Neap:

Trial Examination 2017

VCE Further Mathematics Units 3&4

Written Examination 1

Suggested Solutions

SECTION A – CORE

Data analysis

Question 1 D

The midpoint of each group must be found and multiplied by the frequency. The four figures are then added together and divided by the total frequency (29).

Question 2 E

The mode is the most frequently occurring figure and we have no information about the actual data, so it is impossible to know anything about the mode.

Question 3 E

Recognise the figure for May is 3400.

Use the formula $\textit{deseasonalised figure} = \frac{\textit{actual figure}}{\textit{seasonal index}}$.

Question 4 D

There are three numbers which occur twice so there is no mode.

Question 5 D

The correct scale must be on the vertical axis and a horizontal axis using time must be used.

Question 6 A

Find the gradient using $\frac{y_2 - y_1}{x_2 - x_1} = 0.8$. Then substitute one of the points into the general equation for a linear equation.

Question 7 E

The value of r^2, the coefficient of determination, is 0.25, or 25%. This represents the percentage change; a change in the explanatory variable will change the response variable.

Question 8 B

95% of the population for a normal distribution lies within two standard deviations of the mean that is $45 \pm 2(8)$, so option **B** is the correct solution.

Question 9 C

Both variables are responding to a third variable – the population of the town.

Question 10 C

Since there are only two seasons, the seasonal indices must add to 2. The seasonal index for the dry season is therefore 0.2.

The percentage of rain in the wet season is $\frac{1.8}{0.2} \times 100 = 90\%$.

Question 11 E

Use the median and IQR because the extreme value of 165 will have too big an impact on the calculation of the mean or standard deviation. The mode of 1 is unrepresentative of the figures.

Question 12 B

Calculate the predicted value for $x = 5$ (68.1). The residual is then the real value minus the predicted (1.9).

Question 13 D

Substitute the correct values into $y = a - bx$. a is the y-intercept, b the gradient and the variable has been transformed to x^2.

Question 14 B

$Q_1 = 2$, median = 8 and $Q_3 = 10$. $IQR = 8$. The hurdle for an outlier is $Q_3 + 1.5 \times IQR = 22$.

31 is outside this and therefore an outlier. The data is positively skewed.

Question 15 C

We know the median is 8 from the previous question, so only options **A**, **B** and **C** can be considered. The mode is 10 as it is the figure with the highest frequency, so option **A** is eliminated. To calculate the mean, remember to use the key and find the correct values to add.

Question 16 E

3 represents 10^3, so we are looking for how many months are less than 1000.

$2 + 3 = 5$.

Recursion and financial modelling

Question 17 C

The next term is found by multiplying the previous term by –2. Therefore only options **A**, **C** and **E** can be considered. Option **E** has the n and $n + 1$ around the wrong way and the question asks for the best method of representing the series, so we need to know $t_1 = 10$.

Question 18 C

The interest of 5% is equivalent to 1.05. It is compounding interest over four years so there must be a 1.05^4 term.

Question 19 D

Note that you are asked for t_1 and t_4. Substitute 6 into the expression to find t_1 (14.2). Substitute 14.2 into the expression to find t_2 and repeat until t_4 is found.

Question 20 D

Calculate the depreciation per X-ray by finding the loss in value over the life of the machine and divide by the number of X-rays (100 000). Therefore the value after n X-rays will be $\frac{450\,000 - 20\,000}{100\,000} = 4.3$.

Question 21 C

A is describing the rule for the number of patients infected each day. Only option **C** works for every term.

Question 22 D

$40\ 000 \times 0.8 \times 0.92^3 = \$24\ 918$

Question 23 B

Subtract the deposit from the cost to find the value of credit required. Add a flat interest of 5% for five years. Divide by 60 to find the monthly repayment.

Question 24 C

Using a finance package on your calculator, or otherwise, the balance to be repaid is $25 000, the value after 5 years is $0 and the interest rate is 12% per annum, which is the same as 1% per month. The monthly repayment is $556.11 which is paid for 60 months. $60 \times 556.11 = \$33\ 367$.

SECTION B – MODULES

Module 1 – Matrices

Question 1 **C**

Stream B is row 2. Online campus is column 3.

Thus, the element concerned is E_{23}.

Question 2 **D**

For any permutation matrix, all row and column sums are 1. The order of the matrix has no effect on this.

