VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education**
**2014**

# FURTHER MATHEMATICS

## Written examination 1

**Friday 31 October 2014**

**Reading time: 3.00 pm to 3.15 pm (15 minutes)**
**Writing time: 3.15 pm to 4.45 pm (1 hour 30 minutes)**

## MULTIPLE-CHOICE QUESTION BOOK

**Structure of book**

| *Section* | *Number of questions* | *Number of questions to be answered* | *Number of modules* | *Number of modules to be answered* | *Number of marks* |
|---|---|---|---|---|---|
| A | 13 | 13 | | | 13 |
| B | 54 | 27 | 6 | 3 | 27 |
| | | | | | Total 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

**Materials supplied**
- Question book of 38 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

**Instructions**
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**At the end of the examination**
- You may keep this question book.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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# SECTION A

## Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

## Core: Data analysis

### Question 1

The following ordered stem plot shows the areas, in square kilometres, of 27 suburbs of a large city.

key: 1|6 = 1.6 km$^2$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 6 | 7 | 8 | | | | |
| 2 | 1 | 2 | 4 | 5 | 6 | 8 | 9 | 9 |
| 3 | 0 | 1 | 1 | 2 | 2 | 8 | 9 | |
| 4 | 0 | 4 | 7 | | | | | |
| 5 | 0 | 1 | | | | | | |
| 6 | 1 | 9 | | | | | | |
| 7 | | | | | | | | |
| 8 | 4 | | | | | | | |

The median area of these suburbs, in square kilometres, is

**A.** 3.0
**B.** 3.1
**C.** 3.5
**D.** 30.0
**E.** 30.5

### Question 2

The time spent by shoppers at a hardware store on a Saturday is approximately normally distributed with a mean of 31 minutes and a standard deviation of 6 minutes.

If 2850 shoppers are expected to visit the store on a Saturday, the number of shoppers who are expected to spend between 25 and 37 minutes in the store is closest to

**A.** 16
**B.** 68
**C.** 460
**D.** 1900
**E.** 2400

*Use the following information to answer Questions 3–5.*

The following table shows the data collected from a sample of seven drivers who entered a supermarket car park. The variables in the table are:

- *distance* – the distance that each driver travelled to the supermarket from their home
- *sex* – the sex of the driver (female, male)
- *number of children* – the number of children in the car
- *type of car* – the type of car (sedan, wagon, other)
- *postcode* – the postcode of the driver's home.

| ***Distance*** **(km)** | ***Sex*** **(F = female, M = male)** | ***Number of children*** | ***Type of car*** **(1 = sedan, 2 = wagon, 3 = other)** | ***Postcode*** |
|---|---|---|---|---|
| 4.2 | F | 2 | 1 | 8148 |
| 0.8 | M | 3 | 2 | 8147 |
| 3.9 | F | 3 | 2 | 8146 |
| 5.6 | F | 1 | 3 | 8245 |
| 0.9 | M | 1 | 3 | 8148 |
| 1.7 | F | 2 | 2 | 8147 |
| 2.5 | M | 2 | 2 | 8145 |

**Question 3**

The mean, $\bar{x}$, and the standard deviation, $s_x$, of the variable, *distance*, are closest to

**A.** $\bar{x} = 2.5$ $s_x = 3.3$
**B.** $\bar{x} = 2.8$ $s_x = 1.7$
**C.** $\bar{x} = 2.8$ $s_x = 1.8$
**D.** $\bar{x} = 2.9$ $s_x = 1.7$
**E.** $\bar{x} = 3.3$ $s_x = 2.5$

**Question 4**

The number of categorical variables in this data set is

**A.** 0
**B.** 1
**C.** 2
**D.** 3
**E.** 4

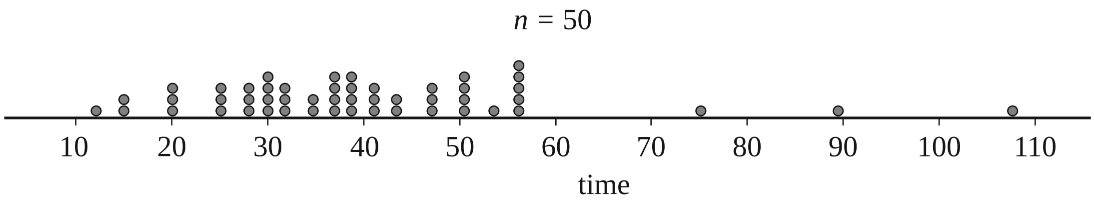
**Question 5**

The number of female drivers with three children in the car is

**A.** 0
**B.** 1
**C.** 2
**D.** 3
**E.** 4

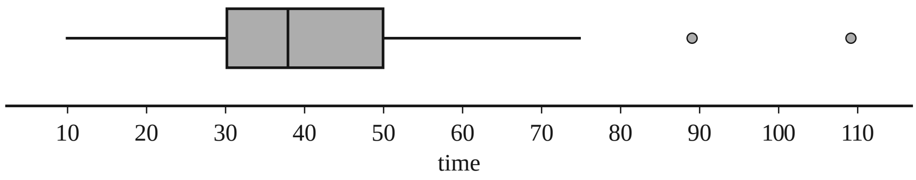
**Question 6**

The dot plot below shows the distribution of the time, in minutes, that 50 people spent waiting to get help from a call centre.

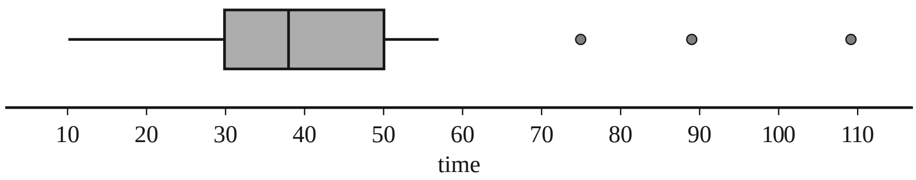



Which one of the following boxplots best represents the data?

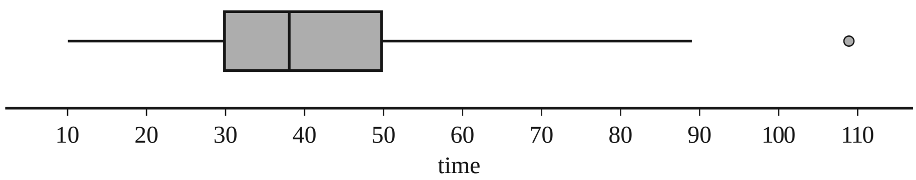
**A.**



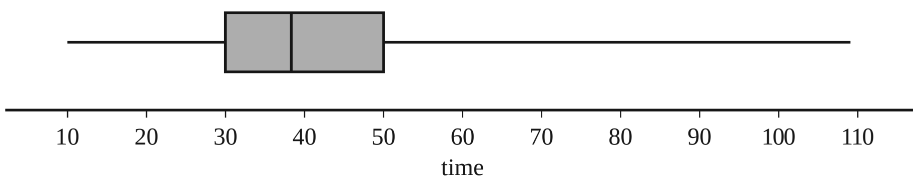

**B.**



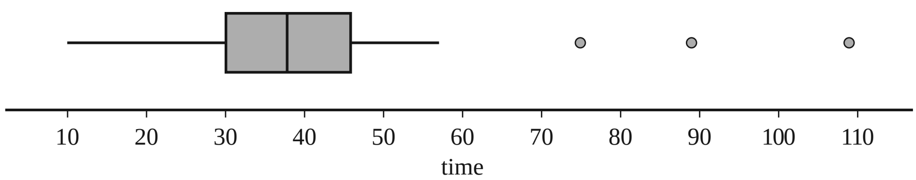

**C.**




**D.**

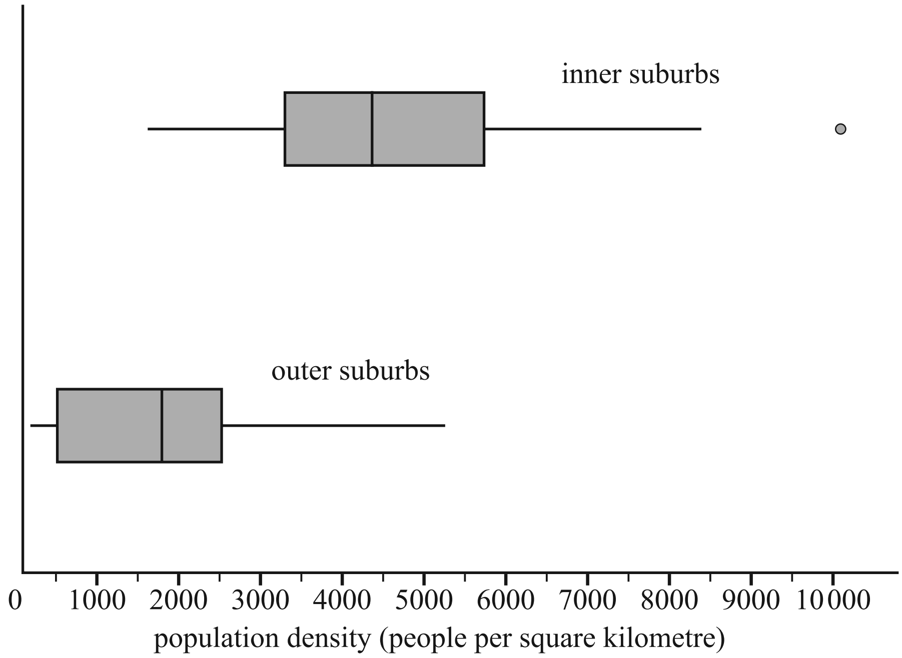



**E.**



**Question 7**

The parallel boxplots below summarise the distribution of population density, in people per square kilometre, for 27 inner suburbs and 23 outer suburbs of a large city.




