Neap:

Trial Examination 2018

VCE Further Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	7	7	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

Students are to write in blue or black pen.
Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 31 pages
Formula sheet
Working space is provided throughout the booklet

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2018 VCE Further Mathematics Units 3&4 Written Examination 2.

SECTION A – CORE

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

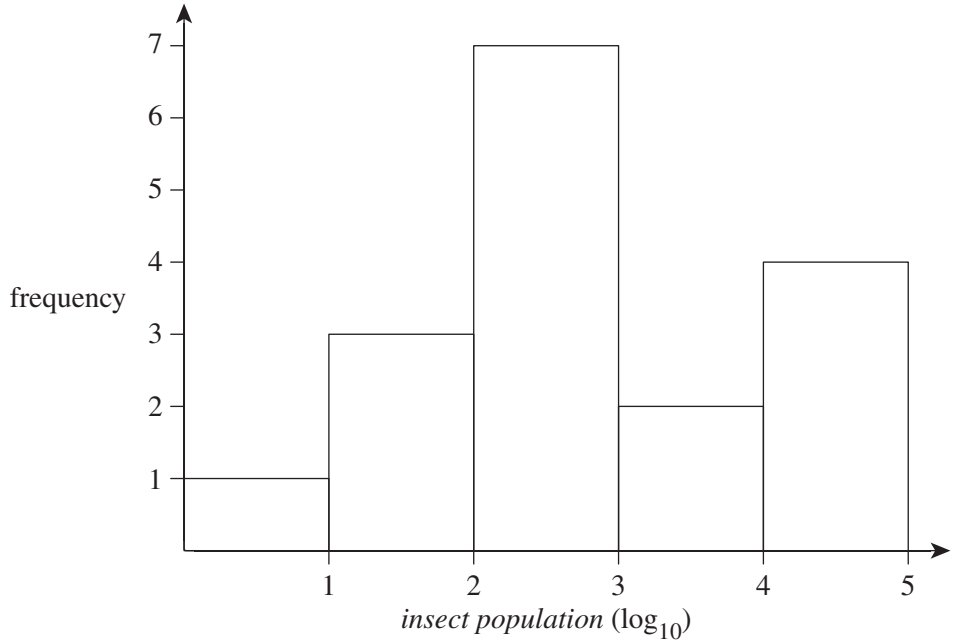
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (9 marks)

A development company is looking at several islands to build a new luxury, family hotel and resort. Variables such as rainfall, maximum and minimum temperature, and insect population are all recorded over a two-year period.

The insect population is measured monthly for one particularly promising location is shown in the graph below.



a. **i.** For how many months has data been recorded? 1 mark

_____

ii. If the insect population is greater than or equal to 10 000 for more than 30% of the time, the island will be deemed as not suitable.

Use relevant mathematics to show this island is considered suitable. 1 mark

_____

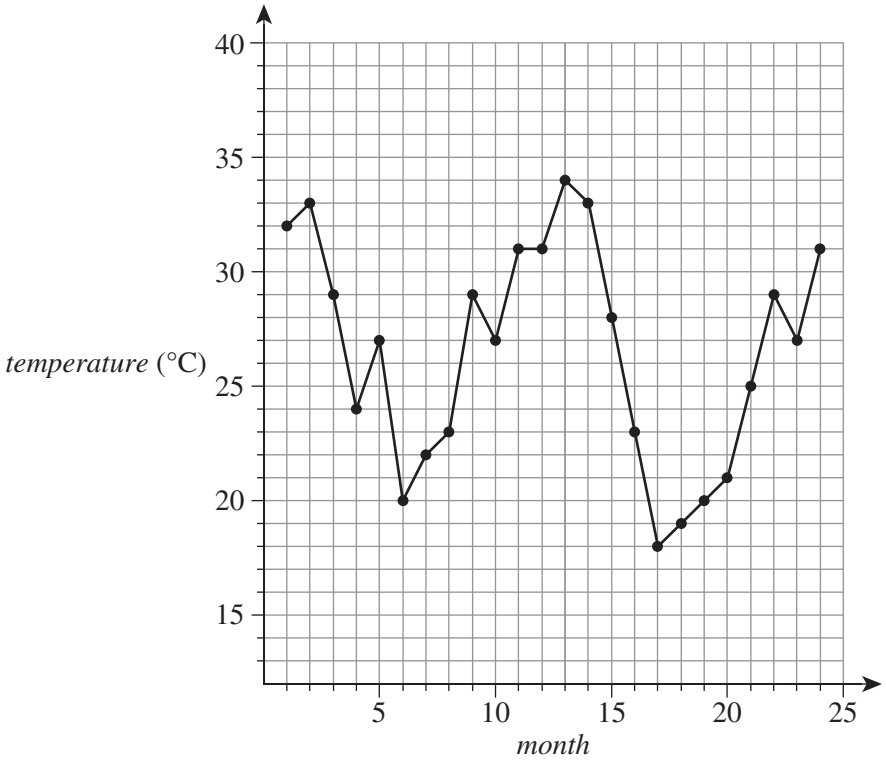
_____

_____

The final choice of location to build the resort is between Amiable Island and Beautiful Island. The table below shows the mean monthly maximum and minimum temperatures recorded on Amiable Island over two years.

