Neap:

Trial Examination 2017

VCE Further Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	6	6	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

Students are to write in blue or black pen.
Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 23 pages.
Formula sheet.
Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – CORE

Instructions for Section A

Answer **all** questions in the spaces provided. Write using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (8 marks)

Peem owns a shoe store and keeps records as shown below of the sales of three different styles and the sizes sold each week. He uses this information to order replacement stock.

	Style		
Size	**Ace**	**Bideman**	**Cavallier**
4	3	0	1
5	8	0	3
6	12	1	7
7	6	2	7
8	2	8	7
9	1	14	2
10	0	11	1
11	1	10	0
12	0	3	0

a. Find the mode, if any, of the size(s) of

i. Ace shoes sold. 1 mark

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ii. Cavallier shoes sold. 1 mark

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b. Calculate the five-number summary for the sales of Bideman shoes. 2 marks

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c. What is the IQR for the sales of Ace shoes? 1 mark

d. Draw a parallel boxplot showing the figures for the Ace and Bideman shoes. Note any data item not within 1.5 × IQR of the nearest quartile must be shown as an outlier. 2 marks

e. Use the boxplots and your calculations to comment upon the sales of Ace and Bideman shoes. 1 mark

Question 2 (7 marks)

The Yadaville Women's Basketball Team consists of 11 friends who play purely for fun. 5 of the girls are available to play every game whilst the other 6 play between 1 and 7 games when they are available.
As the game is purely social, each player has equal court time when they are available but for many weeks they do not play so their average court time is reduced. The following individual statistics were kept over the 10 games.

Player	Total points	Personal fouls	Average minutes on court per game
Alice	3	1	2
Bernice	80	25	35
Cal	0	1	3
Delia	60	20	30
Felicia	10	10	10
Ning	5	4	5
Pang	35	5	35
Shona	50	20	25
Siriporn	0	1	2
Tanya	5	2	6
Tik	65	25	30

a. Complete the following statement: 2 marks

When considering creating a scatterplot for *time on court* and *total points*, the variable

total points is the ' ______________________________ variable because ________________

____

____ '

b. Find the correlation between *total points* and *time on court* to two decimal places. 1 mark

____

____

c. Write the least squares line for *time on court* versus *total points* to two decimal places. 1 mark

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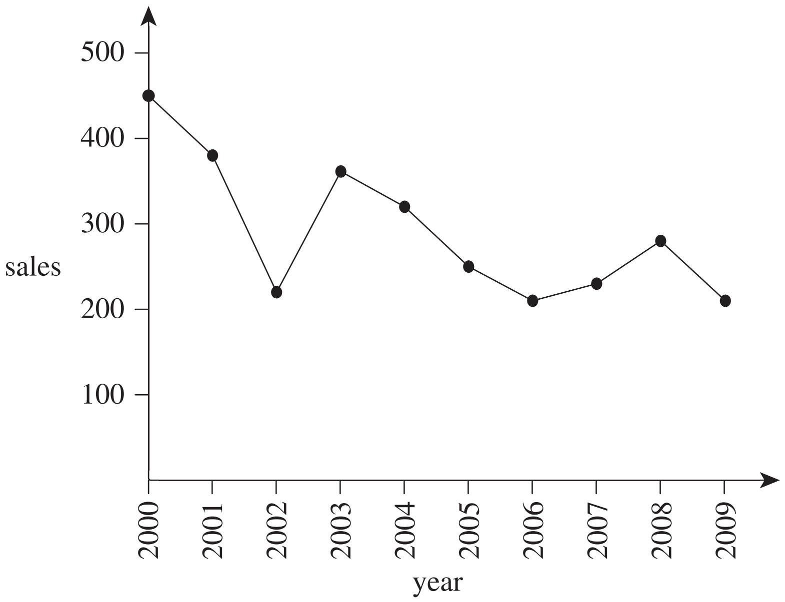
d. Draw a scatterplot for *total points* versus *time on court* on the grid below. Label the graph. 2 marks

e. Consider the relationship between the variables *total points* and *personal fouls*.