Question 3 **C**

Students should write out the matrix equation resulting from the information in the question.

$$\begin{bmatrix} 2 & 5 & 3 \\ 3 & 3 & 3 \\ 5 & 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21 \\ 23 \\ 24 \end{bmatrix}$$

To get the matrix $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, we need to find the inverse of $\begin{bmatrix} 2 & 5 & 3 \\ 3 & 3 & 3 \\ 5 & 4 & 2 \end{bmatrix}$, which is $\frac{1}{24}\begin{bmatrix} -6 & 2 & 6 \\ 9 & -11 & 3 \\ -3 & 17 & -9 \end{bmatrix}$.

Students should ensure that they convert to a fraction so that the decimal result does not confuse them.

Question 4 **B**

$S_{n+1} = TS_n + P$

$$\begin{aligned} S_2 &= TS_1 + P \\ &= \begin{bmatrix} 108 \\ 82 \end{bmatrix} + \begin{bmatrix} 4 \\ -5 \end{bmatrix} \\ &= \begin{bmatrix} 112 \\ 77 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} S_3 &= TS_2 + P \\ &= \begin{bmatrix} 116.2 \\ 72.8 \end{bmatrix} + \begin{bmatrix} 4 \\ -5 \end{bmatrix} \\ &= \begin{bmatrix} 120.5 \\ 67.8 \end{bmatrix} \end{aligned}$$

Question 5 **C**

Students need to produce the matrix resulting from all three methods.

$$D = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, D^2 = \begin{bmatrix} 0 & 0 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, D^2 + D = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Thus the row dominance totals are $\begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \end{bmatrix}$ respectively for methods **1.**, **2.** and **3.**

Method **1.** does not produce a clear winner and thus options **A**, **B** and **D** are not correct. Outer actually has a two-step dominance over Northbourne and thus option **E** is incorrect also. Method **3.** gives difference scores to each of the four schools and thus distinguishes them.

Question 6 **E**

We require states to reoccur every second transition; thus, $T^2 = I$ but $T \neq I.$ We immediately see that options **A** and **B** are invalid as they involve $T = I.$ Some permutation matrices are possible solutions but many are not. One is simply the identity matrix *I* and we know that is invalid. Some satisfy $T^3 = I$ but not $T^2 = I.$ This rules out option **C**. Option **D** has the $T^3 = I$ issue and so is invalid also.

Students can try this in their calculators.

Question 7 **B**

Since we know that *B* is not singular, we know that B^{-1} exists. However, we do not know if A^{-1} exists. This means that option **B** is an incorrect statement. The method given cannot be used as it uses A^{-1} which might not exist.

Check the other options to verify this answer. Option **A** is not incorrect as the inverse of *B* does exist. The statements that matrices *A* and *C* might be singular is clearly true also and thus options **C** and **D** are not the incorrect statements. Finally, any of the matrices could be binary. We have no information about that and so option **E** is not the incorrect statement that we seek either.

Question 8 **C**

The first thing to note is that the table is not organised in a manner consistent with the matrix. We need to swap rows and columns so that South store is the top row and North store is the lower row.

$$\begin{bmatrix} 27 & 61 \\ 37 & 28 \end{bmatrix}$$

Then we can multiply.

$$\begin{bmatrix} 27 & 61 \\ 37 & 28 \end{bmatrix} \begin{bmatrix} 300 \\ 400 \end{bmatrix}$$

Module 2 – Networks and decision mathematics

Question 1 D

For an Eulerian circuit, it is required that each vertex has an even degree. Currently vertices *A* and *B* have degree 3, while all other vertices have an even degree. Thus the change that needs to occur has to change the degree of each of *A* and *B* by 1. We can either add an extra degree or remove 1 joining *A* and *B*. Of these two, only one (removing the *AB* edge) appears in the options.

Question 2 B

A spanning tree must be created. It should be a network

- with no loops or cycles;
- using only the existing edges, and
- it must also be connected.

Only one of the options fulfils all these requirements; options **A** and **E** have cycles within them, option **C** is not connected and option **D** has a connection between *A* and *B*, whereas the original network does not.

