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YEAR 12 *Trial Exam Paper*

2016

FURTHER MATHEMATICS

Written examination 2

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions
- mark allocations
- tips on how to approach the exam

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SECTION A – Core

Data analysis

Question 1a.

Worked solution

$$\frac{52.2}{100} \times 329\,890 = 172\,202.58 \approx 172\,203$$

Mark allocation: 1 mark

- 1 mark for the correct answer
- Working not required for mark to be given

Question 1b.

Worked solution

We can see from the table that there is little difference in the percentage of male and female salary earners in each age group.

If we focus on males, we can see that 51% of salary earners were male in the 15–24 years age group. This is approximately the same as the percentage of males in the 25–34, 35–44, 45–54 and 55–64 years age groups.

Mark allocation: 2 marks

- 1 mark for a statement explaining that there is little difference in the percentage of males and females between age groups
- 1 mark for a statement quoting percentages for males or females, or for quoting percentages within a particular age group

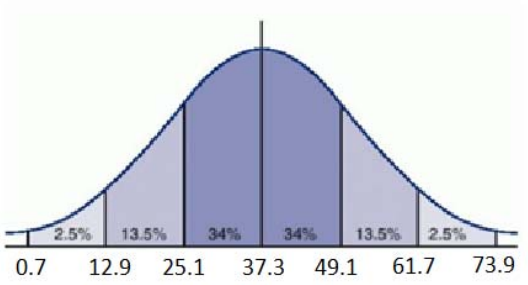
Tip

- *Choose one aspect of the data to compare. Do not attempt to discuss the entire table.*

Question 2a.

Worked solution

Consider the bell curve below.



The age 25.1 years is one standard deviation below the mean and the age 61.7 years is two standard deviations above the mean. Therefore, the percentage between 25.1 years and 61.7 years is

$34 + 34 + 13.5 = 81.5\%$

$$\frac{81.5}{100} \times 22\,507\,617 = 18\,343\,707.9$$

Therefore, 18 343 708 people were aged between 25.1 and 61.7 years.

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 2b.

Worked solution

$$z = \frac{x - \overline{x}}{s_x}$$

$$z = \frac{27 - 37.3}{12.2}$$

$$z = -0.8$$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3a.

Worked solution

This is numerical discrete data.

Mark allocation: 1 mark

- 1 mark for describing data as numerical discrete
- No marks if only numerical, or only discrete is given
- No half marks given

Question 3b.

Worked solution

The median is 30.5.

The median is the middle value. Because there are 34 data values, the middle value is between the 17th and 18th value.

The 17th data value is 30.

The 18th data value is 31.

30.5 is the middle of these numbers.

The range is 22.

The range can be calculated by subtracting the minimum value from the maximum value.

$\text{Range} = \text{Max} - \text{Min}$

$\text{Range} = 44 - 22$

$\text{Range} = 22$

Mark allocation: 1 mark

- Both values must be correct to obtain 1 mark; no half marks given

Question 3c.

Worked solution

Use the statistics function on your CAS calculator to find Q_1 and Q_3.

$Q_1 = 25$ $\quad$ $Q_3 = 32$

$IQR = Q_3 - Q_1$

$IQR = 32 - 25$

$IQR = 7$

Upper fence:

$= Q_3 + 1.5 \times IQR$

$= 32 + 1.5 \times 7$

$= 42.5$

Because the data value of 44 is greater than the upper fence, it is an outlier.

Mark allocation: 2 marks

- 1 mark for correct calculation of upper fence
- 1 mark for a statement explaining why 44 is an outlier

Question 4a.

Worked solution

The explanatory variable is *population*.

Mark allocation: 1 mark

- 1 mark for the correct answer

Tip

- *The explanatory variable is on the x-axis of the scatterplot, and can also be determined from the regression equation.*

Question 4b.

Worked solution

The relationship between *population* and *adult literacy rate* is of moderate strength.

This can be determined using the Pearson's correlation coefficient, *r*.

Mark allocation: 1 mark

- 1 mark for stating moderate

Question 4c.

Worked solution

On average, as the *population* for a country increases by 1 million people, the *adult literacy rate* will increase by 0.8%.

Mark allocation: 2 marks

- 1 mark for correctly stating that the slope of the regression equation is 0.8 AND for a statement indicating that the literacy rate will increase
- 1 mark for an accurate statement, as above or similar, that includes units and correctly uses the variables

Question 4d.i.

Worked solution

Residual value = actual value – predicted value

Predicted value, using the regression equation

$65.7 + 0.8 \times 6.2$

$= 70.66$

Residual value = 46.0 – 70.66

$= -24.66$

Mark allocation: 1 mark

- 1 mark for the correct residual value

Question 4d.ii.

Worked solution

This question is referring to the coefficient of determination, r^2.

$r^2 = (0.531)^2 = 0.2819$

As a percentage, this is 28.2%.

Mark allocation: 1 mark

- 1 mark for the correct answer, written as a percentage

Tip

- *We know that this question is asking for the value of r^2 due to the wording of the question: 'percentage **of the variation in** adult literacy rate of the countries sampled **can be explained by the variation in** population'. This is the standard response for interpretation of the coefficient of determination.*

Question 5a.

Worked solution

Enter the data into a spreadsheet on your calculator and perform the squared transformation to *population* variable.

The equation is $y^2 = 4663.6 + 109.895x$

Written to 3 significant figures, the equation is $y^2 = 4660 + 110x$

$(adult\ literacy\ rate)^2 = 4660 + 110 \times population$

Mark allocation: 2 marks

- 1 mark for the correct slope
- 1 mark for the correct intercept

Question 5b.

Worked solution

Substitute the population value of 11.4 into the equation

$$y^2 = 4660 + 110 \times 11.4$$

Find the value of *y*, which is the adult literacy rate.

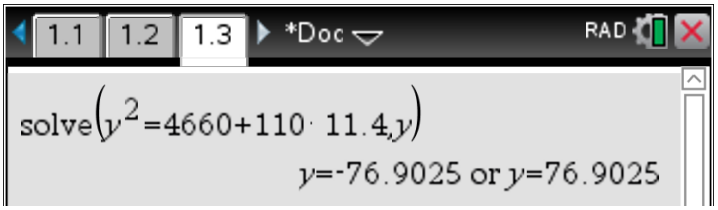
$$y^2 = 4660 + 110 \times 11.4$$
$$y = \sqrt{4660 + 110 \times 11.4}$$
$$y = 76.90$$
$$= 77\%$$

Mark allocation: 1 mark

- 1 mark for the correct answer
- No consequential marks given for an incorrect answer in part a.
- No method marks given

Tip

- *Remember that when a transformation has been performed on the response variable, it is not as simple as just substituting into the equation.*
- *It can be a good idea to use 'solve' on your CAS calculator to find the y-value here.*



- *Be careful with this because you need to make a decision about which answer to use.*
- *A percentage will not be a negative.*

Question 5c.

Worked solution

This is interpolation because it is **within** the data range given in the original data set.

Mark allocation: 1 mark

- 1 mark for stating interpolation or interpolating

Question 6a.

Worked solution

The time series plot shows no general trend with time.

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 6b.

Worked solution

Using the CAS calculator, we find that the equation is $conversion\ rate = 0.966 - 0.00658 \times year$

Mark allocation: 2 marks

- 2 marks for the correct answer
- 1 working mark may be allocated if correct equation is given using 'x' and 'y', rather than the variables

Question 6c.

Worked solution

According to our axis labelling, 2017 will be year 10. Substitute 10 into our regression equation.