Which one of the following statements is **not** true?

**A.** More than 50% of the outer suburbs have population densities below 2000 people per square kilometre.

**B.** More than 75% of the inner suburbs have population densities below 6000 people per square kilometre.

**C.** Population densities are more variable in the outer suburbs than in the inner suburbs.

**D.** The median population density of the inner suburbs is approximately 4400 people per square kilometre.

**E.** Population densities are, on average, higher in the inner suburbs than in the outer suburbs.

**Question 8**

A single back-to-back stem plot would be an appropriate graphical tool to investigate the association between a car's speed, in kilometres per hour, and the

**A.** driver's age, in years.

**B.** car's colour (white, red, grey, other).

**C.** car's fuel consumption, in kilometres per litre.

**D.** average distance travelled, in kilometres.

**E.** driver's sex (female, male).

**Question 9**

The equation of a least squares regression line is used to predict the fuel consumption, in kilometres per litre of fuel, from a car's weight, in kilograms.

This equation predicts that a car weighing 900 kg will travel 10.7 km per litre of fuel, while a car weighing 1700 kg will travel 6.7 km per litre of fuel.

The slope of this least squares regression line is closest to

**A.** –250

**B.** –0.005

**C.** –0.004

**D.** 0.005

**E.** 200

*Use the following information to answer Questions 10 and 11.*

The seasonal indices for the f rst 11 months of the year, for sales in a sporting equipment store, are shown in the table below.

| **Month** | Jan. | Feb. | Mar. | Apr. | May | Jun. | Jul. | Aug. | Sep. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Seasonal index** | 1.23 | 0.96 | 1.12 | 1.08 | 0.89 | 0.98 | 0.86 | 0.76 | 0.76 | 0.95 | 1.12 | |

**Question 10**

The seasonal index for December is

**A.** 0.89

**B.** 0.97

**C.** 1.02

**D.** 1.23

**E.** 1.29

**Question 11**

In May, the store sold \$213 956 worth of sporting equipment.

The deseasonalised value of these sales was closest to

**A.** \$165 857

**B.** \$190 420

**C.** \$209 677

**D.** \$218 322

**E.** \$240 400

**Question 12**

The seasonal index for heaters in winter is 1.25.

To correct for seasonality, the actual heater sales in winter should be

**A.** reduced by 20%.

**B.** increased by 20%.

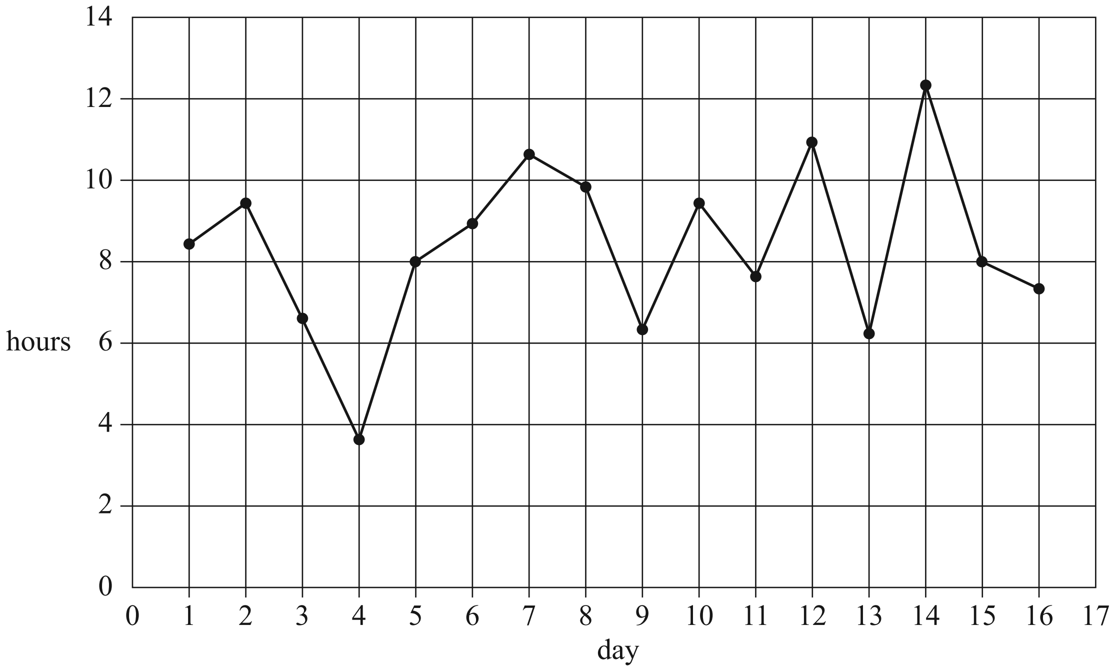
**C.** reduced by 25%.

**D.** increased by 25%.

**E.** reduced by 75%.

**Question 13**

The time series plot below shows the hours of sunshine per day at a particular location for 16 consecutive days.




The three median method is used to f t a trend line to the data.

The slope of this trend line will be closest to

**A.** –0.7

**B.** –0.2

**C.** 0.0

**D.** 0.2

**E.** 0.7

**END OF SECTION A**

## SECTION B

### Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

## Module 1: Number patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

*Use the following information to answer Questions 1 and 2.*

Paul went running every morning from Monday to Sunday for one week.

On Monday, Paul ran 1.0 km.

On Tuesday, Paul ran 1.5 km.

On Wednesday, Paul ran 2.0 km.

The number of kilometres that Paul ran each day continued to increase according to this pattern.

**Question 1**

The number of kilometres that Paul ran on Thursday is

**A.** 2.5
**B.** 3.0
**C.** 3.5
**D.** 4.0
**E.** 5.0

**Question 2**

The total number of kilometres that Paul ran during the week is given by

**A.** the seventh term of an arithmetic sequence with $a = 1$ and $d = 0.5$
**B.** the seventh term of a geometric sequence with $a = 1$ and $r = 0.5$
**C.** the sum of seven terms of an arithmetic sequence with $a = 1$ and $d = 0.5$
**D.** the sum of seven terms of a geometric sequence with $a = 1$ and $r = 0.5$
**E.** the sum of seven terms of a Fibonacci-related sequence with $t_1 = 1$ and $t_2 = 1.5$

**Question 3**

A city has a population of 100 000 people in 2014.

Each year, the population of the city is expected to increase by 4%.

In 2018, the population is expected to be closest to

**A.** 108 000
**B.** 112 000
**C.** 115 000
**D.** 117 000
**E.** 122 000

**Question 4**

On day 1, Vikki spends 90 minutes on a training program.

On each following day, she spends 10 minutes less on the training program than she did the day before.

Let $t_n$ be the number of minutes that Vikki spends on the training program on day $n$.

A difference equation that can be used to model this situation for $1 \le n \le 10$ is

**A.** $t_{n+1} = 0.90t_n \qquad t_1 = 90$

**B.** $t_{n+1} = 1.10t_n \qquad t_1 = 90$

**C.** $t_{n+1} = t_n - 0.10 \qquad t_1 = 90$

**D.** $t_{n+1} = 1 - 10t_n \qquad t_1 = 90$

**E.** $t_{n+1} = t_n - 10 \qquad t_1 = 90$

**Question 5**

Mary plans to read a book in seven days.

Each day, Mary plans to read 15 pages more than she read on the previous day.

