The Mathematical Association of Victoria

FURTHER MATHEMATICS

SOLUTIONS: Trial Exam 2014

Written Examination 2

CORE

Question 1

(a) (i) The median occurs between values 11 and 12 in the ordered ranking.
this value is 50. A1

(ii) Q1 (6th value in ordered data) = 47,
Q3 (17th value in ordered data) = 53,
hence IQR = 53 – 47 = 6 A1

(b) Upper fence limit = Q3 + 1.5 × IQR = 53 + 1.5 × 6 = 53 + 9 = 62 M1
As the value of 63 > upper fence limit, it is an outlier A1

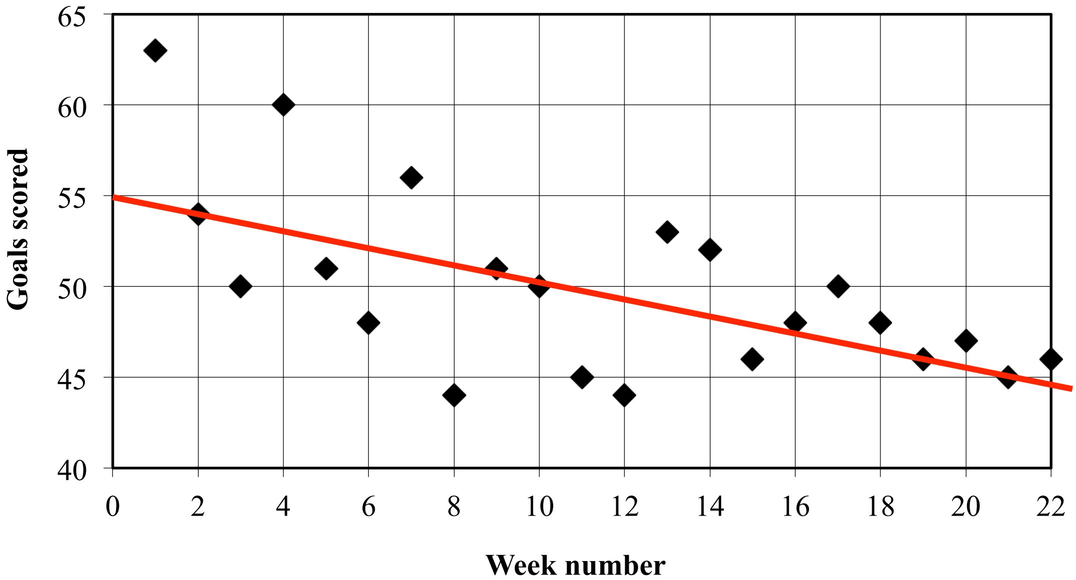
Question 2

No.	1	2	3	4	5	6	7	8	9	10
Value	**1**	**2**	**3**	**3 or 4**	**4**	**5**	**5 or 6 or 7 or 8**	**8**	**9**	**10**

Key values at numbers 3 (3), 5 (4), 6 (5), 8 (8) A1
A CORRECT selection of the alternatives for the other values at numbers 4 and 7. A1

Question 3

(a) A1



Key points (), 55) (21, 45)

© The Mathematical Association of Victoria, 2014

(b) (i) 38.8% of the variation in ***Goals scored*** values can be accounted for the variation in the number of ***Week number***. A1

(ii) $r = \pm\sqrt{0.3883} = -0.623$ (slope of line !) A1

(c) goals scored = 55 – 0.48 × 12 = 49 A1

(d) Residual = actual – predicted = 44 – 49 = – 5 A1

Question 4

(a) $(\textit{fitness score})^2 = 13821.6 + 353.4 \times \textit{age}$ A1

(b) $(\textit{fitness score})^2 = 13821.6 + 353.4 \times 27 = 4279.8$,
$\textit{fitness score} = \sqrt{4279.8} = 65.4$ A1

