VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2014

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

FURTHER MATHEMATICS

Written examination 2

Monday 3 November 2014

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 38 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

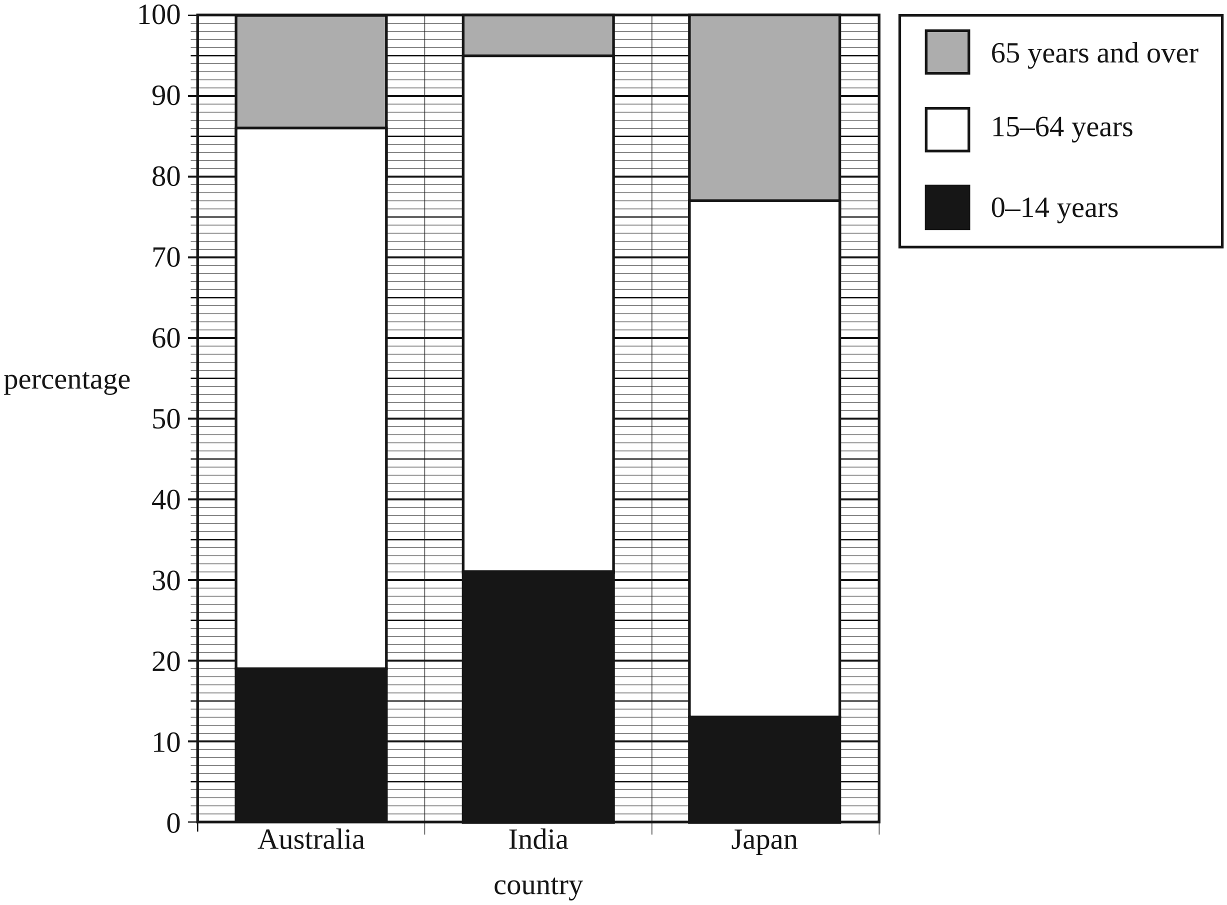
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Diagrams are not to scale unless specified otherwise.

Core

Question 1 (3 marks)

The segmented bar chart below shows the age distribution of people in three countries, Australia, India and Japan, for the year 2010.



Source: Australian Bureau of Statistics, *3201.0 – Population by Age and Sex, Australian States and Territories*, June 2010

a. Write down the percentage of people in Australia who were aged 0–14 years in 2010.
Write your answer, correct to the nearest percentage. 1 mark

b. In 2010, the population of Japan was 128 000 000.

How many people in Japan were aged 65 years and over in 2010? 1 mark

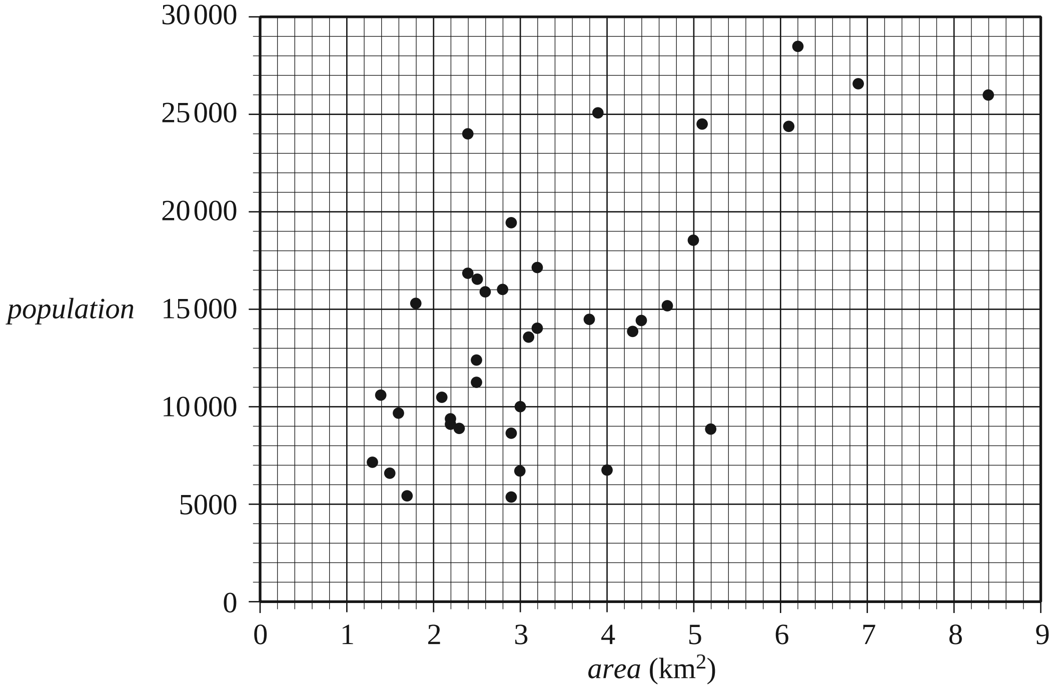
c. From the graph above, it appears that there is no association between the percentage of people in the 15–64 age group and the country in which they live.