Question 3 D

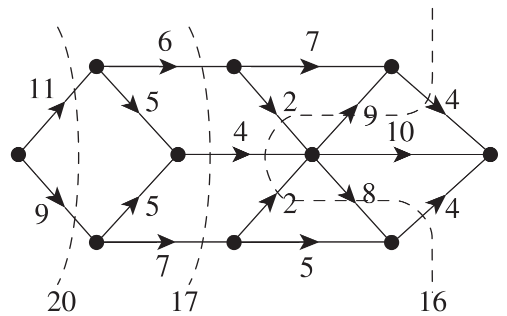
Using Euler formula: $V + F = E + 2$

This can be rearranged as $F = E + 2 - V$.

The vertices (V) are increasing by 1 and the paths (E) are increasing by 4, which means that the faces (F) increase by 3.

Question 4 B

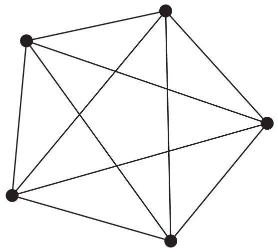
The minimum cut gives the maximum flow. The process thus requires us to try to find the 'cut' across the network that separates the source and the sink which has the smallest flow across it. We need to remember, however, that only flow from the source toward the sink side is counted.



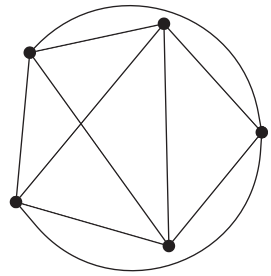
That means that the cut on the right of the above image only has value 16, as the '8' and '9' edges that are cut are going in the opposite direction. Many other cuts are possible, but none are lower than 16.

Question 5 B

First draw the complete network with 5 vertices.



This diagram has many crossed edges, some of which can be fixed by making the edges go around the outside of the vertices as shown below.



From the diagram above, only 1 edge requires removal to make the graph planar.

Question 6 C

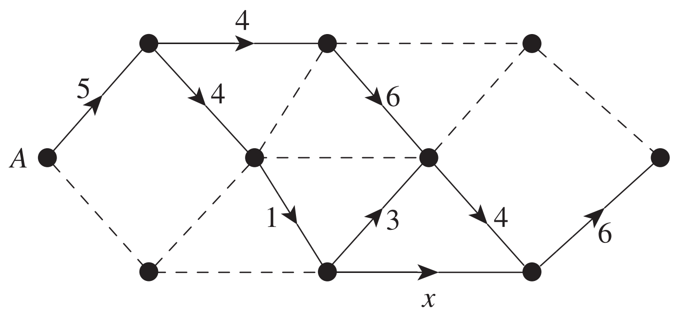
Increasing the duration of *D* must increase the completion time as it is critical. That narrows our options to **C**, **D** and **E**.

If *m* is small, it is quite likely that the completion time will be increased by the same value, as it is likely that *D* remains on the critical path despite the change.

If *m* is a large value, however, it is unlikely that the completion time for the project will be increased by that same value (*m*), as another path will probably become critical instead.

Question 7 C

Dijkstra's method will simplify the diagram as below (although it cannot be completed without knowing *x*).



The solid lines are all that remain. We know that the shortest line will be 23 in length. If we are not required to traverse edge *x*, this will indeed be the length. Thus, we want to avoid traversing the path *x* and take the two edges labelled '3' and '4' instead, which will occur if *x* exceeds 7.

Question 8 C

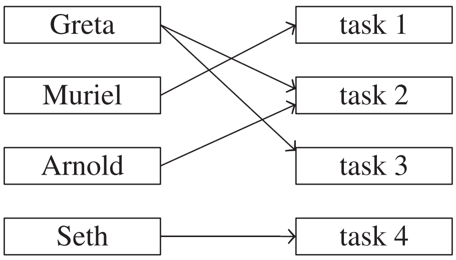
This question can be solved using the Hungarian algorithm. Students may use a shorter trial method.

Even if the Hungarian algorithm is not used, it is clear that task 4 should be allocated to Seth as he is the fastest at this task, whereas he is one of the slowest at all the other tasks.

Greta is fastest at all the other tasks but we can only allocate one to her. With the Hungarian algorithm, after the minima of rows and columns are subtracted, we obtain the following table.

5	0	10
0	15	0
0	5	10

This requires three lines to cover all of the zeroes that we can allocate.



Thus, Arnold must do task 2 (as it is the only one possible in the network above). That leaves task 3 for Greta. Muriel is clearly allocated to task 1.

total time = 85 + 80 + 85 + 90

= 340 minutes

Module 3 – Geometry and measurement

Question 1 D

The complete circle would have area $\pi 6^2 = 36\pi$.