Question 5

mean + 2 standard deviations = 60.6 + 2 × 14.7 = 90
From normal distribution, mean ± 2 standard deviations encompasses 95 %
leaving 2.5 % at each end. M1
2.5 % of 240 = 6
6 netballers had a fitness score above 90. A1

MODULE 1 : NUMBER PATTERNS

Question 1

(a) a = 1.3, d = 1.65 – 1.3 = 0.35
t5 = 1.3 + (5 – 1) × 0.35 = 1.3 + 4 × 0.35 = 2.7 A1

(b) $4 = 1.3 + (n-1) \times 0.35$, giving $n = \dfrac{4 - 1.3 \times 0.35}{0.35} = 8.7$
Hence 9th day A1

(c) Day 3 total = 1.3 + 1.65 + 2 = 4.95
Day 10 total $= \dfrac{10}{2}\left(2 \times 1.3 + 9 \times 0.35\right) = 28.75$ M1
Required distance = 28.75 – 4.95 = 23.8 H1

OR
Day 4 distance = 2 + 0.35 = 2.35 M1
Sum distances days (4 – 10) = 2.35 + 2.70 + 3.05 + 3.40 + 3.75 + 4.1 + 4.45 = 23.8 H1

(d) Day 31 total $= \dfrac{31}{2}\left(2 \times 1.3 + 30 \times 0.35\right) = 203.05$ A1

© The Mathematical Association of Victoria, 2014

(e) A = daily increase = 1.65 – 1.3 = 2.00 – 1.65 = 0.35 A1
For Day 1 $1.3 = 0.35 \times 1 + B$
Giving B = 1.3 – 0.35 = 0.95 A1

Question 2

(a) Day 2 dist = 2.00 × 1.07 = 2.14 A1

(b) Day 31 total $= \dfrac{2\times\left(1.07^{31}-1\right)}{1.07-1}$ = 204.15.. = 204 A1

(c) From 1(d), Jennifer walks 203 km
From 2(b), Katie jogs 204 km M1
Katie furthest by 204 – 203 = 1 km H1

Question 3

Day	Walk (km)	Run (km)
1	0.500	1.200
2	0.650	1.248
3	0.800	1.298
4	0.950	1.350
5	1.100	1.404
6	1.250	1.460
7	1.400	1.518
8	1.550	1.579
9	1.700	1.642
10	1.850	1.708

Show results of calculations for AT LEAST three days (8, 9, 10) M1
Hence Day 9 A1

OR
Distance walked = $150n + 350$, where n = day number
Distance run = $1200 \times 1.04^{n-1}$ (BOTH equations correct) M1
Use calculator to solve $150n + 350 = 1200 \times 1.04^{n-1}$
Gives $n = 8.33..$ Hence Day 9 A1

Question 4

(a) $D_{n+1} = 0.95 \times D_n + 0.450$ $D_1 = 3.0$ (Must have a term value!) A1

(b) If $D_n = 9.0$km, then $D_{n+1} = 0.95 \times 9.0 + 0.450 = 8.55 + 0.45 = 9.0$
The loss of 5% of the previous day's distance is exactly made up by 0.450 km A1

© The Mathematical Association of Victoria, 2014

MODULE 2 : GEOMETRY & TRIGONOMETRY

Question 1

(a) Volume $= 0.5 \times 2 \times 25 \times 110 = 2750$ A1

(b) $DE = \sqrt{25^2 + 2^2} = 25.08$ M1

Total fence $= 110 + 110 + 25.08 + 25.08 = 270.16 = 270.2$ A1

(c) Volume $= 110 \times 25 \times 0.1$ M1

$= 275$ A1

(d) (i) $327^\circ + 90^\circ - 360^\circ = 057^\circ$ A1

(ii) $327^\circ - 180^\circ = 147^\circ$ A1

Question 2

(a) Area paint $= 3(\pi \times 0.45^2 + \pi \times 4.9^2) = 3(0.64 + 75.43) = 228.21$ A1

