2018

VCE

Further Mathematics Trial Examination 2

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STUDENT NUMBER									Letter

VICTORIAN CERTIFICATE OF EDUCATION
2018
FURTHER MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section A - Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	9	9	36
Section B - Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 32 pages.
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Section A – Core

Instructions for Section A

Answer all questions in the spaces provided. Write using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

Question 1 (5 marks)

The number of guests who stayed at a particular motel each month in 2017 is given in the table below.

Month	Jan	Feb	March	April	May	June	July	Aug	Sep	Oct	Nov	Dec
Number guests	600	300	450	600	482	206	154	186	192	320	580	600

a.

(i) What is the median number of people who stayed at this motel in 2017?

1 mark

(ii) In what percentage of months were there at least 300 guests staying at this motel? Give your answer to two decimal places.

1 mark

Question 1(continued)

Mr Troy, the motel owner, buys packets of cereal to serve to his guests for breakfast. Over a long period of time he notices that boxes of cereal labelled 500g have weights that are normally distributed with a standard deviation of 5 g.

b.

(i) What percentage of boxes will be underweight if the mean weight of the boxes of cereal is 495g?

____

1 mark

(ii) In a random sample of 600 boxes how many would be expected to weigh more than 505g if the mean weight of boxes of cereal is 495g?

____

1 mark

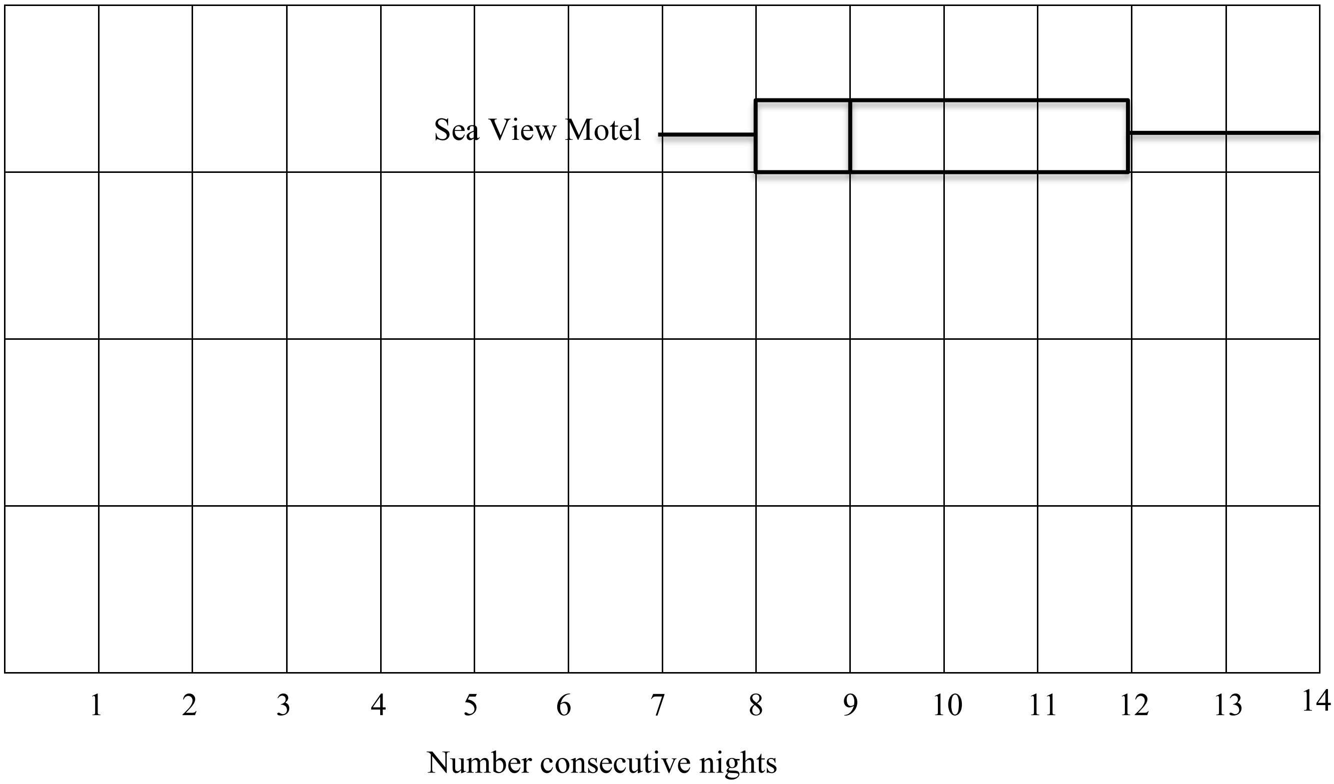
(iii) What would be the mean weight of the boxes of cereal labelled 500g if 0.15% were underweight?

____

1 mark

Question 2 (8 marks)

The box plot below shows the distribution of the number of consecutive nights that guests stayed at the Sea View motel during 2017.



a. What percentage of guests stayed more than 12 consecutive nights at the Sea View motel?

1 mark

Question 2 (continued)

Number of consecutive nights at Mountain View motel				
8	4	1	2	7
12	4	7	14	3
6	3	2	5	2
3	2	3	6	5
1	1	1	4	4

The table above shows the number of consecutive nights that guests stayed at the Mountain View motel.

b.