Explain why, quoting appropriate percentages to support your explanation. 1 mark

Core – continued
TURN OVER

Question 2 (6 marks)

The scatterplot below shows the *population* and *area* (in square kilometres) of a sample of inner suburbs of a large city.



The equation of the least squares regression line for the data in the scatterplot is

$$population = 5330 + 2680 \times area$$

a. Write down the dependent variable. 1 mark

b. Draw the least squares regression line on the **scatterplot above**. 1 mark

(*Answer on the scatterplot above.*)

c. Interpret the slope of this least squares regression line in terms of the variables *area* and *population*. 2 marks

d. Wiston is an inner suburb. It has an area of 4 km^2 and a population of 6690.

The correlation coefficient, r, is equal to 0.668

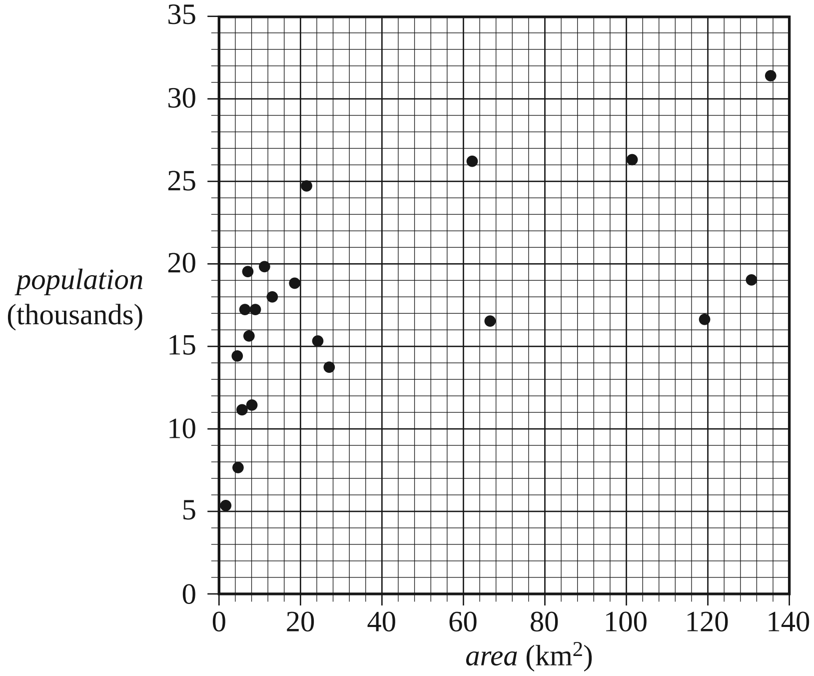
i. Calculate the residual when the least squares regression line is used to predict the population of Wiston from its area. 1 mark

ii. What percentage of the variation in the population of the suburbs is explained by the variation in area?

Write your answer, correct to one decimal place. 1 mark

Question 3 (2 marks)

The scatterplot and table below show the *population*, in thousands, and the *area*, in square kilometres, for a sample of 21 outer suburbs of the same city.



Area (km^2)	*Population* (thousands)
1.6	5.2
4.4	14.3
4.6	7.5
5.6	11.0
6.3	17.1
7.0	19.4
7.3	15.5
8.0	11.3
8.8	17.1
11.1	19.7
13.0	17.9
18.5	18.7
21.3	24.6
24.2	15.2
27.0	13.6
62.1	26.1
66.5	16.4
101.4	26.2
119.2	16.5
130.7	18.9
135.4	31.3

In the outer suburbs, the relationship between *population* and *area* is non-linear.

A **log** transformation can be applied to the variable *area* to linearise the scatterplot.

a. Apply the **log** transformation to the data and determine the equation of the least squares regression line that allows the population of an outer suburb to be predicted from the logarithm of its area.

Write the slope and intercept of this regression line in the boxes provided below.

Write your answers, correct to one decimal place. 1 mark

$$population = \boxed{} + \boxed{} \times \log_{10}(area)$$

b. Use this regression equation to predict the population of an outer suburb with an area of 90 km^2.