(b) Litres paint $= \dfrac{2 \times 228.21}{12} = 38.04$

Number of cans $= \dfrac{38.04}{4} = 9.51 = 10$ A1

Question 3

(a) AB $= \sqrt{7^2 + 17^2} = 18.38 = 18$ A1

(b) AC $= \sqrt{17^2 + 23^2 - 2 \times 17 \times 23 \times \cos 93^\circ} = 29.31$ A1

(c) angle $= \tan^{-1}\left(\dfrac{7}{23}\right) = 17^\circ$ A1

(d) Height $= 32 \times \tan 15^\circ = 8.57 = 8.6$ A1

Question 4

Ratio radii $= \dfrac{\sqrt[3]{6044}}{\sqrt[3]{4540}} = 1.1$ M1

Hence multiplying factor for area $= 1.1^2 = 1.21$ A1

MODULE 3 : GRAPHS & RELATIONS

Question 1

(a) $R = 3.5x + 2.2y$ A1

(b) 87 hotdogs A1

$579.50 = 3.5x + 2.2\times125$
$x = \frac{579.50-2.2\times125}{3.5} = 87$ hotdogs

(c) $P = 2.3x + 1.4y$ A1

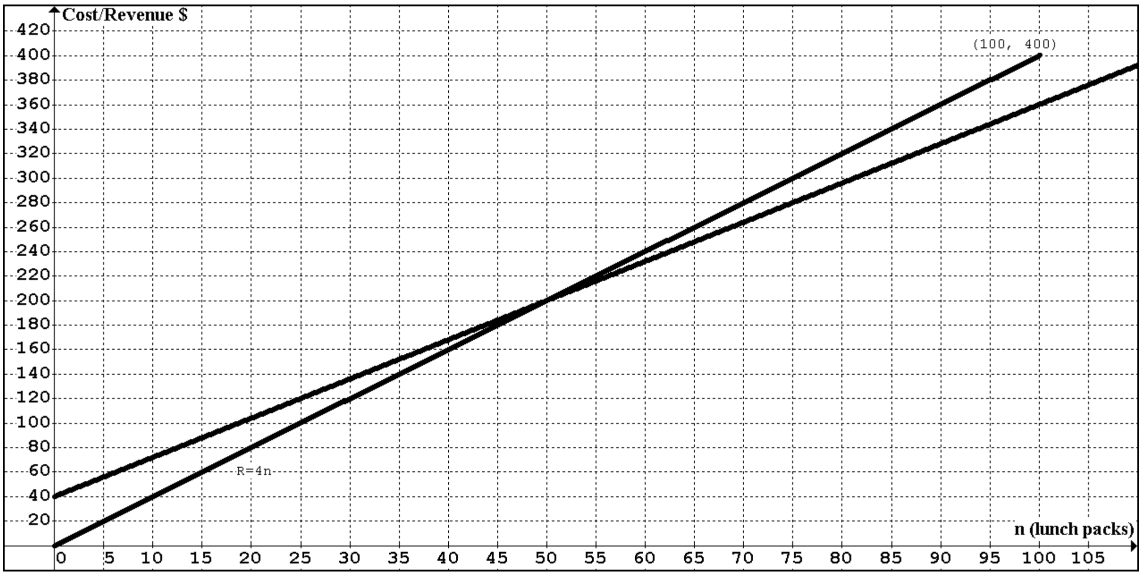
$P = R - C$
$P = 3.5x + 2.2y - (1.2x + 0.8y)$
$P = 2.3x + 1.4y$

Question 2

(a) $b = 40$ A1

(b) The cost of the delivery is \$40. A1

(c)



Note: the line must not extend beyond the point (100, 400). A1

(d) 51 lunch packs A1

The break-even point is where $R = C$.
$4n = 3.5n + 40$
$n = 50$
Therefore a profit would first be made when the canteen sold 51 lunch packs.

© The Mathematical Association of Victoria, 2014

Question 3

(a) \$20 if they have 2 deliveries of between 60 and 100 lunch packs (for example 75 in each delivery) A1

(b) $a = -0.5$ A1
$b = 60$ A1

The line passes through (40, 40), (60, 30) and (100, 10).