(i) What is the interquartile range for the number of consecutive nights that guests stay at the Mountain View motel?

___

1 mark

(ii) Show that the 14 night consecutive stay at the Mountain View motel is an outlier.

___

___

1 mark

(iii) On the graph on Page 4, draw a box plot to show the distribution of the number of consecutive nights guests spent at the Mountain View motel.

2 marks

Question 2 (continued)

c. Use the information from both boxplots to compare the centre and spread of the two distributions.

(i) Centre

1 mark

(ii) Spread

1 mark

d. Use the information from the boxplots to explain why the number of consecutive nights spent at each of these motels is associated with the motel. Quote the values of appropriate statistics in your answer.

1 mark

Question 3 (6 marks)

The shop attached to the Sea View motel sells ice creams. The table below shows the number of ice creams sold and the maximum temperature on the day of the sale.

Maximum Temperature ^{0}C	Number ice creams sold
12	200
8	150
20	250
24	200
30	350
15	150
6	50
18	250
22	300
10	100

a.

(i) Which variable in the above table is the explanatory variable?

____

1 mark

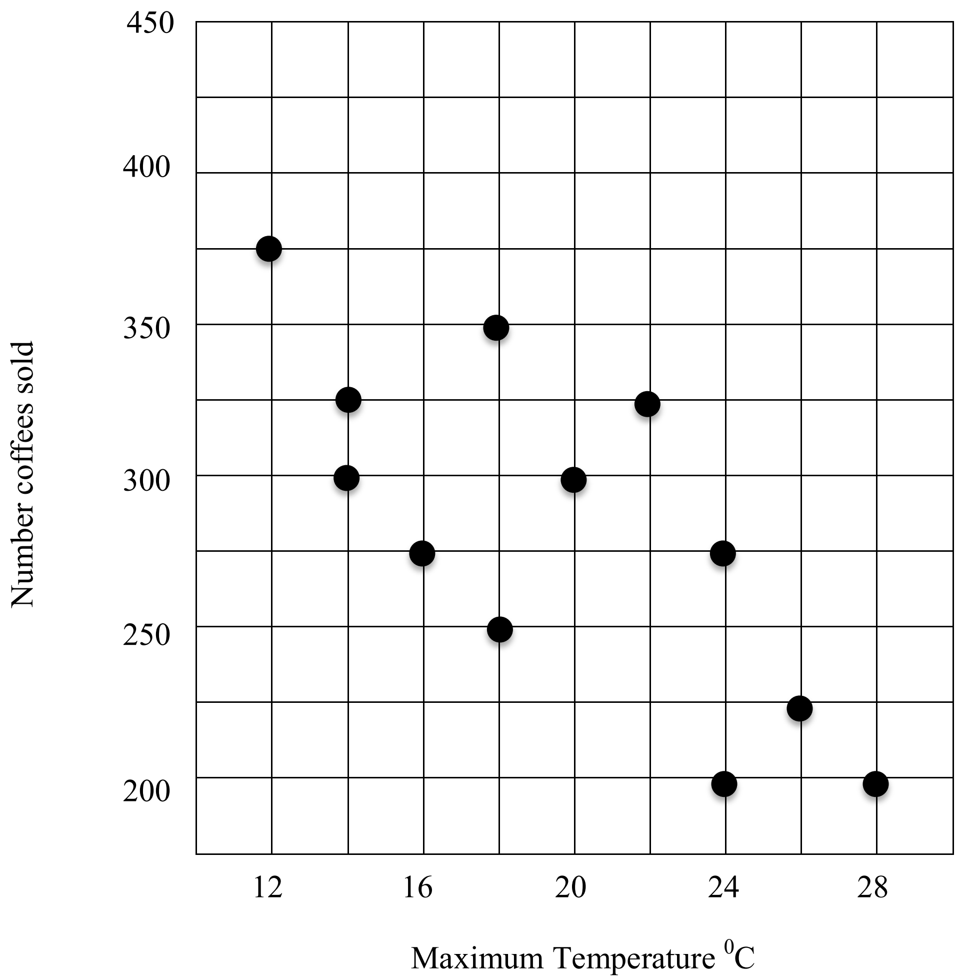
(ii) Determine the equation for the least squares line that can be used to predict the number of ice creams sold from the maximum daily temperature. Give all values in this equation to one decimal place.

____

1 mark

Question 3 (continued)

b. The scatterplot for the number of coffees sold at this shop and the maximum temperature on that day is shown below.



(i) Draw the graph of this least squares line on the scatterplot above.

1 mark

(ii) Interpret the slope of the regression line in terms of the variables number of coffees sold and the maximum temperature of the day.

____

____

1 mark

Question 3 (continued)

b.

(iii) Determine the residual value if the least squares line is used to predict the number of coffees sold when the maximum temperature is 20^0C.

___

1 mark

(iv) What is the percentage variation in the number of coffees sold that is explained by the variation in the maximum temperature of the day? Give your answer to three significant figures.

