Neap:

Trial Examination 2016

VCE Further Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	6	6	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

Students are to write in blue or black pen.

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 23 pages.

Formula sheet.

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2016 VCE Further Mathematics Units 3&4 Written Examination 2.

SECTION A – CORE

Instructions for Section A

Answer **all** questions in the spaces provided. Write using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (4 marks)

Fai is investigating the possibility of buying a home with two bungalows on the property that are currently rented to guests for short holidays. The following information was provided by the current owner.

Year	Number of guests	Profit ($)	Nights rented
2010	2450	27 000	300
2011	2200	25 600	280
2012	3100	34 340	360
2013	1880	20 850	200
2014	2850	29 800	310
2015	2960	32 500	345

a. Find the median profit per year. 1 mark

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b. Which year had the highest mean profit per night rented? 1 mark

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c. **i.** Find the missing 3-point moving mean. 1 mark

Year	2010	2011	2012	2013	2014	2015
Nights rented	300	280	360	200	310	345
3-point moving mean			280	290	205	

ii. Why is it preferable to use a 3-point moving mean rather than a 4-point moving mean? 1 mark

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Question 2 (13 marks)

Fai wants to see how guest numbers impact the profit, so she analyses the relationship between them.

a. Complete the following statement. 2 marks

The profit is the ______________________ variable because ____________________________

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b. Draw a scatterplot of the *profit ($)* versus *number of guests* on the following grid, labelling the graph. 3 marks

c. Use your calculator or otherwise to find each of the quantities below. Round your answer to two decimal places.

i. correlation coefficient 1 mark

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ii. least square regression equation 2 marks

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d. Use the given rounded values to complete the table of values. 2 marks

	2010	**2011**	**2012**	**2013**	**2014**	**2015**
Number of guests	2450	2200	3100	1880	2850	2960
Actual profit ($)	27 000	25 600	34 340	20 850	29 800	32 500
Predicted profit ($)	27 106	24 569	33 704	21 321	31 166	32 383
Residual						

e. Draw a residual plot. 2 marks

f. Comment upon the assumption of linearity. 1 mark

Question 3 (7 marks)

Looking more carefully at the data on the number of guests staying for 2013, 2014 and 2015, the following information is obtained.

	Q1	**Q2**	**Q3**	**Q4**	**Total guests**
2013	600	450	300	530	1880
2014	1200	700	200	750	2850
2015	1300	660	300	700	2960
Total	3100	1810	800	1980	

a. Which quarter has the greatest range of values? 1 mark

b. Plot a time series of the data for the number of guests staying each quarter on the grid below. 2 marks

c. Describe, if one exists, the pattern. 1 mark

Fai wants to take a holiday and shut down the accommodation for one quarter each year.

d. In which quarter should she take her holiday to minimise the loss of guests? 1 mark

e. Calculate the seasonal index for the first quarter to two decimal places. 2 marks

Recursion and financial modelling

Question 4 (3 marks)

The property is on the market for \$800 000 and Fai will need to take out an investment loan to pay for it.

The loan she is offered is at 4.8% per annum compounding monthly. Each month Fai will pay the interest only on the loan.

a. Calculate Fai's total annual repayment to the nearest dollar. 1 mark

Based on the average profits in previous years, Fai could expect a profit of \$35 000 for her first year of operation. The growth in profit can be modelled by the equation $P_{n+1} = 1.04P_n$, with $P_0 = 35\ 000$.

b. Based upon this model, after how many years will Fai first make a profit of more than \$39 000? 2 marks

Question 5 (4 marks)

Fai has been saving over the last three years to purchase an investment property. She begins with a balance of \$50 000 and earns 5.2% interest at the end of each year. She also deposits an extra \$35 000 into the investment after the interest has been added.

a. Complete the table below to predict the value of Fai's investment after 3 years. 2 marks

Year	Deposit (\$)	Interest earned (\$)	Balance (\$)
0	50 000	0.00	50 000
1	35 000		
2	35 000		
3	35 000		

b. At the end of the second year, Fai can only afford to deposit \$15 000 instead of her normal \$35 000.