$m = \dfrac{10-40}{100-40} = -0.5 = a$

$y = mx + c$ using m = −0.5 and point (40,40)

$40 = -0.5 \times 40 + c$

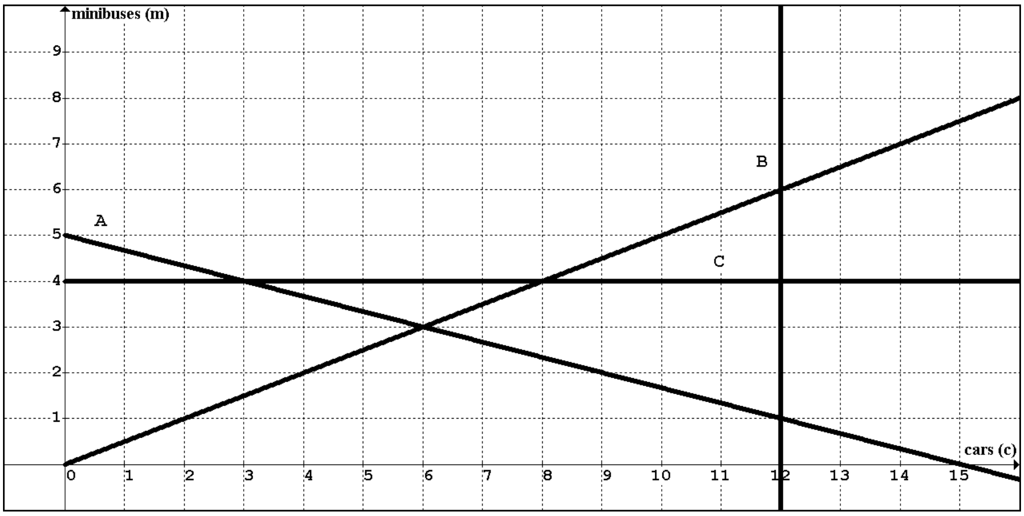
$c = 60 = b$

Question 4

(a) A1

Line 1	C
Line 2	A
Line 3	B

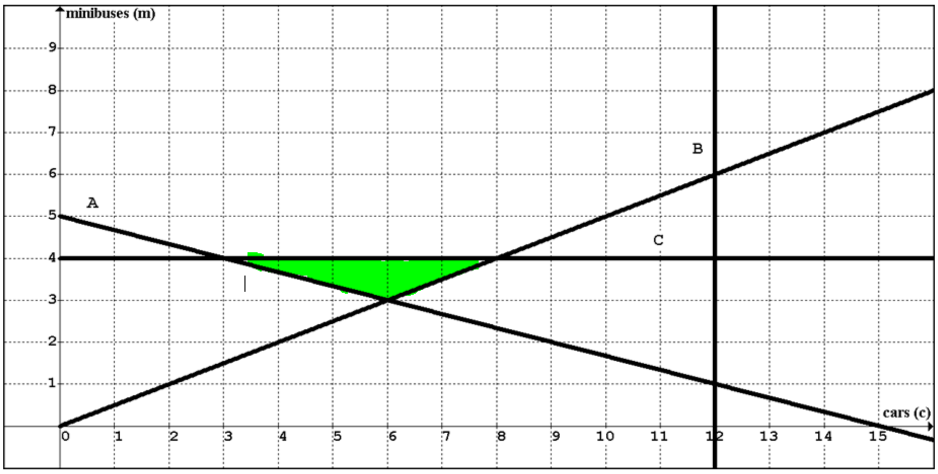
(b) $c \leq 12$ A1



A1

© The Mathematical Association of Victoria, 2014

(c) (3, 4), (4,4), (5, 4), (6,4), (7,4), (8, 4) and (6, 3)



Accept either region shaded OR points OR both A1

(d) A maximum of \$90 A1

The gradient of the objective function must be negative, but closer to zero than the gradient of $4c + 12m = 60$.
The gradient of $4c + 12m = 60$ is given by transposing the equation to $m = -\frac{c}{3} + 5$.
The gradient of this line is therefore, $gradient = -\frac{1}{3}$.
The objective function would be $C = ac + 270m$, where a is the cost of each car. This equation can be transposed to $m = -\frac{a}{270} + \frac{C}{270}$.

$$-\frac{a}{270} > -\frac{1}{3}$$

$$\frac{a}{270} < \frac{1}{3}$$

$a < \frac{1}{3} \times 270 < 90$ Therefore the amount available for each car must be less than \$90.