Write your answer, correct to the nearest one thousand people. 1 mark

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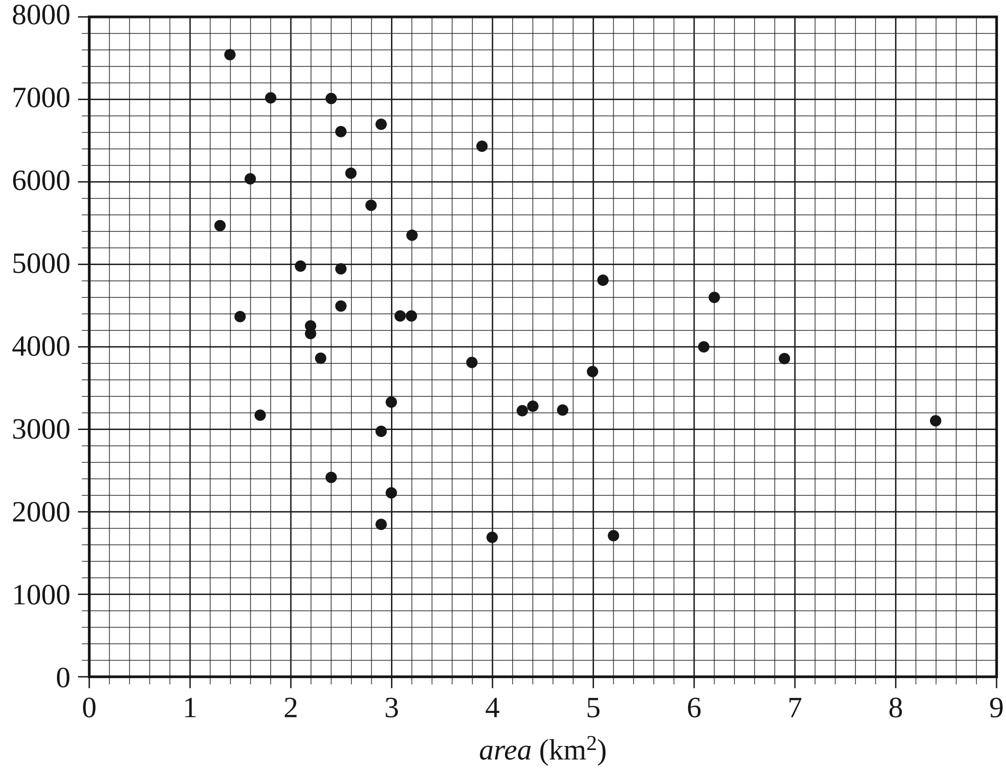
Question 4 (4 marks)

The scatterplot below shows the *population density*, in people per square kilometre, and the *area*, in square kilometres, of 38 inner suburbs of the same city.



For this scatterplot, $r^2 = 0.141$

a. Describe the association between the variables *population density* and *area* for these suburbs in terms of strength, direction and form. 1 mark

b. The mean and standard deviation of the variables *population density* and *area* for these 38 inner suburbs are shown in the table below.

	***Population density* (people per km^2)**	***Area* (km^2)**
Mean	4370	3.4
Standard deviation	1560	1.6

i. One of these suburbs has a population density of 3082 people per square kilometre.

Determine the standard *z*-score of this suburb's population density.
Write your answer, correct to one decimal place. 1 mark

Assume the areas of these inner suburbs are approximately normally distributed.

ii. How many of these 38 suburbs are **expected** to have an area that is two standard deviations or more above the mean?
Write your answer, correct to the nearest whole number. 1 mark

iii. How many of these 38 inner suburbs **actually** have an area that is two standard deviations or more above the mean? 1 mark

Module 1: Number patterns

Question 1 (4 marks)

Land in a wildlife reserve contains both grassland and desert.

Each year, some grassland becomes desert.

Let L_n be the expected area of grassland in the wildlife reserve, in square kilometres, at the end of year n.

The change in the area of grassland in the wildlife reserve, from year to year, is modelled by the difference equation

$$L_{n+1} = 0.99L_n \qquad L_{2014} = 20\,000$$

a. How many square kilometres of grassland are expected to be in the wildlife reserve at the end of 2014? 1 mark

b. What percentage of grassland is expected to become desert each year? 1 mark

c. Show that 19 800 km^2 of grassland are expected to be in the wildlife reserve at the end of 2015. 1 mark

d. What area of grassland, in square kilometres, is expected to become desert between the end of 2015 and the end of 2016? 1 mark

Question 2 (4 marks)

A conservation group plans to convert some of the desert to grassland each year.

The table below shows the area of desert, in square kilometres, that will be converted to grassland in each of the years 2014, 2015 and 2016.

	2014	**2015**	**2016**
Area of desert that will be converted to grassland (km^2)	0.8	0.68	0.578

The area of desert that will be converted to grassland each year forms a geometric sequence.

a. Write down a calculation to show that the common ratio for this geometric sequence is 0.85 — 1 mark

b. What area of desert, in square kilometres, will be converted to grassland in 2019?

Write your answer, correct to two decimal places. — 1 mark

c. By the end of 2018, what total area of desert, in square kilometres, will have been converted to grassland?

Write your answer, correct to two decimal places. — 1 mark

d. In what year will the total area of desert converted to grassland first exceed 4 km^2? — 1 mark

Question 3 (7 marks)

Some of the land in the wildlife reserve is an elephant habitat.

Let H_n be the area of the elephant habitat, in square kilometres, at the end of year n.

The change in the area of the elephant habitat, from year to year, can be modelled by the difference equation

$$H_{n+1} = 0.85H_n + 500 \qquad H_{2014} = 14\,000$$

a. Write down a calculation to show that the expected area of the elephant habitat at the end of 2015 is 12 400 km^2. 1 mark

The elephant habitat contains a population of elephants.

Let E_n be the number of elephants in the elephant habitat at the end of year n.

The change in the number of elephants, from year to year, can be modelled by the difference equation

$$E_{n+1} = 1.06E_n \qquad E_{2014} = 5000$$

The elephant habitat will be overpopulated if there are less than 2 km^2 for each elephant.

b. Show that the elephant habitat is expected to be overpopulated in 2016. 2 marks

To prevent the elephant habitat from becoming overpopulated, a number of elephants will be moved to other reserves at the end of each year.

Let P_n be the number of elephants in the elephant habitat at the end of year n.

Let k be the number of elephants that are moved to other reserves at the end of each year.