The book contains 1155 pages.

The number of pages that Mary will need to read on the first day, if she is to finish reading the book in seven days, is

**A.** 112
**B.** 120
**C.** 150
**D.** 165
**E.** 180

**Question 6**

Consider the following sequence.

$$2, 1, 0.5 \ldots$$

Which of the following difference equations could generate this sequence?

**A.** $t_{n+1} = t_n - 1 \qquad t_1 = 2$

**B.** $t_{n+1} = 3 - t_n \qquad t_1 = 2$

**C.** $t_{n+1} = 2 \times 0.5^{n-1} \qquad t_1 = 2$

**D.** $t_{n+1} = -0.5t_n + 2 \qquad t_1 = 2$

**E.** $t_{n+1} = 0.5t_n \qquad t_1 = 2$

**Question 7**

The first term of a Fibonacci-related sequence is $p$.

The second term of the same Fibonacci-related sequence is $q$.

The difference in value between the fourth and fifth terms of this sequence is

**A.** $p - q$
**B.** $q - p$
**C.** $p + q$
**D.** $p + 2q$
**E.** $2p + 3q$

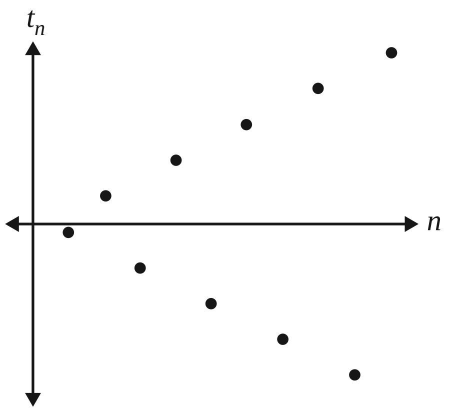
**Question 8**

The first term of a geometric sequence is $a$, where $a < 0$.

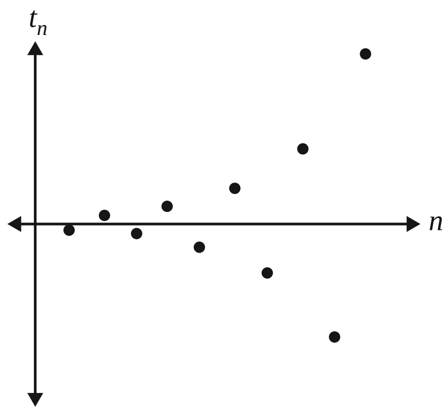
The common ratio of this sequence, $r$, is such that $r < -1$.

Which one of the following graphs best shows the first 10 terms of this sequence?

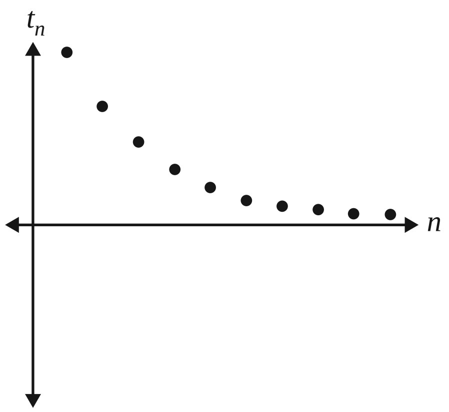
**A.**



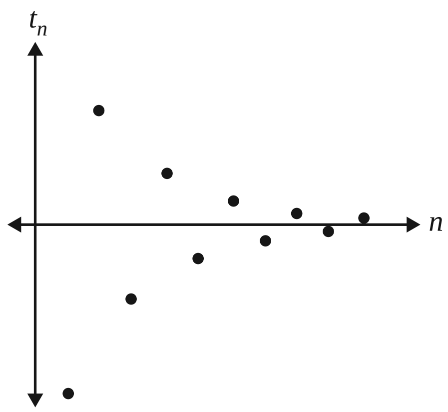

**B.**



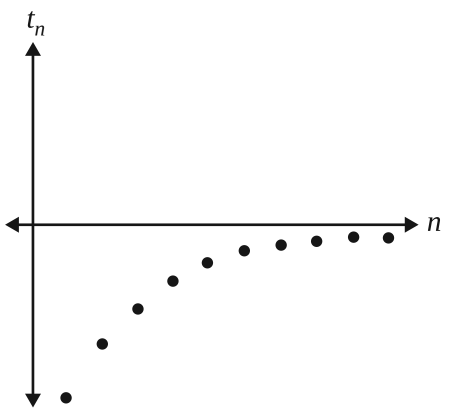

**C.**




**D.**




**E.**



**Question 9**

Sam takes a tablet containing 200 mg of medicine once every 24 hours.

Every 24 hours, 40% of the medicine leaves her body. The remaining 60% of the medicine stays in her body.

Let $D_n$ be the number of milligrams of the medicine in Sam's body immediately after she takes the *n*th tablet.

The difference equation that can be used to determine the number of milligrams of the medicine in Sam's body immediately after she takes each tablet is shown below.

$$D_{n+1} = 0.60D_n + 200 \qquad D_1 = 200$$

Which one of the following statements is **not** true?

**A.** The number of milligrams of the medicine in Sam's body never exceeds 500.

**B.** Immediately after taking the third tablet, 392 mg of the medicine is in Sam's body.

**C.** The number of milligrams of the medicine that leaves Sam's body during any 24-hour period will always be less than 200.

**D.** The number of milligrams of the medicine that leaves Sam's body during any 24-hour period is constant.

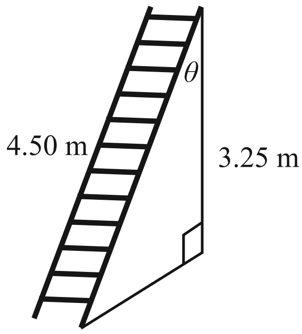
**E.** If Sam stopped taking the medicine after the fifth tablet, the amount of the medicine in her body would drop to below 200 mg after a further 48 hours.

## Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

The top of a ladder that is 4.50 m long rests 3.25 m up a wall, as shown in the diagram below.




The angle, $\theta$, that the ladder makes with the wall is closest to

**A.** 36°

**B.** 44°

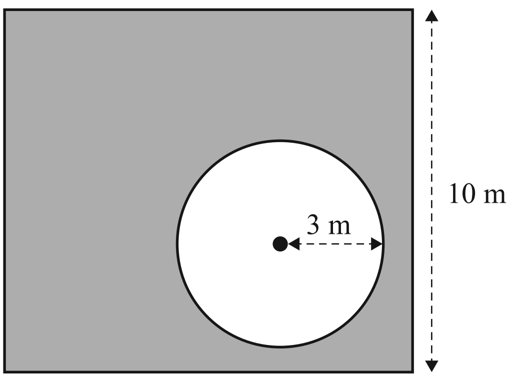
**C.** 46°

**D.** 50°

**E.** 54°

**Question 2**

A circular pool is located in a square lawn, as shown below.




The sides of the square lawn are 10 m in length.

The pool has a radius of 3 m.

The area of the lawn surrounding the pool, in square metres, is closest to

**A.** 21

**B.** 59

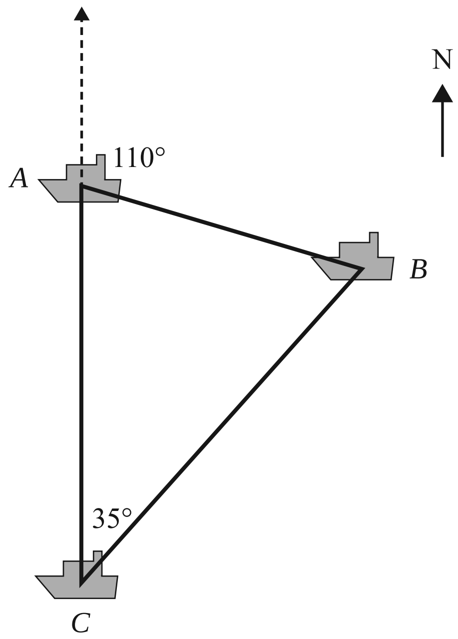
**C.** 72

**D.** 81

**E.** 128

**Question 3**

The diagram below shows the location of three boats, *A*, *B* and *C*.




Boat *B* is on a bearing of 110° from boat *A*.

Boat *B* is also on a bearing of 035° from boat *C*.

Boat *A* is due north of boat *C*.