By how much will this reduce the balance after 3 years if all other things remain the same? 2 marks

Question 6 (5 marks)

To make this investment work, Fai is budgeting for an increase of 5% in the guest numbers from the preceding year and an additional 100 guests. This should occur over the next 10 years starting with a base number of 3000 in the first year.

a. Write a recursive relation to predict N_n, the number of guests after n years of operating. 2 marks

b. How many guests has Fai budgeted upon for the fourth year? 2 marks

c. By using this model, after how many years will the guest numbers be expected to exceed 5000? 1 mark

END OF SECTION A

SECTION B – MODULES

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules. Writing using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

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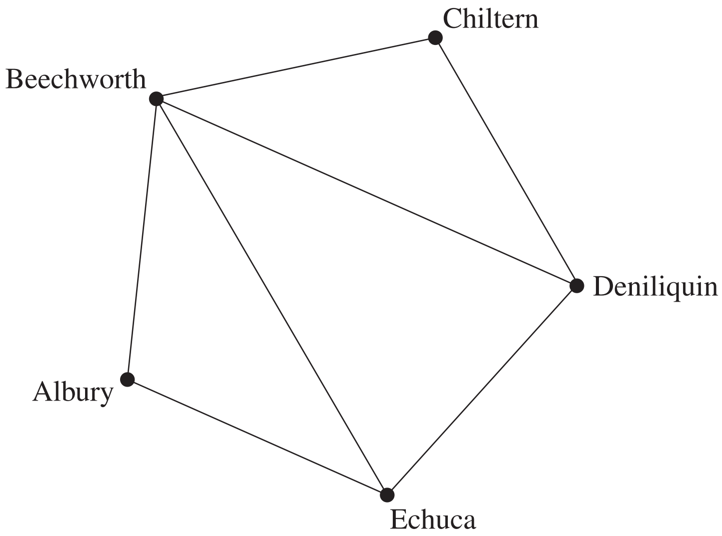
Module 1 – Matrices

Question 1 (9 marks)

Filomena and Febs are opening a new travel agency, 'North Victoria Travel'.

There are 5 attractions in North Victoria that must be connected. They are shown in the diagram below.



A matrix is produced for the network using the first letter of each town, but some values have been omitted.

$$\begin{array}{c} \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & b \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & a & 0 & c \\ 1 & e & 0 & 1 & d \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \end{array}$$

a. State the values of a, b, c, d and e. 2 marks

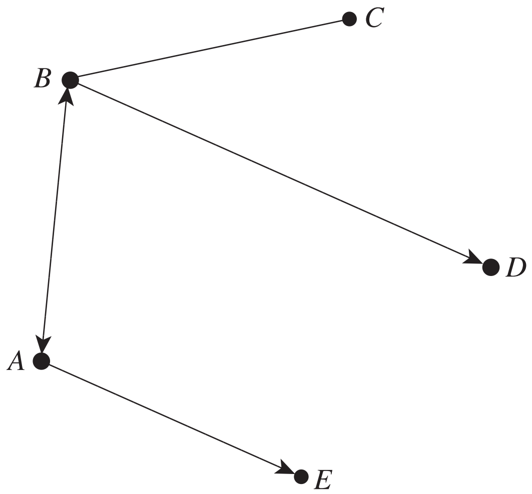
b. Write down the row sum for row 3. 1 mark

c. State what your answer to part **b.** represents physically. 1 mark

In some situations it is preferable to travel along roads in a certain direction for tour purposes, as some attractions need to be viewed before others. Filomena redesigns the matrix to take this into account.

$$\begin{array}{c} \textit{from town} \\ F = \begin{array}{c} \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \quad \textit{to town} \end{array}$$

d. Complete the diagram below. A double-ended arrow indicates that either direction is acceptable. A single-ended arrow shows the direction in which the tour bus should proceed. 1 mark



e. Write the sum of all rows as a column matrix, C; that is, the sum of the first row of F becomes $C_{1,\,1}$ and the sum of the second row of F becomes $C_{2,\,1}$ and so on. 1 mark

The local tourism authorities are interested in determining the likely travel through each town. A single-order traffic matrix might not be sufficient as roads from busy towns will carry more tourist buses into a town than others. They decide that a second-order traffic matrix, F^2 should be used.

f. Calculate F^2. 1 mark

$$\begin{bmatrix} & \\ & \end{bmatrix}$$

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The tourism authority wants to multiply by a matrix G so that F^2G gives a 5×1 matrix with the sums of all the rows of F^2. The tourism authority will use this matrix to evaluate tourism traffic.