The change in the number of elephants, from year to year, can be modelled by another difference equation

$$P_{n+1} = 1.06P_n - k \qquad P_{2014} = 5000$$

c. How many elephants must be moved to other reserves at the end of each year if the number of elephants in the elephant habitat is to remain constant? 1 mark

d. How many elephants must be moved to other reserves at the end of each year if the number of elephants in the elephant habitat at the end of 2015 is to be 100 elephants fewer than at the end of 2014? 1 mark

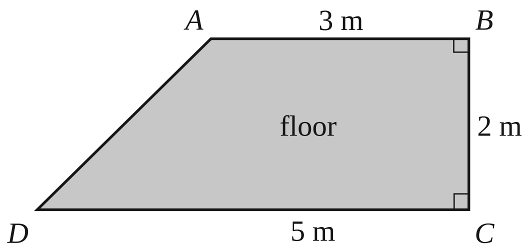
e. What is the minimum number of elephants that should be moved to other reserves at the end of each year to ensure that the elephant habitat is not overpopulated in 2016? 2 marks

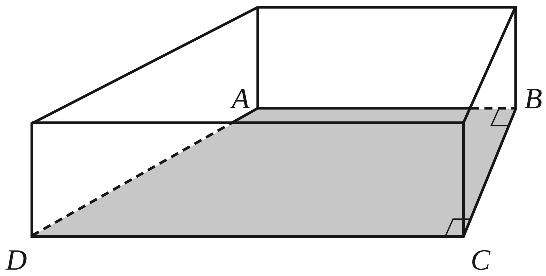
Module 2: Geometry and trigonometry

Question 1 (2 marks)

The floor of a chicken coop is in the shape of a trapezium.

The floor, *ABCD*, and the chicken coop are shown below.





$AB = 3$ m, $BC = 2$ m and $CD = 5$ m.

a. What is the area of the floor of the chicken coop?
Write your answer in square metres. 1 mark

b. What is the perimeter of the floor of the chicken coop?
Write your answer in metres, correct to one decimal place. 1 mark

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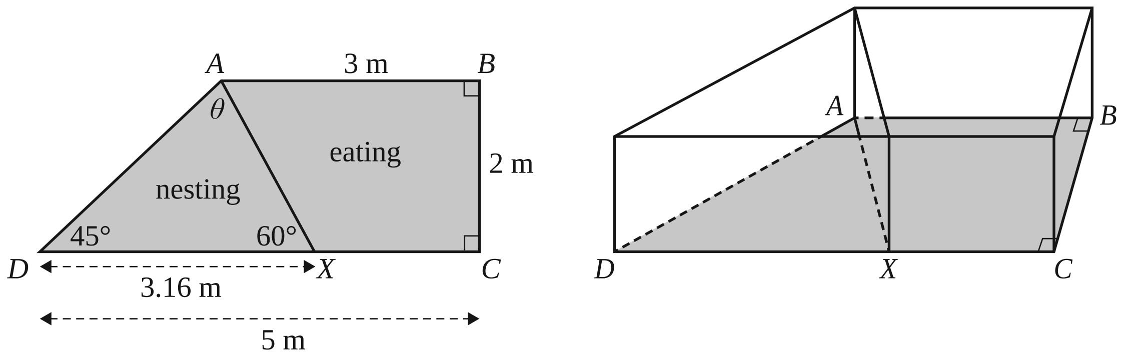
Question 2 (6 marks)

The chicken coop has two spaces, one for nesting and one for eating.

The nesting and eating spaces are separated by a wall along the line AX, as shown in the diagrams below.



$DX = 3.16$ m, $\angle ADX = 45°$ and $\angle AXD = 60°$.

a. Write down a calculation to show that the value of θ is 75°. 1 mark

b. The sine rule can be used to calculate the length of the wall AX.
Fill in the missing numbers below. 1 mark

$$\frac{AX}{\sin \boxed{}^{\circ}} = \frac{\boxed{}}{\sin \boxed{}^{\circ}}$$

c. What is the length of AX?
Write your answer in metres, correct to two decimal places. 1 mark

d. Calculate the area of the floor of the nesting space, ADX.
Write your answer in square metres, correct to one decimal place. 1 mark

The height of the chicken coop is 1.8 m.
Wire mesh will cover the roof of the eating space.
The area of the walls along the lines *AB*, *BC* and *CX* will also be covered with wire mesh.

e. What total area, in square metres, will be covered by wire mesh?
Write your answer, correct to the nearest square metre. 2 marks

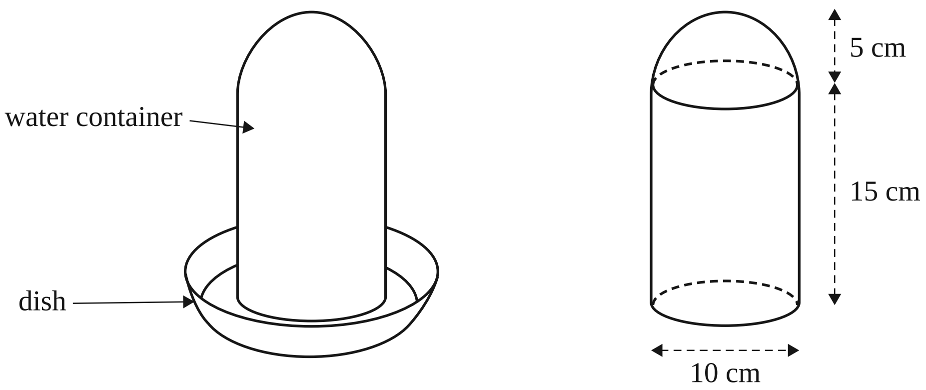
Question 3 (5 marks)

The chicken coop contains a circular water dish.

Water flows into the dish from a water container.

The water container is in the shape of a cylinder with a hemispherical top.

The water container and the dish are shown in the diagrams below.



The cylindrical part of the water container has a diameter of 10 cm and a height of 15 cm.

The hemisphere has a radius of 5 cm.

a. What is the surface area of the hemispherical top of the water container?
Write your answer, correct to the nearest square centimetre. 1 mark

b. What is the maximum volume of water that the water container can hold?
Write your answer, correct to the nearest cubic centimetre. 2 marks

The eating space of the chicken coop also has a feed container.

The feed container is similar in shape to the water container.

The volume of the water container is three-quarters of the volume of the feed container.

The surface area of the water container is 628 cm^2.

c. What is the surface area of the feed container?
Write your answer, correct to the nearest square centimetre. 2 marks

Question 4 (2 marks)

One of the chickens escapes into a neighbouring field through an open gate.

The chicken's owner is 50 m due north of the gate, searching for the chicken.

The chicken is 40 m from the gate on a bearing of 295°.

What is the bearing of the chicken from its owner?

Write your answer, correct to the nearest degree.