The angle *ABC* is

**A.** 35°

**B.** 65°

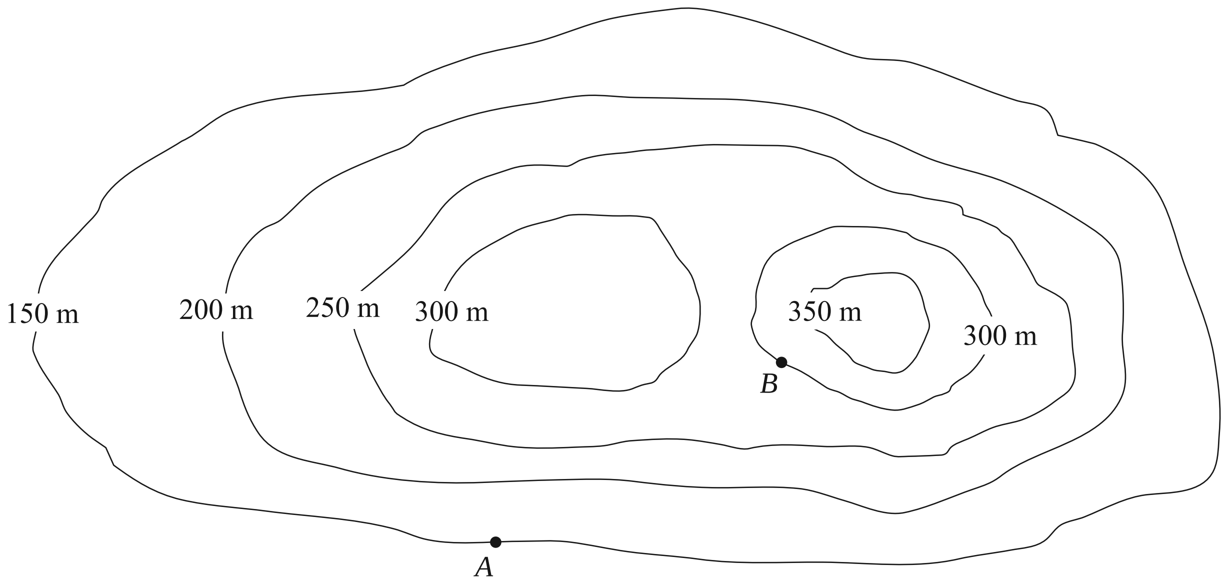
**C.** 70°

**D.** 75°

**E.** 110°

**Question 4**

The contour map shown below has contours at 50 m intervals.




The angle of depression of point *A* from point *B* is 23°.

The horizontal distance between point *A* and point *B*, in metres, is closest to

**A.** 64

**B.** 127

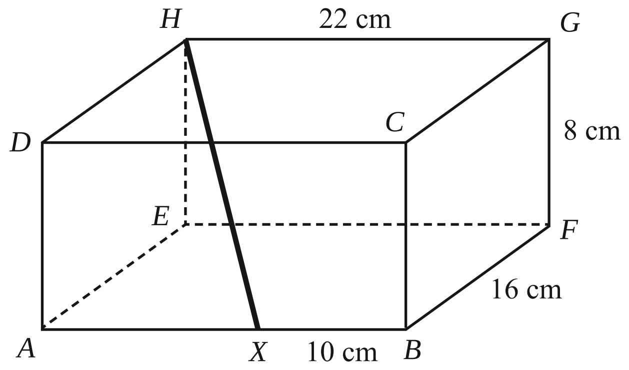
**C.** 353

**D.** 384

**E.** 707

**Question 5**

A rectangular box, *ABCDEFGH*, is 22 cm long, 16 cm wide and 8 cm high, as shown below.




A thin rod is resting in the box. One end of the rod sits at *X* and the other end of the rod sits at *H*.

The point *X* lies on the line *AB* at a distance of 10 cm from *B*.

The length of the rod, in centimetres, is closest to

**A.** 17.89

**B.** 18.87

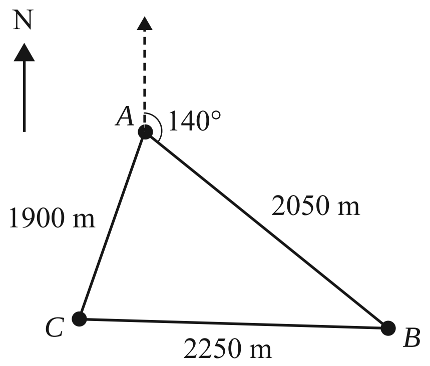
**C.** 20.00

**D.** 21.54

**E.** 26.83

*Use the following information to answer Questions 6 and 7.*

A cross-country race is run on a triangular course. The points $A$, $B$ and $C$ mark the corners of the course, as shown below.




The distance from $A$ to $B$ is 2050 m.

The distance from $B$ to $C$ is 2250 m.

The distance from $A$ to $C$ is 1900 m.

The bearing of $B$ from $A$ is 140°.

**Question 6**

The bearing of $C$ from $A$ is closest to

**A.** 032°

**B.** 069°

**C.** 192°

**D.** 198°

**E.** 209°

**Question 7**

The area within the triangular course $ABC$, in square metres, can be calculated by evaluating

**A.** $\sqrt{3100 \times 1200 \times 1050 \times 850}$

**B.** $\sqrt{3100 \times 2250 \times 2050 \times 1900}$

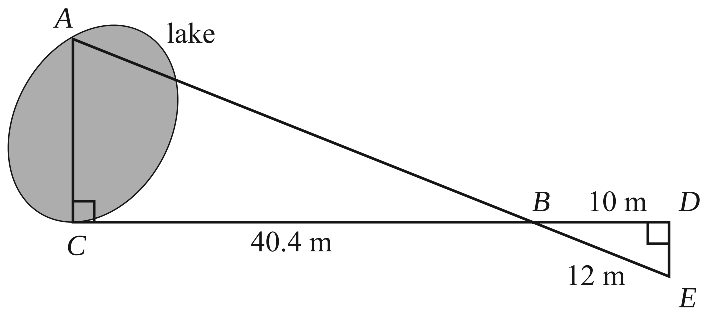
**C.** $\sqrt{6200 \times 4300 \times 4150 \times 3950}$

**D.** $\frac{1}{2} \times 2050 \times 2250 \times \sin(140°)$

**E.** $\frac{1}{2} \times 2050 \times 2250 \times \sin(40°)$

**Question 8**

The distance, $AC$, across a small lake can be calculated using the measurements shown in the diagram below.



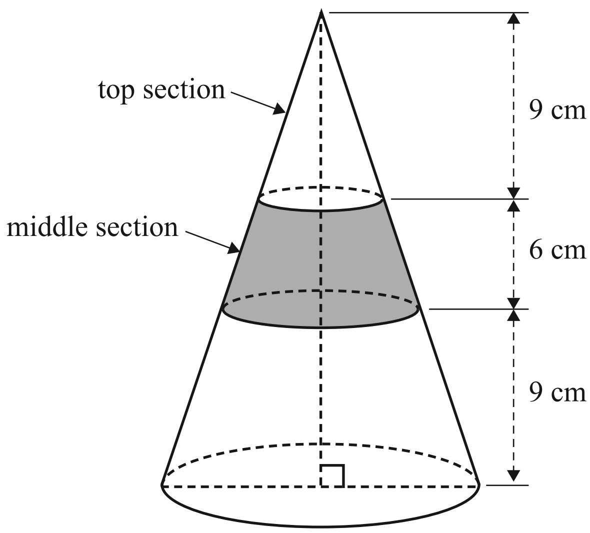

In this diagram, $BCA$ and $BDE$ are right-angled triangles, where $CB$ = 40.4 m, $BD$ = 10 m and $BE$ = 12 m.

The distance between the points $A$ and $C$, in metres, is closest to

**A.** 22.4
**B.** 26.8
**C.** 33.6
**D.** 48.5
**E.** 177.8

**Question 9**

The middle section of a cone is shaded, as shown in the diagram below.




The surface area of the unshaded top section of the cone is 180 $cm^2$.

