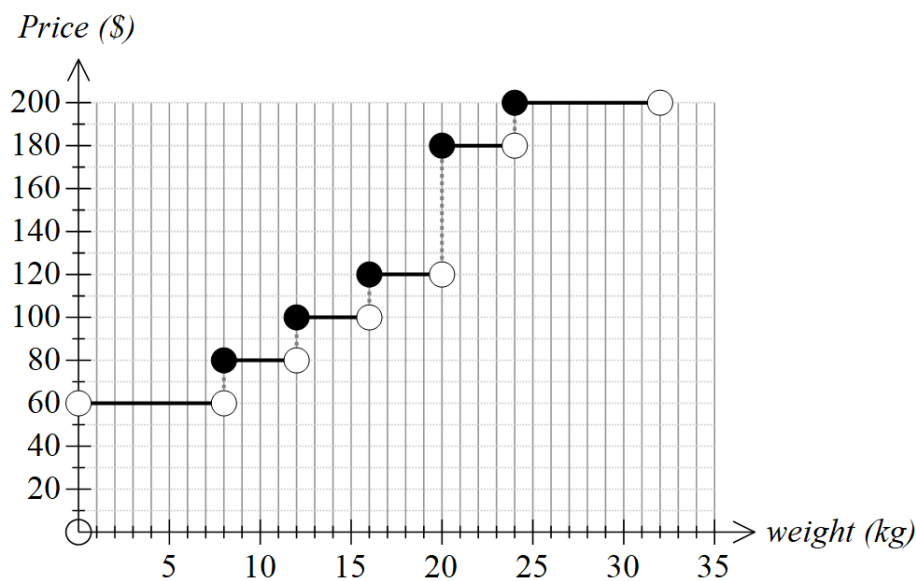
Question 1

The equation of the horizontal line passing through the point (3, -2) is

A. $y = \dfrac{-2}{3}x$

B. $y = \dfrac{-3}{2}x$

C. $x = 3$

D. $y = -2$

E. $y = 3x - 2$

Question 2

An airline charges for excess luggage based on the weight in kilograms.

The graph below can be used to determine the cost, in dollars, of excess luggage up to 32kg.

Two friends are travelling together, their total excess luggage, is 40kg.
To minimise the charges, they should split the luggage so that

A. One person carries 30kg and the other carries 10kg

B. One person carries 28kg and the other carries 12kg

C. One person carries 25kg and the other carries 15kg

D. One person carries 24kg and the other carries 16kg
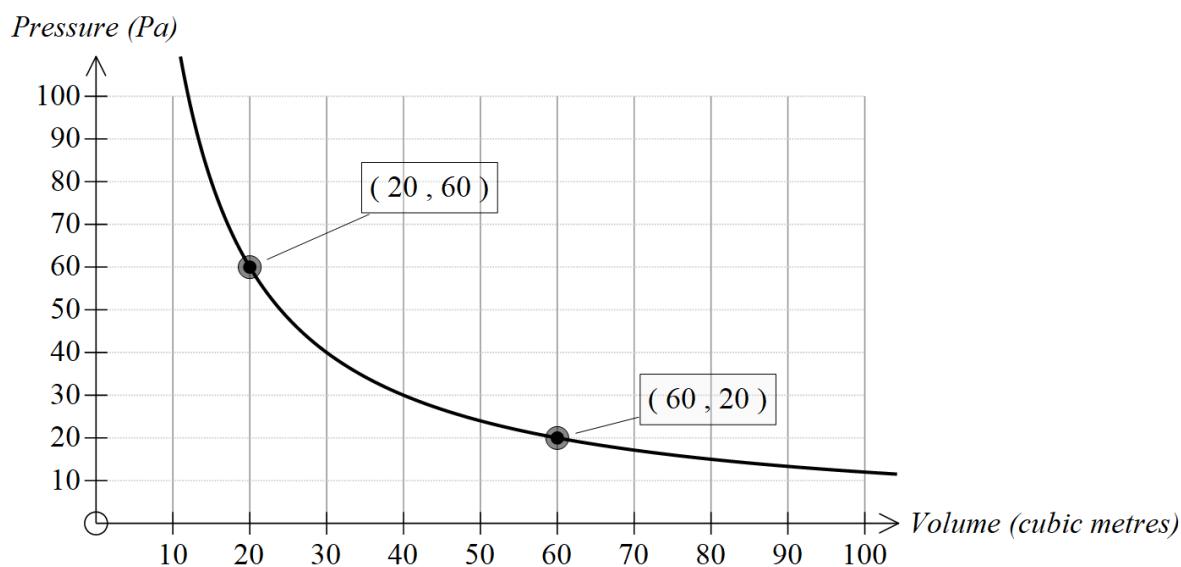
E. Each person carries 20 kg

Question 3

A real estate agent's monthly salary is made up of a base pay plus commission as a percentage of sales. The base pay is $1600, and the commission is 2.5% of total sales.