END OF MODULE 2

Module 3: Graphs and relations

Question 1 (8 marks)

Fastgrow and Booster are two tomato fertilisers that contain the nutrients nitrogen and phosphorus.

The amount of nitrogen and phosphorus in each kilogram of Fastgrow and Booster is shown in the table below.

	1 kg of Booster	**1 kg of Fastgrow**
Nitrogen	0.05 kg	0.05 kg
Phosphorus	0.02 kg	0.06 kg

a. How many kilograms of phosphorus are in 2 kg of Booster? 1 mark

b. If 100 kg of Booster and 400 kg of Fastgrow are mixed, how many kilograms of nitrogen would be in the mixture? 1 mark

Arthur is a farmer who grows tomatoes.

He mixes quantities of Booster and Fastgrow to make his own fertiliser.

Let x be the number of kilograms of Booster in Arthur's fertiliser.

Let y be the number of kilograms of Fastgrow in Arthur's fertiliser.

Inequalities 1 to 4 represent the nitrogen and phosphorus requirements of Arthur's tomato field.

Inequality 1	$x \geq 0$
Inequality 2	$y \geq 0$
Inequality 3 (nitrogen)	$0.05x + 0.05y \geq 200$
Inequality 4 (phosphorus)	$0.02x + 0.06y \geq 120$

Arthur's tomato field also requires at least 180 kg of the nutrient potassium.

Each kilogram of Booster contains 0.06 kg of potassium.

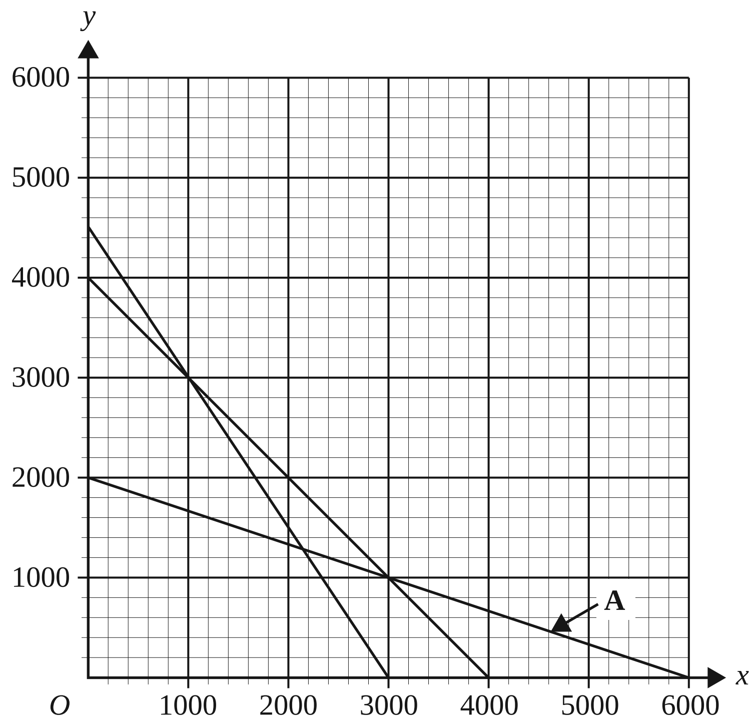
Each kilogram of Fastgrow contains 0.04 kg of potassium.

c. Inequality 5 represents the potassium requirements of Arthur's tomato field.

Write down Inequality 5 in terms of x and y. 1 mark

Inequality 5 (potassium) ______________________

The lines that represent the boundaries of Inequalities 3, 4 and 5 are shown in the graph below.



d. **i.** Using the graph above, write down the equation of line A. 1 mark

ii. On the **graph above**, shade the region that satisfies Inequalities 1 to 5. 1 mark

(*Answer on the graph above.*)

Arthur would like to use the least amount of his own fertiliser to meet the nutrient requirements of his tomato field and still satisfy Inequalities 1 to 5.

e. **i.** What weight of his own fertiliser will Arthur need to make? 1 mark

ii. On the **graph above**, show the point(s) where this solution occurs. 2 marks

(*Answer on the graph above.*)

Question 2 (3 marks)

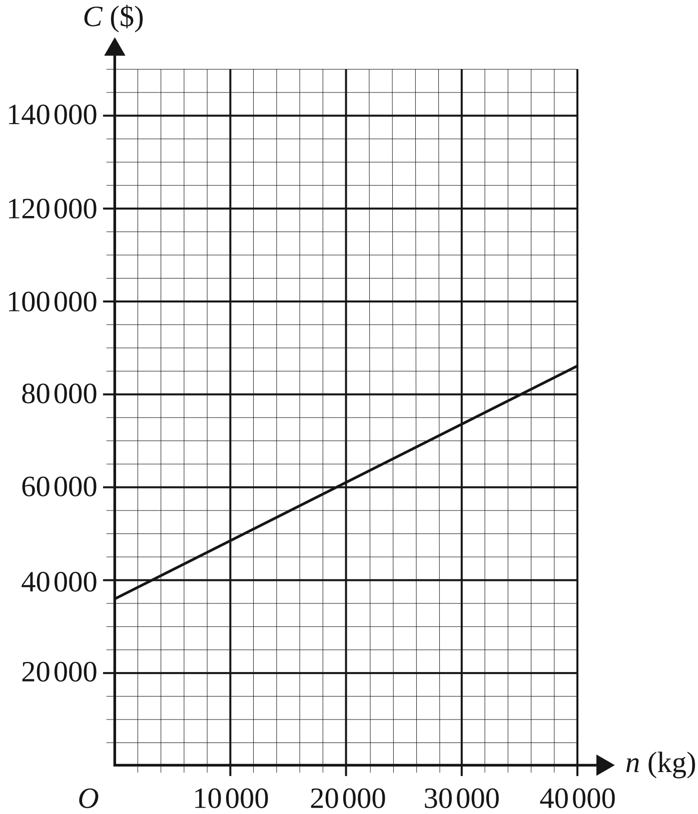
The cost, C, in dollars, of producing n kilograms of tomatoes is given by

$$C = 1.25n + 36\,000 \qquad 0 \le n \le 40\,000$$

The revenue, R, in dollars, from selling n kilograms of tomatoes is given by

$$R = 3.5n \qquad 0 \le n \le 40\,000$$

The cost, C, for the production of n kilograms of tomatoes is graphed below.



a. On the **graph above**, draw the revenue equation line, $R = 3.5n$. 1 mark

(*Answer on the graph above.*)

b. What profit will Arthur make if he sells a total of 20 000 kg of tomatoes? 2 marks

Question 3 (4 marks)

A shop owner bought 100 kg of Arthur's tomatoes to sell in her shop.