$conversion\ rate = 0.966 - 0.00658 \times 10$

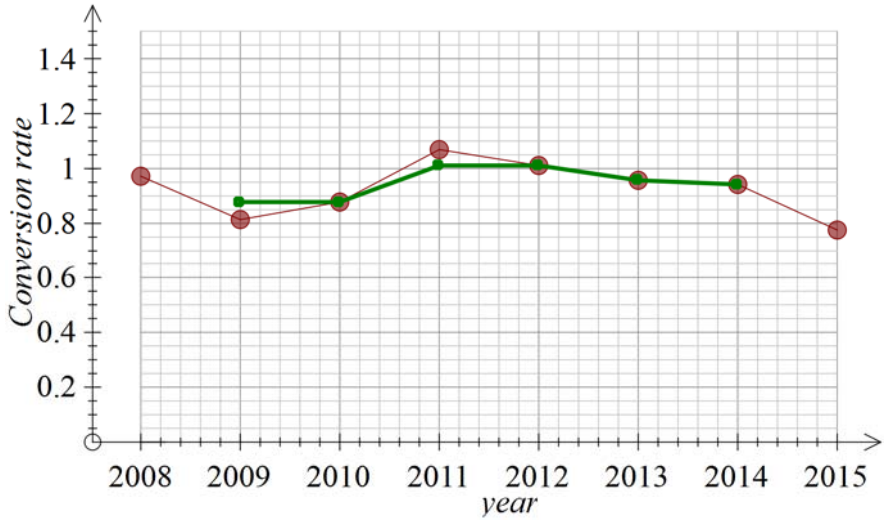
$conversion\ rate = 0.90$

Mark allocation: 1 mark

- 1 mark for the correct answer
- No consequential marks given for incorrect equation from part b. used in part c.

Question 6d.

Worked solution



The three-median smoothed value for 2009 is 0.878.

This is found by identifying the median of the points for 2008, 2009 and 2010.

0.972 0.814 0.878

Write them in ascending order and find the middle value.

0.814 (0.878) 0.972

Mark allocation: 1 mark

- 1 mark for the correct answer
- The data point must be connected to the original time series graph with a straight line

Question 7a.

Worked solution

Using the recurrence relation

$V_0 = 56\,000$

$V_1 = V_0 - 5880$
$V_1 = 56\,000 - 5880$
$V_1 = 50\,120$

$V_2 = V_1 - 5880$
$V_2 = 50\,120 - 5880$
$V_2 = 44\,240$

$V_3 = V_2 - 5880$
$V_3 = 44\,240 - 5880$
$V_3 = 38\,360$

Mark allocation: 1 mark

- 1 mark for a correct series of calculations

Question 7b.

Worked solution

Repeat the recurrence relation until the value of the printing machine reaches or goes below $20 000.

$V_3 = 38360$
$V_4 = 32480$
$V_5 = 26600$
$V_6 = 20720$
$V_7 = 14840$

As seen from the iterations, the value of the printing machine drops below $20 000 after 7 years.

Mark allocation: 1 mark

- 1 mark for the correct answer of 7 years

Tip

- *Note that although the value of the printing machine is close to $20 000 after 6 years, David will only consider selling his printing machine after it reaches or drops below $20 000. Therefore, an extra year of use is necessary.*

Question 7c.

Worked solution

$V_7 = 56\,000 - 0.15 \times 7 \times 35\,000$

$V_7 = 19\,250$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 7d.

Worked solution

The general recurrence relation for reducing balance depreciation is

$$V_n = R^n \times V_0, \quad \text{where} \quad R = 1 - \frac{r/n}{100}$$

The value of variables in this relation are

$V_0 = 56\ 000$

$$R = 1 - \frac{12.5/4}{100} = 0.96875 = 0.97$$

Therefore, the recurrence relation is

$$V_n = 0.97^n \times 56\ 000$$

Mark allocation: 2 marks

- 2 marks if the relation is correct
- If incorrect, 1 mark can be given for the following:
  - correct value for R
  - correct general form for relation

Question 8a.

Worked solution

$$R = 1 + \frac{r/n}{100}$$

$$1.008 = 1 + \frac{r/12}{100}$$

Solve for r

$$r = 9.6\%$$

Mark allocation: 1 mark

- 1 mark for the correct answer (percentage sign not needed)

Question 8b.

Worked solution

$$V_{18} = 1.008^{18} \times 115\,000$$

$$V_{18} = 132\,736$$

Mark allocation: 1 mark

- 1 mark for the correct answer

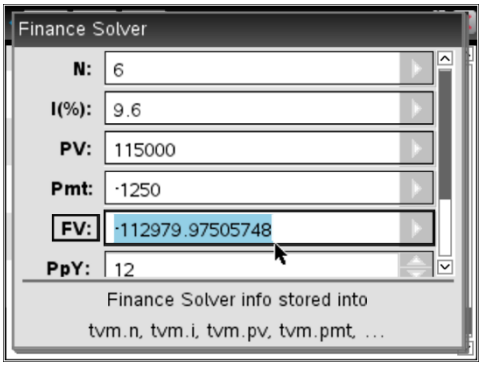
Question 8c.

Worked solution

An amortisation table can be helpful in these situations:

Payment number	Payment	Interest	Principal reduction	Balance of loan
0	–	–	–	115 000.00
1	1250.00	920.00	330.00	114 670.00
2	1250.00	917.36	332.64	114 337.36
3	1250.00	914.70	335.30	114 002.06
4	1250.00	912.02	337.98	113 664.08
5	1250.00	909.31	340.69	113 323.39
6	1250.00	906.59	343.41	112 979.98

Alternatively, the financial solver in the CAS calculator can be used.



Mark allocation: 1 mark

- 1 mark for the correct answer
- A consequential mark can be given here for an answer used from part a.

Question 8d.

Worked solution

$$r_{effective} = \left(\left(1+\frac{r/n}{100}\right)^{n} - 1\right) \times 100$$

$$r_{effective} = \left(\left(1+\frac{9.6/12}{100}\right)^{12} - 1\right) \times 100$$

$$r_{effective} = 10.03\%$$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 9a.

Worked solution

Use the financial solver in your CAS calculator, to calculate the payment.

$N = 12$

$I = 6.9\%$

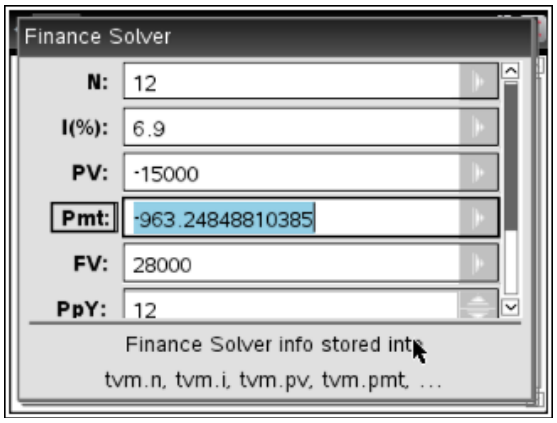
$PV = -15\,000$

$Pmt = ?$

$FV = 28\,000$

$PpY = 12$

$CpY = 12$



The monthly payment made by Rita will need to be $963.25.

Mark allocation: 1 mark

- 1 mark for the correct answer
- No marks given if the number is written as negative
- No marks given if answer is not written in dollars and cents

Question 9b.

Worked solution

Rita invested $15 000 and will make 12 payments of $916.49.

$15\,000 + 12 \times 916.49 = \$25\,997.88$

Rita's final balance was $28 000.

Interest earned = $28\,000 – 25\,997.88 = 2002.12$

Rita's account earned $2002.12 interest.