However, the actual area of the sector is 32. Thus we have a $\frac{32}{36\pi}$ fraction of a circle.

The subtended angle is thus $\frac{32}{36\pi} \times 360 = \frac{320}{\pi}$

$= 101.86$

Question 2 B

Each standard muffin requires $\frac{2.0}{30} = 66.7 \text{ g}$ of flour.

Each deluxe muffin will require $66.7 \times 1.25^3 = 130.2$ g of flour.

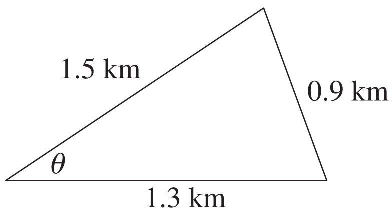
Thus we can make $\frac{2000}{130.2} = 15$ deluxe muffins.

Question 3 C

The circumference at latitude 12° is $2\pi \times 6400\cos(12°)$. However, we do not have a full circle here. Depending on which way one travels, there are either 265° or 95° of longitude separating the two points. Thus $\frac{95°}{360°} \times 2\pi \times 6400\cos(12°)$ is the distance.

Question 1 A

We will need to find an internal angle and then use the area formula.



Use cosine rule:

$$\cos\theta = \frac{1.3^2 + 1.5^2 - 0.9^2}{2 \times 1.3 \times 1.5}$$

$$= 0.802$$

$$\therefore \theta = 40.69°$$

Now find area:

$$\text{area} = \frac{1}{2} \times 1.3 \times 1.5\sin(40.69°)$$

$$= 0.582$$

Alternative method:

$$s = \frac{1.3 + 1.5 + 0.9}{2}$$

$$= 1.85$$

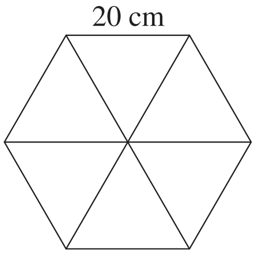
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{1.85(1.85 - 1.3)(1.85 - 1.5)(1.85 - 0.9)}$$

$$= 0.582$$

Question 5 **B**

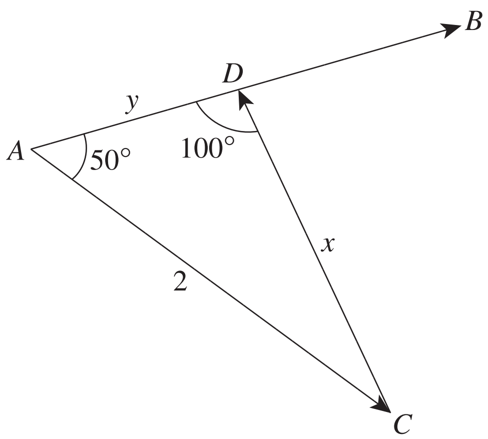
With perimeter 120 cm, each side of the hexagon must have length 20 cm. 6 triangles can be formed as shown below, all meeting in the centre.



Every angle in the centre must be 60° (so they add to 360°). All other angles must be 60° so that all triangle angles sum to 180°.

This means that all of the 6 triangles must be equilateral, and so each side is 20 cm in length. The line across the middle is 2 of these line segments added together. Therefore it is 40 cm.

Question 6 **C**



$$\frac{x}{\sin(50°)} = \frac{2}{\sin(100°)}$$

$$x = \frac{2\sin(50°)}{\sin(100°)}$$

$$= 1.56$$

$$\frac{y}{\sin(30°)} = \frac{2}{\sin(100°)}$$

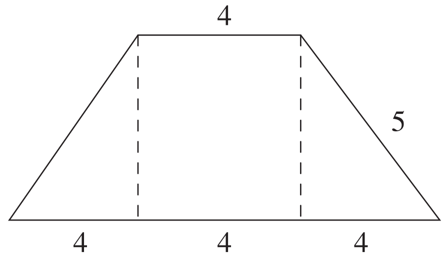
$$y = \frac{2\sin(30°)}{\sin(100°)}$$

$$= 1.02$$

Thus Muriel travels 0.98 km to *D* while Martha travels 1.56 km. Martha travels 0.58 km further.

Question 7 A

The volume can be found either through the area of a trapezium or by dividing the base into triangles and a rectangle. In either case, the height of the trapezium must be determined.