© The Mathematical Association of Victoria, 2014

MODULE 4 : BUSINESS-RELATED MATHEMATICS

Question 1

(a) $\dfrac{2695}{11} = \$245$ A1

(b) $\$500 + 48 \times \$70 = \$3860$ A1

(c) $48 \times \$70 - (\$2695 - \$500) = \$1165$ A1

(d) Rate $= \dfrac{100 \times 1165}{2195 \times 4} = 13.268\ldots = 13.3\%$ A1

(e) 2013 price $= 1.035 \times 2695 = \$2789.33$
2014 price $= 1.025 \times 2789.33 = \$2859.06 = \$2859$ A1

Question 2

(a) Amount $= \dfrac{\$1150 \times 12}{0.0575} = \$240\ 000$ A1

(b) $240 000 (It is a perpetuity – NO change in principal over time) A1

Question 3

(a) Value $= 27\ 000 - 4 \times 0.14 \times 27\ 000 = 11\ 880$ A1

(b) Value $= 27000\left(1 - \dfrac{18}{100}\right)^4 = 12\ 207.29$ A1

(c) Using $V = P - nrP$, where $P = \$27\ 000$, $r = 14\%$, n = number of years
$\$5000 = \$27\ 000 - n \times 0.14 \times \$27\ 000$
Giving $n = \dfrac{\$27000 - \$5000}{0.14 \times \$27000} = 5.82\ldots$ Hence 6 years A1

Question 4

(a) (i) A1

No. of instalment periods/payments	Interest rate p. a.	Present value	Future Value	Payments per year	Compounds per year
180	5.95	540000	0	12	12

Accept – 540 000

(ii) \$4542.25*180 – \$540 000 = \$277 605 A1

(b)

No. of instalment periods/payments	Interest rate p. a.	Present value	Payment	Payments per year	Compounds per year
90	5.95	540000	– 4542.25	12	12

Solve for Future Value = 329 122 A1

(c)

Interest rate p. a.	Present value	Payment	Future Value	Payments per year	Compounds per year
5.95	540000	– 4542.25	0	12	12

Solve for N = 29.7 M1

To repay \$125 000 will take 30 months (2.5 years)
90 payments is 7.5 years
Total time is 7.5 + 2.5 = 10 years. A1