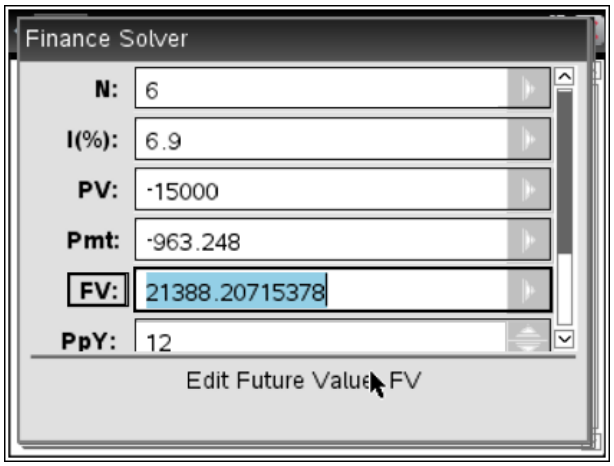
Mark allocation: 1 mark

- 1 mark for the correct answer
- The dollar sign is not required, but the answer must be correct to 2 decimal places.

Question 9c.

Worked solution

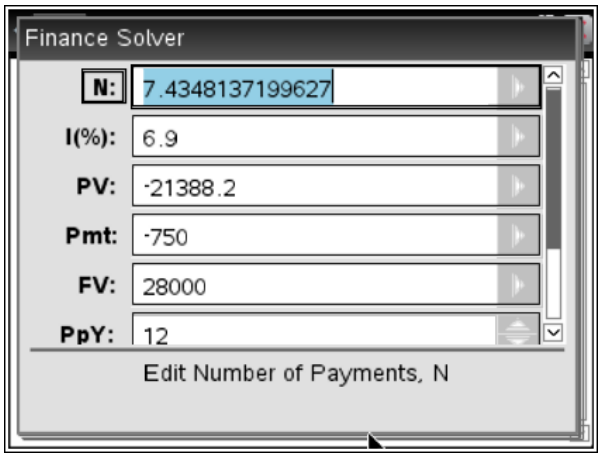
First, we need to calculate the value of Rita's investment after 6 months.



The value of Rita's loan after 6 months is $21 388.21.

Now, we enter this amount into *PV*, change the *Pmt* to 750 and change the *FV* to 28 000.

This will allow us to calculate *N* under these new conditions.



It will take Rita a further 8 payments to reach her goal.

This means that, in total, it will take Rita 14 months to reach her goal.

Note that 7 payments will not be quite enough and therefore Rita will need an additional month.

Mark allocation: 1 mark

- 1 mark for the correct answer (14 months)

SECTION B – Modules

Module 1 – Matrices

Question 1a.

Worked solution

The sum of the elements in row 3 represent the number of friendships that Carly has within the group of girls.

OR

Carly has 3 friendships within the group.

Mark allocation: 1 mark

- 1 mark for a correct statement regarding Carly's friendships within the group

Question 1b.

Worked solution

It is not necessary for a person to be friends with themselves.

Also, there are no loops in the diagram.

Mark allocation: 1 mark

- 1 mark for correctly identifying that a 1 in row 1, column 1 (for example) would mean that Abbey is friends with herself and that this is not necessary (or equivalent statement)

Question 2a.

Worked solution

The missing element in row 1, column 3 is 0 (because Abbey did not order any pizza)

The missing element in row 3, column 2 is 1 (because Diane ordered 1 pizza)

Mark allocation: 1 mark

- 1 mark for both elements correct

Question 2b.

Worked solution

Using matrices to solve simultaneous equations:

Our initial equation is $AX = B$

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} m \\ s \\ p \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 5.5 \end{bmatrix}$$

Next, we should rearrange the equation, using the inverse of A, to solve for matrix X

$$X = A^{-1}B \qquad \begin{bmatrix} m \\ s \\ p \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 5 \\ 5.5 \end{bmatrix}$$

Use the calculator to perform this matrix multiplication

$$\begin{bmatrix} m \\ s \\ p \end{bmatrix} = \begin{bmatrix} 2 \\ 2.5 \\ 3 \end{bmatrix}$$

Therefore, the price of one slice of pizza is $3.00.

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3a.i.

Worked solution

The element in row 3, column 2 tells us that 10% of the students who have selected a Science workshop this week will select an English workshop next week.

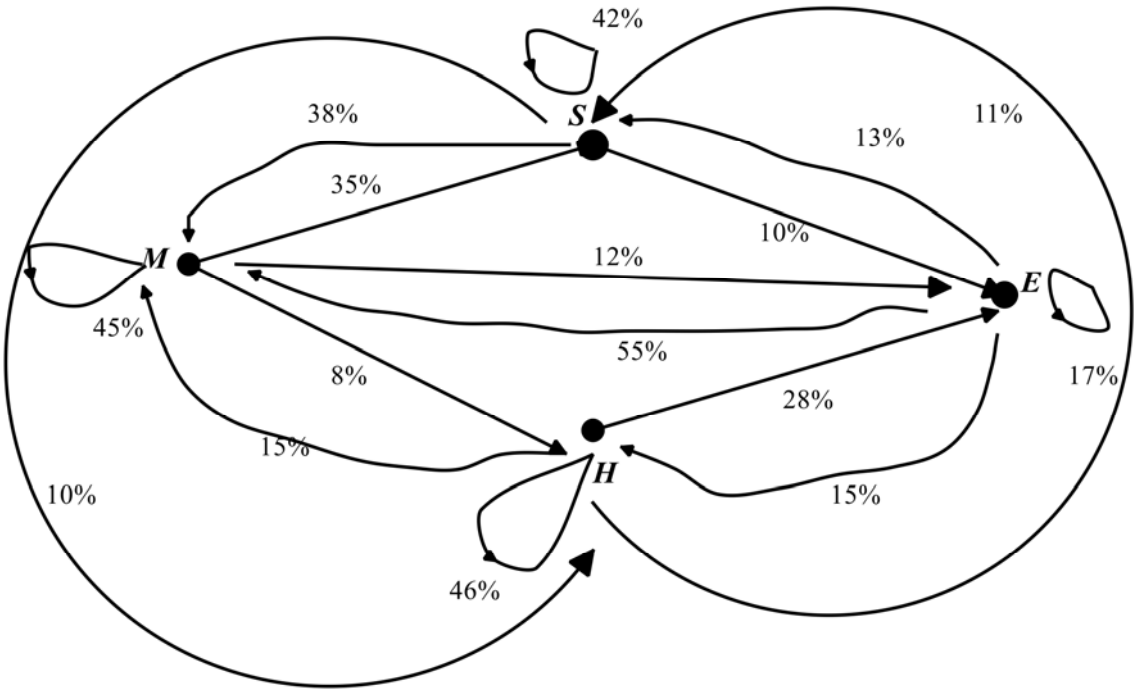
(Or an equivalent statement using 0.1 as the proportion of students)

Mark allocation: 1 mark

- 1 mark for a correct statement

Question 3a.ii.

Worked solution



Mark allocation: 2 marks

- 1 mark for correctly placed lines, with arrows included
- 1 mark for percentages correctly listed
- Proportions (decimals) not accepted

Question 3b.i.

Worked solution

$$S_1 = T \times S_0$$

$$S_1 = \begin{bmatrix} 0.45 & 0.38 & 0.55 & 0.15 \\ 0.35 & 0.42 & 0.13 & 0.11 \\ 0.12 & 0.10 & 0.17 & 0.28 \\ 0.08 & 0.10 & 0.15 & 0.46 \end{bmatrix} \times \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 120.1 \\ 82.4 \\ 46.65 \\ 50.85 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 120 \\ 82 \\ 47 \\ 51 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for the correct state matrix (decimal or written to the nearest whole number)

Question 3b.ii.

Worked solution

$$S_5 = T^5 \times S_0$$

$$S_5 = \begin{bmatrix} 0.45 & 0.38 & 0.55 & 0.15 \\ 0.35 & 0.42 & 0.13 & 0.11 \\ 0.12 & 0.10 & 0.17 & 0.28 \\ 0.08 & 0.10 & 0.15 & 0.46 \end{bmatrix}^5 \times \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix}$$

$$S_5 = \begin{bmatrix} 119.03 \\ 90.40 \\ 43.89 \\ 46.68 \end{bmatrix}$$

$$S_5 = \begin{bmatrix} 119 \\ 90 \\ 44 \\ 47 \end{bmatrix}$$

The number of students predicted to choose the English workshop in the fifth week is closest to 44.