___

1 mark

Question 4 (5 marks)

Month	Jan	Feb	March	April	May	June	July	Aug	Sep	Oct	Nov	Dec
Bananas sold (kg)	710	480	450	300	620	490	400	320	780	550	480	360

The above table shows the number of kilograms of bananas sold by the shop each month in a certain year.

a. What is the four-mean number of kilograms of bananas sold, centred on July?

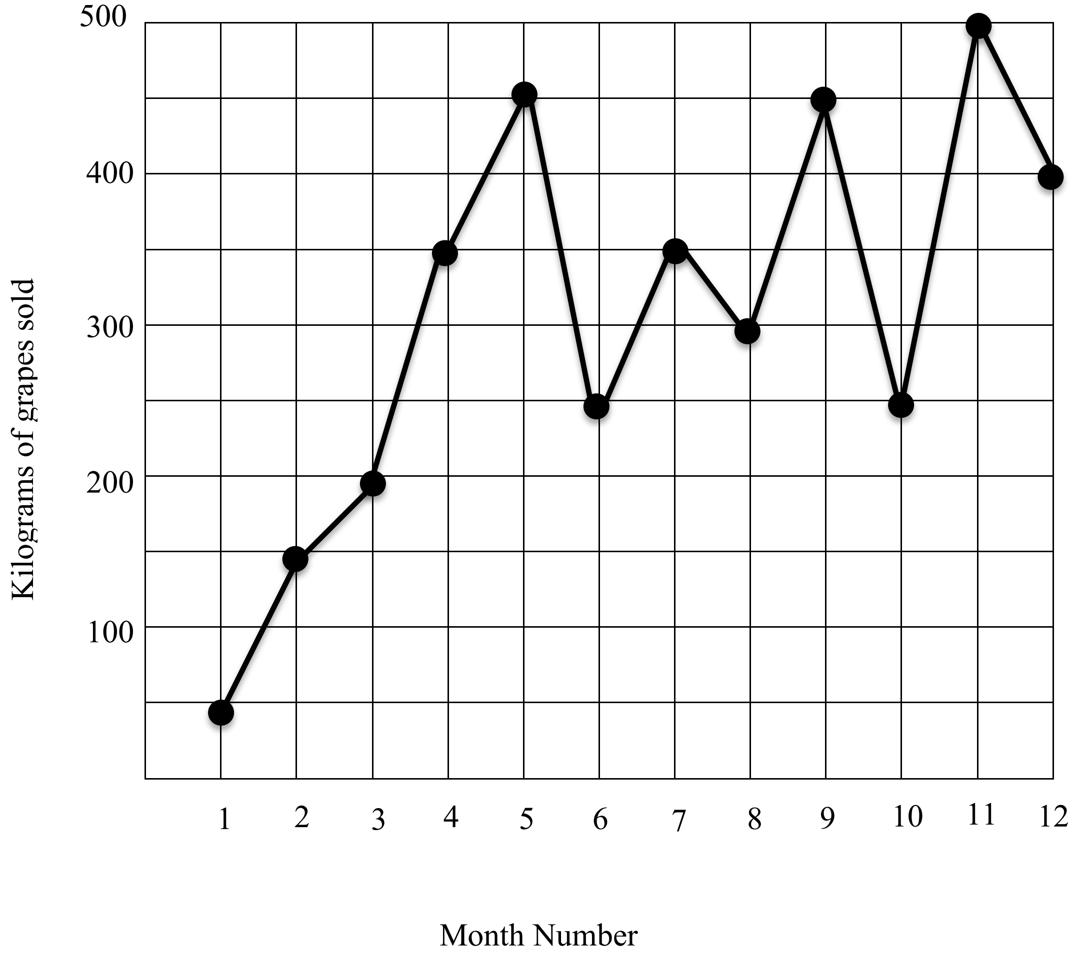
___

___

2 marks

Question 4 (continued)

The time-series plot below shows the number of kilograms of grapes sold each month by this shop. January = 1, February = 2 and so on.



b.

(i) Apply a three-median smoothing to the time series plot above. Show your points for this smoothing line with crosses, (X).

2 marks

(ii) Describe the general pattern in the number of kilograms of grapes sold each month that is revealed by the smoothed pattern.

___

___

1 mark

Recursion and financial modelling

Question 5 (5 marks)

Sophie started work at A.J. Smith's in 2006. Her annual salary, A_n increased by \$1200 each year until in 2010 it was \$84800

a.

(i) What was her annual salary, A_0, for the year 2006?

____

1 mark

(ii) Write down a recurrence relation, which will model Sophie's salary, A_n, after n years.

____

1 mark

(iii) What salary will Sophie receive in her 7^{th} year working for this firm?

____

1 mark

b. After working for the firm for 10 years, Sophie receives a 5% increase in salary for the next five years.

(i) Write down a recurrence relation to model the value of her new salary structure, V_{n+1}

____

1 mark

(ii) What is her salary during the fifth year of this new salary structure? Give your answer to the nearest cent.

____

1 mark

Question 6 (4 marks)

Sophie buys a car for \$52000. She has saved some money but needs to borrow ¾ of the money from the bank at a rate of 4% per annum compounding quarterly. She agrees to repay the money in equal quarterly repayments over 8 years.

a.

(i) What is the quarterly interest rate she is charged by the bank?