Which of the variables, if either, is the respondent variable? Explain your answer. 1 mark

Question 3 (9 marks)

Jaxson keeps records of his sales over a ten-year period as shown in the time series plot below.



Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Sales	450	380	220	360	320	250	210	230	280	210

a. Use a 3-point moving mean to sketch the smoothed curve on the time series graph. 2 marks

(Answer on the time series graph above.)

b. Describe the trend, if any, in the data over time. 1 mark

___

Jaxson sees his business is in trouble, and so he looks for a new business to invest in. He considers the company Instachat, which has shown great growth over the last year as shown in the table below.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Value in \$100 000s	1.25	1.58	2.00	2.51	3.16	3.98	5.01	6.31	7.94	10.00	12.59	15.85

c. Find both the least squares line and the correlation coefficient to two decimal places, assume the relationship is linear. 1 mark

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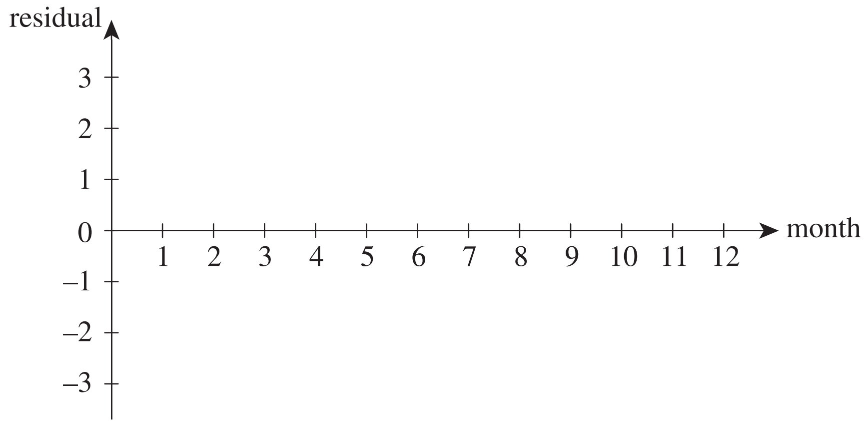
d. On the axes below, sketch a residual plot. 2 marks



e. Comment upon the assumption of linearity. 1 mark

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f. Apply an x^2 transformation to the data. Use the results to explain whether the transformation has improved the fit of the relationship to the data. 2 marks

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Recursion and financial modelling

Question 4 (2 marks)

Ben loves solving maths problems. His friend Josef sets him a couple of fun questions.

a. What is unusual about the following relationship? 1 mark

$$t_{n+1} = 3t_n - 4,\ t_0 = 2$$

____

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b. Write the first five terms for $t_{n+2} = t_{n+1} + t_n$, $t_1 = 3$, $t_2 = 4$. 1 mark

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Question 5 (8 marks)

Ben and Josef are starting their own business manufacturing personalised custom-made footballs. They predict in the first month they will make a small profit of $9000 but then increase their profit by 10% each month until the profit in month n, P_n, is at least $50 000.

a. Write a recursive relationship to describe the predicted growth in profit. 1 mark

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b. What is the predicted profit at the end of the third month? 1 mark

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c. How many complete months will it take for the profit to go above the $50 000 target? 1 mark

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The stitching machine for the footballs costs $150 000. Two different methods for calculating the depreciation are considered.

- Method 1: a flat 15% of the original purchase price
- Method 2: reducing balance of 20% per annum

d. Write an expression to calculate the value of the machine after n years for each of the methods. 2 marks

Method 1:

value = ___

Method 2:

value = ___

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e. Draw a graph of both methods on the same set of axes for the first 6 years. 2 marks

f. Using the graph from **part e.**, find an approximate time for when the value of the machine has depreciated to the same value using both methods. 1 mark

Question 6 (2 marks)

On January 1, Ben makes an investment of $250 000 which earns 6% per annum interest compounding monthly. He withdraws $2000 a month as income on the final day of the month, immediately after the interest has been added.