Mark allocation: 1 mark

- 1 mark for answer of 44 that is not written as a matrix

Question 3b.iii.

Worked solution

We are looking for the steady-state matrix.

This means we must find 2 consecutive matrices that give the same elements.

$$S_{15} = T^{15} \times S_0 = \begin{bmatrix} 0.45 & 0.38 & 0.55 & 0.15 \\ 0.35 & 0.42 & 0.13 & 0.11 \\ 0.12 & 0.10 & 0.17 & 0.28 \\ 0.08 & 0.10 & 0.15 & 0.46 \end{bmatrix}^{15} \times \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix} = \begin{bmatrix} 119.074 \\ 90.5136 \\ 43.834 \\ 46.5785 \end{bmatrix}$$

$$S_{16} = T^{16} \times S_0 = \begin{bmatrix} 0.45 & 0.38 & 0.55 & 0.15 \\ 0.35 & 0.42 & 0.13 & 0.11 \\ 0.12 & 0.10 & 0.17 & 0.28 \\ 0.08 & 0.10 & 0.15 & 0.46 \end{bmatrix}^{16} \times \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix} = \begin{bmatrix} 119.074 \\ 90.5136 \\ 43.834 \\ 46.5785 \end{bmatrix}$$

We have reached the steady state matrix.

Therefore, in the long term, the number of students expected to select the Humanities workshop is closest to 47 students.

Mark allocation: 2 marks

- 1 mark for the correct answer of 47 students in Humanities workshop

Question 3c

Worked solution

$$S_1 = \begin{bmatrix} 0.41 & 0.32 & 0.45 & 0.15 \\ 0.32 & 0.40 & 0.13 & 0.11 \\ 0.12 & 0.10 & 0.17 & 0.28 \\ 0.08 & 0.10 & 0.15 & 0.45 \end{bmatrix} \times \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix} = \begin{bmatrix} 105 \\ 77.7 \\ 46.65 \\ 50.3 \end{bmatrix} = \begin{bmatrix} 105 \\ 78 \\ 47 \\ 50 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for the correct matrix

Question 3d.

Worked solution

Multiply T by S_0

$$\begin{bmatrix} 0.41 & 0.32 & 0.45 & 0.15 \\ 0.32 & 0.40 & 0.13 & 0.11 \\ 0.12 & 0.10 & 0.17 & 0.28 \\ 0.08 & 0.10 & 0.15 & 0.45 \end{bmatrix} \times \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix} + \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix}$$

$$\begin{bmatrix} 105 \\ 77.7 \\ 46.65 \\ 50.3 \end{bmatrix} + \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix}$$

Subtract the answer from the original numbers of students.

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 110 \\ 70 \\ 65 \\ 55 \end{bmatrix} - \begin{bmatrix} 105 \\ 77.7 \\ 46.65 \\ 50.3 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 5 \\ -7.7 \\ 18.35 \\ 4.7 \end{bmatrix}$$

$a = 5$, $b = -8$, $c = 18$, $d = 5$

Mark allocation: 1 mark

- 1 mark for correct answers for a, b, c and d rounded to the nearest whole number

Explanatory notes

The meaning of these letters is that the teachers need the following to maintain their workshop numbers:

5 new Maths enrolments

8 fewer Science enrolments

18 new English enrolments

5 new Humanities enrolments

Module 2 – Networks and decision mathematics

Question 1a.

Worked solution

Vertex *D* has 4 edges connected to it.

Therefore, the degree of vertex *D* is 4.

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 1b.

Worked solution

An Eulerian trail is possible when the network has exactly 2 vertices of odd degree, with the remaining vertices having an even degree. The network shown has 2 odd vertices (Start and *E*), and the rest are even. This means that an Eulerian trail is possible.

The trail will start and finish at the odd vertices.

Therefore, one example of an Eulerian trail is

Start – *F* – First aid – *D* – First aid – *E* – *D* – *B* – *C* – *A* – Start – *E*

Mark allocation: 1 mark

- 1 mark for a correct Eulerian trail, starting at Start and finishing at *E*
- Note that there is more than one possible answer here

Question 1c.i.

Worked solution

This is referring to an Eulerian circuit.

This is only possible if all vertices within the network are of even degree.

This network has 2 odd vertices, Start and *E* and therefore does not satisfy the conditions for an Eulerian circuit.

Mark allocation: 1 mark

- One mark for a correct explanation containing reference to the need for all vertices to be even and a reference to the vertices Start and *E* being odd

Question 1c.ii.

Worked solution

Now that we have all vertices with even degree, an Eulerian circuit is possible.

One possible circuit is

Start – *F* – First aid – *D* – First aid – *E* – *D* – *B* – *C* – *A* – Start – *E* – Start

Mark allocation: 1 mark

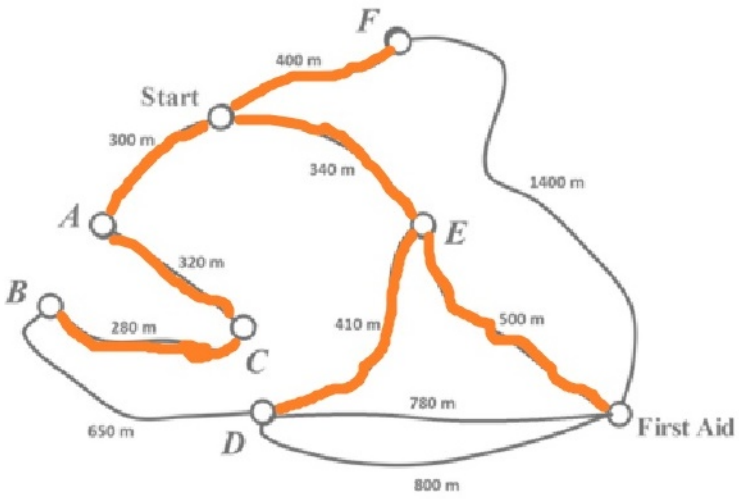
- 1 mark for the correct answer

Question 1d.i.

Worked solution

This is referring to a minimum spanning tree.

The minimum spanning tree is shown below



Mark allocation: 1 mark

- 1 mark for correct edges highlighted (alternatives may exist)

Question 1d.ii.

Worked solution

Length of minimum spanning tree is equal to:

400 + 300 + 280 + 340 + 410 + 500 = 2230 m

Mark allocation: 1 mark

- 1 mark given for correct length of minimum spanning tree

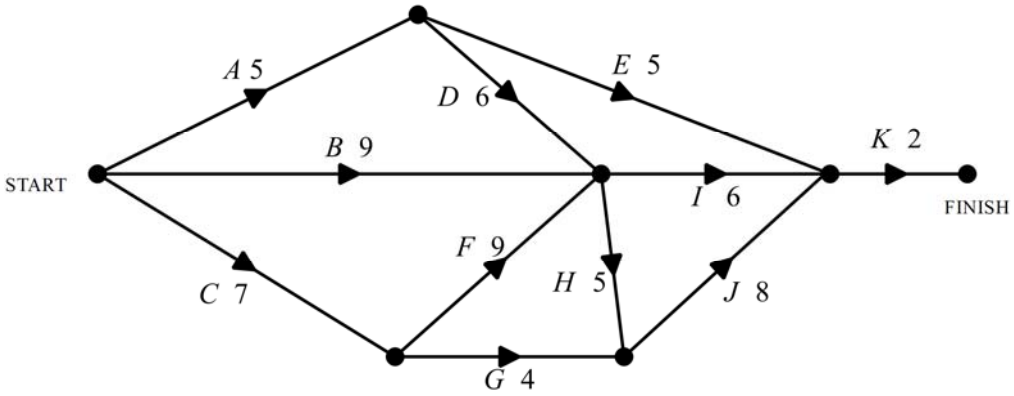
Question 2a.