MODULE 5 : NETWORKS AND DECISION MATHEMATICS

Question 1

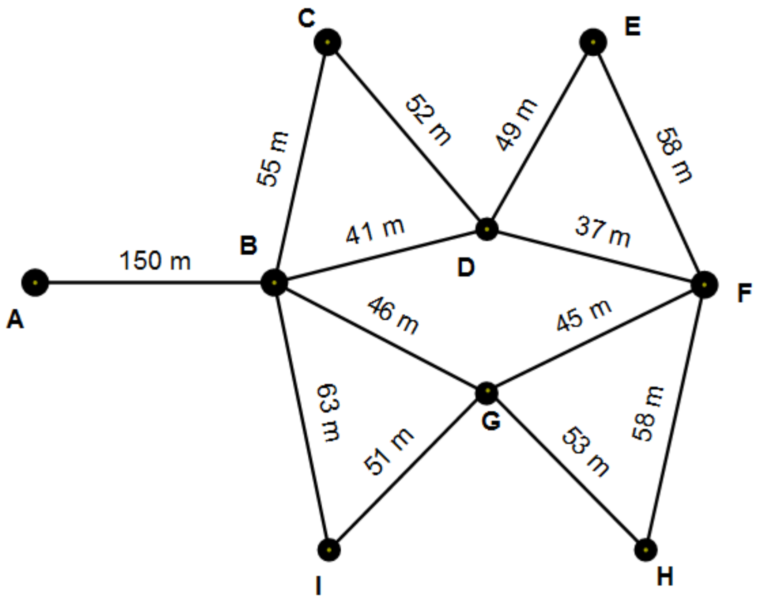
(a) 5 A1

(b) Euler Path A1

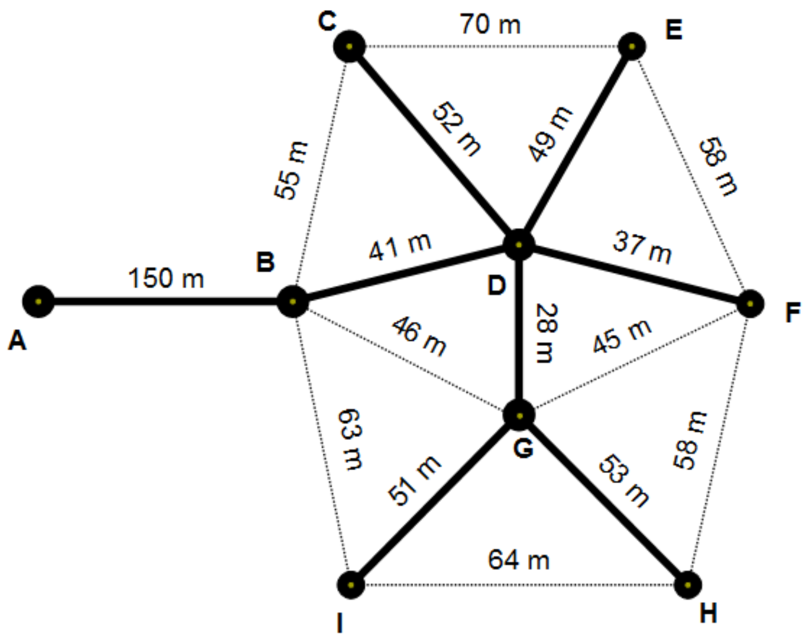
(c) An Euler Path requires that the degree of every vertex is even except 2. There are 8 odd degree vertices in this network. All vertices except F have an odd degree. A1

(d) 3 A1

Removal of DG, CE and IH would mean that the degree of all vertices are even except A (degree of one) and B (degree of 5), so now a path could start at A and finish at B.



(e) As shown in bold below. A spanning tree M1
Tree as shown (minimal spanning tree) A1



© The Mathematical Association of Victoria, 2014

Question 2

(a)

$$\begin{array}{c} \\ R \\ H \\ Y \\ M \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 4 & 0 & 5 \\ 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 2 \end{bmatrix} \end{array}$$

A1

The following matrix may be shown as a working step

$$\begin{array}{c} \\ R \\ H \\ Y \\ M \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 4 & 3 & 7 \\ 2 & 0 & 3 & 2 \\ 0 & 1 & 3 & 3 \\ 1 & 0 & 5 & 4 \end{bmatrix} \end{array}$$

(b)

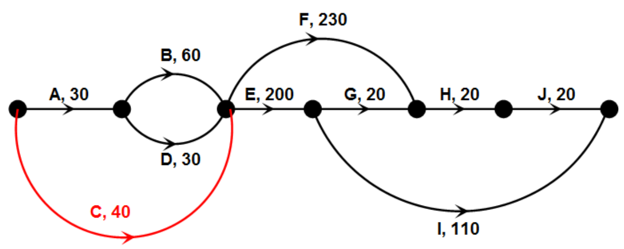
Event	Allocation 1 : Selected Player	Allocation 2 : Selected Player
Run (6 km) (R)	Charles	Andrew
Hurdles (1 km) (H)	David	David
Cycle (10 km) (Y)	Andrew	Charles
Medicine Ball (1 km) (M)	Bruce	Bruce