Month (Year 1)	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Maximum (°C)	32	33	29	24	27	20	22	23	29	27	31	31
Minimum (°C)	6	8	12	15	16	14	12	17	13	15	14	10
Month (Year 2)	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Maximum (°C)	34	33	28	23	18	19	20	21	25	29	27	31
Minimum (°C)	8	12	13	9	15	16	16	14	15	12	11	9

The time series for the maximum temperature is shown in the graph below.



b. **i.** Comment on any pattern or trend that you can identify. 1 mark

_____

_____

ii. Sketch the smoothed curve using the three-median method. 2 marks

(Answer on the time series graph above.)

c. The mean maximum temperatures recorded on Beautiful Island over the same time are shown below.

15, 22, 23, 23, 24, 25, 25, 26, 27, 28, 28, 29, 29, 30, 30, 31, 31, 32, 32, 33, 33, 33, 34, 34

i. Draw a boxplot showing the data above. 2 marks

ii. Prove that the figure '15' is not an outlier. 2 marks

Question 2 (7 marks)

Eventually Beautiful Island is chosen as the preferred location and the design process begins. A hotel with up to 400 family-sized rooms can be built on the site. The development company commissions a study to predict the percentage occupancy rate (the number of rooms occupied) from the number of rooms the hotel builds. The estimations are shown below.

Number of rooms	10	50	100	150	200	250	300	350	400
Occupancy percentage	98	65	50	48	43	41	38	36	33

a. Complete the statement below. 1 mark

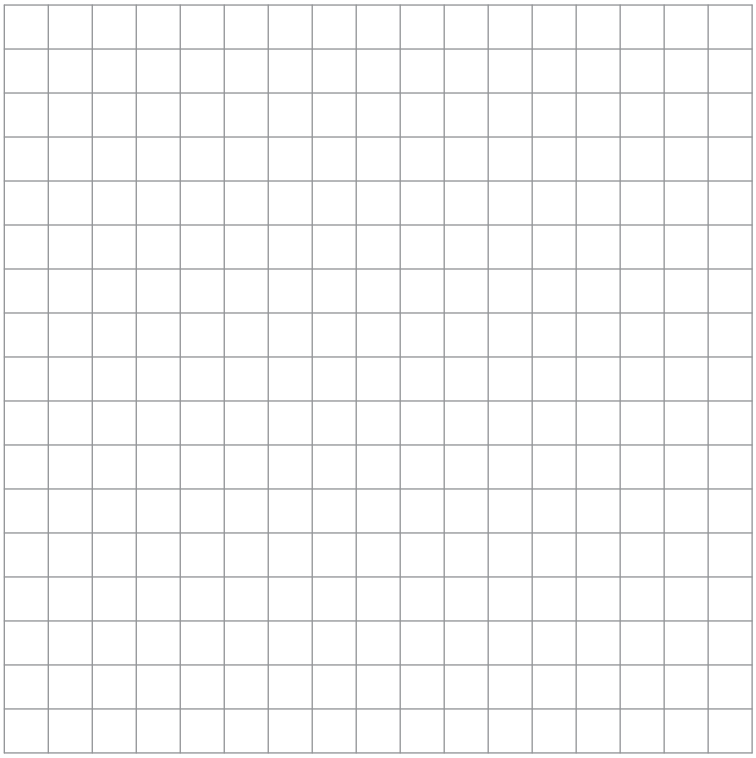
The _____ is the explanatory variable because

_____

_____

_____

b. Sketch a scatterplot for *number of rooms* versus *occupancy percentage* on the grid below. Label the graph. 2 marks



c. What does the scatterplot sketched in **part b.** show about the relationship between the *number of rooms* and the *occupancy percentage*? 1 mark

_____

_____

d. Use your calculator, or otherwise, to find the correlation coefficient.
Round your answer to two decimal places. 1 mark

_____

e. **i.** Apply a log x transformation and recalculate the correlation coefficient. 1 mark

_____

ii. What conclusion can be made from the two correlation coefficients calculated in **part d.** and **part e. i.**? 1 mark

_____

_____

_____

Question 3 (3 marks)

The projections indicate that if a 400-room hotel is built, the mean occupancy will be 125 rooms with a standard deviation of 40. Assume that the occupancy is normally distributed.

a. What percentage of the time will the occupancy exceed 205 rooms? 1 mark

_____

_____

_____

b. Calculate the number of nights per year that the occupancy will be below 85 rooms.
Round your answer to the nearest whole number. 1 mark

_____

_____

c. The standardised number of rooms occupied, z, on a particular night is 1.8.
Calculate how many rooms are actually occupied. 1 mark

