2017 VCE Further Mathematics 1 (NHT) examination report

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Core – Data analysis

Question	Answer
1	D
2	E
3	C
4	D
5	C
6	D
7	C
8	A
9	D
10	B
11	E
12	C
13	B
14	D
15	A
16	A
17	E
18	B
19	A
20	B
21	D
22	C
23	A
24	C

Question 2

$$\frac{6+7+8+9+8}{52} = 0.730... \approx 73\%$$

Question 5

On a log scale, a number equal to 10^n would be shown as n.

Hence, a population density of 10 is shown on the horizontal log scale as 1 since $10 = 10^1$.

The frequencies where population density was less than 10 compared to frequencies where log(population) is less than 1

= 2 + 5 = 7

Question 9

For a beak length of 10.5 mm, the value of $z = 2$.

The 69–95–99.7% rule indicates that approximately 95% of the beak lengths will be within two standard deviations either side of the mean. Therefore, 5% of the data is beyond two standard deviations either side of the mean, with 2.5% at either end of the bell curve.

Question 10

Find the approximate gradient of the line using two points on the graph.

For example, using (7, 7.35) and (10, 9), the gradient = $\frac{9-7.35}{10-7} = 0.55$

While there was no option that had a gradient of 0.55, there were four options with a gradient of 0.56. Since the question asked for the *closest* equation, accept a gradient of 0.56.

There were only two options where the gradient was 0.56 **and** the response variable was *width*. These were options B and C.

Option C showed an intercept of 7.4, which was approximately the value shown on the graph when *width* = 7. But the *intercept* must be where the line cuts the vertical axis when the explanatory variable = 0. Therefore the answer was option B.

It should be noted that the least squares line cannot be reliably extrapolated for beak lengths less than 7 mm, and it is obviously meaningless to use it to conclude that a beak with zero length must be 3.5 mm wide.

Question 12

The equation is best determined by using the linear regression functionality of technology, noting that the explanatory variable is in the **second** column of the table.

The scale on the graph was not sufficiently accurate to be able to read data points there.

Question 13

The graph shows seasonality, since there is an annual cycle of peaks and troughs in sales. A trend is not evident since there is no general upward or downward direction of the sales over the four years.

Question 14

To determine a four-mean smoothed value, find the centre (average) of two calculated four-mean values.

First, find the mean of the 2015 values for Quarter 1, Quarter 2, Quarter 3 and Quarter 4

$$= \frac{9.6 + 14.5 + 8.6 + 5.3}{4} = 9.5$$

Next, find the mean of the 2015 values for Quarter 2, Quarter 3 and Quarter 4 and the 2016 value for Quarter 1

$$= \frac{14.5 + 8.6 + 5.3 + 10.3}{4} = 9.675$$

The centre of these two values is $\frac{9.5 + 9.675}{2} = 9.5875$, which is closest to option D = 9.6.

Question 15

The seasonal index for Sunday must first be calculated from the understanding that the average seasonal index is one.

Let x be the seasonal index for Sunday

$$\therefore \frac{0.65 + 0.60 + 0.74 + 0.82 + 1.12 + 1.45 + x}{7} = 1$$

$$\therefore x = 1.62$$

The deseasonalised earnings for $3839 on Sunday is given by

$$\frac{\text{actual value}}{\text{seasonal index}} = \frac{3839}{1.62} = 2369.7\ldots \approx \$2370$$

Question 16

The seasonal index for Wednesday is 1 – 0.74 = 0.26 or 26% below the expected average of one.

Core – Recursion and financial modeling

Question 17

$A_0 = 2$

$A_1 = 3 \times A_0 - 3$

$= 3 \times 2 - 3 = 3$

and

$A_2 = 3 \times A_1 - 3$

$= 3 \times 3 - 3 = 6$

Now have the first three terms of the sequence shown only as option E.

Question 18

To increase an amount by 3.1% multiply (1 + 3.1%) or 1.031

Question 19

This models an investment that grows by a constant $80 each year. The options only allow for this addition to come from interest and the constant value indicates it is simple interest.

The rate of interest per annum is given by

$$\frac{80}{2000} = 0.04 = 4\%$$

Question 20

The entire interest earned is withdrawn whenever it is credited to a perpetuity. The account balance neither increases nor decreases.

Therefore, the interest earned on $80 000 is $3000 per year at a rate given by

$$\frac{3000}{80000} = 0.0375 = 3.75\%$$

Question 21

First work out the interest rate per payment from the details for the first payment:

$$\text{Rate} = \frac{720}{180000} = 0.004 = 0.4\% \text{ each repayment period}$$

Use this rate on the balance after the first repayment.

Interest = $0.4\% \times 179870.00 = 719.48$

Question 22

N=	36
I%=	3.6
PV=	–420 000
PMT=	**2499.99…**
FV=	372 934.71
P/Y= C/Y=	12

Question 23

Reducing balance means that $P_{n+1} = r \times P_n$ where $r < 1$

This leaves only option A.