g. Write down matrix G. 1 mark

h. Which town has the most traffic if this method is employed? 1 mark

Question 2 (3 marks)

Febs investigates the tourism preferences of Melbourne families travelling to North Victoria. They are asked to choose 1 of the 5 towns as a favourite and this preference is tested every year. Febs notes that the transition matrix below governs how preferences change over 1 year.

$$T = \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0.90 & 0.10 & 0.10 & 0.05 & 0.05 \\ 0.05 & 0.80 & 0 & 0 & 0.05 \\ 0 & 0 & 0.75 & 0 & 0 \\ 0.05 & 0.05 & 0.10 & 0.95 & 0.05 \\ 0 & 0.05 & 0.05 & 0 & 0.85 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array}$$

a. What proportion of those favouring Beechworth (B) in any year change their preference to Echuca (E) the next year? 1 mark

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In a November 2015 survey, the following preferences were noted.

Town	Albury	Beechworth	Chiltern	Deniliquin	Echuca
No. favouring in 2015	900	500	700	600	1200

b. What will be the predicted numbers for the November 2016 survey? 1 mark

Town	Albury	Beechworth	Chiltern	Deniliquin	Echuca
No. favouring in 2016					

c. Determine the long-term annual tourist survey numbers for each town. 1 mark

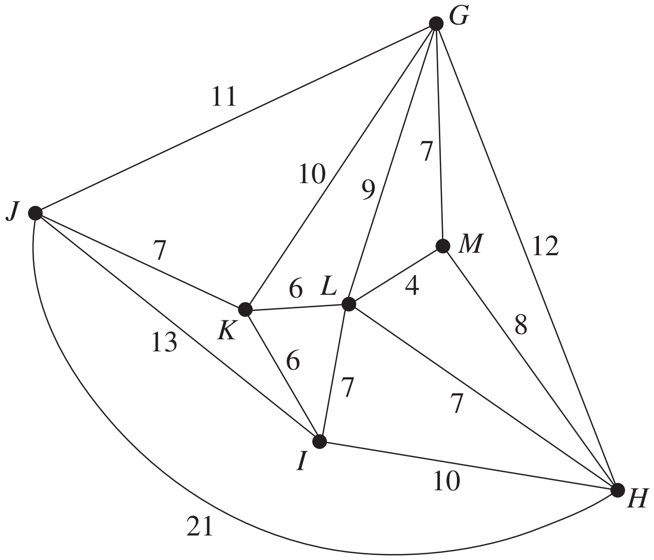
Town	Albury	Beechworth	Chiltern	Deniliquin	Echuca
No. favouring long-term					

END OF MODULE 1

Module 2 – Networks and decision mathematics

Question 1 (5 marks)

A shire is installing a system of gas pipelines. There are a series of towns that must be supplied. It is preferred that the total length of pipeline be kept to a minimum. The lengths of pipes are shown in the network diagram below. The edges represent the pipes between towns and the numbers on the edges are lengths of pipes in km.



a. What is the least length for installed pipelines between *J* and *H*? 1 mark

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All of the pipelines need to be inspected, except those from *L* to *M* that have already been checked. A machine is used to detect whether the underground pipeline below is leaking. It is hoped that the machine can start at one town and proceed along all pipelines exactly once, yet visit them all.

b. State (with reasons) whether this is possible, and where the machine should start and finish. 2 marks

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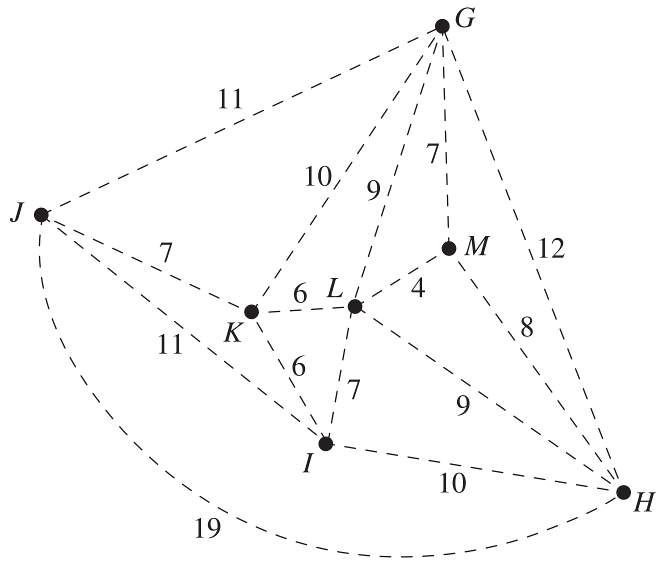
All towns must be connected to every other town by one upgraded gas pipeline. The upgraded pipelines will follow the path of the existing pipelines, but not all will be required. A minimal spanning tree should be used.