____

1 mark

(ii) How much interest does she pay altogether? Give your answer to the nearest cent.

____

1 mark

b.

(i) After three years of repayments, Sophie is in a situation where she can repay \$2000 a quarter. How much money will she owe at the end of the three years before she starts repaying \$2000 per quarter? Give your answer to the nearest cent.

____

1 mark

(ii) By how many months will her repayments be reduced? Give your answer to one decimal place.

____

1 mark

Question 7 (3 marks)

A. J. Smith purchases a van for use in the business. The van costs \$38000 and is depreciated using the unit cost method based on the number of kilometres travelled. The van has a scrap value of \$6000 after 8 years and travels 50000 kilometres per year.

a. By how much is the value of the van depreciated per kilometre?

____

1 mark

b. Write an equation for the value of the van, V_n, after n years of travel.

____

1 mark

c. How many kilometres would the van have travelled when it was worth \$2000?

____

1 mark

Section B – Modules

Instructions for Section B

Select two modules and answer **all** questions within the selected modules.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Contents Page

Module 1 - Matrices

Question 1 (4 marks)

The price of tomatoes at three supermarkets, **A**pex, **B**est and **C**are, is different in three different suburbs, **R**osemount, **S**urrey and **T**emple. Matrix Y shows the price per kilogram of tomatoes in each of the different stores. The prices are given in dollars.

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ Y = & \begin{bmatrix} 3.70 & 3.60 & 3.40 \\ 4.20 & 4.00 & 4.10 \\ 8.60 & 6.20 & 7.00 \end{bmatrix} \begin{array}{c} R \\ S \\ T \end{array} \end{array}$$

If the elements of Y are $y_{i,j}$

a. What is the element $y_{2,3}$?

____

1 mark

b. Using the information given above, where could a person buy tomatoes at the cheapest price?

____

1 mark

The three supermarkets have a sale of their tomatoes. Apex discounts its tomatoes by 5%, Best gives a discount of 10% and the discount at Care is 20%.

c. A consumer watchdog wishes to compare the total sale price of these tomatoes in each suburb.

Their researchers multiply matrix, Y, by matrix $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

What are the values of a, b and c where these pronumerals are numbers between 0 and 1?

____

1 mark

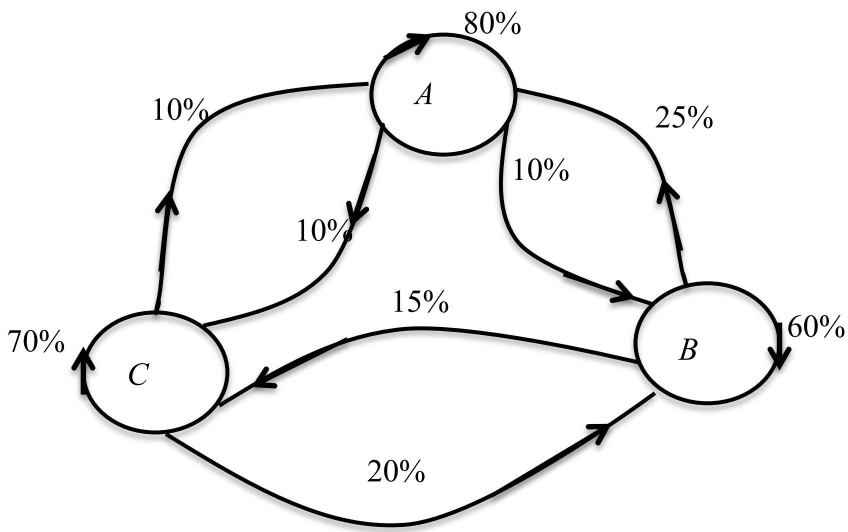
Question 1 (continued)

d. What price would a shopper pay during this sale if he bought a kilogram of tomatoes in each of the stores in Temple?

1 mark

Question 2 (3 marks)

People in Rosemount change the supermarket that they go to within their suburb each week. The transition diagram below shows how these people are expected to change their choice of supermarket.



a. Of the people from Rosemont who choose the Best supermarket one week, what percentage of them is expected to NOT choose Care supermarket the next week?

1 mark

Question 2 (continued)

Matrix W lists the number of people who will choose each supermarket in week one.

$$W = \begin{array}{c} \begin{bmatrix} 16000 \\ 14000 \\ 10000 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \end{array}$$

b. Find the values of a, b and c in the following calculation that could be used to find the number of people who would use the Best supermarket next week.

$$a \times 16000 + b \times 14000 + c \times 10000 = 12000$$

____

1 mark

c. How many people would be expected to shop at either Apex or Care in week four?