By completing the table, or otherwise, find the values of A, the interest paid at the end of January and B, the balance at the end of April.

Date	Deposit	Withdrawal	Interest	Balance
January 1	$250 000			$250 000
January 31		$2000	A	
February 28		$2000		
April 31		$2000		B

END OF SECTION A

SECTION B – MODULES

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules. Write using blue or black pen.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
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Contents **Page**

Module 1 – Matrices

Question 1 (5 marks)

Spanish Island Koala Sanctuary are analysing their koala populations. In September 2016 they had 2000 baby koalas (B), 1500 immature koalas (I), 3000 adult koalas (A) and 0 dead koalas (D). They estimate that:

- each adult koala produces 0.4 baby koalas per year.
- 20% of baby koalas die each year, with the rest becoming immature koalas.
- 10% of immature koalas die each year, with the rest becoming adult koalas.
- 15% of adult koalas die each year and the rest remain adults.
- there are 0 dead koalas.

A transition matrix is produced to describe this situation, but it is incomplete.

$$T = \begin{array}{c} \begin{matrix} B & I & A & D \end{matrix} \\ \begin{bmatrix} 0.00 & & 0.40 & 0.00 \\ & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.90 & & \\ 0.20 & 0.10 & 0.15 & 1.00 \end{bmatrix} \end{array} \begin{matrix} B \\ I \\ A \\ D \end{matrix} \qquad S_0 = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

a. Fill in the gaps in the **transition matrix, *T*, above** and also write down the initial state matrix, S_0. 2 marks

(Answer on the transition matrix and the initial state matrix above.)

In fact, the actual proportions are slightly different and the accurate matrix T is given by

$$T = \begin{array}{c} \begin{matrix} B & I & A & D \end{matrix} \\ \begin{bmatrix} 0.00 & 0.00 & 0.45 & 0.00 \\ 0.75 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.85 & 0.80 & 0.00 \\ 0.25 & 0.15 & 0.20 & 1.00 \end{bmatrix} \end{array} \begin{matrix} B \\ I \\ A \\ D \end{matrix}$$

b. Use T to find the number of baby, immature and adult koalas for September 2017. 1 mark

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The park rangers believe that populations exceeding 7000 koalas are unsustainable on the island.

c. Determine the likely koala populations in 2026 and thus determine if the population will become unsustainable or die out in that time. 2 marks

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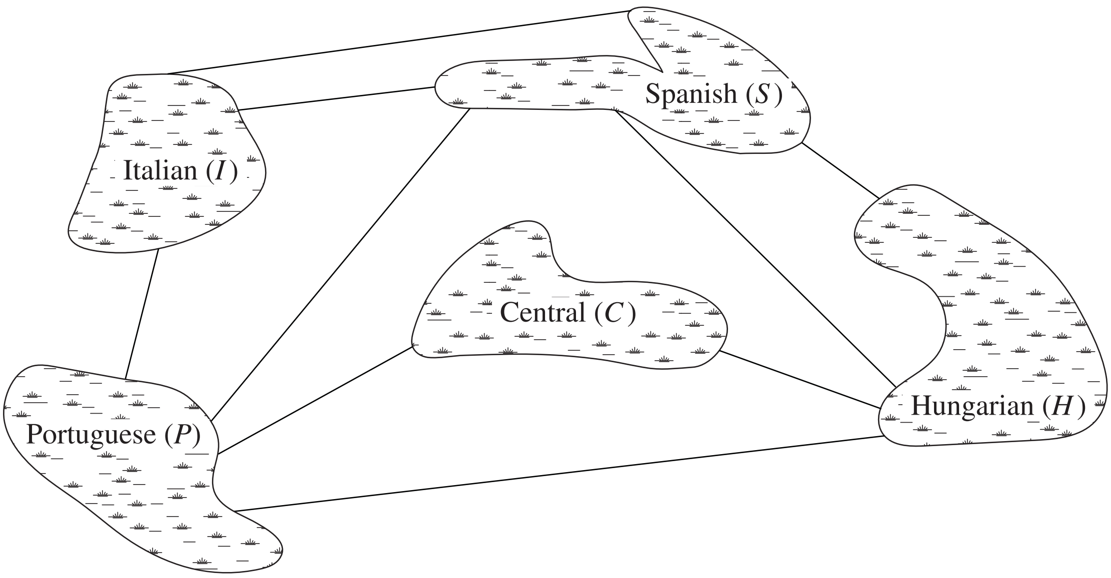
Question 2 (5 marks)