The surface area of the middle section of the cone, in square centimetres, is

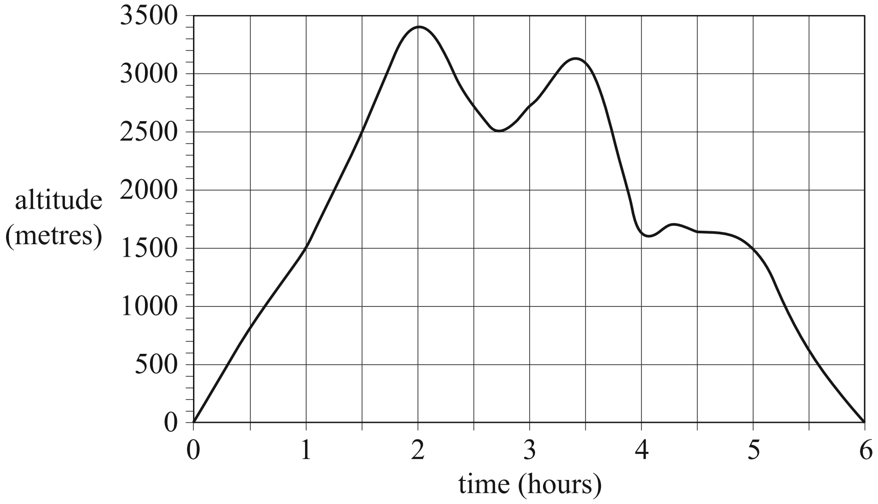
**A.** 80
**B.** 120
**C.** 300
**D.** 320
**E.** 500

## Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

The graph below shows the altitude, in metres, of a balloon over a six-hour flight.




Over the six-hour period, the length of time, in hours, where the altitude of the balloon was at least 1500 m is

**A.** 3
**B.** 4
**C.** 5
**D.** 6
**E.** 7

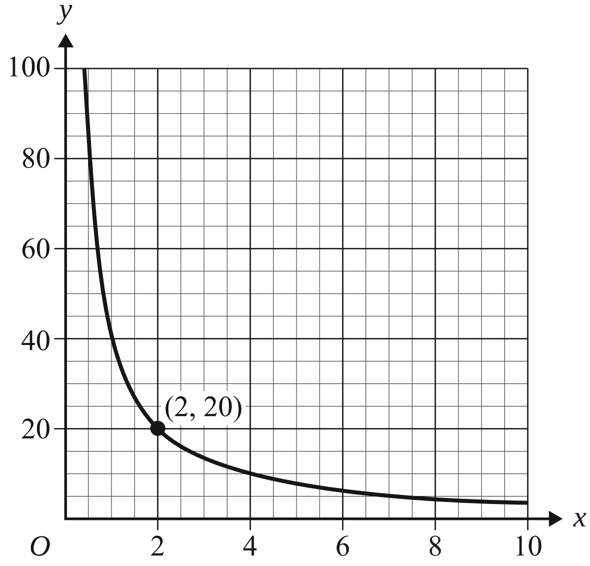
**Question 2**

The vertical line that passes through the point (3, 2) has the equation

**A.** $x + y = 5$
**B.** $xy = 6$
**C.** $3y = 2x$
**D.** $y = 2$
**E.** $x = 3$

**Question 3**

The point (2, 20) lies on the graph of $y = \frac{k}{x}$, as shown below.




The value of $k$ is

**A.** 5

**B.** 10

**C.** 20

**D.** 40

**E.** 80

**Question 4**

A line passes through the points (–1, 1) and (3, 5).

Another point that lies on this line is

**A.** (0, 1)

**B.** (1, 3)

**C.** (2, 6)

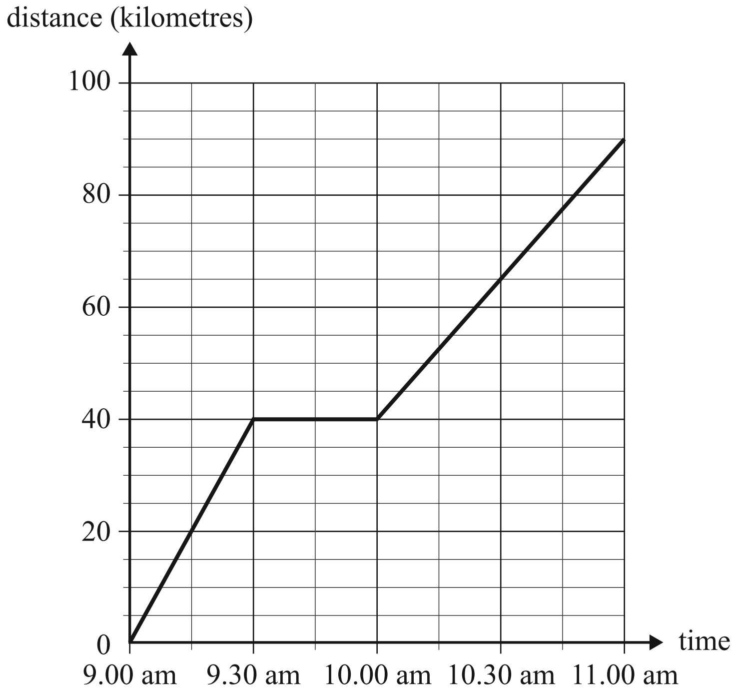
**D.** (3, 4)

**E.** (4, 7)

**Question 5**

The distance–time graph below shows a train's journey between two towns.

During the journey, the train stopped for 30 minutes.




The average speed of the train, in kilometres per hour, for the journey is closest to

**A.** 45

**B.** 50

**C.** 60

**D.** 65

**E.** 80

**Question 6**

The Domestics Cleaning Company provides household cleaning services.

For two hours of cleaning, the cost is \$55.

For four hours of cleaning, the cost is \$94.

The rule for the cost of cleaning services is

$$cost = a + b \times hours$$

where $a$ is a fixed charge, in dollars, and $b$ is the charge per hour of cleaning, in dollars per hour.

Using this rule, the cost for five hours of cleaning is

**A.** \$19.50

**B.** \$97.50

**C.** \$99.50

**D.** \$113.50

**E.** \$121.50

**Question 7**

Consider the following statements that relate to the solution of linear programming problems.

Which one of the following statements is **true**?

**A.** Only one point can be a solution.

**B.** No point outside the feasible region can be a solution.

**C.** To have a solution, the feasible region must be bounded.

**D.** Only the corner points of a feasible region can be a solution.

**E.** Only the corner points with integer coordinates can be a solution.

**Question 8**

The constraints of a linear programming problem are given by the following set of inequalities.

$$
\begin{aligned}
x + y &\le 8 \\
3x + 5y &\le 30 \\
x &\ge 0 \\
y &\ge 0
\end{aligned}
$$

The coordinates of the points that define the boundaries of the feasible region for this linear programming problem are

**A.** (0, 0), (0, 6), (3, 5), (8, 0)

**B.** (0, 0), (0, 6), (5, 3), (8, 0)

**C.** (0, 0), (0, 6), (5, 3), (10, 0)

**D.** (0, 0), (0, 8), (5, 3), (8, 0)

**E.** (0, 0), (0, 8), (5, 3), (10, 0)

**Question 9**

Xavier and Yvette share a job.

Yvette must work at least twice as many hours as Xavier.

They must work at least 40 hours each week, in total.

Xavier must work at least 10 hours each week.

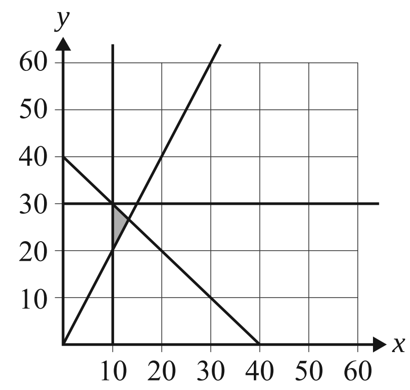
Yvette can only work for a maximum of 30 hours each week.

Let *x* represent the number of hours that Xavier works each week.

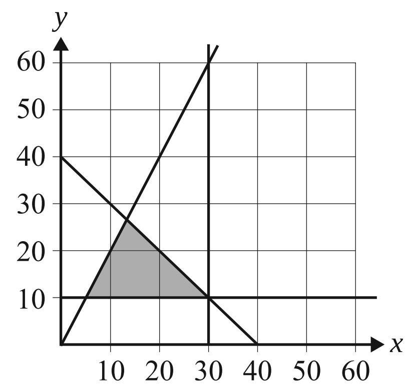
Let *y* represent the number of hours that Yvette works each week.

In which one of the following graphs does the shaded area show the feasible region defined by these conditions?