Worked solution

The predecessors of activity *H* are *B*, *D* and *F*.

Also, activity *H* must come before activity *J*.

Activity *H* is shown on the diagram below.



Mark allocation: 1 mark

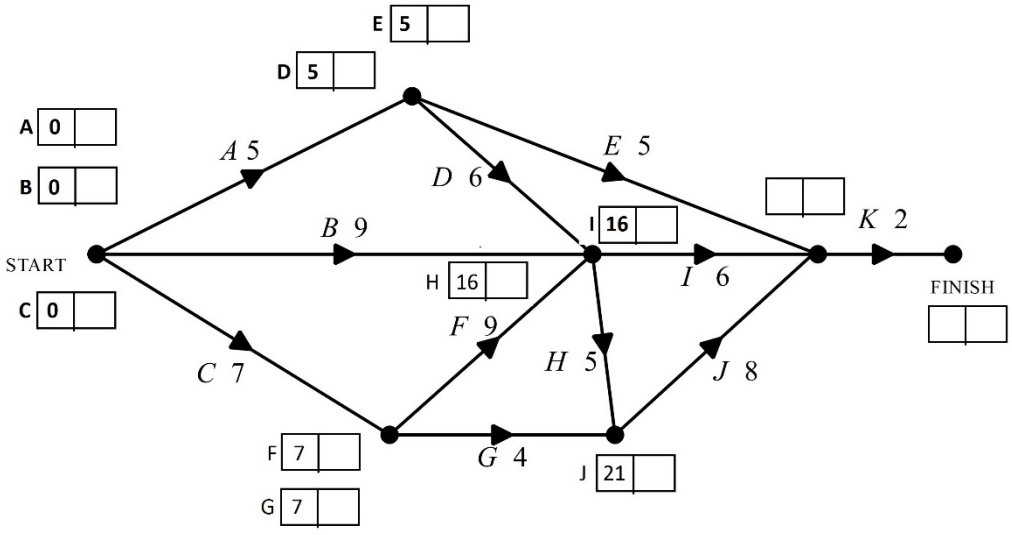
- 1 mark given for correct placement of activity *H*
- Must have the arrow pointing in the correct direction and the label and duration

Question 2b.

Worked solution

The earliest start time for activity *J* is 21 weeks.

This can be found by using forward tracking (shown below).



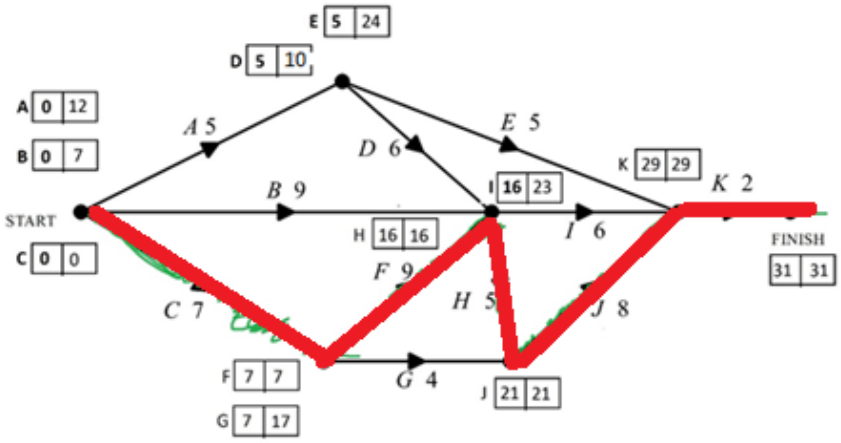
Mark allocation: 1 mark

- 1 mark for correct answer of 21 (weeks)

Question 2c.

Worked solution

The critical path is $C - F - H - J - K$, and is shown on the diagram below.



Mark allocation: 1 mark

- 1 mark for the correct critical path with activities written in order

Explanatory notes

The critical path is always made up of the activities that have no float or slack time.

Calculating the earliest start time and latest start time for each activity will enable you to find the critical path.

Pay attention to the predecessors of activities when looking at the earliest start time. For example, the earliest for activity *J* may initially appear to be 7 + 4 = 11 from following the path from *C* to *G* to *J*. However, we need to consider the fact that activity *H* must be completed before activity *J* can begin, and therefore we need to also consider the predecessors of *H*.

Question 2d.

Worked solution

Slack (float) time = earliest start time – latest start time

For activity B, this is 7 – 0 = 7

Mark allocation: 1 mark

- 1 mark for the correct answer of 7 (weeks)

Tip

- *A common incorrect answer will be 14. This comes from incorrectly finding a latest start time of 14. When backtracking along the network, we should subtract from the lowest number, 16, not from 23.*

Question 2e.

Worked solution

Activity *D* is not on the critical path and has a slack time of 5 weeks. If the delay is no longer than this, the project can still be completed in the minimum completion time of 31 weeks.

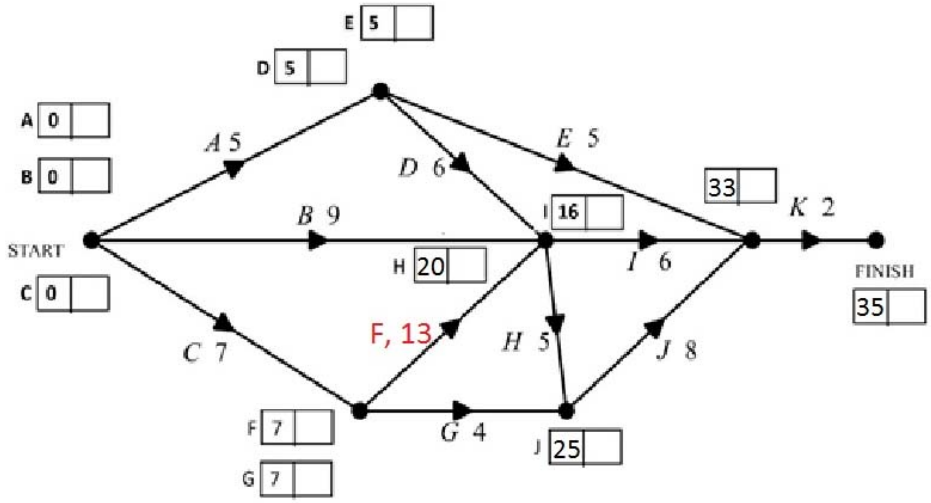
Mark allocation: 1 mark

- 1 mark for the statement above or similar

Question 2f.

Worked solution

The new minimum completion time is 35 weeks.



Mark allocation: 1 mark

- 1 mark for 35 (weeks)

Module 3 – Geometry and measurement

Question 1a.

Worked solution

We are calculating the volume of a rectangular prism $V = l \times w \times h$

We must first ensure that all of the measurements are in the same units; that is, we need to convert 450 mm to metres by dividing by 10 000.

$$450 \text{ mm} \div 10\,000 = 0.045 \text{ m}$$

Volume of concrete, in metres, equals

$V = l \times w \times h$

$V = 6 \times 2 \times 0.045$

$V = 0.54 \text{ m}^3$

Mark allocation: 1 mark

- 1 mark for the correct answer of 0.54; units are not required
- This is an exact answer and rounding does not apply

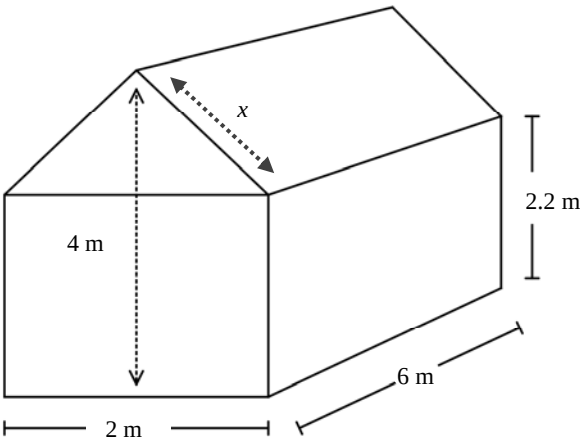
Tip

- *The step where we convert the measurement in millimetres to metres is easy to overlook. Be careful when answering this type of question and read units carefully.*

Question 1b.