_____

_____

_____

_____

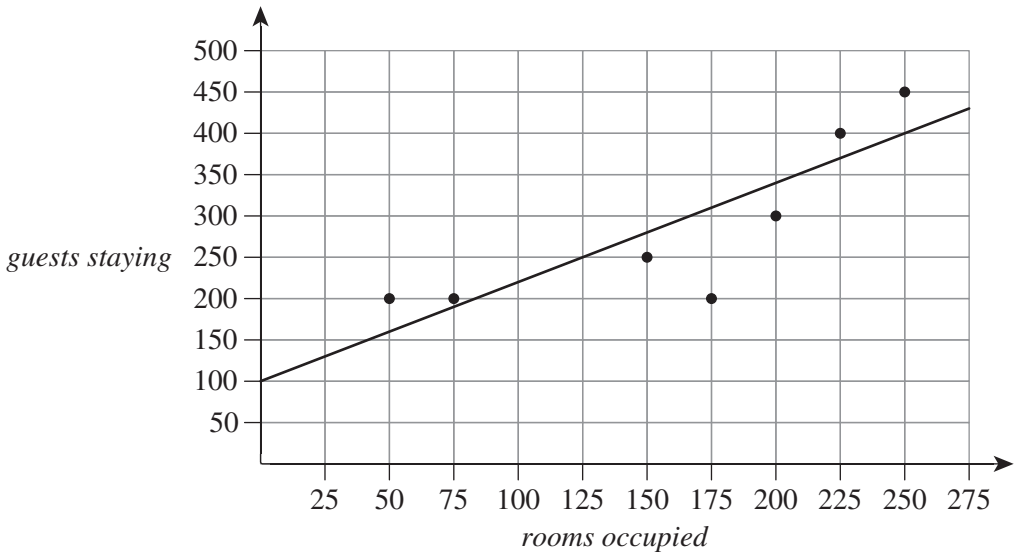
Question 4 (5 marks)

Since the predicted occupancy rates for a 400-room hotel are not high enough to be profitable, a decision is made to build a hotel with 250 family-sized rooms.

Some of the predictions for the number of guests staying compared to the number of occupied rooms are shown in the table below.

Rooms occupied	50	75	150	175	200	225	250
Guests staying	200	200	250	200	300	400	450

On the following graph, a line of best fit has been added by eye.



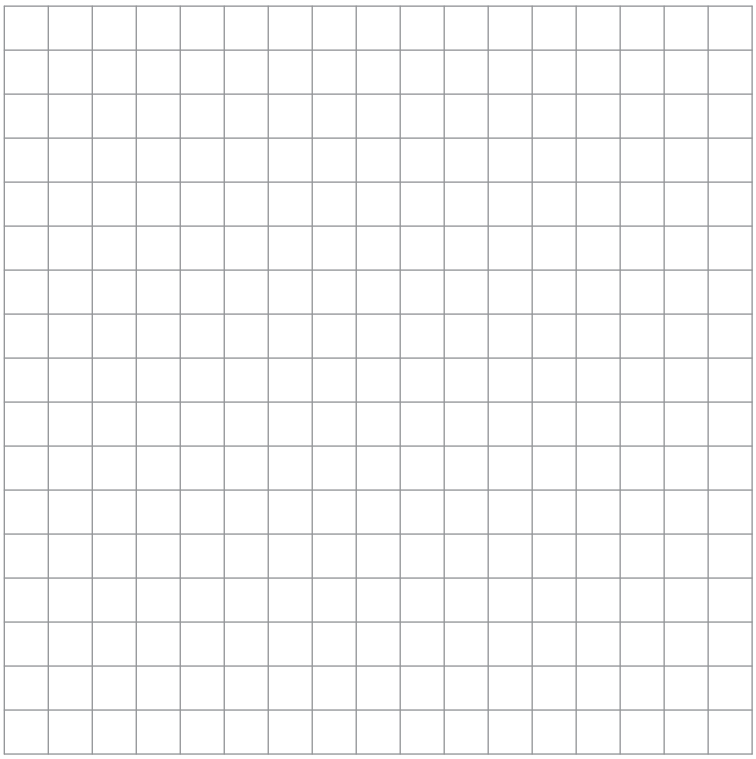
a. Find the equation of the line of best fit as shown on the above graph. Do not use your calculator to find the regression equation. 2 marks

_____

_____

_____

b. Draw a residual plot on the grid below. Label the graph. 2 marks



c. Referring to the graph in **part b.**, what conclusion does the residual plot lead to? 1 mark

_____

_____

Recursion and financial modelling

Question 5 (3 marks)

The purchase of the island property and the building of the resort will cost $7 000 000, all of which is borrowed. The loan is at 5.2% per annum compounding monthly. For the first five years, only the interest will be paid on the loan.

a. Calculate the monthly repayment. 1 mark

_____

_____

b. In the first year, the profit from accommodation, meals and hotel services is expected to be $300 000, growing by 15% per annum over the first five years.

i. Write a recursive equation to model this income. 1 mark

_____

_____

ii. Given the profit in the first year was $300 000, in which year does the profit first exceed $400 000? 1 mark

_____

_____

Question 6 (4 marks)

Michelle is a potential full-time resident of the resort and has been told that her rent will be $30 000 per year, payable monthly. This cost will increase each year according to $t_{n+1} = 1.06t_n$.

a. At what percentage rate is Michelle's rent increasing annually? 1 mark

_____

_____

Michelle has an investment of $650 000 which currently earns 4.1% per annum, compounding monthly, and she hopes that this income stream will cover her rent.

b. How much interest will Michelle earn in the first year? 1 mark

_____

_____

c. After paying her rent, the value of Michelle's investment decreases more in the second year than in the first.