c. **i.** Draw the minimal spanning tree on the diagram below. 1 mark

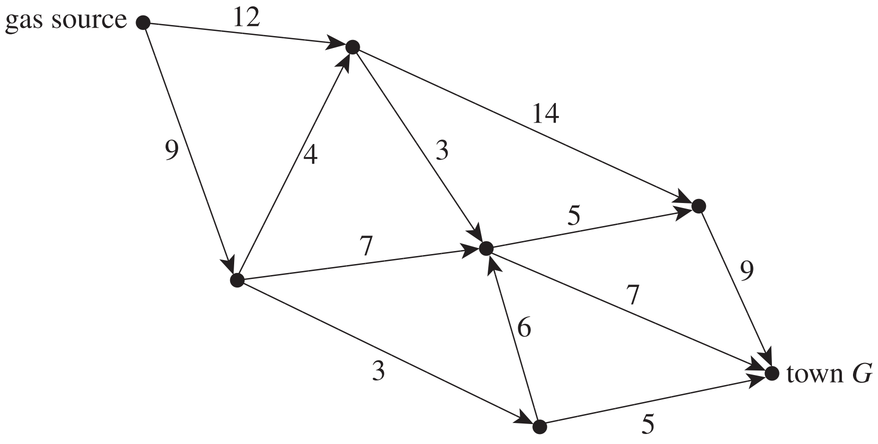


ii. A new town will be added to this network. It is town *P*, 4 km from *I*, 5 km from *H* and 4 km from *L*.

How much longer would the total pipelines need to be to connect *P* in addition to the existing towns? You may choose different pipes to those you used in part **c. i.** 1 mark

Question 2 (3 marks)

The diagram below shows the gas flow from the source to town *G*. The edges represent pipes and the numbers are the maximum flow through that pipe in GL per hour.



a. Determine the maximum flow from the gas source to town *G*. 1 mark

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b. One pipe may be increased in maximum flow by 4 GL.

On the diagram above, indicate which pipe should be improved and state the new maximum gas flow. 2 marks

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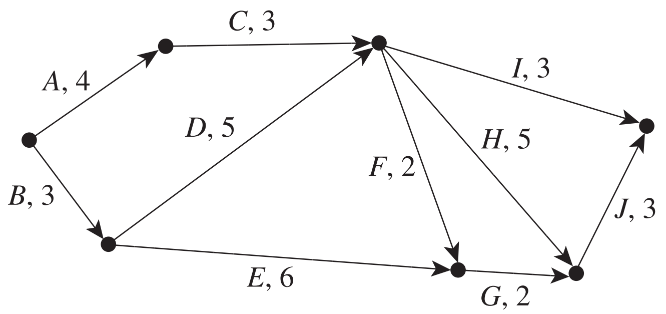
Question 3 (4 marks)

A series of activities are required in order to survey, construct and lay gas pipelines. These are summarised in the table below.

Activity	Predecessors	Duration	Earliest starting time (EST)
A	none	4	0
B	none	3	0
C	*A*	3	4
D	*B*	5	3
E	*B*	6	3
F		2	
G	*F*, *E*	4	10
H	*C*, *D*	5	8
I	*G*, *H*	3	8
J	*C*, *D*	3	

This information is shown as a network below.



a. Determine the values of the missing table items; that is, the ESTs for *F* and *J*. 1 mark

b. Find the latest starting time (LST) of activity *E*. 2 marks

c. Write down, in order, the activities on the critical path. 1 mark

END OF MODULE 2

Module 3 – Geometry and measurement

Question 1 (4 marks)

The 'Tour of Transylvania' bike race is over several legs. It starts at Augherton (A), passes through Barkville (B) and then on to Crankburg (C). The details of the towns are given below.

Town	Latitude	Longitude
Augherton	17 S	136 E
Barkville	17 S	140 E
Crankburg	19 S	140 E

a. Find the distance from Crankburg to Barkville to the nearest km. 1 mark

b. Find the time difference between Augherton and Barkville in minutes. 2 marks

c. It is noted that Augherton and Barkville are 427 km apart.