Worked solution

Additional measurements are shown on the diagram below.



To calculate the amount of tin required, we calculate the total surface area (TSA).

TSA of roof = TSA for triangular prism (without base)

TSA of roof = $(bh + bl + l\sqrt{b^2 + h^2}) - bl$

For the triangular prism $l = 6, b = 2, h = 4 - 2.2 = 1.8$

$\text{TSA} = (2 \times 1.8 + 2 \times 6 + 6 \times \sqrt{1^2 + 1.8^2}) - 2 \times 6$

TSA = 28.3 m^2

Rounded to the nearest square metre, this is 28 m^2.

Mark allocation: 1 mark

- 1 mark for the correct answer, rounded to the nearest whole number

Question 1c.

Worked solution

The total area of tin required for the shed is:

$$28.3 \text{ (from previous question)} + 2 \times 2.2 \times 2 + 2 \times 2.2 \times 6 = 63.5$$

Scale factor = $\dfrac{\text{smaller shed}}{\text{larger shed}} = \dfrac{28.3}{63.5} = 0.4394$

Because we are considering an area, our scale factor has been squared.

Therefore, scale factor = $\sqrt{\dfrac{28.3}{63.5}} = 0.667 \approx \dfrac{2}{3}$

Mark allocation: 2 marks

- To obtain 2 marks the following must be shown:
  - 1 mark can be allocated for calculating the correct ratio of $\dfrac{28.3}{63.5}$

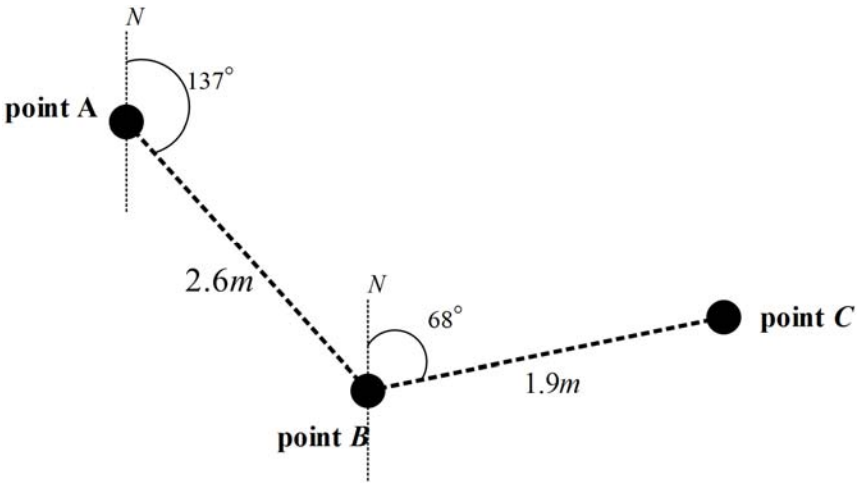
    OR
  - 1 mark can be given for taking the square root of a ratio found.
  - The final answer of $\dfrac{2}{3}$ (or its decimal equivalent) must be found to get the second mark and must follow mathematical reasoning that achieves this result.

Tip

- *It can sometimes be difficult to gain full marks for 'show that' questions.*
- *Make sure that you look at the number of marks allocated for the question. If the question is worth 2 marks, this means that there are two steps or pieces of working that are required to get the final answer.*
- *Check over your answer, and make sure that you can see the final answer required and one other piece of working.*

Question 2a.

Worked solution



Mark allocation: 1 mark

- 1 mark for a line drawn to point *C*, which includes both angle and distance
- Point *C* must be clearly labelled.

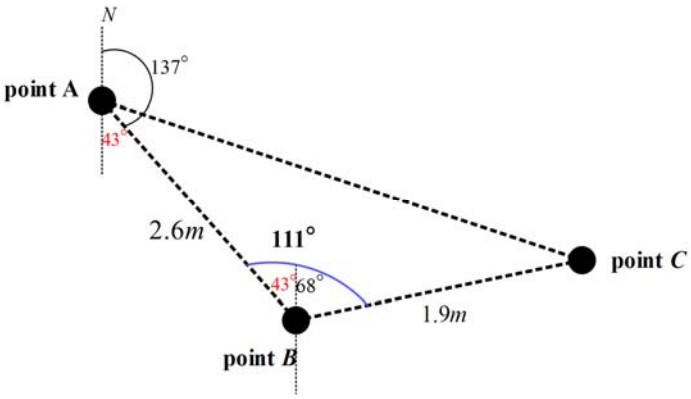
Question 2b.

Worked solution

Step 1:

We can identify vertically opposite angles.

We know that the first angle is 43 because we subtract 137 from 180 at point *A*. Therefore, the second angle can be placed at point B (see diagram below).



We can also find that the angle at *B* within the triangle is 111 (43 + 68).

Step 2:

We can use the cosine rule to find the distance between point *A* and point *C*

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 1.9^2 + 2.6^2 - 2\times 1.9\times 2.6\times \cos(111)$$
$$b = \sqrt{1.9^2 + 2.6^2 - 2\times 1.9\times 2.6\times \cos(111)}$$
$$b = 3.7297$$
$$b \approx 3.7m$$

Step 3:

We can now find the angle *BAC* in the triangle

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{1.9}{\sin A} = \frac{3.7}{\sin(111^\circ)}$$
$$\sin A = \frac{1.9\times \sin(111^\circ)}{3.7}$$
$$A = 27.4679$$
$$A = 27^\circ$$

Step 4:

The bearing of point *C* from point *A* is equal to (180 – 27 – 43)° = 110°.

Mark allocation: 2 marks

- 2 marks for correct calculation of the bearing 110°
- 1 working mark can be given for finding the value of side *b* or angle *A*.

Question 3a.

Worked solution

We use the length of arc formula:

$$s = \frac{\pi r \theta}{180}$$

$$s = \frac{\pi \times 6400 \times 35}{180}$$

$$s = 3909.54 \text{ km}$$

$$s = 3910 \text{ km}$$

The distance from Canberra to the South Pole is 3910 km.

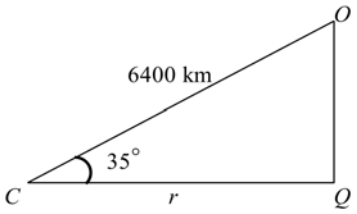
Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3b.i.

Worked solution

Canberra and Adelaide have the same latitude.



Using trigonometry

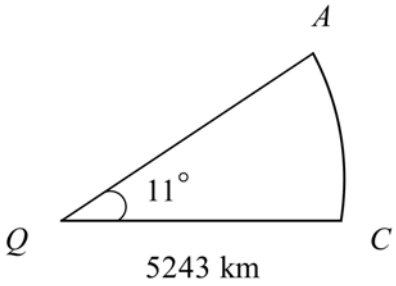
$$r = 6400 \times \cos 35 \approx 5243 \text{ km}$$

Mark allocation: 1 mark

- 1 mark for the correct answer, rounded to the nearest whole number.

Question 3b.ii.