Give two reasons for this. 2 marks

_____

_____

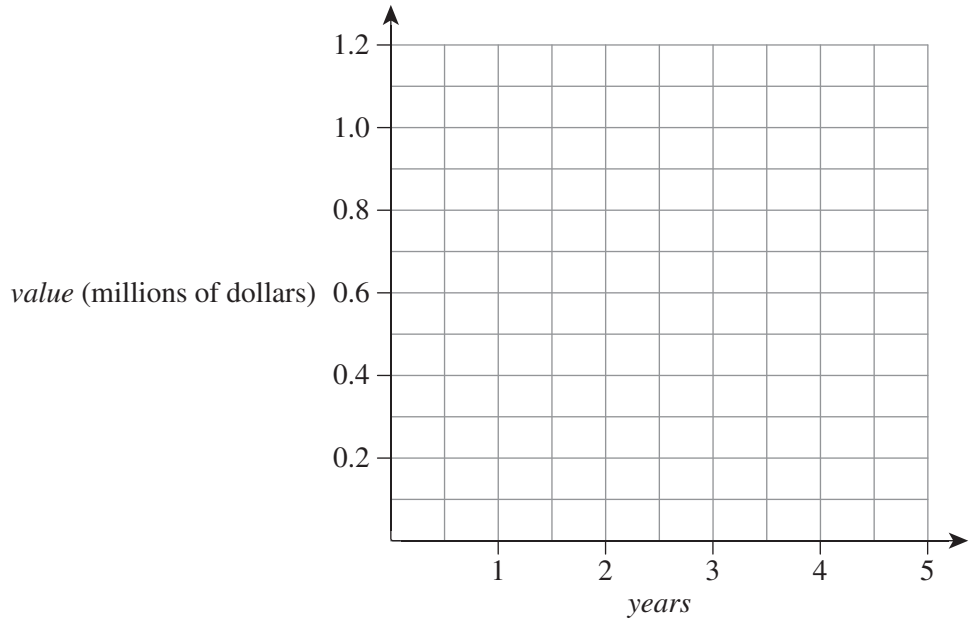
_____

_____

Question 7 (5 marks)

The air-conditioning system for the resort costs $1 200 000 and will be depreciated over five years. Initially two different methods of depreciating this asset are considered; a flat rate of 20% or decreasing the value by 25% per annum.

a. Sketch the graphs of both of these depreciation methods on the set of axes below. 2 marks



b. Between which two years does the flat depreciation method first become the lower value? 1 mark

_____

_____

It is also possible to depreciate the air-conditioning using the unit cost method. After five years of using the air-conditioners for an average of one million hours per year, the scrap value will be $100 000.

c. Calculate the unit depreciation cost per hour of the air-conditioning system.

Round your answer to the nearest cent. 1 mark

_____

_____

_____

_____

_____

d. Using the unit cost depreciation method, calculate the value after four years. 1 mark

_____

_____

END OF SECTION A

SECTION B – MODULES

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Contents **Page**

Module 1 – Matrices

Question 1 (5 marks)

Tanya opens a new business. To help her four employees *W*, *X*, *Y* and *Z* get to know each other, she organises a games night. The results are shown in the matrix below. A '1' in row 1, column 2 indicates a win for employee *W* over employee *X*.

$$\begin{array}{c} \\ W \\ X \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{cccc} W & X & Y & Z \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

a. Why does the diagonal from the top left to the bottom right contain only zeroes? 1 mark

_____

_____

b. A mistake has been made in the recording of the results for the game between *W* and *Z*.
Find the mistake and explain why it is a mistake? 1 mark

_____

_____

Tanya's shop sells women's jeans (*J*), dresses (*D*), tops (*T*) and snacks (*S*). The sales for the first four weeks are shown in the matrix below.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} J & D & T & S \end{array} \\ \begin{bmatrix} 8 & 20 & 9 & 100 \\ 10 & 30 & 20 & 140 \\ 20 & 20 & 15 & 150 \\ 10 & 30 & 20 & 180 \end{bmatrix} \end{array}$$

c. How many dresses were sold in the four weeks? 1 mark

_____

_____

The profit in dollars on each item is given by the matrix below.

$$\begin{matrix} J \\ D \\ T \\ S \end{matrix} \begin{bmatrix} 12 \\ 10 \\ 8 \\ 1.20 \end{bmatrix}$$

d. Calculate the total profit over the first four weeks. 2 marks

_____

_____

_____

_____

_____

_____

Question 2 (3 marks)

Tanya expands and a larger second store is opened. After twelve months, the sales at each store reach equilibrium. Unfortunately the individual monthly profit from each store cannot be calculated since the monthly figures have accidentally been combined and scrambled.

However, it is known that the profit over three months from the first store plus five months of profit from the second store is equal to $27 500. The profit over six months from the first store plus ten months of profit from the second store is equal to $55 000.

a. Explain why this information is insufficient to find the profit each month. 1 mark

_____

_____

Another set of figures is discovered. Four months profit from the first store plus three months from the second store is equal to $22 000.

b. Use this information to complete the following.