Spanish Island is only one of a series of islands in Wildlife Park. Visitors to the park need to travel by boat between the islands as shown below. The boat routes are represented by the straight lines between the islands. Islands may be connected by a single route, multiple routes or no direct route at all.



A matrix can be used to show these connections. The first row of the matrix (Spanish Island) is complete but most of the others are not.

$$B = \begin{array}{c} \\ \end{array}\begin{array}{c} \begin{array}{ccccc} S & I & C & P & H \end{array} \\ \left[\begin{array}{ccccc} 0 & 2 & 0 & 1 & 2 \\ & & & & \\ & & 0 & & \\ & & & 0 & 1 \\ & & & 1 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ S \\ I \\ C \\ P \\ H \end{array}$$

a. Fill in the gaps in the **matrix above.** 2 marks

(Answer on the matrix above.)

b. Explain what the matrix B^2 would represent. 1 mark

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c. Thus determine the number of ways to travel from Spanish Island to Portuguese Island using either one or two boats. 2 marks

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Question 3 (2 marks)

The owners of Wildlife Park use matrices to store information about their visitors. Each row applies to a single island and each column to a single day of the week (Monday to Friday). The resulting matrix is shown below.

$$\begin{array}{c} \begin{matrix} M & Tu & W & Th & F \end{matrix} \\ \begin{bmatrix} 112 & 96 & 102 & 118 & 145 \\ 43 & 41 & 45 & 60 & 72 \\ 25 & 27 & 29 & 41 & 50 \\ 61 & 60 & 58 & 68 & 83 \\ 58 & 50 & 51 & 78 & 109 \end{bmatrix} \begin{matrix} S \\ I \\ C \\ P \\ H \end{matrix} \end{array}$$

There is a small fee for visiting each island as given in the table below.

	Spanish	**Italian**	**Central**	**Portuguese**	**Hungarian**
Fee	2.00	1.60	1.20	1.50	2.00

a. Set up a matrix product that will allow the total taking for each day to be calculated. 1 mark