Worked solution



We use the length of arc formula

$$s = \frac{\pi r \theta}{180}$$

$$s = \frac{\pi \times 5243 \times 11}{180}$$

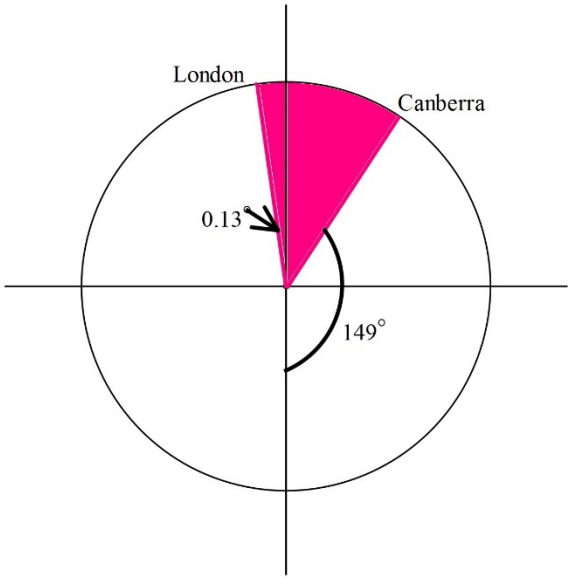
$$s = 1006.53 \text{ km}$$

$$s = 1007 \text{ km}$$

Question 3c.

Worked solution

A diagram can help in these types of questions.



We need to find the difference in longitude between London and Canberra.

This is equal to 31.13

There is 4 minutes time difference for each 1° difference in longitude.

31.3 ÷ 4 = 7.78

Therefore, London is approximately 8 hours behind Canberra.

If Jason calls his cousin at 9.00 am Canberra time, it will be 1.00 am in London.

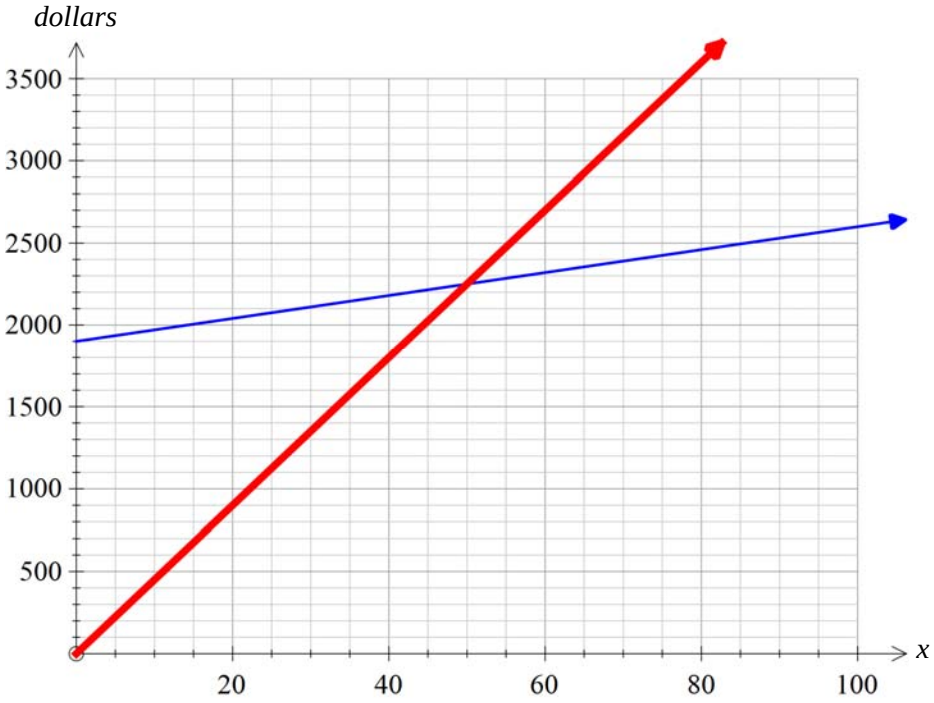
Mark allocation: 2 marks

- Full marks given for an answer of 1.00 am
- If this is incorrect, a working mark can be given for one of the following:
  - an appropriate diagram
  - finding the time difference in longitude between the two cities
  - dividing any time difference by 4 minutes.

Module 4 – Graphs and relations

Question 1a.

Worked solution



The bold line above represents the revenue function.

Mark allocation: 1 mark

- 1 mark for the correct straight line

Question 1b.

Worked solution

The gradient of the revenue function is 45.

This means that Laura sells each of her dresses for $45.

Mark allocation: 2 marks

- 1 mark for the correct gradient
- 1 mark for a correct description as above (or equivalent)
- In this case, it is important that the gradient is interpreted in terms of dollars.

Question 1c.

Worked solution

From the graph, we can see that the break-even point (where the cost and revenue functions intersect or are equal) is at $x = 50$. This means that 50 dresses must be produced for Laura to break even.

We can check this using the equations.

We find the point where the equations are equal

$$R = C$$
$$45x = 7x + 1900$$
$$38x = 1900$$
$$x = 50$$

Mark allocation: 1 mark

- 1 mark for the correct answer of 50

Question 1d.

Worked solution

An increase of 5% means that the cost function must be multiplied by 1.05

$$C_2 = 1.05 \times (7x + 1900)$$
$$C_2 = 7.35x + 1995$$

We must now find where this cost function is equal to the revenue function for Laura's small business.

$$R = C_2$$
$$45x = 7.35x + 1995$$
$$37.65x = 1995$$
$$x = 52.988$$
$$x \approx 53$$

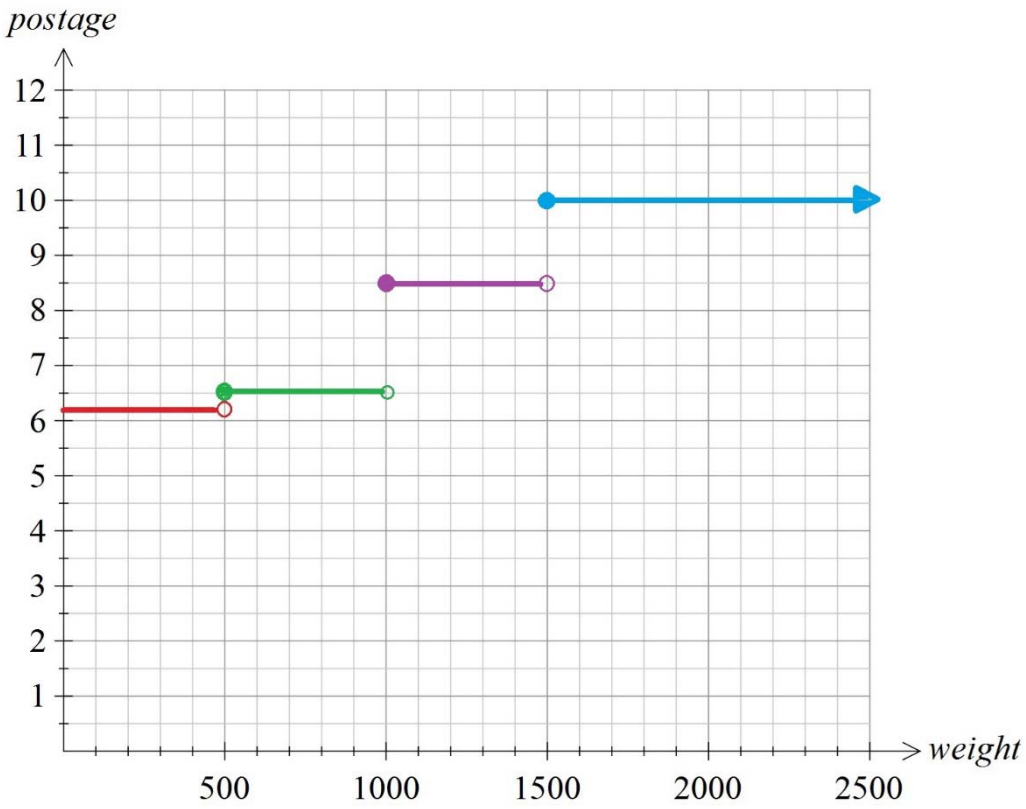
Laura must now sell 53 dresses to break even after the increase in the cost of making her dresses.