Let P_1 = the monthly profit of the first store and P_2 = the monthly profit of the second store. 1 mark

$$\begin{bmatrix} \_\_\_\_\_ & \_\_\_\_\_ \\ \_\_\_\_\_ & \_\_\_\_\_ \end{bmatrix} \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix} = \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix}$$

c. Solve to find the values of P_1 and P_2. 1 mark

_____

_____

_____

_____

Question 3 (4 marks)

Tanya decides to open an online store offering the same items and this disrupts her monthly sales figures across both stores. The total sales for the first month at each store are shown below.

$$S_1 = \begin{matrix}\text{store 1}\\ \text{store 2}\\ \text{online}\end{matrix}\begin{bmatrix}850\\1510\\340\end{bmatrix}$$

The total sales figures change according to this transition matrix.

$$T = \begin{bmatrix}0.2 & 0.3 & 0.4\\ 0.3 & 0.4 & 0.4\\ 0.5 & 0.3 & 0.2\end{bmatrix}$$

a. Find S_3. 1 mark

_____

_____

_____

Tanya wants a long term prediction for how the three stores will be trading in five years.

b. Perform the necessary matrix calculations to see if a steady state has been reached after five years. Comment on the results you find. 2 marks

_____

_____

_____

_____

_____

c. The new business plan assumes an increase of 30 sales per month in stores 1 and 2, and an increase of 50 online sales.

Complete S_{n+1} for the above business plan, below. 1 mark

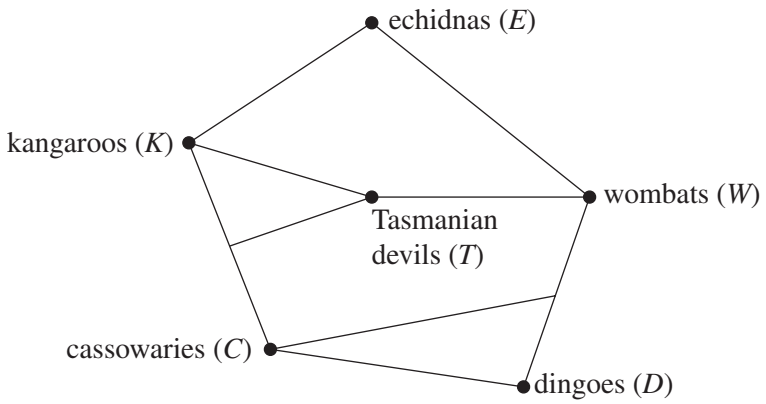
$$S_{n+1} = \begin{bmatrix}\_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\\ \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\\ \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_\end{bmatrix} S_n + \begin{bmatrix}\_\_\_\_\_\\ \_\_\_\_\_\\ \_\_\_\_\_\end{bmatrix}$$

END OF MODULE 1

Module 2 – Networks and decision mathematics

Question 1 (2 marks)

The map below shows the paths connecting the Australian animal exhibits at a zoo.



a. Starting at the kangaroos, which exhibits can be reached directly by using only one path? 1 mark

_____

b. The map is represented by a graph with the exhibits represented by vertices and the paths represented by edges.

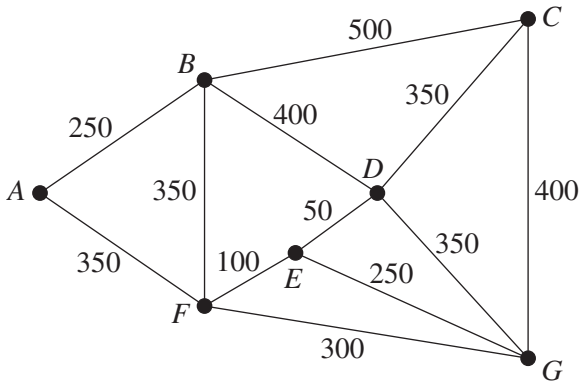
Between which two pairs of vertices are there multiple edges? 1 mark

_____

_____

Question 2 (2 marks)

The zoo has a large area containing wild cat exhibits. The graph below shows the paths connecting these exhibits at the zoo with the lengths of the paths shown in metres.

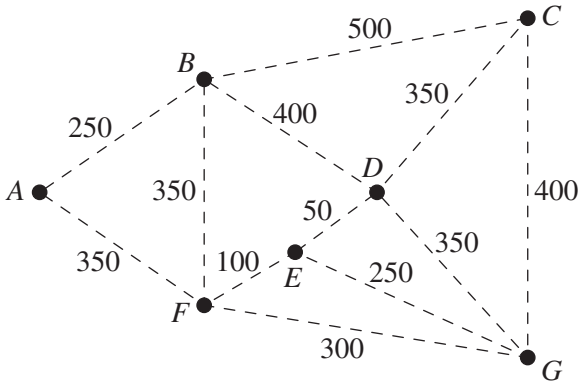


In winter the zoo is open during the night. Electric cabling needs to be installed to connect all of the enclosures in this area for a light and sound show. The cable is run along the edge of the paths.

a. Determine the minimum length of cable required. 1 mark

_____

b. In the diagram below, draw how the cable should be arranged to minimise the total length. 1 mark



Question 3 (3 marks)

Four workers at the zoo are allocated tasks in order to have the wild cat exhibits ready for the night time opening. Alvita (*A*), Bronte (*B*), Cy (*C*) and Dean (*D*) must together complete the four tasks in the minimum possible time. The estimated times in minutes for each worker to complete each task are shown in the table below.