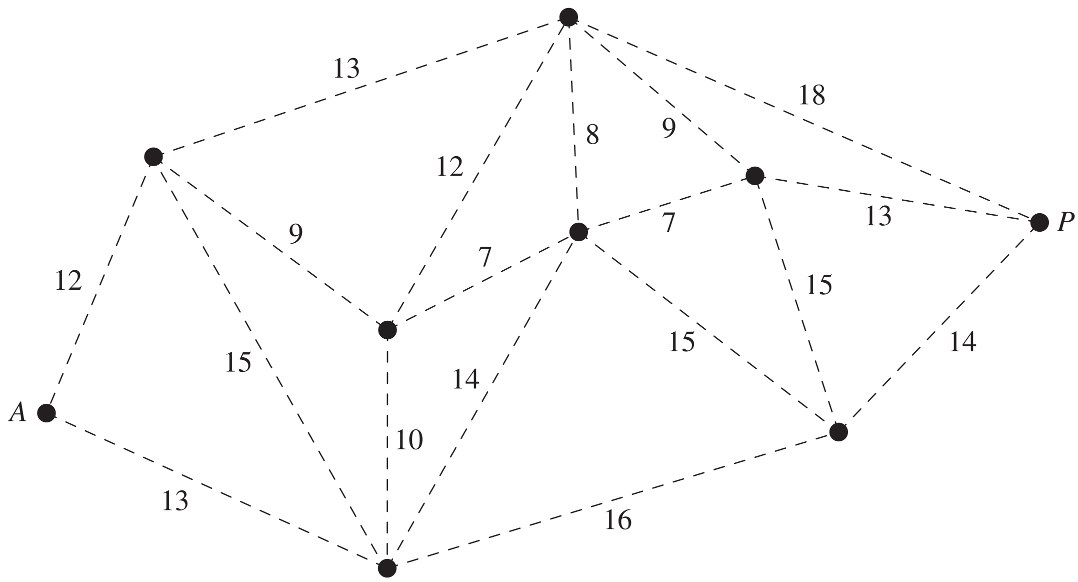
b. Write down the total takings for Monday to Friday. 1 mark

END OF MODULE 1

Module 2 – Networks and decision mathematics

Question 1 (5 marks)

'Sustain Us' are a company looking to set up an alternative energy grid featuring only renewable sources. Nine towns are designed to be connected to this grid. These towns and the distances between them in kilometres are shown below.



a. The company wants to connect all of the towns but in such a way as to reduce the total length of cabling between towns.

Find the minimal spanning trees and show this on the **diagram above**. 1 mark

(Answer on the diagram above.)

b. Complete the following statement: 1 mark

The network shown on the diagram in **part a.** can be referred to as the '______________

______________________' of the original network.

c. What is the minimum length of cabling required to link all towns? 1 mark

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The company wants to find the shortest distance between key locations *A* and *P*.

d. Using Dijkstra's algorithm, find this shortest distance, assuming that all of the dotted-line paths shown above are able to be used. 2 marks

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Question 2 (4 marks)

Four new sets of solar installations exist to be distributed among four towns. As there are differences between the locations, the amount of power that can be generated by each depends on where they are placed, as shown in the table below.

Daily energy	Walkerville	Xenonton	Yarckmon	Zamboling
installation *P*	9.5	9.0	8.3	7.2
installation *Q*	8.4	8.2	6.5	5.7
installation *R*	6.2	5.8	4.8	3.6
installation *S*	5.1	3.1	3.5	2.0

The arrangement that maximises total electricity generation is optimal. The electricity board decide that they need to place this data into a matrix, but before attempting to start the Hungarian algorithm, the electricity authority decides to subtract all these figures from 10. Thus for the Walkerville installation, one result appears in the matrix as 10 – 9.5 = 0.5.

a. Explain why this subtraction process is necessary and what would happen if this step was omitted. 1 mark

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The resulting matrix is as shown below.

$$\begin{bmatrix} 0.5 & 1.0 & 1.7 & 2.8 \\ 1.6 & 1.8 & 3.5 & 4.3 \\ 3.8 & 4.2 & 5.2 & 6.4 \\ 4.9 & 6.9 & 6.5 & 8.0 \end{bmatrix}$$

The lowest value in each row is subtracted from each row and the lowest value in each column is subtracted from each column. The result is shown below (including all zeroes) but is incomplete.

$$\begin{bmatrix} 0.0 & 0.3 & 0.0 & 0.0 \\ 0.0 & 0.0 & & 0.4 \\ 0.0 & 0.2 & 0.2 & 0.3 \\ 0.0 & 1.8 & & \end{bmatrix}$$

b. Fill in the non-zero gaps on the **matrix above**. 1 marks

(Answer on the matrix above.)