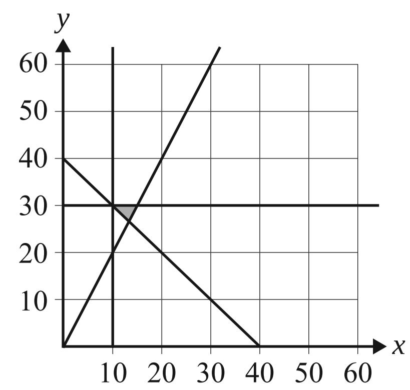
**A.**



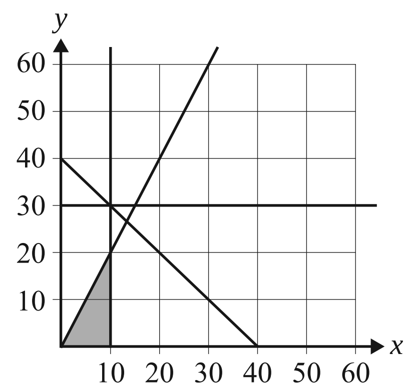

**B.**



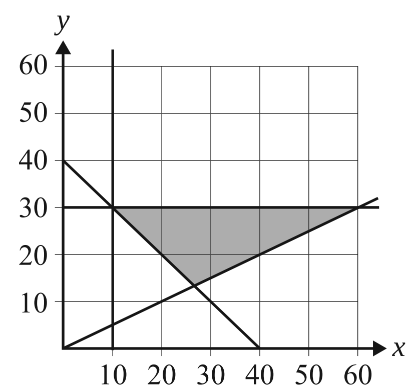

**C.**




**D.**




**E.**




**END OF MODULE 3**

## Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

This month, a business charges $1500 to install a water tank.

Next month, the charge will increase by 3.5%.

The charge next month will be

**A.** $45.00
**B.** $52.50
**C.** $1545.00
**D.** $1552.50
**E.** $1950.00

**Question 2**

An internet car market site charges $120 to advertise a car for sale.

The car is sold for $15 000.

The $120 charge as a percentage of the selling price of the car is

**A.** 0.008%
**B.** 0.08%
**C.** 0.80%
**D.** 1.20%
**E.** 1.25%

**Question 3**

Amy invests $15 000 for 150 days.

Interest is calculated at the rate of 4.60% per annum, compounding daily.

Assuming that there are 365 days in a year, the value of her investment after 150 days is closest to

**A.** $15 279
**B.** $15 284
**C.** $15 286
**D.** $15 690
**E.** $16 776

**Question 4**

The cost of hiring a plasterer is $86.00 per hour plus GST of 10%.

The cost of hiring a plasterer for four hours, including GST, is

**A.** $120.40
**B.** $309.60
**C.** $344.00
**D.** $352.60
**E.** $378.40

**Question 5**

A bank approves a \$90 000 loan for a customer.

The loan is to be repaid fully over 20 years in equal monthly payments.

Interest is charged at a rate of 6.95% per annum on the reducing monthly balance.

To the nearest dollar, the monthly payment will be

**A.** \$478

**B.** \$692

**C.** \$695

**D.** \$1409

**E.** \$1579

**Question 6**

A loan of \$1000 is to be repaid with six payments of \$180 per month.

The effective annual rate of interest charged is closest to

**A.** 8.0%

**B.** 13.7%

**C.** 16.0%

**D.** 27.4%

**E.** 30.9%

**Question 7**

New furniture was purchased for an office at a cost of \$18 000.

Using flat rate depreciation, the furniture will be valued at \$5000 after four years.

The expression that can be used to determine the value of the furniture, in dollars, after one year is

**A.** $18\,000 - (4 \times 5000)$

**B.** $18\,000 - \left(\dfrac{18\,000 - 5000}{4}\right)$

**C.** $18\,000 - \dfrac{5000}{4}$

**D.** $\dfrac{18\,000}{4} - 5000$

**E.** $18\,000 \times 0.726$

**Question 8**

Robert invested \$6000 at 4.25% per annum with interest compounding quarterly.

Immediately after interest is paid at the end of each quarter, he adds \$500 to his investment.

The value of Robert's investment at the end of the third quarter, after his \$500 has been added, is closest to

**A.** \$6193

**B.** \$7569

**C.** \$7574

**D.** \$7709

**E.** \$8096

**Question 9**

Leslie borrowed \$35 000 from a bank.

Interest is charged at the rate of 4.75% on the reducing monthly balance.

The loan is to be repaid with 47 monthly payments of \$802.00 and a final payment that is to be adjusted so that the loan will be fully repaid after exactly 48 monthly payments.

Correct to the nearest cent, the amount of the final payment will be

**A.** \$0.39

**B.** \$3.57

**C.** \$802.00

**D.** \$802.39

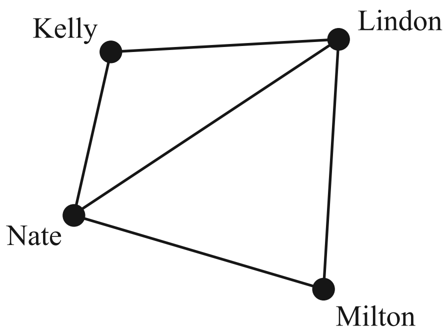
**E.** \$805.57

## Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

The graph below shows the roads connecting four towns: Kelly, Lindon, Milton and Nate.




A bus starts at Kelly, travels through Nate and Lindon, then stops when it reaches Milton.

The mathematical term for this route is

**A.** a loop.

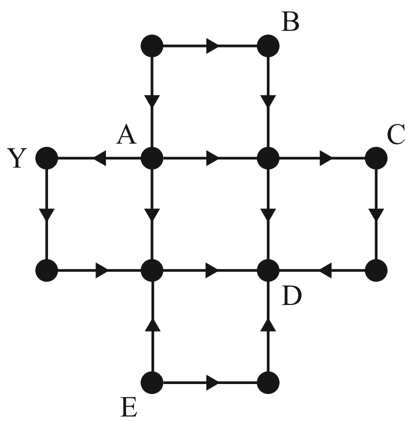
**B.** an Eulerian path.

**C.** an Eulerian circuit.

**D.** a Hamiltonian path.

**E.** a Hamiltonian circuit.

**Question 2**




In the directed graph above, the only vertex with a label that can be reached from vertex Y is

**A.** vertex A

**B.** vertex B

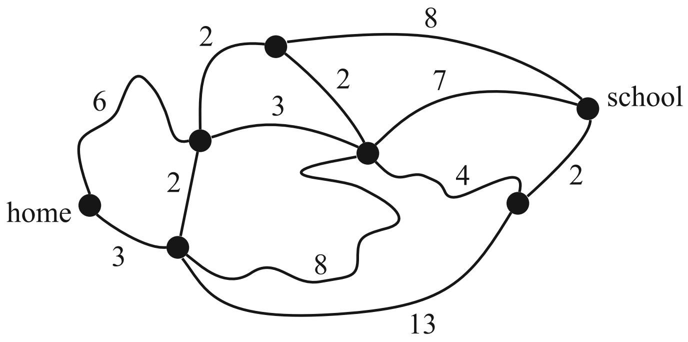
**C.** vertex C

**D.** vertex D

**E.** vertex E

**Question 3**

The diagram below shows the network of roads that Stephanie can use to travel between home and school.




The numbers on the roads show the time, in minutes, that it takes her to ride a bicycle along each road.

Using this network of roads, the shortest time that it will take Stephanie to ride her bicycle from home to school is

**A.** 12 minutes

**B.** 13 minutes

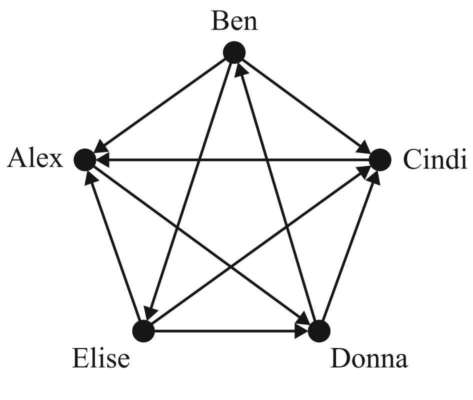
**C.** 14 minutes

**D.** 15 minutes

**E.** 16 minutes

**Question 4**

The directed graph below shows the results of a chess competition between five players: Alex, Ben, Cindi, Donna and Elise.




Each arrow indicates the winner of individual games. For example, the arrow from Alex to Donna indicates that Alex beat Donna in their game.

The sum of their one-step and two-step dominances is calculated to give each player a dominance score. The dominance scores are then used to rank the players.