	Task 1	**Task 2**	**Task 3**	**Task 4**
A	50	80	70	80
B	60	40	80	70
C	50	70	60	90
D	40	30	70	80

After row and column reductions are performed, the table becomes as below.

	Task 1	**Task 2**	**Task 3**	**Task 4**
A	0	30	10	0
B	20	0		0
C	0	20	0	10
D	10	0	30	20

a. Determine the entry that should appear in the shaded cell. 1 mark

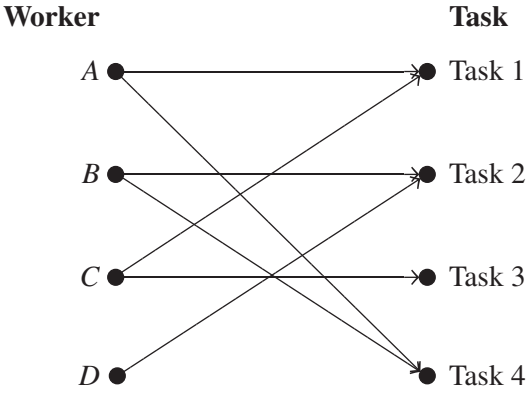
_____

b. Explain why it is possible to allocate tasks so as to minimise the total possible time to complete the four tasks. 1 mark

_____

_____

c. After row and column reductions are performed, the following bipartite graph is obtained.

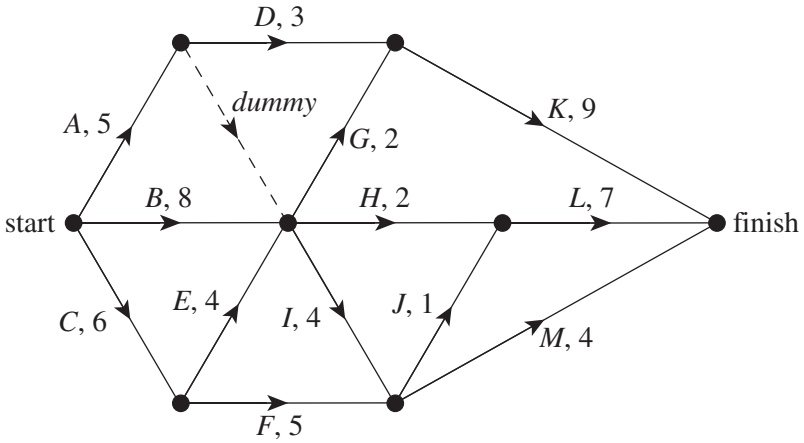


Complete the following table with the number of the task that each person should be allocated in order to minimise the total overall time. 1 mark

Worker	Task
A	
B	
C	
D	

Question 4 (5 marks)

The hippopotamus enclosure requires renovations prior to the night openings. The renovation project has activities *A* to *M*. The activity network for this project is shown below with the numbers on the edges showing the duration in days.



a. Write down the immediate predecessors of activity *H*. 1 mark

_____

b. Determine the critical path for this project. 1 mark

_____

c. Which non-critical activities could have their duration increased by 5 days without delaying the project. 1 mark

_____

_____

d. The overall time for this project can be reduced by putting extra resources into activity *C* or activity *L*. Either activity can be reduced by 2 days.

i. Determine the critical path or paths that will result in the minimum completion time for the project if activity *C* or activity *L* is reduced by 2 days. 1 mark

_____

_____

_____

_____

ii. What is the minimum completion time for this project in this case? 1 mark

_____

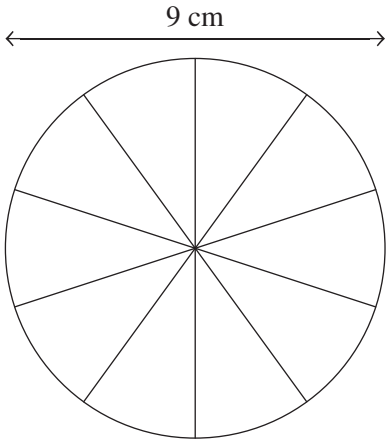
END OF MODULE 2

Module 3 – Geometry and measurement

Question 1 (2 marks)

James Roberts produces a variety of products for the hospitality industry in his factory ‘Easy Catering'.

One of his products is a conical spice holder for the kitchens. The spice holder contains 10 equal segments. The base of the cone is illustrated in the diagram below.



a. Three of the segments are to be colour coded green around the edge.

How far around the circumference of the circular lid will be coloured green?

Round your answer to two decimal places. 1 mark

_____

_____

b. Given that the spice holder has a height of 21 cm, calculate the volume of the spice holder.