c. Thus allocate installations to towns. 2 marks

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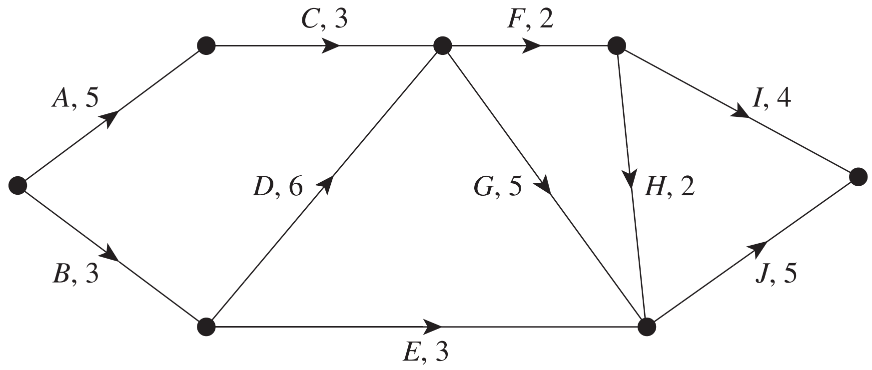
Question 3 (3 marks)

The construction of the solar network is a project comprising tasks *A* to *J* as shown in the project diagram below. Each town's duration, in days is shown.



a. Determine the earliest completion time. 1 mark

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b. Determine the critical path. 1 mark

___

c. What is the greatest time that any activity can be increased in duration by without extending the earliest completion time? 1 mark

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END OF MODULE 2

Module 3 – Geometry and measurement

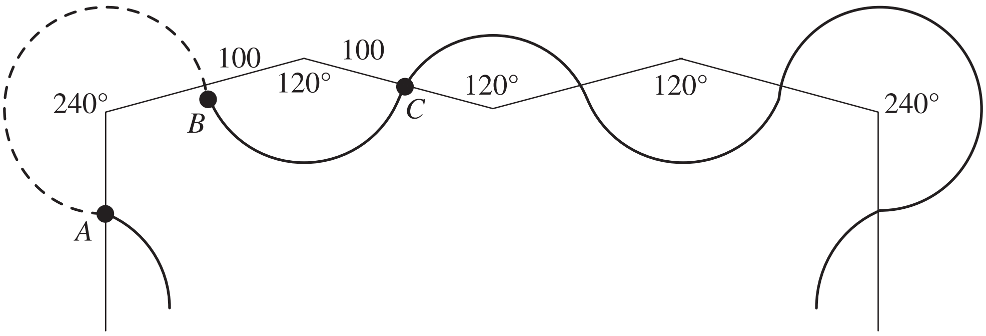
Question 1 (2 marks)

'True Light Travel' organises tours to major tourist locations worldwide. Their 'North and South' tour involves tourists travelling between Kathmandu (27°N, 85°E), Easter Island (27°S, 109°W) and the Thar Desert (27°N, 71°E).

If tourists travel from the Thar Desert to Easter Island, how far is this leg of the trip to the nearest km? Explain your reasoning by comparing the two locations as they are located on the Earth.

Question 2 (10 marks)

To improve the popularity of their tours, the company decides to build a museum of ancient Thar Desert culture. The museum will have a traditional Thar stone wall made of joining circular arcs.