____

1 mark

Question 3 (5 marks)

Each week at Care supermarket, the staff is moved about the four different sections, groceries, vegetables, dairy and meat. The transition matrix, T, shows the way in which this is done.

$$\begin{array}{cc} & \textit{this week} \\ \textit{next week} & \begin{bmatrix} 0.5 & 0.2 & 0.1 & 0.05 \\ 0.2 & 0.6 & 0.15 & 0.1 \\ 0.25 & 0.1 & 0.7 & 0.05 \\ 0.05 & 0.1 & 0.05 & 0.8 \end{bmatrix} \begin{matrix} G \\ V \\ D \\ M \end{matrix} \end{array}$$

Let S_n be the state matrix for the number of staff expected to work in each section of the supermarket in week n.

$$S_1 = \begin{matrix} G \\ V \\ D \\ M \end{matrix} \begin{bmatrix} 16 \\ 12 \\ 8 \\ 5 \end{bmatrix}$$

The matrix used by the supermarket to move staff is $S_{n+1} = TS_n$

a. How many staff will be moved from week one to week two?

____

1 mark

b. What is S_2, the state matrix for week two?

$$\begin{matrix} G \\ V \\ D \\ A \end{matrix} \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \end{bmatrix}$$

1 mark

Question 3 (continued)

c. Of the staff working in vegetables in week three, what percentage of them worked in dairy in week two? Give your answer to one decimal place.

___

___

___

2 marks

d. What is the minimum number expected to work in the grocery section in any of the first six weeks?

___

___

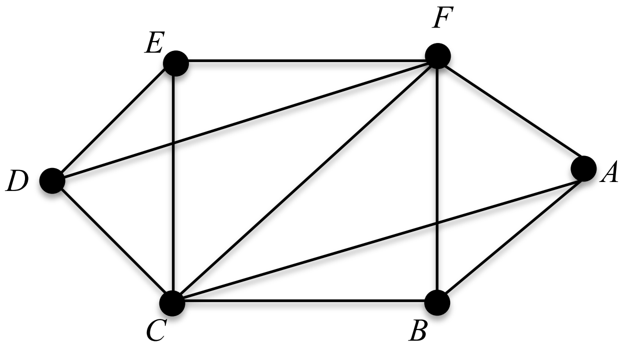
1 mark

End of Module 1: Matrices

Module 2: Networks and decision mathematics

Question 1 (5 marks)

A factory, F, supplies coffee beans to five cafes, A, B, C, D and E in Jamestown.
The network diagram below shows the roads that connect the cafes and the factory.



a. Complete the following adjacency matrix for this network to show the roads linking the various cafes and the factory.

$$\begin{array}{cc} & \begin{array}{cccccc} F & A & B & C & D & E \end{array} \\ \begin{array}{c} F \\ A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \end{array}$$

1 mark

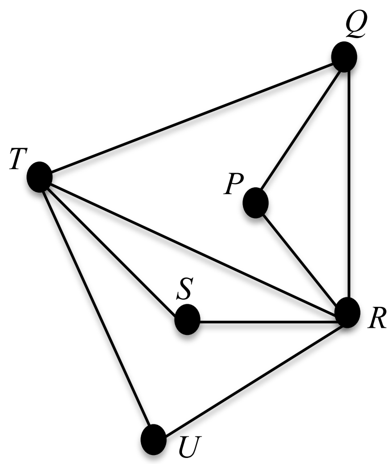
Question 1 (continued)

b. The driver wishes to leave the factory, visit each of the cafes just once and then return to the factory. Show the different ways that this could be done.

___

2 marks

c. The council of Jamestown wants to remove fallen leaves from the streets joining prominent community buildings. The streets and the buildings, P, Q, R, S, T and U are shown in the network below.



(i) Is it possible for the council worker to start at building, T, and clear the leaves from each street by only moving along each street once? Give a reason for your answer.

___

___

1 mark

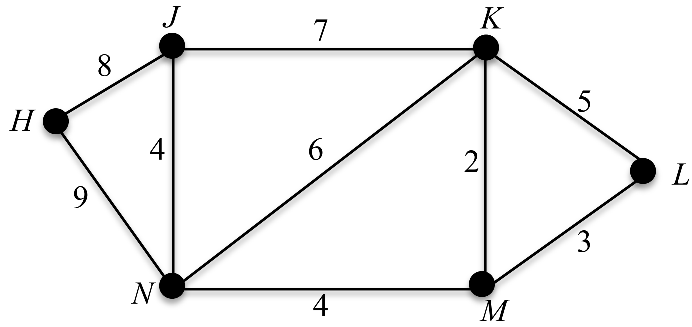
(ii) Use Euler's formula to show that this is a planar graph.

___

1 mark

Question 2 (2 marks)

Restaurants H, J, K, L, M and N in Jamestown need to be connected to the Internet by cable.
The distances in kilometres for the cable between these restaurants are shown in the network below.



a. Draw the spanning tree that has minimal length for the network in the space below.