Either allocation A1
Second allocation A1

(c) There are 2 possible allocations because Charles is 3 minutes longer in either of the run or the cycling than Andrew, so it makes no difference which one he does. A1

Question 3

(a) Activity must connect start of A to end of B and D (start of E and F), must have an arrow as shown. A1



(b) 400 minutes A1

400 minutes along path ABEI (30+60+200+110 = 400)

(c) 340 minutes A1

340 minutes after the start (400 – 60 = 340)

(d) 320 minutes. A1
Only required reductions are B, H and I with a new critical path of ABFHJ

© The Mathematical Association of Victoria, 2014

(e) $140 A1

Activities B, H and I all need to be reduced. Activities C and D should not be paid for because reducing these activities has no effect on the length of time as neither of them are on the critical path before or after reduction.

MODULE 6 : Matrices

Question 1

(a) $F = \begin{bmatrix} 40 & 60 & 80 & 30 \end{bmatrix}$ A1

(b) The number of columns in F is 4, which is the same as the number of rows in N, so the product FN exists. $FN = (1\times4)\times(4\times2)$ results in a (1×2) matrix. A1

(c) $FN = M = \begin{bmatrix} 6550 & 5540 \end{bmatrix}$ A1

(d) The element m_{12} represents the total amount of money taken in annual fees from the female members of the club. A1

(e) $G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ A1

Question 2

(a)

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} C \\ P \\ H \end{bmatrix} = \begin{bmatrix} 226 \\ 206 \\ 139 \end{bmatrix}$$ A1

(b)

$$D = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 2 \\ 4 & -2 & -3 \end{bmatrix}$$ A1

(c) $-2\times226 + 1\times206 + 2\times139 = \$32$ A1

Question 3

(a) Each week players move to a different training component and none repeat the same activity in consecutive weeks. A1

(b) 40 players will do football skills A1

$$\begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.5 & 0 & 0.6 \\ 0.5 & 0.8 & 0 \end{bmatrix}^2 \times \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 30 \\ 30 \\ 40 \end{bmatrix}$$

(c) Must show 2 equal and subsequent state matrices rounding to $\begin{bmatrix} 23 \\ 36 \\ 41 \end{bmatrix}$ A1

$$\begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.5 & 0 & 0.6 \\ 0.5 & 0.8 & 0 \end{bmatrix}^{20} \times \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.5 & 0 & 0.6 \\ 0.5 & 0.8 & 0 \end{bmatrix}^{21} \times \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 23.42.. \\ 36.04.. \\ 40.54.. \end{bmatrix} \approx \begin{bmatrix} 23 \\ 36 \\ 41 \end{bmatrix}$$

Question 4

(a) 39 firsts, 56 seconds and 25 thirds A1

$$P_3 = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.4 \\ 0 & 0.2 & 0.6 \end{bmatrix} \times \begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix} + \begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix} = \begin{bmatrix} 39 \\ 56 \\ 25 \end{bmatrix}$$

(b) 40 players at each level. A1

$$\begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix} - \begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix} = \begin{bmatrix} 40 \\ 48 \\ 32 \end{bmatrix}$$ M1

$$\begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.4 \\ 0 & 0.2 & 0.6 \end{bmatrix}^{-1} \times \begin{bmatrix} 40 \\ 48 \\ 32 \end{bmatrix} = \begin{bmatrix} 40 \\ 40 \\ 40 \end{bmatrix}$$

(c) $x = -3,\ \ y = 2,\ \ z = 1$ A1

$$\begin{bmatrix} 39 \\ 56 \\ 25 \end{bmatrix} - \begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

Alternative solution:

$$\begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.4 \\ 0 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix} = \begin{bmatrix} 41 \\ 51 \\ 28 \end{bmatrix}$$

Hence

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix} - \begin{bmatrix} 41 \\ 51 \\ 28 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$