Question 24

Calculate the rate of interest using $4418.80 as a PV for 18 months to give an FV of $4862.80

N=	18
I%=	**6.4000…**
PV=	–4418.80
PMT=	0
FV=	4862.80
P/Y= C/Y=	12

Use this rate to find the PV needed to give a FV of $4418.80 six months later.

N=	6
I%=	6.4
PV=	**–4280.000…**
PMT=	0
FV=	4418.80
P/Y= C/Y=	12

Module 1 – Matrices

Question	Answer
1	C
2	A
3	A
4	D
5	C
6	E
7	E
8	D

Question 2

The only matrix product to give a 1×1 matrix as a result was option A.

Question 5

The relevant equations are $5p + 4m = 31$ and $4p + 5m = 32$

This can be written as the matrix equation

$$\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} p \\ m \end{bmatrix} = \begin{bmatrix} 31 \\ 32 \end{bmatrix}$$

To solve this, pre-multiply both sides by the inverse of the square matrix.

Therefore

$\begin{bmatrix} p \\ m \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 31 \\ 32 \end{bmatrix}$ and so the first matrix equation is suitable.

The matrix equation can also be written as

$$\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} p \\ m \end{bmatrix} = \begin{bmatrix} 32 \\ 31 \end{bmatrix}`$$

Therefore

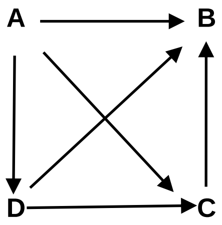
$$\begin{bmatrix} p \\ m \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 32 \\ 31 \end{bmatrix}$$

$$= -\frac{1}{9}\begin{bmatrix} 4 & -5 \\ -5 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 32 \\ 31 \end{bmatrix}$$

and so the third matrix equation is also suitable.

The other two options are unsuitable as one is not defined for multiplication and the other has an inverse matrix that does not match the values in the column matrix.

Question 7



If Amanda has 2 two-step dominances over Ben, she must have one via Carlos and one via Darius. This means that Darius beat Ben (option E) and Carlos beat Ben.

If Amanda then has a two-step dominance over Carlos, it must be via Darius, and not via Ben since he lost to Carlos.

If Ben had beaten Amanda, then both Carlos and Darius would have had two-step dominances over Amanda, neither of which show up in the matrix. Hence Amanda beat Ben.

The diagram above summarises these conclusions.

Question 8

The defined product of a row matrix × a column matrix produces a 1×1 matrix.

The product of the three matrices is required to give a 1×1 matrix. Therefore either:

- (a) the product of the first two matrices must be a row matrix and the third matrix must then be a column matrix
- (b) the first matrix must be a row matrix and the product of the last two matrices must then be a column matrix.

Looking at pairs of the given matrices, the product:

- *AR* is not defined
- *CA* is not defined
- *CR* is not defined
- *RC* can give a 1×1 matrix but then matrix *A* cannot be further included as a third matrix
- *AC* can give a column matrix
- *RA* can give a row matrix.

Using conclusion (a) above, *RA* × *C* is a row matrix × an $n \times n$ matrix ≠ a 1×1 matrix.

Using conclusion (b) above, *R* × *AC* is a row matrix × a column matrix = a 1×1 matrix (option D).

Module 2 – Networks and decision mathematics

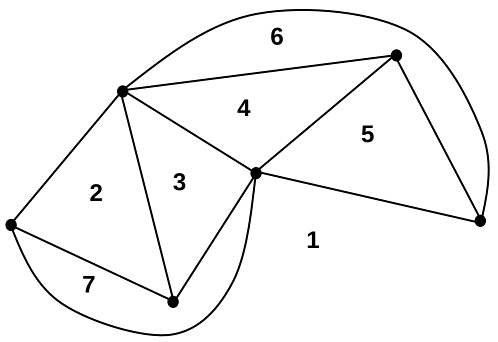
Question	Answer
1	A
2	C
3	C
4	E
5	D
6	B
7	C
8	B

Question 2

A *cycle* is a *path* that returns to its starting vertex without repeating edges or vertices. This is impossible in the second graph.

Question 3

Redraw two edges that appear to cross over another edge, as shown below. Then count the faces as shown below.