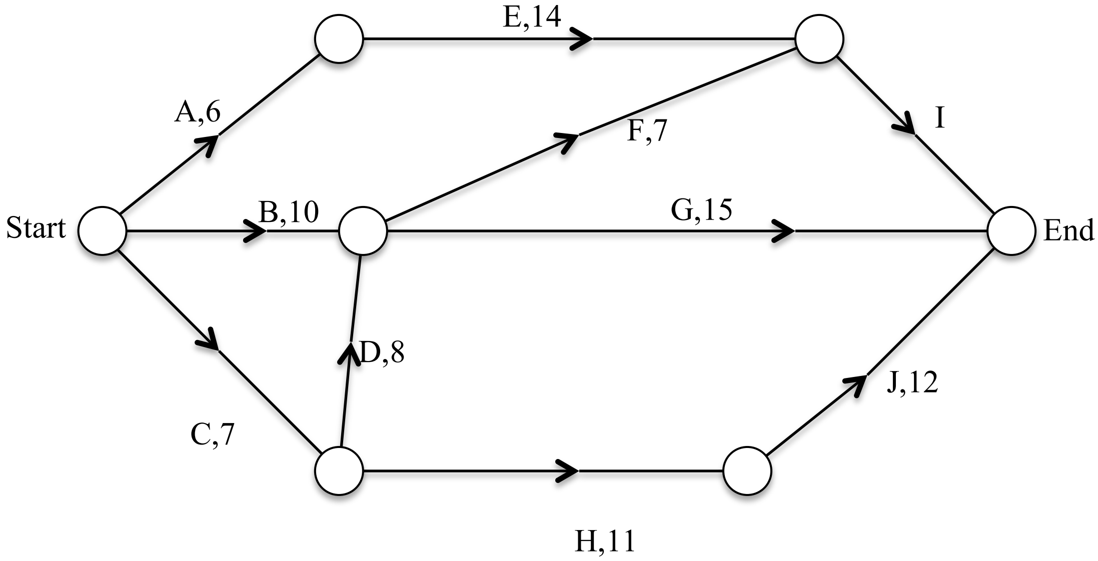
1 mark

b. What is the minimum length of cable required for this job?

____

1 mark

Question 3 (5 marks)



The above network shows the activities that must be completed before the cable for the Internet is ready for use.

a. List all the activities must be completed before activity G can begin?

____

1 mark

The minimum time for the project to be completed is 31 days.

b. What is the minimum time that activity I can take to complete?

____

1 mark

Question 3 (continued)

If the cable company employs more workers, the project can be completed in a shorter time. This is good for the company, because it receives a bonus for every day saved. However, it does cost more money to employ extra workers, and these workers can only do specific activities. **Table 1** shows the details of this information.

Table 1

Activity	Maximum Reduction (days)	Cost /employee/day ($)	Bonus/day saved ($)
A	4	1000	4000
D	3	2000	4000
F	5	1000	4000
G	6	1000	4000

c. What is the shortest time that the cable could now be laid?

1 mark

d. If it takes one extra employee for every day of reduction in time for an activity, how many new workers should be employed to save as much time as possible?

1 mark

e. What is the maximum amount that the company will be able to save by employing extra people?

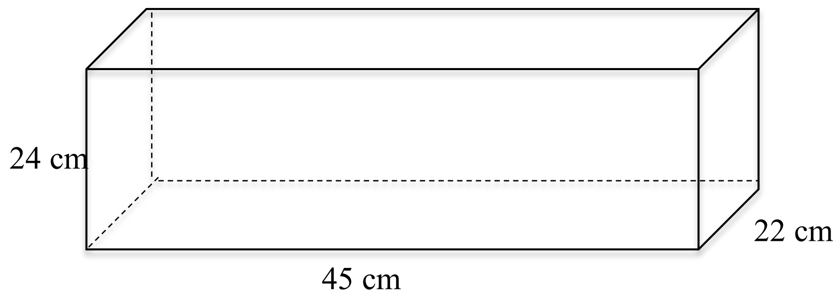
1 mark

End of Module 2: Networks and decision mathematics

Module 3: Geometry and measurement

Question 1 (3 marks)

The Gordon family is going on a picnic. The container used to carry their food is shown below.

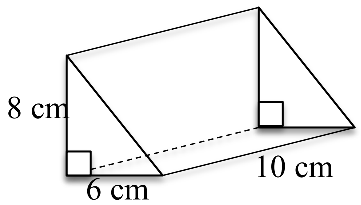


a. What is the volume of this container?

____

1 mark

A smaller container, used to hold sandwiches, is shown below.



b. What is the surface area of this container?

____

____

1 mark

c. What is the length of the longest bread stick that can be packed in the food container? Give your answer to one decimal place.

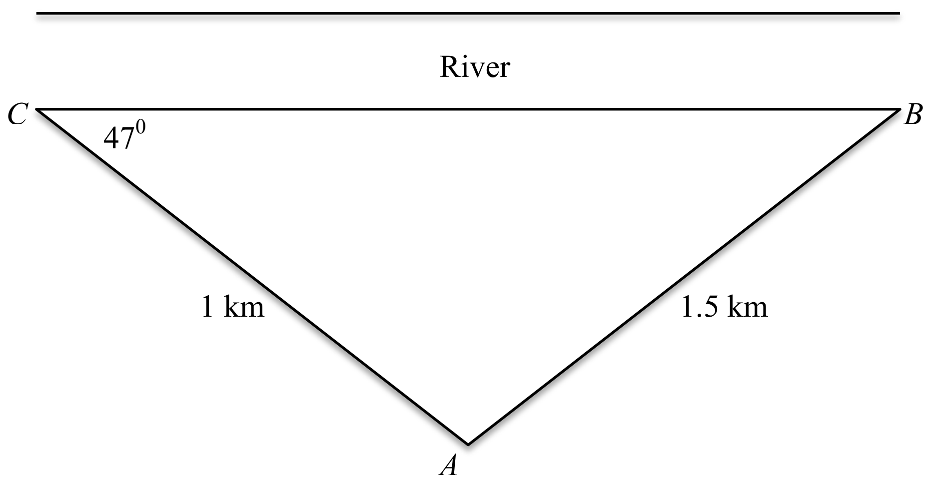
____

____

1 mark

Question 2 (3 marks)

The triangular shaped picnic ground, *ABC*, is on the banks of a river.