If we ignore the curvature of the Earth, what is the distance between Augherton and Crankburg? 1 mark

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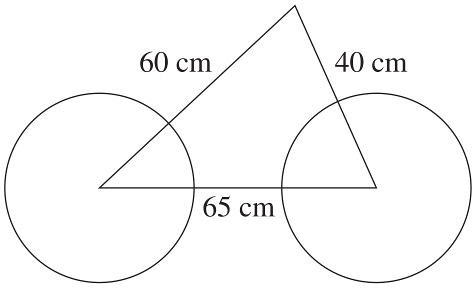
Question 2 (8 marks)

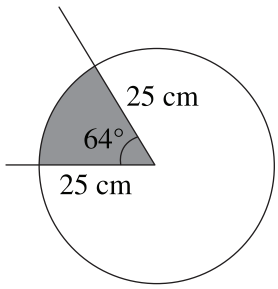
Trans Cycles design bicycles for the race. Part of the design is shown below.



Three metal rods form a triangle as shown.

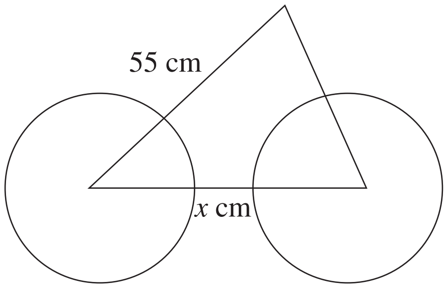
a. Find the smallest angle within this triangle to the nearest degree. 2 marks

The design is altered slightly. Each wheel has a radius of 25 cm. The wheel on the right has an angle of 64° where the two rods meet at its centre.



b. Find the area of the sector formed by the wheel and these two rods. 2 marks

The manager of Trans Cycles decides to vary the design slightly so that the rod that was 60 cm becomes 55 cm long. The third rod (previously 65 cm) will be x cm. The angle between the 65 cm and the x cm rods is 40°. The manager asks his designer to draw up the design. He expects a design that has all acute angles within it as shown below.



The designer draws a triangle with an obtuse angle and the wheels very close together.

c. Show by calculations that both the designer's and manager's designs are possible within the rules. 2 marks

d. Find the separation between the wheel centres for both the manager's intended design and what the designer drew. 2 marks

END OF MODULE 3

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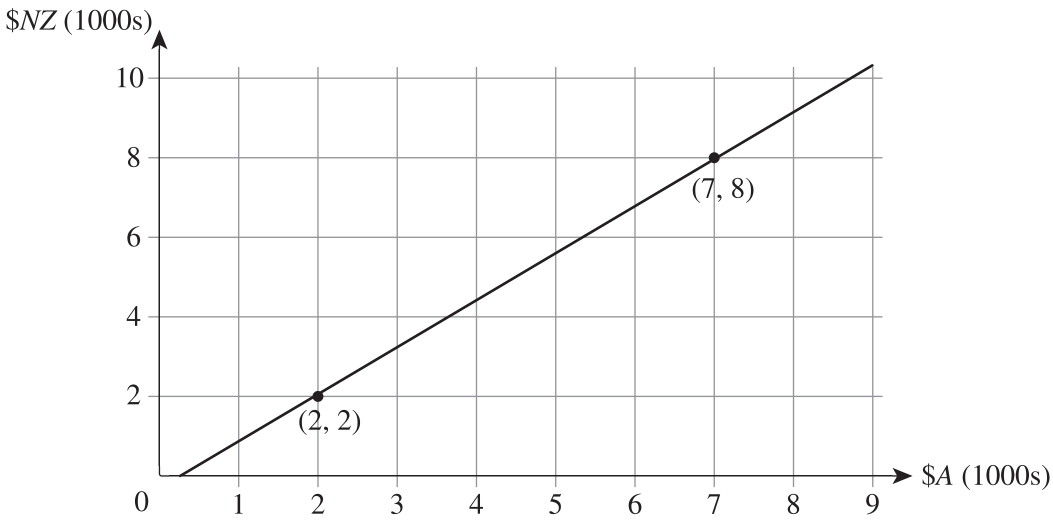
Module 4 – Graphs and relations

Question 1 (5 marks)

'New Age' currency exchange changes Australian dollars (*$A*) into New Zealand dollars (*$NZ*). The graph below shows the amount of New Zealand dollars which can be obtained for various Australian dollar amounts.



a. How many New Zealand dollars can be obtained for $4000 Australian? 1 mark

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b. Cameron is deciding whether he will convert $4000 or $5000 Australian into New Zealand currency.