Question 5

The paths are:

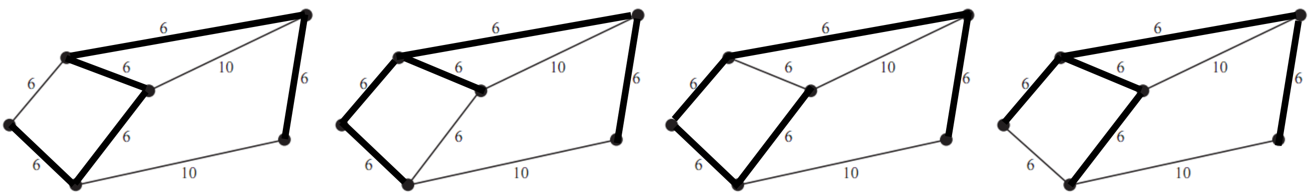
- *CGJOP*
- *CFINOP*
- *CFIMP*
- *CFIKLP*

Question 6

The critical path is *CFIKLP* and is 19 day long.

Because activity *I* is on the critical path, it cannot be delayed and must start as soon as activities *C* and *F* are completed, taking a combined 4 days.

Question 7



Question 8

Initial allocation = 19 minutes

Duty	Name	Time
1	Abbey	6
2	Dinh	3
3	Cathal	4
4	Barb	6

New allocation = 20 minutes

Duty	Name	Time
1	Dinh	5
2	Abbey	5
3	Cathal	4
4	Barb	6

Initial allocation total time = 19 minutes.

Correction to Dinh for Duty 2 gives total time = 20 minutes.

An increase of one minute.

Module 3 – Geometry and measurement

Question	Answer
1	D
2	B
3	E
4	E
5	D
6	E
7	A
8	C

Question 1

$$\frac{58}{360} \times \pi \times 12^2 = 72.884\ldots$$

Question 2

$$\tan^{-1}\left(\frac{20}{100}\right) = 11.309\ldots$$

Question 3.

The greatest angle less than 180° between lines of longitude is required.

This is 38 + 84 = 122° between Moscow and Atlanta.

Question 4

Side walls + end walls + roof

$$2 \times 75 \times 96 \quad + 2\times\left(75\times 80+\frac{1}{2}\times 80\times\left(\sqrt{52^2-40^2}\right)\right) \quad +2 \times 52 \times 96$$

= 39 042.119…

Question 5

$$\sin(y)=\frac{8\times\sin(x)}{12}$$

$$=\frac{8\times 2/5}{12}=\frac{16}{60}=\frac{4}{15}$$

Question 6

Angle between the two cities along the same meridian = 25 + 35 = 60°

Distance along the meridian $=\frac{60}{360}\times 2\pi\times 6400=6702.06...$

Question 7

The radius of the parallel at latitude 50° N is given by 6400 × sin 40°

An angle between the cities along the 50° N parallel is 85 + 112 = 197°

But since this is greater than 180°, the shortest distance between the cities is found from the smaller angle between the two meridians. This angle is 360 – 197 = 163°

Distance between the two cities = $\frac{163}{360}\times 2\pi\times\left(6400\times\sin 40^\circ\right)$

Question 8

From the bearing of *SQ* and the isosceles nature of triangle *PSQ*, it follows that ∠*PSQ* = 120°

Therefore ∠*QPS* = ∠*PQS* = 30°

In triangle *PSR*:

$\angle PSR = \sin^{-1}\left(\frac{100\times\sin 30}{80}\right)=38.6821...$ **but** this is less than ∠*PSQ* (120°)

Therefore, the ambiguous case for sine must be used

That is, ∠*PSR* = 180 – 38.6821… = 141.3179…

Then ∠*QSR* = ∠*PSR* – ∠*PSQ*

= 141.3179… – 120 = 21.3179…°

For triangle *QSR*:

∠*SQR* = 180 – ∠*SQP* = 180 – 30 = 150°

and so $QR = \dfrac{80 \times \sin 21.3179}{\sin 150} = 58.166...$ (closest to option C = 58°)

Module 4 – Graphs and relations

Question	Answer
1	C
2	B
3	E
4	D
5	D
6	A
7	B
8	A

Question 2

Let n be the number of games Jenny played

$12n + 130 = 262$

so $n = 11$

Question 3

120 + 210 + 230 = 560

Question 4

Gradient = $\dfrac{10-12}{3-0} = -\dfrac{2}{3}$

The equation for the line is $y = -\dfrac{2}{3}x + 12$

The x-intercept is at $0 = -\dfrac{2}{3}x + 12$

Question 6

Let p = number of paperback books and h = number of hardback books

Must solve simultaneously:

$$p + h = 12$$

$$8p + 14h = 126$$

Question 7

Since all the points along AB give the maximum value of Z, the function Z must have the same gradient as line AB.

Line AB is the boundary for Inequality 1, and has a gradient of 0.8

The only option that also has a gradient of 0.8 is option B.

Question 8

With two hoses, the tank would have been filled in $\frac{600}{400} \times 15 = 22.5$ minutes

The tank is actually filled 9 minutes later at 22.5 + 9 = 31.5 minutes.

Therefore *a* = 31.5

The required rate, or gradient, = $\frac{600-400}{31.5-15} = 12.1...$ minutes (closest to option A).