Using symmetry, it can be seen that the triangles on each side have base 4 m. Thus the height can be found.

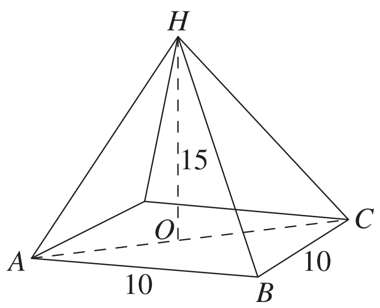
$$h = \sqrt{5^2 - 4^2}$$
$$= 3 \text{ m}$$

$$\text{Thus area of base} = \frac{1}{2}(4 + 12)3$$
$$= 24 \text{ m}^2$$

The volume will just be this area multiplied by the perpendicular depth.

$$\text{volume} = 24 \times 10$$
$$= 240 \text{ m}^3$$

Question 8 D



The longest length will be from one of the corners of the base to the point.

$$\text{length } AC = \sqrt{10^2 + 10^2}$$
$$\cong 14.14 \text{ cm}$$

$$\text{length } AO = 7.07 \text{ cm}$$

$$\text{length } AH = \sqrt{7.07^2 + 15^2}$$
$$= \sqrt{275}$$
$$\cong 16.58 \text{ cm}$$

Module 4 – Graphs and relations

Question 1 D

The answer can be found in the section of the graph with the steepest gradient upwards. This is the section from 16 September 2016 to 15 October 2016.

Question 2 D

The process of finding the incorrect point can be found by trying each individual point. For example, for option **A**, $3x - 2y = 3(2) - 3(0) = 6$, which is correct.

The same process also provides a correct answer for options **B**, **C** and **E**.

For option **D**, $3x - 2y = 3(-2) - 2(3) = -12 \neq 6$.

Note: A faster method would be to note that two of the responses have the same y-value (options ***C*** *and* ***D****). Both cannot be correct and so only options* ***C*** *and* ***D*** *need to be checked.*

Question 3 A

Option **A** requires us to calculate average speed. The bus travelled for 4.5 hours and the distance was 240 km; therefore the average speed was $\frac{240}{4.5} = 53.3$ km/h. Thus, option **A** is an untrue statement.

Option **B** is correct as the bus stops between 2:30 am and 3:00 am.

Option **C** is also correct as the gradient is 60 initially.

Option **D** is correct as the bus was only moving for 4 hours (despite the journey being 4.5 hours).

Option **E** is correct as the speed is always 60 km/h when the bus is moving.

Question 4 D

At 3:00 am the bus is 120 km from its destination but has only 1.5 hours to get there in time. It must travel at $\frac{120}{1.5} = 80$ km/h.

Thus the correct response is option **D**.

Question 5 B

$2p + 3w = 14$

$3p + 4w > 19$

Option **B** is the only option to satisfy the first condition and give a cost above $19 for the second.
Option **A** fails on the second condition. Options **C** and **D** fail on the first condition but pass the second.
Option **E** passes the first condition but fails on the second.

Question 6 C

If the objective function is not parallel to any constraint, there must be one distinct answer. Thus one answer exists for Mary's problem. It is likely that multiple solutions exist for Jamie's problem, however, as every point on the constraint parallel to the objective function bounding the feasible region is a solution. Even if there are no points other than the endpoints of this line segment that have integer coordinates, the endpoints are enough to achieve multiple solutions.

Question 7 D

The change being made alters one of the constraints. The shallowest (least steep) line is moved upward but its gradient is not changed. The only way that this alters the solution is if point *B* is no longer in the feasible region.

Find point *B*:

$8x + 3y = 54$ and $4x + 3y = 48$ meet at *B*.

Subtracting:

$$4x = 6$$
$$\therefore x = 1.5$$

$$6 + 3y = 48$$
$$\therefore y = 14$$

Check the new constraint:

$$\begin{aligned} 2x + 3y &= 2(1.5) + 3(14) \\ &= 45 \geq 44 \end{aligned}$$

Thus *B* is still in the feasible region. No change has occurred to the point and thus no change has occurred to the minimum wage cost.

Question 8 A

$y \geq 2x$

$x + y \leq 50$

$y \leq 40$

$x \geq 15$

Each of the graphs in options **A** to **E** show the line bounding the regions. In all but one case, however, the wrong side of at least one of these test boundaries has been chosen.