How many extra New Zealand dollars does he get for converting $5000 instead of $4000 Australian? 2 marks

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c. Is the graph a straight line? Explain how you can tell, showing calculations. 2 marks

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Question 2 (7 marks)

The success of 'New Age' will depend on how they sell their other services. They also sell managed funds. A basic package includes 5 tech shares and 4 resource shares. An advanced package includes 4 tech shares and 12 resource shares. This month they have 5000 tech shares and 6000 resource shares to allocate to either package. Let x be the number of basic packages produced and y be the number of advanced packages.

a. One of the constraints is $4x + 12y \leq 6000$.

What is the other constraint? 1 mark

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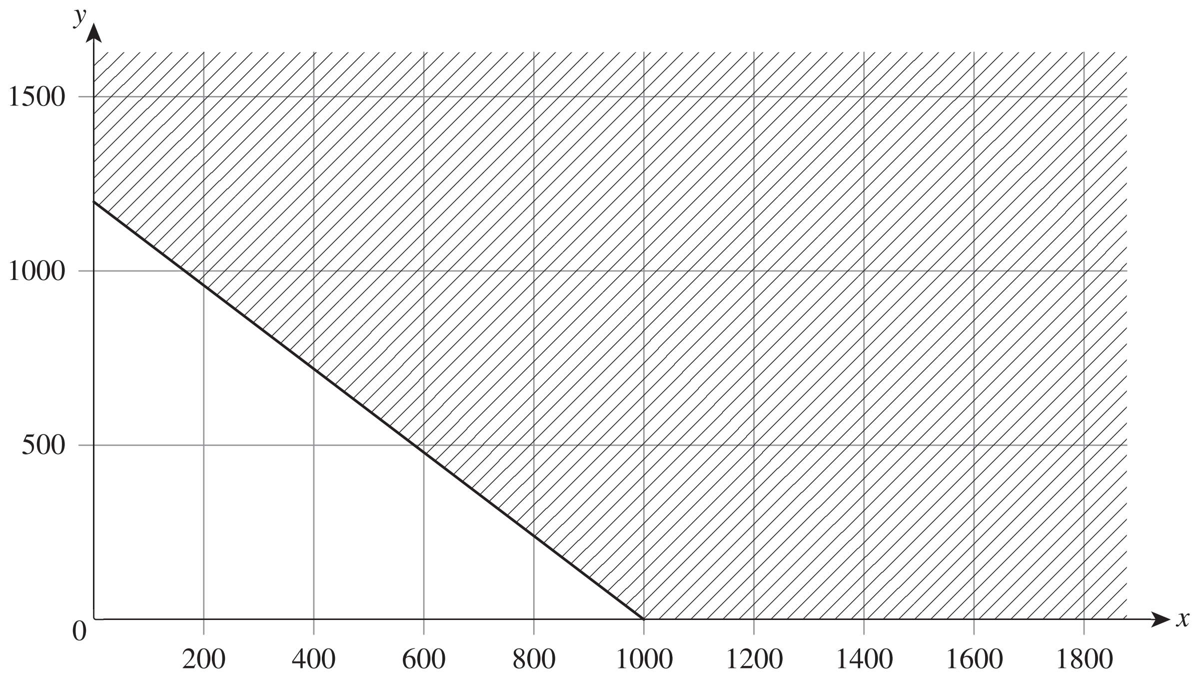
b. Complete the graph below for which the feasible region is shown unshaded. 1 mark



c. 'New Age' can make a profit of \$100 from each basic package and \$120 from each advanced package.

Determine the objective function, P, for total profit. 1 mark

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d. Complete the table below, filling in all gaps. 2 marks

Point	Coordinates	Value of ***P***
A (*y*-intercept)	(0, 500)	\$60 000
B (intersect)		
C (*x*-intercept)	(1000, 0)	

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e. Thus state the number of each package that should be made and the associated maximum profit. 2 marks

END OF QUESTION AND ANSWER BOOKLET