The ranking of the players in this competition, from highest to lowest dominance score, is

**A.** Ben, Elise, Donna, Alex, Cindi

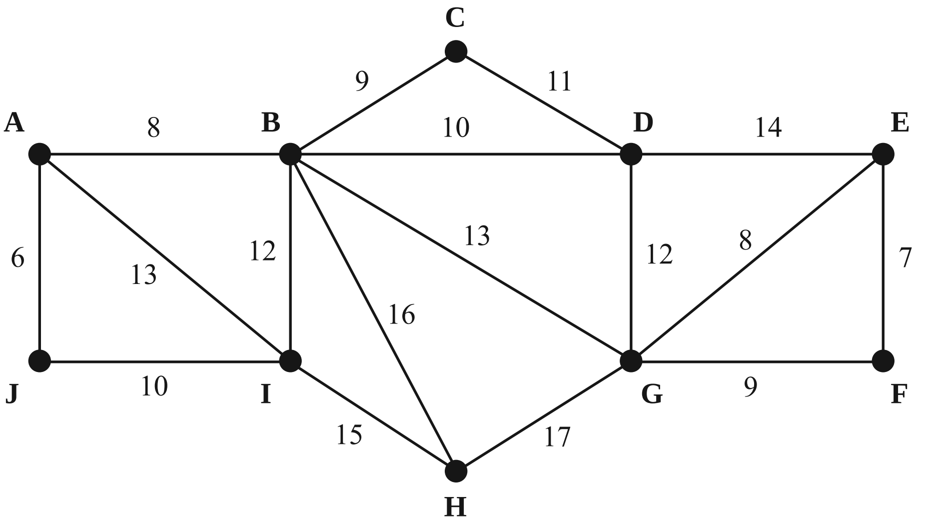
**B.** Ben, Elise, Cindi, Donna, Alex

**C.** Ben, Elise, Donna, Cindi, Alex

**D.** Elise, Ben, Donna, Alex, Cindi

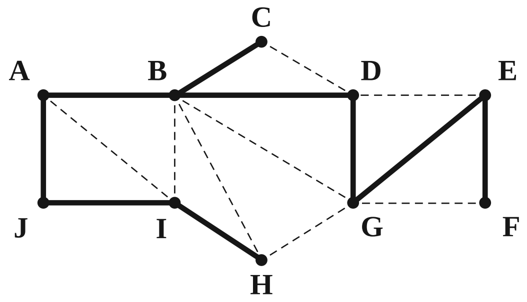
**E.** Elise, Ben, Donna, Cindi, Alex

### Question 5

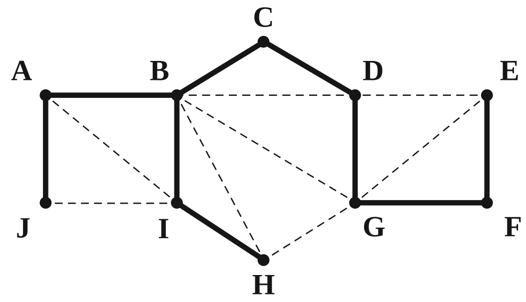



Which one of the following is the minimal spanning tree for the weighted graph shown above?

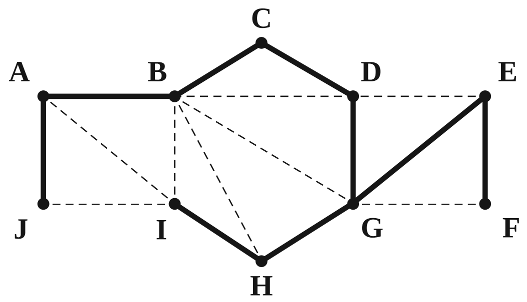
**A.**



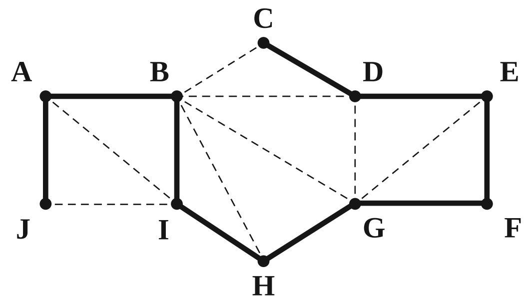

**B.**



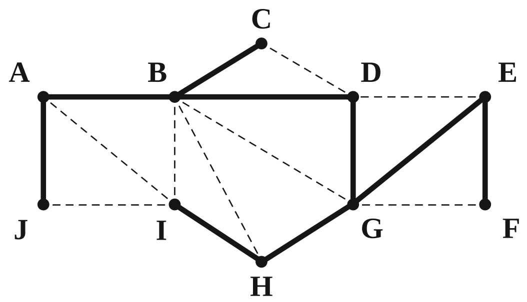

**C.**




**D.**

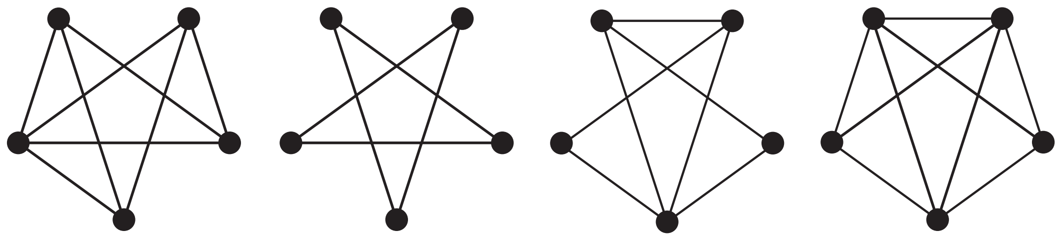



**E.**



*Use the following information to answer Questions 6 and 7.*

Consider the following four graphs.



**Question 6**

How many of these four graphs have an Eulerian circuit?

**A.** 0
**B.** 1
**C.** 2
**D.** 3
**E.** 4

**Question 7**

How many of these four graphs are planar?

**A.** 0
**B.** 1
**C.** 2
**D.** 3
**E.** 4

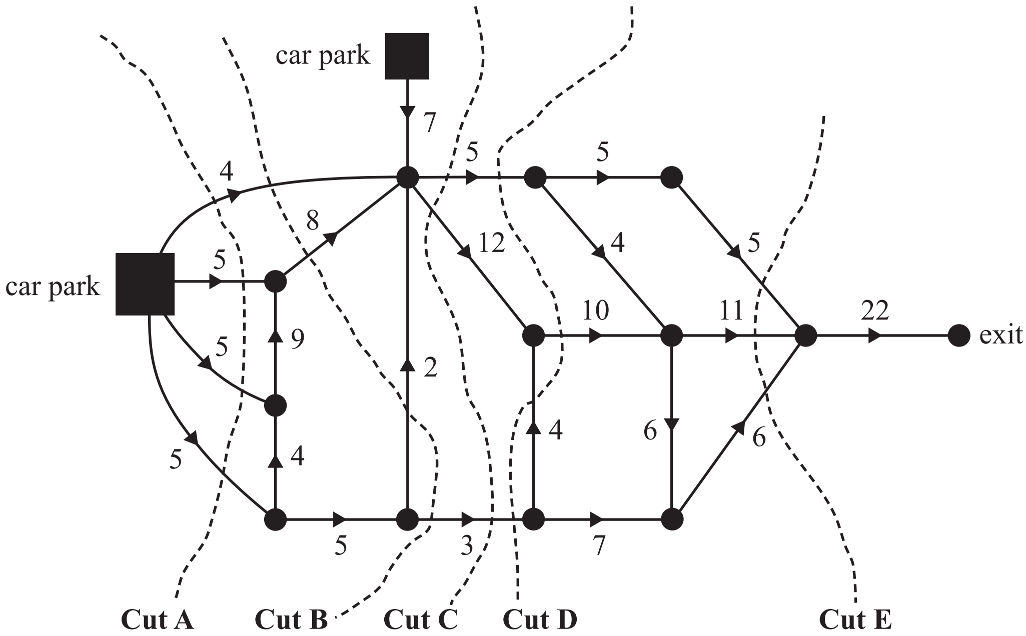
**Question 8**

Which one of the following statements about critical paths is **true**?

**A.** There can be only one critical path in a project.
**B.** A critical path always includes at least two activities.
**C.** A critical path will always include the activity that takes the longest time to complete.
**D.** Reducing the time of any activity on a critical path for a project will always reduce the minimum completion time for the project.
**E.** If there are no other changes, increasing the time of any activity on a critical path will always increase the completion time of a project.

**Question 9**

A network of tracks connects two car parks in a festival venue to the exit, as shown in the directed graph below.




The arrows show the direction that cars can travel along each of the tracks and the numbers show each track’s capacity in cars per minute.

Five cuts are drawn on the diagram.