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 1e.

Worked solution



Mark allocation: 1 mark

- 1 mark for correctly placing line, including open circle at right-hand end

Question 1f.

Worked solution

The weight of 2.3 kg = 2300 g.

This falls into the final interval, and therefore, the parcel will cost $10 for postage.

Mark allocation: 1 mark

- 1 mark for $10

Question 1g.

Worked solution

We are trying to find where $P_2 < P_1$.

Therefore, we solve this inequality by considering the first interval on the original postage function, for weight of 0 < w < 500

$P_2 < 6.20$

$2.5 + 0.35w < 6.2$

$0.35w < 3.7$

$w < 10.57$ g

$w < 11$ g

Therefore, the second postage function will be cheaper where a parcel weighs less than 11 g.

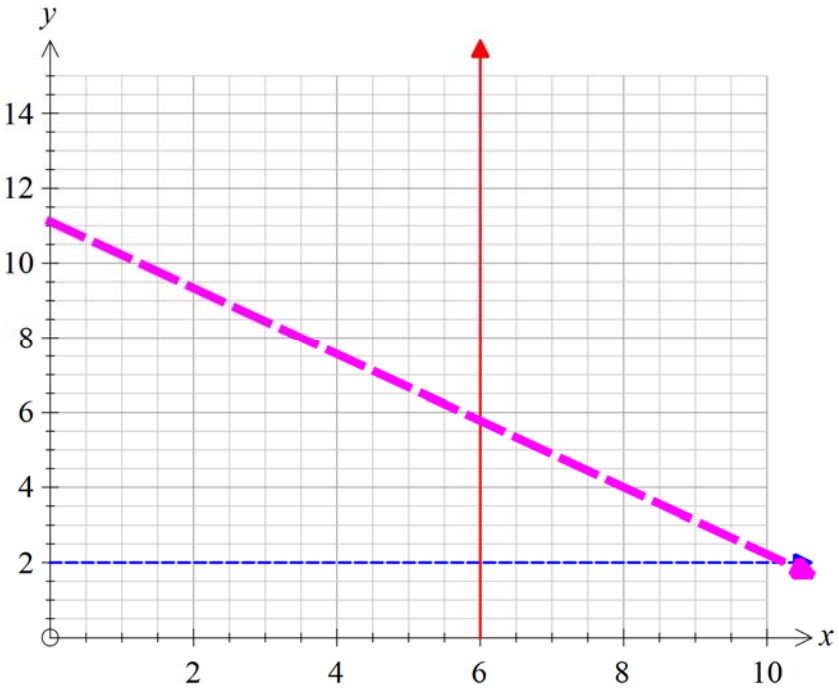
Note: It is unlikely that this will occur, and therefore, the alternative suggestion for calculating postage is not a possible option if Laura wishes to keep her postage costs low for her customers.

Mark allocation: 1 mark

- 1 mark for w < 11 g
- Mark not given if w = 11 g is written as the answer

Question 2a.

Worked solution



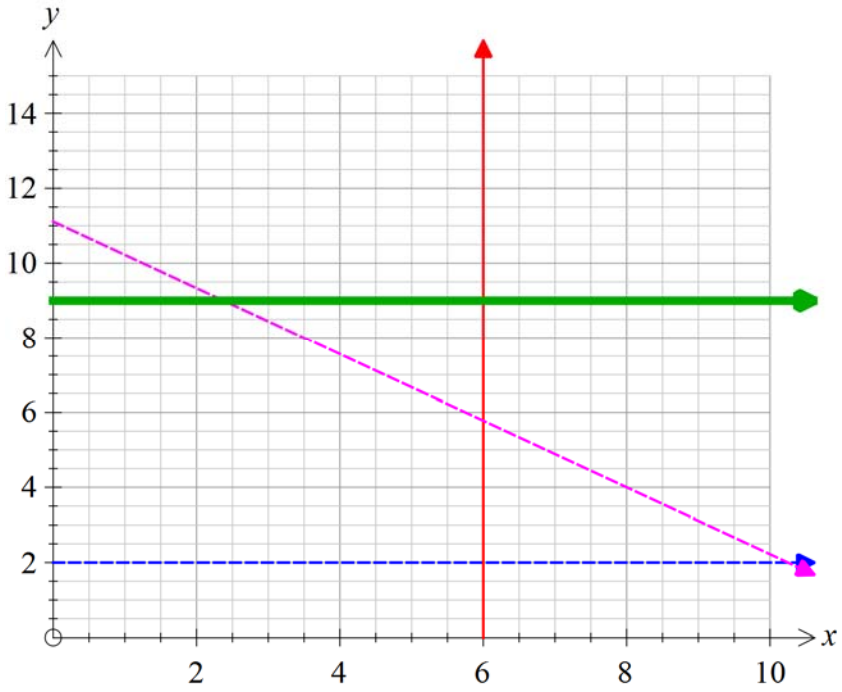
Mark allocation: 1 mark

- 1 mark for the correct sketch of inequality 3
- Line must be dotted

Question 2b.

Worked solution

Inequality 4 $y \leq 9$

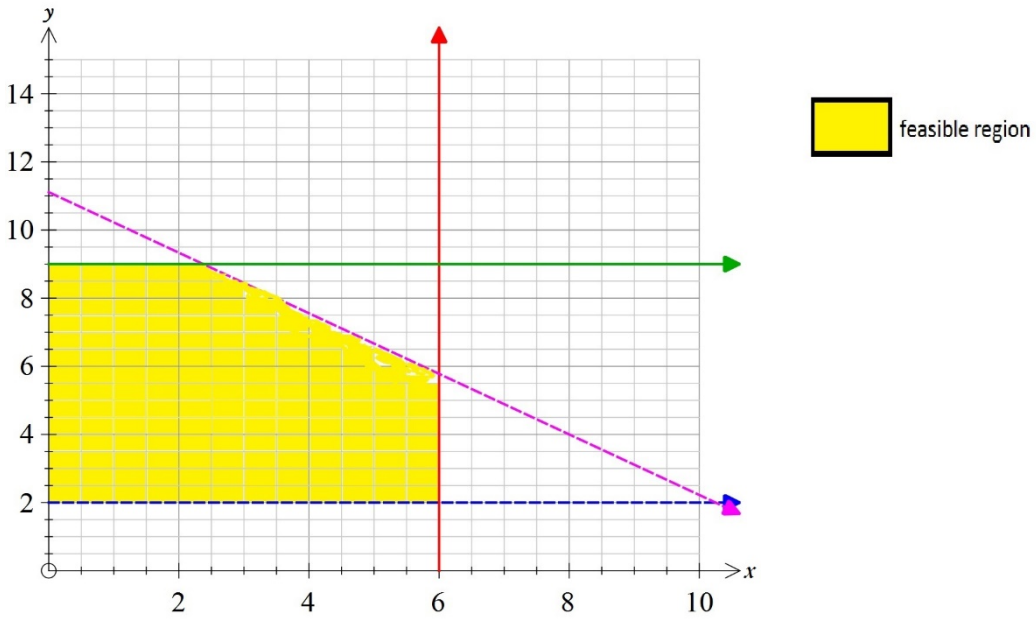


Mark allocation: 1 mark

- 1 mark for both the correct inequality and the correct sketch on the original graph.

Question 2c.

Worked solution



Mark allocation: 1 mark

- 2 marks for the correct shaded region with a key included
- 1 mark can be given for
  - ➢ correct shading without key
  - ➢ part of required shaded region accurate and a key included

END OF WORKED SOLUTIONS