She bought the tomatoes for \$3.50 per kilogram.

The shop owner will offer a discount to her customers based on the number of kilograms of tomatoes they buy in one bag.

The revenue, in dollars, that the shop owner receives from selling the tomatoes is given by

$$\text{revenue} = \begin{cases} 5.4n & 0 < n \le 2 \\ 10.8 + 4(n-2) & 2 < n \le 10 \\ a + 2(n-10) & 10 < n < 100 \end{cases}$$

where n is the number of kilograms of tomatoes that a customer buys in one bag.

a. What is the revenue that the shop owner receives from selling 8 kg of tomatoes in one bag? 1 mark

b. Show that a has the value 42.8 in the revenue equation above. 1 mark

c. Find the maximum number of kilograms of tomatoes that a customer can buy in one bag, so that the shop owner never makes a loss. 2 marks

END OF MODULE 3
TURN OVER

Module 4: Business-related mathematics

Question 1 (5 marks)

The adult membership fee for a cricket club is \$150.

Junior members are offered a discount of \$30 off the adult membership fee.

a. Write down the discount for junior members as a percentage of the adult membership fee. 1 mark

Adult members of the cricket club pay \$15 per match in addition to the membership fee of \$150.

b. If an adult member played 12 matches, what is the total this member would pay to the cricket club? 1 mark

If a member does not pay the membership fee by the due date, the club will charge simple interest at the rate of 5% per month until the fee is paid.

Michael paid the \$150 membership fee exactly two months after the due date.

c. Calculate, in dollars, the interest that Michael will be charged. 1 mark

Module 4 – Question 1 – continued

The cricket club received a statement of the transactions in its savings account for the month of January 2014.

The statement is shown below.

Date	Details	Deposit	Withdrawal	Balance
01 Jan. 2014	Brought Forward			\$58 950.00
08 Jan. 2014	Match Fees	\$750.00		\$59 700.00
17 Jan. 2014	Withdrawal		\$	\$42 700.00
23 Jan. 2014	Membership Fees	\$4500.00		\$47 200.00
31 Jan. 2014	Interest	\$125.12		\$47 325.12

d. **i.** Calculate the amount of the withdrawal on 17 January 2014. 1 mark

ii. Interest for this account is calculated on the minimum balance for the month and added to the account on the last day of the month.

What is the annual rate of interest for this account?

Write your answer, correct to one decimal place. 1 mark

Question 2 (4 marks)

A sponsor of the cricket club has invested $20 000 in a perpetuity.

The annual interest from this perpetuity is $750.

The interest from the perpetuity is given to the best player in the club every year, for a period of 10 years.

a. What is the annual rate of interest for this perpetuity investment? 1 mark

b. After 10 years, how much money is still invested in the perpetuity? 1 mark

c. The average rate of inflation over the next 10 years is expected to be 3% per annum.

i. Michael was the best player in 2014 and he considered purchasing cricket equipment that was valued at $750.

What is the expected price of this cricket equipment in 2015? 1 mark

ii. What is the 2014 value of cricket equipment that could be bought for $750 in 2024?

Write your answer, correct to the nearest dollar. 1 mark

Question 3 (4 marks)

The cricket club had invested \$45 550 in an account for four years.

After four years of compounding interest, the value of the investment was \$60 000.

a. How much interest was earned during the four years of this investment? 1 mark

Interest on the account had been calculated and paid quarterly.

b. What was the annual rate of interest for this investment?

Write your answer, correct to one decimal place. 1 mark

The \$60 000 was re-invested in another account for 12 months.

The new account paid interest at the rate of 7.2% per annum, compounding monthly.

At the end of each month, the cricket club added an additional \$885 to the investment.

c. **i.** The equation below can be used to determine the account balance at the end of the first month, immediately after the \$885 was added.

Complete the equation by filling in the boxes. 1 mark

$$\text{account balance} = 60\,000 \times \left(1 + \boxed{}\right) + \boxed{}$$

ii. What was the account balance at the end of 12 months?

Write your answer, correct to the nearest dollar. 1 mark

Question 4 (2 marks)

The cricket club borrowed \$400 000 to build a clubhouse.

Interest is calculated at the rate of 4.5% per annum, compounding monthly.

The cricket club will make monthly repayments of \$2500.

After a number of monthly repayments, the balance of the loan will be reduced to \$143 585.33

What percentage of the next monthly repayment will reduce the balance of the loan?

Write your answer, correct to the nearest percentage.

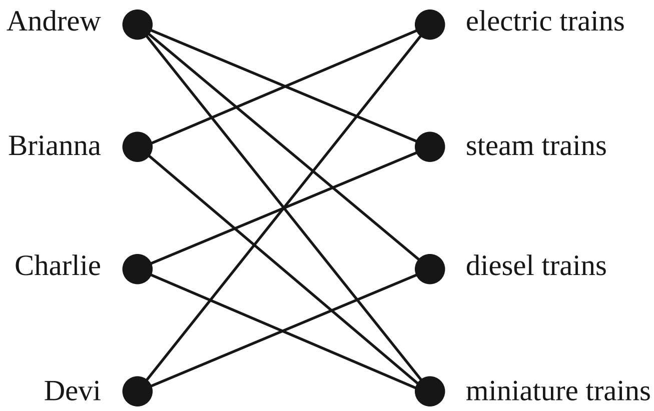
END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1 (2 marks)

Four members of a train club, Andrew, Brianna, Charlie and Devi, have joined one or more interest groups for electric, steam, diesel or miniature trains.