The maximum number of cars per minute that will reach the exit is given by the capacity of

**A.** Cut A

**B.** Cut B

**C.** Cut C

**D.** Cut D

**E.** Cut E

## Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \text{ is equal to}$$

**A.** $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 15 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

**B.** $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

**C.** $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

**D.** $\begin{bmatrix} 0 & 0 & 4 & 0 \\ 4 & 1 & 1 & 9 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 9 & 0 \end{bmatrix}$

**E.** $\begin{bmatrix} 0 & 0 & 2 & 0 \\ 2 & 1 & 1 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$

**Question 2**

$$\begin{aligned} y - z &= 8 \\ 5x - y &= 0 \\ x + z &= 4 \end{aligned}$$

The system of three simultaneous linear equations above can be written in matrix form as

**A.** $\begin{bmatrix} 0 & 1 & -1 \\ 0 & 5 & -1 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}$

**B.** $\begin{bmatrix} 0 & 1 & -1 \\ 5 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}$

**C.** $\begin{bmatrix} 1 & -1 \\ 5 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}$

**D.** $\begin{bmatrix} 0 & 5 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}$

**E.** $\begin{bmatrix} 0 & 5 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}$

**Question 3**

Regular customers at a hairdressing salon can choose to have their hair cut by Shirley, Jen or Narj.

The salon has 600 regular customers who get their hair cut each month.

In June, 200 customers chose Shirley ($S$) to cut their hair, 200 chose Jen ($J$) to cut their hair and 200 chose Narj ($N$) to cut their hair.

The regular customers' choice of hairdresser is expected to change from month to month as shown in the transition matrix, $T$, below.

$$\begin{array}{c} \text{this month} \\ \begin{array}{ccc} S & J & N \end{array} \end{array}$$
$$T = \begin{bmatrix} 0.75 & 0.10 & 0.10 \\ 0.10 & 0.75 & 0.15 \\ 0.15 & 0.15 & 0.75 \end{bmatrix} \begin{array}{l} S \\ J \\ N \end{array} \text{ next month}$$

In the long term, the number of regular customers who are expected to choose Shirley is closest to

**A.** 150

**B.** 170

**C.** 185

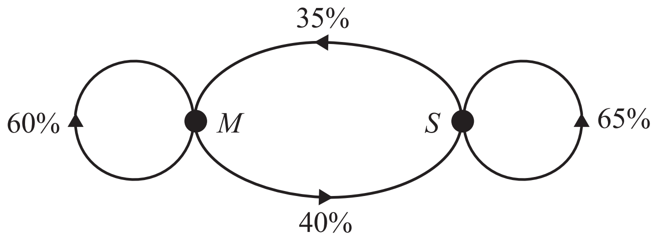
**D.** 195

**E.** 200

**Question 4**

Two hundred and fifty people buy bread each day from a corner store. They have a choice of two brands of bread: Megaslice (*M*) and Superloaf (*S*).

The customers' choice of brand changes daily according to the transition diagram below.




On a given day, 100 of these people bought Megaslice bread while the remaining 150 people bought Superloaf bread.

The number of people who are expected to buy each brand of bread the next day is found by evaluating the matrix product

**A.**

$$\begin{array}{cc} M & S \end{array}$$
$$\begin{bmatrix} 0.60 & 0.40 \\ 0.35 & 0.65 \end{bmatrix}\begin{bmatrix} 100 \\ 150 \end{bmatrix}\begin{array}{l} M \\ S \end{array}$$

**B.**

$$\begin{array}{cc} M & S \end{array}$$
$$\begin{bmatrix} 0.60 & 0.40 \\ 0.65 & 0.35 \end{bmatrix}\begin{bmatrix} 100 \\ 150 \end{bmatrix}\begin{array}{l} M \\ S \end{array}$$

**C.**

$$\begin{array}{cc} M & S \end{array}$$
$$\begin{bmatrix} 0.60 & 0.35 \\ 0.40 & 0.65 \end{bmatrix}\begin{bmatrix} 100 \\ 150 \end{bmatrix}\begin{array}{l} M \\ S \end{array}$$

**D.**

$$\begin{array}{cc} M & S \end{array}$$
$$\begin{bmatrix} 0.65 & 0.40 \\ 0.35 & 0.60 \end{bmatrix}\begin{bmatrix} 100 \\ 150 \end{bmatrix}\begin{array}{l} M \\ S \end{array}$$

**E.**

$$\begin{array}{cc} M & S \end{array}$$
$$\begin{bmatrix} 0.60 & 0.65 \\ 0.40 & 0.35 \end{bmatrix}\begin{bmatrix} 100 \\ 150 \end{bmatrix}\begin{array}{l} M \\ S \end{array}$$

**Question 5**

Students from Year 7 and Year 8 in a school sold trees to raise funds for a school trip.

The number of large, medium and small trees that were sold by each year group is shown in the table below.

| Year group | Large | Medium | Small |
|---|---|---|---|
| 7 | 52 | 78 | 61 |
| 8 | 45 | 56 | 81 |

The large trees were sold for \$32 each, the medium trees were sold for \$26 each and the small trees were sold for \$18 each.

A matrix product that can be used to calculate the amount, in dollars, raised by each year group by selling trees is

**A.**

$$\begin{bmatrix} 52 & 78 & 61 \\ 32 & 26 & 18 \end{bmatrix} \begin{bmatrix} 45 \\ 56 \\ 81 \end{bmatrix}$$

**B.**

$$\begin{bmatrix} 7 & 52 & 78 & 61 \\ 8 & 45 & 56 & 81 \end{bmatrix} \begin{bmatrix} 32 \\ 26 \\ 18 \\ 0 \end{bmatrix}$$

**C.**

$$\begin{bmatrix} 32 & 26 & 18 \end{bmatrix} \begin{bmatrix} 52 & 45 \\ 78 & 56 \\ 61 & 81 \end{bmatrix}$$

**D.**

$$\begin{bmatrix} 32 & 26 & 18 \end{bmatrix} \begin{bmatrix} 52 & 78 & 61 \\ 45 & 56 & 81 \\ 0 & 0 & 0 \end{bmatrix}$$

**E.**

$$\left( \begin{bmatrix} 52 \\ 78 \\ 61 \end{bmatrix} + \begin{bmatrix} 45 \\ 56 \\ 81 \end{bmatrix} \right) \times \begin{bmatrix} 32 & 26 & 18 \end{bmatrix}$$

**Question 6**

The order of matrix $X$ is $3 \times 2$.

The element in row $i$ and column $j$ of matrix $X$ is $x_{ij}$ and it is determined by the rule

$$x_{ij} = i + j$$

The matrix $X$ is

**A.** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

**B.** $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}$

**C.** $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

**D.** $\begin{bmatrix} 1 & 2 \\ 3 & 3 \\ 4 & 4 \end{bmatrix}$

**E.** $\begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$

**Question 7**

A transition matrix, $T$, and a state matrix, $S_2$, are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$$

If $S_2 = TS_1$, the state matrix $S_1$ is

**A.** $\begin{bmatrix} 200 \\ 250 \\ 150 \end{bmatrix}$

**B.** $\begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$

**C.** $\begin{bmatrix} 300 \\ 0 \\ 300 \end{bmatrix}$

**D.** $\begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$

**E.** undefined

**Question 8**

Wendy will have lunch with one of her friends each day of this week.

Her friends are Angela ($A$), Betty ($B$), Craig ($C$), Daniel ($D$) and Edgar ($E$).

On Monday, Wendy will have lunch with Craig.

Wendy will use the transition matrix below to choose a friend to have lunch with for the next four days of the week.

$$\begin{array}{c} \text{today} \\ \begin{array}{ccccc} A & B & C & D & E \end{array} \end{array}$$

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} \quad \text{tomorrow}$$

The order in which Wendy has lunch with her friends for the next four days is

**A.** Angela, Betty, Craig, Daniel

**B.** Daniel, Betty, Angela, Craig

**C.** Daniel, Betty, Angela, Edgar

**D.** Edgar, Angela, Daniel, Betty

**E.** Edgar, Daniel, Betty, Angela

**Question 9**

$A$ and $B$ are square matrices such that $AB = BA = I$, where $I$ is an identity matrix.

Which one of the following statements is **not** true?

**A.** $ABA = A$

**B.** $AB^2A = I$

**C.** $B$ must equal $A$

**D.** $B$ is the inverse of $A$

**E.** both $A$ and $B$ have inverses

**END OF MULTIPLE-CHOICE QUESTION BOOK**

# FURTHER MATHEMATICS

## Written examinations 1 and 2

### FORMULA SHEET

**Directions to students**

| |
|---|
| Detach this formula sheet during reading time. |
| This formula sheet is provided for your reference. |

# Further Mathematics formulas

## Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares regression line: $y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

## Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, |r| < 1$

## Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: $\pi r^2$

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

## Module 3: Graphs and relations

**Straight-line graphs**

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

## Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$, where $R = 1 + \dfrac{r}{100}$

hire-purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

## Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

## Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

**END OF FORMULA SHEET**