If the monthly salary was $8400, the total sales were

A. $400000

B. $334000

C. $272000

D. $2720

E. $1810

Question 4

The results of a science experiment are displayed in the graph below

The relationship between *Pressure* and *Volume* is

A. $Pressure = 1.25\,(Volume)$

B. $Pressure = 0.75\,(Volume)^2$

C. $Pressure = 0.15\,(Volume)^3$

D. $Pressure = \dfrac{1200}{Volume}$

**E.** $Pressure = \dfrac{70000}{(Volume)^2}$

Question 5

The daily cost, C, in dollars, of making speciality cupcakes, is given by $C = 2.4n + 425$, where n is the number of cupcakes. The cupcakes sell for \$4.50 each.

The minimum number of cupcakes that need to be sold to 'break even' is:

A. 62
B. 95
C. 177
D. 202
E. 203

Question 6

The shaded region of the graph below shows the feasible region for a linear programming problem.
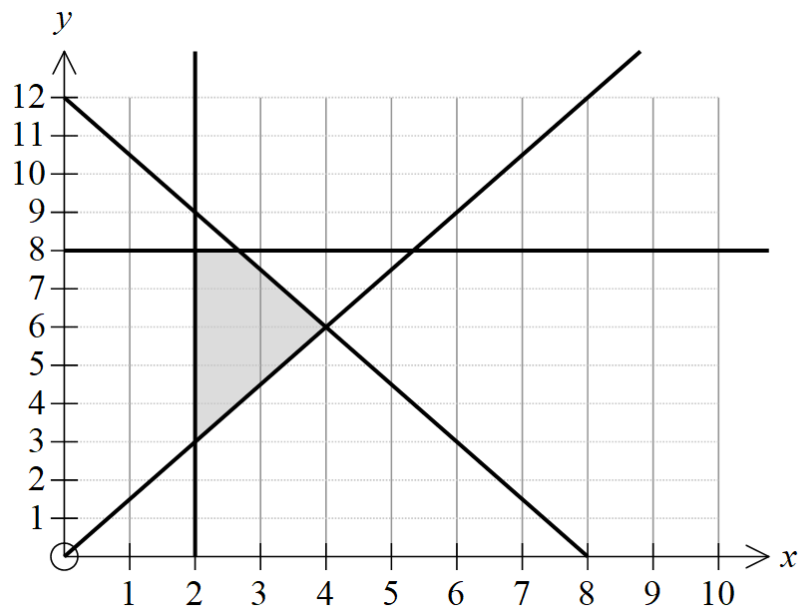
The objective function for this problem is $Z = x + y$. The maximum value of Z is

A. 5

B. 10

C. $\dfrac{32}{3}$

D. $\dfrac{21}{2}$

E. 11

Question 7

The feasible region of a linear programming problem is defined by the following constraints

$$8x + 5y \leq 40$$
$$y \leq 6$$
$$y \geq 2x + 1$$

Which of the following points do **not** lie in the feasible region?

A. (2, 5)
B. (1, 6)
C. (1, 5)
D. (1, 4)
E. (0, 6)

Question 8

Steen is a recycling manufacturer who wants a specific mix that is **at least** 18% chromium and 16% to nickel for his product.

He can get two different metals to combine for this product

- Metal X contains 30% chromium and 10% nickel
- Metal Y contains 10% chromium and 20% nickel

A suitable combination of the two metals would be

A. 50 tonnes of X and 50 tonnes of Y
B. 20 tonnes of X and 80 tonnes of Y
C. 40 tonnes of X and 60 tonnes of Y
D. 60 tonnes of X and 40 tonnes of Y
E. 80 tonnes of X and 20 tonnes of Y

END OF MULTIPLE - CHOICE QUESTION BOOKLET