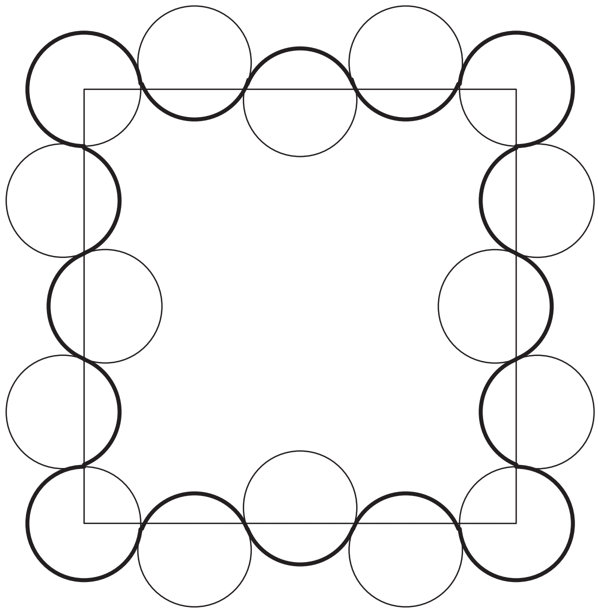


Each arc section has radius 100 metres. The subtended angle is 120°, except at the corners where the angle is 240°.

a. Determine the length of wall section shown by the dashed line to the nearest metre. 1 mark

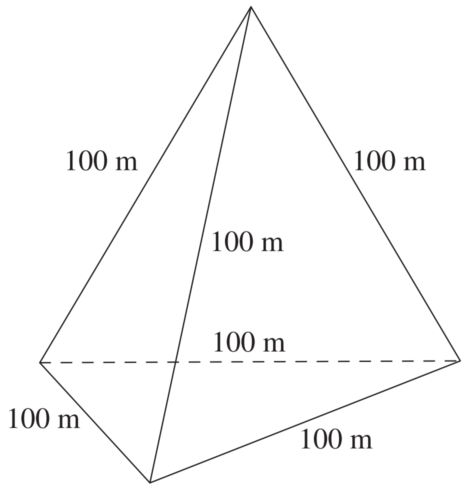
b. Given that C is due east of B, find the bearing of B from A. 1 mark

‘True Light Travel’ want to determine the approximate area of the compound. They form a square by joining the centres of all the corner circles.



c. Find the area of the square formed to the nearest square metre. 3 marks

In the centre of the museum is a tetrahedral tower. The base and all three sides are identical equilateral triangles. Each triangle has side length 100 metres.



d. Determine the area of each triangle. 1 mark

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___

e. Determine the volume of the tetrahedron. 2 marks

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A model is being designed for the tower. A scale factor of 1 : 1000 will be used.

f. Determine the surface area and volume of the model. 2 marks

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END OF MODULE 3

Module 4 – Graphs and relations

Question 1 (4 marks)

'Party On Party Supplies' produces food such as party pies and sausage rolls. They are looking to expand their operations to include mini-pizzas to their range. For this they require a new machine, costing $4500. When the machine operates, it costs an additional $80 to make 1000 mini-pizzas.

a. Write down an equation for the cost of buying the machine and making *n* pizzas. 1 mark

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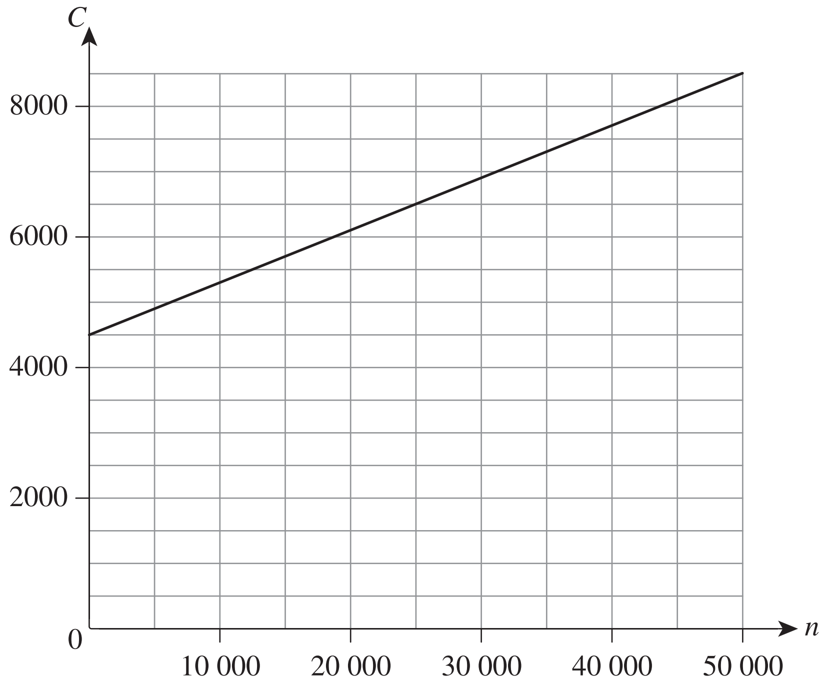
Due to being a major buyer of products, 'Party On Party Supplies' are given an additional discount from their supplier, and the result is shown in the graph below.