The edges of the bipartite graph below show the interest groups that these four train club members have joined.



a. How many of these four members have joined the steam trains interest group? 1 mark

b. Which interest group have both Brianna and Charlie joined? 1 mark

Question 2 (4 marks)

Planning a train club open day involves four tasks.

Table 1 shows the number of hours that each club member would take to complete these tasks.

Table 1

Task	Andrew	Brianna	Charlie	Devi
publicity	13	12	10	10
finances	9	10	11	11
equipment	8	12	11	10
catering	9	10	11	8

The Hungarian algorithm will be used to allocate the tasks to club members so that the total time taken to complete the tasks is minimised.

The first step of the Hungarian algorithm is to subtract the smallest element in each row of Table 1 from each of the elements in that row.

The result of this step is shown in Table 2 below.

a. Complete Table 2 by filling in the missing numbers for Andrew. 1 mark

Table 2

Task	Andrew	Brianna	Charlie	Devi
publicity	3	2	0	0
finances		1	2	2
equipment		4	3	2
catering		2	3	0

After completing Table 2, Andrew decided that an allocation of tasks to minimise the total time taken was not yet possible using the Hungarian algorithm.

b. Explain why Andrew made this decision. 1 mark

Table 3 shows the final result of all steps of the Hungarian algorithm.

Table 3

Task	**Andrew**	**Brianna**	**Charlie**	**Devi**
publicity	4	2	0	1
finances	0	0	1	2
equipment	0	3	2	2
catering	1	1	2	0

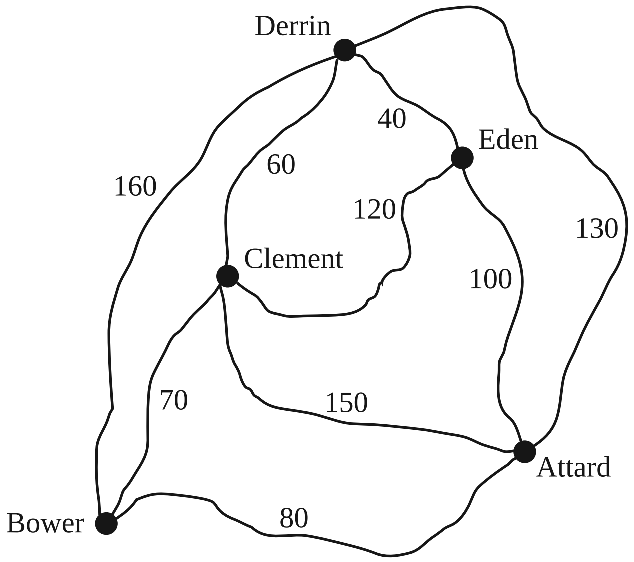
c. **i.** Which task should be allocated to Andrew? 1 mark

ii. How many hours in total are used to plan for the open day? 1 mark

Question 3 (4 marks)

The diagram below shows a network of train lines between five towns: Attard, Bower, Clement, Derrin and Eden.

The numbers indicate the distances, in kilometres, that are travelled by train between connected towns.



Charlie followed an Eulerian path through this network of train lines.

a. **i.** Write down the names of the towns at the start and at the end of Charlie's path. 1 mark

ii. What distance did he travel? 1 mark

Brianna will follow a Hamiltonian path from Bower to Attard.

b. What is the shortest distance that she can travel? 1 mark

The train line between Derrin and Eden will be removed. If one other train line is removed from the network, Andrew would be able to follow an Eulerian circuit through the network of train lines.

c. Which other train line should be removed?

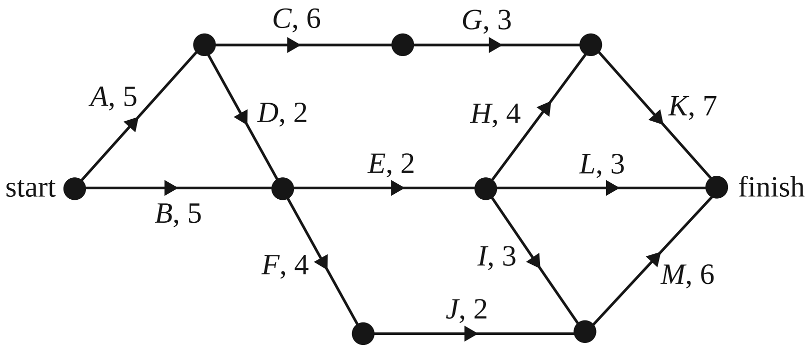
In the boxes below, write down the pair of towns that this train line connects. 1 mark

between [ ] and [ ]

Question 4 (5 marks)

To restore a vintage train, 13 activities need to be completed.

The network below shows these 13 activities and their completion times in hours.



a. Determine the earliest starting time of activity *F*. 1 mark

The minimum time in which all 13 activities can be completed is 21 hours.

b. What is the latest starting time of activity *L*? 1 mark

c. What is the float time of activity *J*? 1 mark

Just before they started restoring the train, the members of the club needed to add another activity, *X*, to the project.

Activity *X* will take seven hours to complete.

Activity *X* has no predecessors, but must be completed before activity *G* starts.

d. What is the latest starting time of activity *X* if it is not to increase the minimum completion time of the project? 1 mark

Activity *A* can be crashed by up to four hours at an additional cost of $90 per hour.

This may reduce the minimum completion time for the project, including activity *X*.

e. Determine the least cost of crashing activity *A* to give the greatest reduction in the minimum completion time of the project. 1 mark

END OF MODULE 5

TURN OVER

Module 6: Matrices

Question 1 (6 marks)

A small city is divided into four regions: Northern (*N*), Eastern (*E*), Southern (*S*) and Western (*W*).