$\angle BCA = 47^0$, $AC = 1$ km and $AB = 1.5$ km

a. What is the size of $\angle ABC$? Give your answer to the nearest degree.

1 mark

b. What is the length of *BC*? Give your answer to the nearest kilometre.

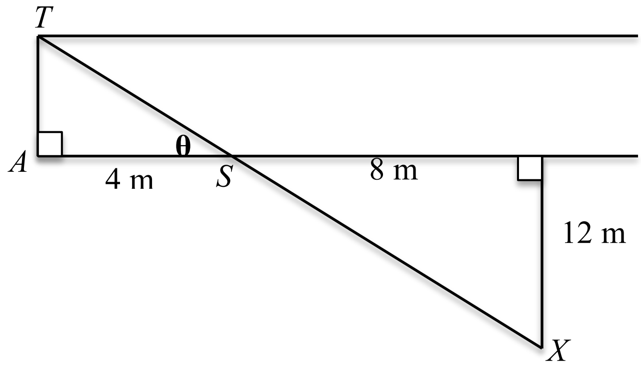
1 mark

c. What is the area of the picnic ground? Give your answer to two significant figures.

1 mark

Question 3 (4 marks)

Mr. Gordon is interested to know the width of the river. He notices a tree, *T*, on the other bank of the river and lines himself up directly across from this tree. He then walks 4 m along his side of the river and inserts a stake, *S*. He walks another 8 metres and then walks 12 metres away from the river but perpendicular to it until he reaches *X*.



a. What is the distance across the river, *AT*?

1 mark

b. What is the size of angle θ? Give your answer to the nearest degree.

1 mark

Question 3 (continued)

c. What is the distance from the bottom of the tree to the stake? Give your answer to one decimal place.

1 mark

d. If the tree is 5 m high, what is the angle of elevation from X to the top of the tree? Give your answer to the nearest degree.

1 mark

Question 4 (2 marks)

Two boats pass on the river. Boat A, travelling due west has a speed of 10 km/hr. Boat B, travelling at 3 km /hr. has a bearing of 110^0 T.

a. What is the angle between the two boats 10 minutes after they pass each other?

1 mark

b. What is the distance between the boats 10 minutes after they pass each other? Give your answer to the nearest kilometre. Ignore any currents in the water.

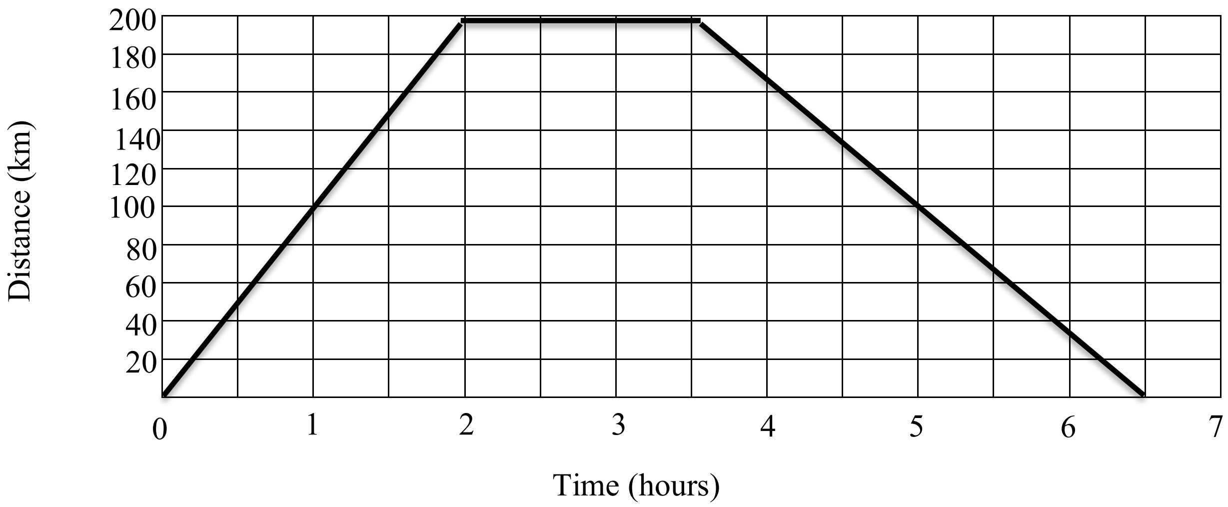
1 mark

End of Module 3: Geometry and measurement

Module 4: Graphs and relations

Question 1 (3 marks)

Sam and Josie work for the same company and travel to country towns to sell dishwashers.
Sam's journey to a country town from his office on one particular day is shown on the graph below.



a. What is Sam's speed on his way to the country town?