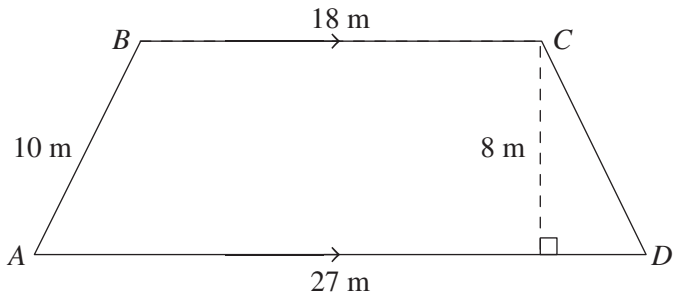
Round your answer to zero decimal places. 1 mark

_____

_____

Question 2 (2 marks)

The diagram below shows the floor plan of a production room in the 'Easy Catering' factory. *AD* and *BC* are parallel walls



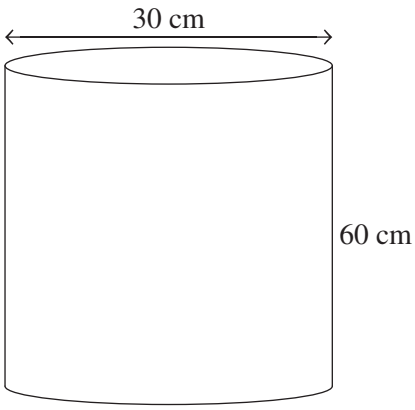
A security camera will be fixed to point *D* in order to sweep the room.

Calculate the angle *CDA*.

Round your answer to one decimal place.

Question 3 (3 marks)

James Roberts also sells beverages in cylindrical kegs. They decide to double the amount of material used in the production of the kegs. This will double the surface area of the kegs.



The kegs are mathematically similar.

a. If the original kegs held 30 L, how many litres will the new kegs hold?
Round your answer to one decimal place. 2 marks

_____

_____

_____

_____

Another beverage in the range is sold in the original cylindrical keg as shown in the above diagram.

b. Calculate the surface area of the keg.
Round your answer to one decimal place. 1 mark

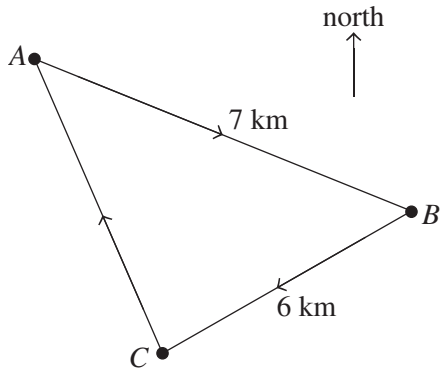
_____

_____

Question 4 (3 marks)

Leroy is competing in the annual 'Easy Catering' team gathering. Employees take part in a race as part of a team-building exercise.

He starts at point *A*, runs a course to points *B* and *C*, and finally returns to point *A*. This route is shown in the diagram below.



a. Show that Leroy has to travel 7.5 km on the final leg of the race.

Round your answer to one decimal place. 1 mark

_____

_____

b. Calculate the bearing for the final leg of the race. Use 7.5 km as the length of the final leg.

Give your answer to the nearest degree. 2 marks

_____

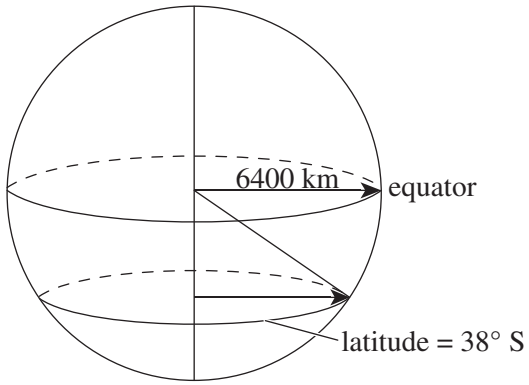
_____

_____

_____

Question 5 (2 marks)

Edward is responsible for international sales for 'Easy Catering' and is in Geelong (38° S, 144° E). Matilda, a potential new client, is in Auckland (38° S, 174° E).



Find the distance along the parallel of latitude 38° S from Geelong to Auckland.

Round your answer to the nearest ten kilometres.

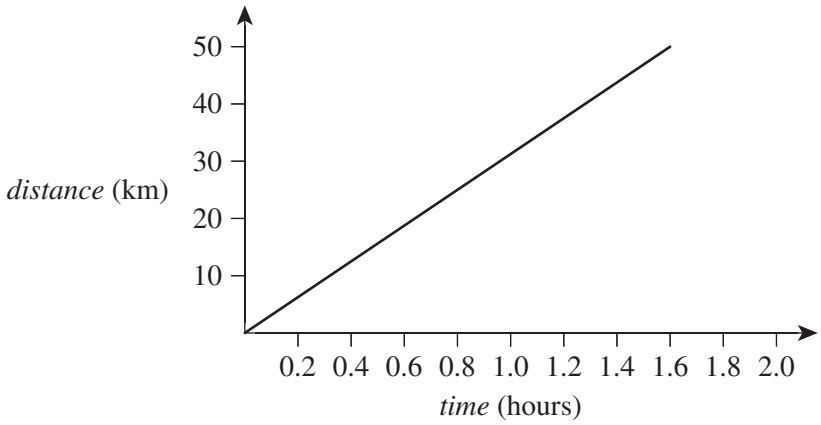
END OF MODULE 3

Module 4 – Graphs and relations

Question 1 (3 marks)

John and Priti are competing in a cycle race from Bendigo to Castlemaine. The route consists of a 50 km cycle to Castlemaine and then a 50 km return trip to Bendigo.

The graph below shows John's progress on the first leg. The slope represents John's speed in km/h.



a. How fast in km/h does John travel on the first leg of the race?