The mini-pizzas will be sold at $1 for three.

b. **i.** Plot this line on the **graph above**. 1 mark

(Answer on the graph above.)

ii. Write down the equation of this line on the line drawn in **part i.** 1 mark

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___

c. Thus determine how many mini-pizzas must be sold to break even. Answer to the nearest 1000 pizzas. 1 mark

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Question 2 (8 marks)

'Party On Party Supplies' offers two main party-packages – the standard and deluxe. On any given weekend they have the resources to run ten parties. They are also restricted by their staff resources. A standard party requires two staff members while a deluxe party requires five staff members. The company only has thirty staff members. The manager, in analysing their options, writes down a series of constraints on x, the number of standard, and y, the number of deluxe, parties, but omits one.

$$x \geq 0$$

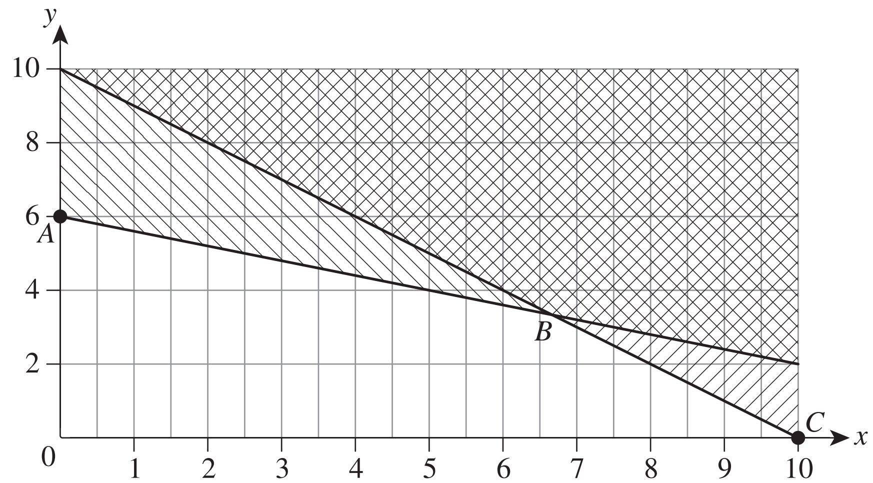
$$y \geq 0$$

$$x + y \leq 10$$

a. Write down the remaining constraint. 1 mark

___

The constraints are plotted and the partial result is shown below. A different constraint is omitted this time, however.



The constraints meet at three points: $A(0, 6)$, B and $C(10, 0)$.

b. Find the third point, B, correct to two decimal places. 1 mark

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The profit to be gained is given by the equation $P = 120x + 235y$.

c. From this equation, write down the profit that is achieved per package. 1 mark

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It is only possible to have a whole number of each party each weekend.

d. Write down all the possible points along the upper boundaries (that is, between points *A* and *C*) at which the maximum profit could be achieved. 2 marks

e. Determine the objective function (profit) value at each of these points and thus determine the best number of each party to offer to maximise profit. 2 marks

f. A manager at 'Party On' decides to use the point *B* on the graph to calculate the maximum profit, even if it does not have integer values.

By how much do they overestimate the maximum profit? 1 mark

END OF QUESTION AND ANSWER BOOKLET