1 mark

Josie leaves from the same office, one hour after Sam. She travels at 100 km/hr for one hour, to arrive at her destination, where she remains for one hour. She then drives back to the office at 50 km/hr.

b. How long was Josie away from the office?

1 mark

c. Show Josie's Journey on the above graph.

1 mark

Question 2 (3 marks)

Sam receives an annual salary of \$50000 for selling dishwashers and other white goods. Josie's annual salary for the same work is \$44660 plus a 20% commission on each sale of a dishwasher. Each dishwasher sells for \$890.

a. Write the equation for Josie's salary, P, when she sells n dishwashers in the year.

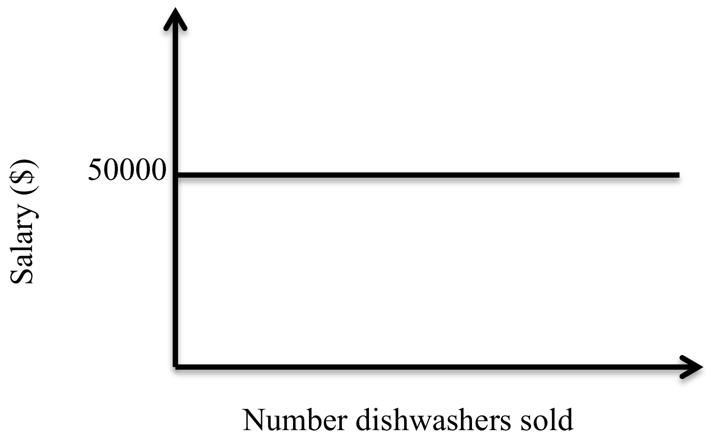
____

1 mark

b. How many dishwashers must Josie sell in a year to earn the same salary as Sam?

____

1 mark



c. On the graph above, sketch the equation for Josie's salary, showing two relevant points.

1 mark

Question 3 (6 marks)

The factory produces two models of dishwasher, standard and deluxe. These dishwashers have to pass through two different work areas. The table below shows the processing time at each work area.

Work Area	Unit Processing Time (days)		Total time available (days)
	Standard	Deluxe	
A	2	8	400
B	2	4	320

If the number of standard dishwashers produced = x and the number of deluxe dishwashers produced = y, then the following constraints exist. The fourth constraint is missing.

$$2x + 8y \le 400$$
$$y \ge 0$$
$$x \ge 80$$

a. Write down the missing constraint.

___

1 mark

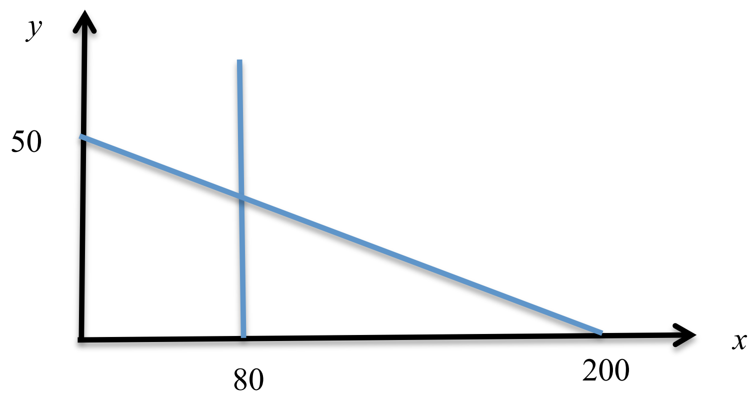
b. Explain the meaning of the constraint $x \ge 80$

___

1 mark

Question 3 (continued)

c. Complete the graph below showing the missing constraint.



1 mark

d. Shade the feasible region on this graph.

1 mark

The profit on a standard dishwasher is \$130 and the profit on a deluxe dishwasher is \$280

e. How many of each type of dishwasher should be made to maximise the profit on the dishwashers?

____

____

____

____

1 mark

f. What is this maximum profit?

____

1 mark

End of Module 4 : Graphs and relations

End of 2018 VCE Further Mathematics Trial Examination 2
Question and Answer Book

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FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score:	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line:	$y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value:	residual value = actual value – predicted value
seasonal index:	seasonal index$=\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core: Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1: Matrices

determinant of a 2×2 matrix:	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix:	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$
recurrence relation:	$S_0 =$ initial state, $\quad S_{n+1} = TS_n + B$

Module 2: Networks and decision mathematics

Euler's formula:	$v + f = e + 2$

Module 3: Geometry and measurement

area of a triangle:	$A = \frac{1}{2}bc\sin(\theta^0)$
Heron's formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$
sine rule:	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule:	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle:	$2\pi r$
length of an arc:	$r \times \frac{\pi}{180} \times \theta^0$
area of a circle:	πr^2
area of sector	$\pi r^2 \times \frac{\theta^0}{360}$
volume of a sphere:	$\frac{4}{3}\pi r^3$
surface area of a sphere:	$4\pi r^2$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a prism:	area of base $\times$ height
volume of a pyramid:	$\frac{1}{3}$ $\times$ area of base $\times$ height

Module 4: Graphs and relations

gradient (slope) of a straight line:	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line:	$y = mx + c$

END OF FORMULA SHEET