Round your answer to two decimal places. 1 mark

_____

_____

b. John has a 10 km lead when Priti begins her return trip to Bendigo. John is travelling at 17 km/h and Priti is travelling at 26 km/h.

How long will it take Priti to catch John?

Round your answer to the nearest minute. 2 marks

_____

_____

_____

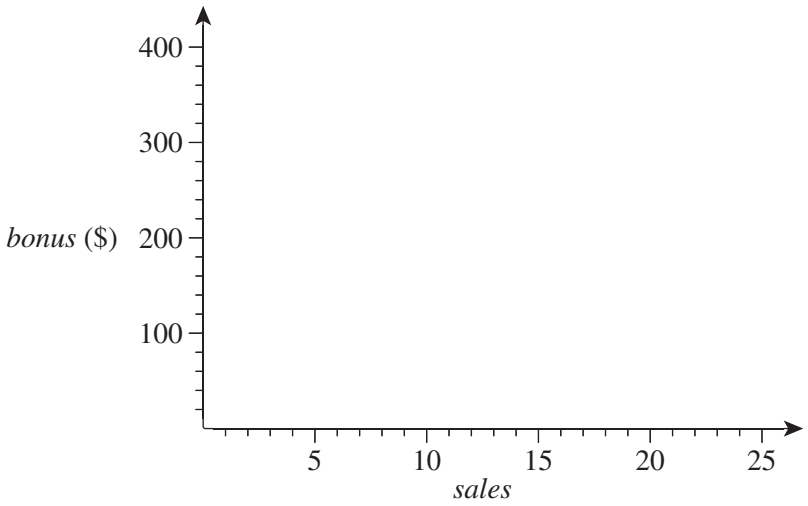
_____

Question 2 (4 marks)

John works for a domestic refrigeration retailer called 'Fantastic Fridges'. John earns a bonus payment for the sales he makes in a calendar month. His bonus increases as he makes more sales. His bonus agreement is shown in the following table.

Sales	Bonus ($)
1–9	80
10–12	140
13–16	240
17–20	320
21+	400

a. Represent this information on the axes below 1 mark



Priti joined the company before John on the previous bonus scheme. She receives $15 per sale.

b. How many sales would John have to make to receive the same bonus per sale as Priti? 1 mark

_____

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Priti launches a radio advertising campaign for 'Fantastic Fridges'. The cost, in dollars, of running the campaign can be found using the equation $C = 800 + 300n$, where C is the cost and n is the number of weeks of radio advertisements. The extra sales revenue from the campaign is predicted to be $500 per week.

c. If Priti runs the advertising campaign for 11 weeks, what would be the cost? 1 mark

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d. How many weeks would Priti have to run the advertising campaign in order to break even? 1 mark

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Question 3 (5 marks)

One of the suppliers to 'Fantastic Fridges' is 'Cool Runnings'. 'Cool Runnings' produces both fridge and freezer units. Each month, up to 120 fridges or freezers can be produced.

The inequalities below represent constraints on the number of each per month. The number of fridges produced each month is represented by x, and the number of freezers produced each month is represented by y.

Constraint 1 $x \geq 0$

Constraint 2 $y \geq 0$

Constraint 3 $x + y \leq 120$

Constraint 4 $y \leq \frac{x}{3}$

a. Interpret Constraint 4 in terms of the number of fridges and the number of freezers produced each month. 1 mark

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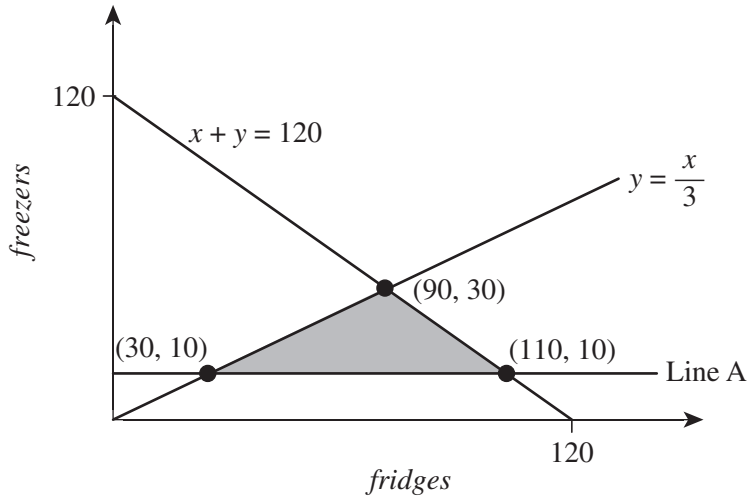
A fifth constraint based on production numbers is added and is labelled as 'Line A' in the following graph.



b. Write down the inequality that represents Constraint 5. 1 mark

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The profit, P, that the company can make from the sale of fridges and freezers is given by $P = 130x + 150y$.

c. Find the maximum profit that 'Cool Runnings' make from the sale of fridges and freezers. 1 mark

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d. The company wants to increase maximum profits to \$18 000. Using the same constraints, the new profit, Q, that is made is given by $Q = mx + ny$.

If the maximum profit of \$18 000 is given by selling 100 fridges and 20 freezers, then find the values of m and n. 2 marks

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END OF QUESTION AND ANSWER BOOKLET