The number of adult males (*M*) and the number of adult females (*F*) living in each of the regions in 2013 is shown in matrix *V* below.

$$V = \begin{array}{cc} M & F \end{array} \\ \begin{bmatrix} 1360 & 1460 \\ 1680 & 1920 \\ 900 & 1060 \\ 1850 & 1770 \end{bmatrix} \begin{array}{c} N \\ E \\ S \\ W \end{array}$$

a. Write down the order of matrix *V*. 1 mark

b. How many adult males lived in the Western region in 2013? 1 mark

c. In terms of the population of the city, what does the sum of the elements in the second column of matrix *V* represent? 1 mark

An election is to be held in the city.

All of the adults in each of the regions of the city will vote in the election.

One of the election candidates, Ms Aboud, estimates that she will receive 45% of the male votes and 55% of the female votes in the election.

This information is shown in matrix *P* below.

$$P = \begin{bmatrix} 0.45 \\ 0.55 \end{bmatrix} \begin{array}{c} M \\ F \end{array}$$

d. Explain, in terms of rows and columns, why the matrix product $V \times P$ is defined. 1 mark

The product of matrices V and P is shown below.

$$V \times P = \begin{bmatrix} 1360 & 1460 \\ 1680 & 1920 \\ 900 & 1060 \\ 1850 & 1770 \end{bmatrix} \times \begin{bmatrix} 0.45 \\ 0.55 \end{bmatrix} = \begin{bmatrix} w \\ 1812 \\ 988 \\ 1806 \end{bmatrix}$$

e. Using appropriate elements from the matrix product $V \times P$, write a calculation to show that the value of w is 1415. 1 mark

f. How many votes does Ms Aboud expect to receive in the election? 1 mark

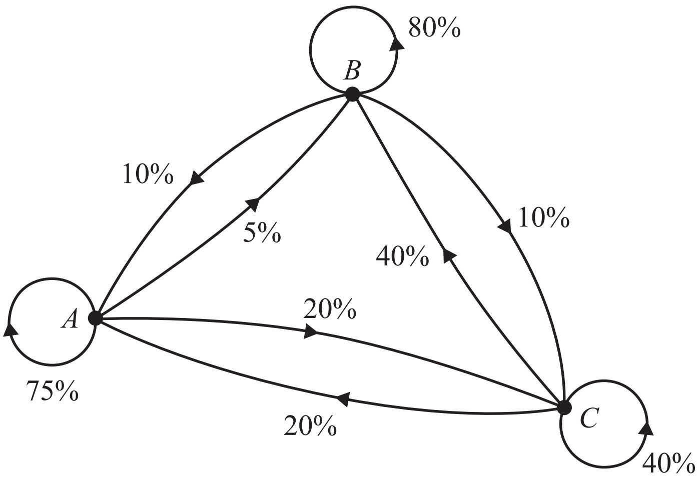
Question 2 (6 marks)

There are three candidates in the election: Ms Aboud (A), Mr Broad (B) and Mr Choi (C).

The election campaign will run for six months, from the start of January until the election at the end of June.

A survey of voters found that voting preference can change from month to month leading up to the election.

The transition diagram below shows the percentage of voters who are expected to change their preferred candidate from month to month.



a. **i.** Of the voters who prefer Mr Choi this month, what percentage are expected to prefer Ms Aboud next month? 1 mark

ii. Of the voters who prefer Ms Aboud this month, what percentage are expected to change their preferred candidate next month? 1 mark

In January, 12 000 voters are expected in the city. The number of voters in the city is expected to remain constant until the election is held in June.

The state matrix that indicates the number of voters who are expected to have a preference for each candidate in January, S_1, is given below.

$$S_1 = \begin{bmatrix} 6000 \\ 3840 \\ 2160 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

b. How many voters are expected to change their preference to Mr Broad in February? 1 mark

The information in the transition diagram has been used to write the transition matrix, T, shown below.

$$T = \begin{array}{c} \text{this month} \\ \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.75 & 0.10 & 0.20 \\ 0.05 & 0.80 & 0.40 \\ 0.20 & 0.10 & 0.40 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \end{array} \text{ next month}$$

c. **i.** Evaluate the matrix $S_3 = T^2 S_1$ and write it down in the space below.
Write the elements, correct to the nearest whole number. 1 mark

$$S_3 = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

ii. What information does matrix S_3 contain? 1 mark

d. Using matrix T, how many votes would the winner of the election in June be expected to receive?
Write your answer, correct to the nearest whole number. 1 mark

Question 3 (3 marks)

Mr Choi may need to withdraw from the election at the end of May.

Matrix T, shown below, shows the percentage of voters who change their preferred candidate, from month to month, **before** Mr Choi would withdraw from the election.

$$\begin{array}{cc} & \text{this month} \\ & \begin{array}{ccc} A & B & C \end{array} \\ T = & \begin{bmatrix} 0.75 & 0.10 & 0.20 \\ 0.05 & 0.80 & 0.40 \\ 0.20 & 0.10 & 0.40 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \quad \text{next month} \\ C \end{array}$$

Matrix T_1, shown below, shows the percentage of voters who change their preferred candidate, from May to June, **after** Mr Choi would withdraw from the election.

$$\begin{array}{cc} & \text{May} \\ & \begin{array}{ccc} A & B & C \end{array} \\ T_1 = & \begin{bmatrix} 0.75 & 0.15 & 0.6 \\ 0.25 & 0.85 & 0.4 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \quad \text{June} \\ C \end{array}$$

Consider the voters who preferred Mr Broad in May and who were expected to prefer Mr Choi in June.

a. What percentage of these voters are now expected to prefer Mr Broad in June? 1 mark

The state matrix that indicates the number of voters who are expected to have a preference for each candidate in January, S_1, is given below.

$$S_1 = \begin{bmatrix} 6000 \\ 3840 \\ 2160 \end{bmatrix} \begin{array}{l} A \\ B \\ C \end{array}$$

b. If Mr Choi withdraws, how many votes is Mr Broad expected to receive in the election in June?

Write your answer, correct to the nearest vote. 2 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Further Mathematics formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares regression line: $y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}$, $r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}$, $|r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight-line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$, where $R = 1 + \dfrac{r}{100}$

hire-purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET