The Mathematical Association of Victoria
FURTHER MATHEMATICS
SOLUTIONS: Trial Exam 2014

Written Examination 1

SECTION A: Core--Data analysis

1.	A	**2.**	E	**3.**	C	**4.**	D	**5.**	E
6.	B	**7.**	E	**8.**	B	**9.**	D	**10.**	D
11.	E	**12.**	C	**13.**	B				

SECTION B: MODULES

Module 1: Number Patterns

1.	D	**2.**	D	**3.**	A	**4.**	D	**5.**	A
6.	B	**7.**	B	**8.**	C	**9.**	D		

Module 2: Geometry and trigonometry

1.	D	**2.**	C	**3.**	B	**4.**	A	**5.**	B
6.	A	**7.**	C	**8.**	D	**9.**	B		

Module 3: Graphs and relations

1.	A	**2.**	B	**3.**	B	**4.**	D	**5.**	D
6.	A	**7.**	B	**8.**	D	**9.**	C		

Module 4: Business-related mathematics

1.	A	**2.**	D	**3.**	D	**4.**	D	**5.**	C
6.	B	**7.**	C	**8.**	D	**9.**	D		

Module 5: Networks and decision mathematics

1.	B	**2.**	A	**3.**	C	**4.**	D	**5.**	D
6.	E	**7.**	E	**8.**	B	**9.**	C		

Module 6: Matrices

1.	D	**2.**	B	**3.**	E	**4.**	B	**5.**	B
6.	A	**7.**	D	**8.**	E	**9.**	D		

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Worked solutions--Core: Data analysis

Question 1

2, 2, m, 6, 9, 11, 11, 14, n, 21

10 ordered data values are listed.
5 values in lower half, so 3rd value is $Q_1 = m$ and 5 values in upper half, so 8th value is $Q_3 = 14$

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 14 - m \end{aligned}$$

Answer A

Question 2

$$\bar{x} = \frac{\Sigma x}{n}$$

$$10 = \frac{2+2+m+6+9+11+11+14+n+21}{10}$$

$$100 = 76 + m + n$$

$$24 = m + n$$

m can only have values from 2 to 6

If n = 14 then m =10 (not possible)
If n = 15 then m = 9 (not possible)
If n = 16 then m = 8 (not possible)
If n = 17 then m = 7 (not possible)

If n = 18 then m =6. This option satisfies all the constraints.

Answer E

Question 3

When the mean value is below the median then the data is negatively skewed

Answer C

Question 4

Note: Histograms display univariate data
Time series plots and scatterplots display two numerical variables,
Back to back stem-plots display one numerical variable and one categorical variable with two levels only.

Bivariate data is given with one numerical variable (time in hours) and one categorical variable (exam performance with 3 levels). This can be displayed with parallel boxplots.

Answer D

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Question 5

$$z = \frac{x - \bar{x}}{s} = \frac{82.5 - 75}{3} = 2.5$$

Answer E

Question 6

$$m = \frac{rS_y}{S_x} = \frac{0.8 \times 6}{3} = 1.6$$

$$\begin{aligned} c &= \bar{y} - m\bar{x} \\ &= 70 - 1.6 \times 75 \\ &= -50 \end{aligned}$$

$$D = 1.6s - 50$$

Answer B

Question 7

Consider each statement

A. The point (90, 31.4) is situated below the regression line so the residual is negative – **True**

B. The slope of -0.66 means that there is a decrease of 0.66 kg/cm^2 of pressure for every increase in 1 cm^3 volume- **True**

C. The regression line sits above the data point (70, 40.5). This means that the line over-predicts the pressure with a volume of 70cm^3. – **True**

D. The volume of 80 cm^3 is within the data set so this is interpolation **– True**

E. $(-0.9745)^2 = 89.8\%$ of the variation in pressure is ***explained*** by the variation in volume **– False**

Note: A change in one variable ***does not cause*** the other variable to change.

Answer E

Question 8

The shape of the scatterplot suggests that the data can be linearised by compressing the x-scale or compressing the y-scale. This is achieved by applying logarithmic or reciprocal transformations to either axis.

Answer B

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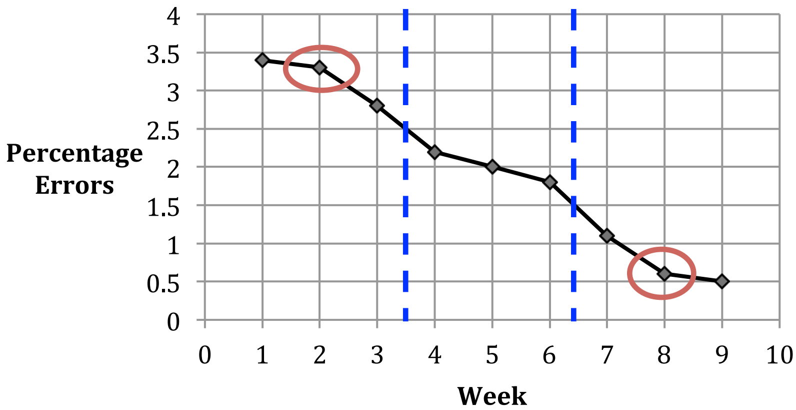
Question 9



Divide the 9 points in three groups.

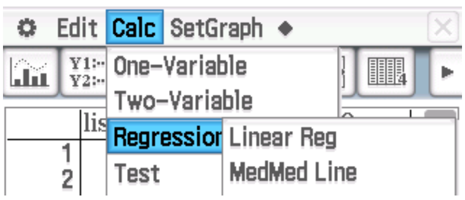
The median point in the left group $(x_L, y_L) = (2, 3.3)$

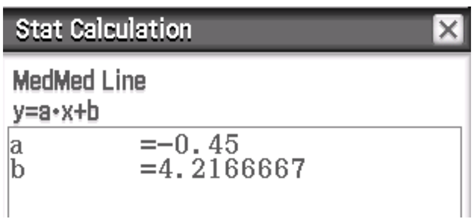
The median point in the right group $(x_R, y_R) = (8, 0.6)$

$$m = \frac{y_R - y_L}{x_R - x_L} = \frac{3.3 - 0.6}{2 - 8} = -0.45$$

Althernatively

Enter data in the calculator and find the equation of the three median line.





Answer D

Question 10

Seasonal variation with an increasing trend

Answer D

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Question 11

Quarter	1	2	3	4	5	6	7	8	9	10	11	12
Number of life saving incidents	10	7	12	15	12	9	15	20	16	14	21	24

$$\frac{15+12+9+15}{4} = 12.75 \qquad \frac{12+9+15+20}{4} = 14$$

Find the average of the values 12.75 and 14 to achieve the centre $\frac{12.75+14}{2}$

Answer E

Question 12

Summer	Autumn	Winter	Spring
	1	1	1.20

Seasonal indices sum is 4. This means that for 2013 summer S.I. = 0.8

For seasonal indices to be the same they must all equal 1
This means summer must increase by 0.2

$$\%\,\text{increase} = \frac{\text{increased value}}{\text{original value}} \times 100\%$$
$$= \frac{0.2}{0.8} \times 100\%$$
$$= 25\%$$

Answer C

Question 13

$$\textit{Deseasonalised sales} = 4.24 \times \textit{quarter number} + 1601.29$$
$$= 4.24 \times 27 + 1601.29$$
$$= 1715.77$$

$$\textit{Seasonal value} = 1715.77 \times 0.72 = 1235.3544$$

Answer B

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Module 1: Number patterns

Question 1

$t_4 = 20$ and $t_5 = 10$

$r = \frac{t_5}{t_4} = \frac{10}{20} = \frac{1}{2}$

$t_4 = 20$

$ar^3 = 20$

$a \times \left(\frac{1}{2}\right)^3 = 20$

$\frac{a}{8} = 20$

$a = 160$

Alternatively

$\times\frac{1}{2}$

160 , **80** , **40** , 20 , 10

$\times 2$ $\times 2$ $\times 2$ $\times 2$

Answer D

Question 2

Evaluating 11 + 13 + 15 + …+ 33

Method 1: Using algebra

$a = 11$, $d = 2$, $t_n = 33$

sub in $t_n = a + (n-1)d$

$33 = 11 + (n-1)2$

solving gives $n = 12$

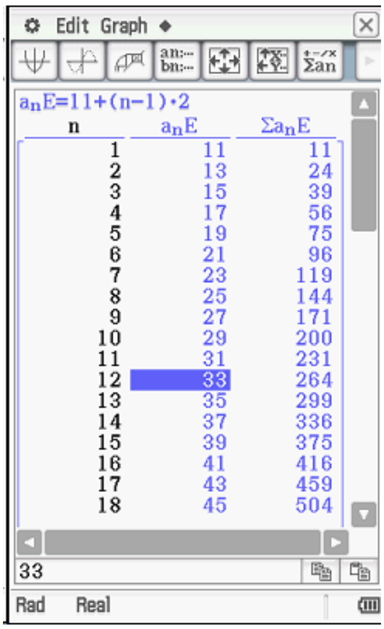
sub in $S_n = \frac{n}{2}(a + l)$

$S_{12} = \frac{12}{2}(11 + 33) = 264$

Method 2: Using Calculator

$t_n = a + (n-1)d$ where $a = 11$, $d = 2$

Enter the general expression $t_n = 11 + (n-1)2$ in the calculator scroll down to where $t_n = 33$ and read the sum



Answer D

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Question 3

For the sequence -3, 4, 11, 18, . . .

$a = -3$ and $\mathrm{d} = t_3 - t_2 = t_2 - t_1 = 11 - 4 = 4 - -3 = 7$

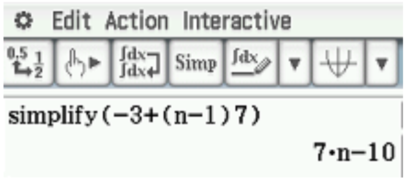
$t_n = a + (n-1)d$

$t_n = -3 + (n-1)7$

expanding gives

$t_n = -3 + 7n - 7$

$= 7n - 10$



Answer A

Question 4

$t_{n+1} = t_n + 2$, $t_1 = 2$ generates the sequence 2, 4, 6, 8, 10…. (not A)

$t_{n+1} = 3t_n - 2$, $t_1 = 2$ generates the sequence 2, 4, 10, 28, … (not B)

$t_{n+1} = 2t_n - 4$, $t_1 = 2$ generates the sequence 2, 0, -4, … (not C)

$t_{n+2} = t_{n+1} + t_n$, $t_1 = 2, t_2 = 4$ **generates the sequence 2, 4, 6, 10, 16, ….**

$t_{n+2} = t_{n+1} + t_n + 2$, $t_1 = 2, t_2 = 4$ generates the sequence 2, 4, 6, 12, 20, …(not E)

Answer D

Question 5

$S_1 = t_1 = a = 1$

$S_2 = t_1 + t_2$

$= a + ar$

$3 = 1 + r$

so $r = 2$

$t_4 = ar^3$

$= 1 \times 2^3$

$= 8$

$S_3 = t_1 + t_2 + t_3$

$= a + ar + ar^2$

$= 1 + 2 + 4$

$= 7$

Answer A

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Question 6

$t_n = 2t_{n-1} - t_{n-2}$

If $t_4 = 7$ and $t_3 = 6$ then

$$t_4 = 2t_3 - t_2$$
$$7 = 2\times 6 - t_2$$
$$7 = 12 - t_2$$
$$t_2 = 5$$

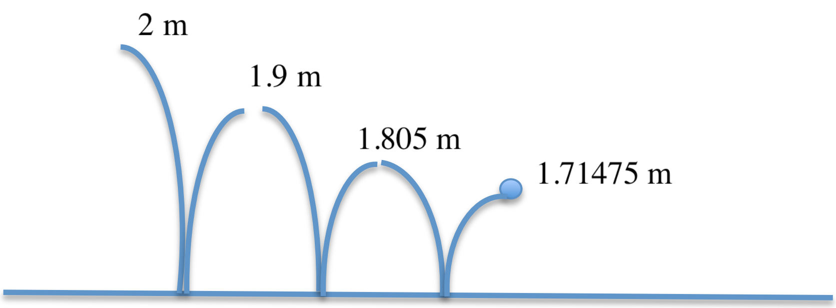
Answer B

Question 7

First rebound height is $0.95\times 2 = 1.9$

Second rebound height is $0.95\times 1.9 = 1.805\,\text{m} = 181\,\text{cm}$

Third rebound height is $0.95\times 1.805 = 1.71475\,\text{m} = 171\,\text{cm}$



Answer B

Question 8

Height **after** the nth bounce means that $H_0 = 2$ and $H_1 = 1.9$

Possible equations are $H_n = 0.95H_{n-1},\ H_0 = 2$ or $H_n = 0.95H_{n-1},\ H_1 = 1.9$

Answer C

Question 9

Initial distance plus distances travelled up plus distance travelled down

$a = 1.9$ and $r = 0.95$

$$\text{Total distance travelled} = 2 + 2\times S_\infty$$
$$= 2 + 2\times\frac{a}{1-r}$$
$$= 2 + 2\times\frac{1.9}{1-0.95}$$
$$= 78$$

Answer D

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Module 2: Geometry and trigonometry

Question 1

x is the vertically opposite co-interior angle of 65^o

$x = 180^\circ - 65^\circ = 115^\circ$

Answer D

Question 2

let $a = 5, b = 8$ *and* $c = 9$

$$\begin{aligned} s &= \frac{a+b+c}{2} = \frac{5+8+9}{2} = 11 \\ Area &= \sqrt{11(11-5)(11-8)(11-9)} \\ &= \sqrt{11 \times 6 \times 3 \times 2} \\ &= \sqrt{396} \end{aligned}$$

Answer C

Question 3

Volume of cube = $9 \times 9 \times 9 = 729\,\text{cm}^3$

Height of Pyramid = $19 - 9 = 10\,\text{cm}$

Volume of Pyramid = $\frac{1}{3} \times 9 \times 9 \times 10 = 270\,\text{cm}^3$

Total Volume = $729 + 270 = 999\,\text{cm}^3$

Answer B

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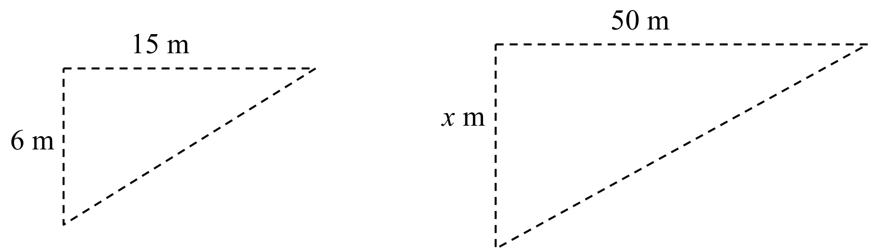
Question 4

Using similar triangles



$$\frac{x}{6} = \frac{50}{15}$$
$$\therefore x = \frac{50 \times 6}{15}$$

Answer A

Question 5

Total Surface area = Hemisphere + Circular Base

Note : diameter = 4 cm , therefore the radius = 2 cm

$$\begin{aligned} TSA &= 2\pi r^2 + \pi r^2 \\ &= 3\pi r^2 \\ &= 3\pi(2)^2 \\ &= 3\pi \times 4 \\ &= 12\pi \end{aligned}$$

Answer B

Question 6

Area ratio is $a^2 : b^2 = 1:4$
Length ratio is $a : b = 1:2$

The diameter of the original paperweight is 4 cm

original : enlarged
$1:2$
$4:x$

so the diameter is twice as large 8 cm

Answer A

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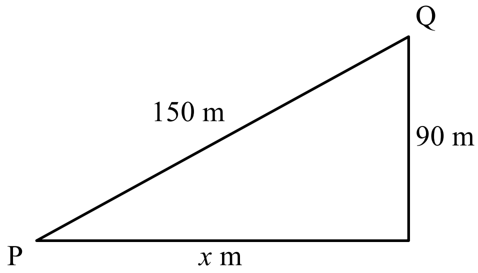
Question 7

The profile of the hill from P to Q is shown where PQ = 150 m



From the contours, the vertical distance = 120 – 30 = 90 m

To determine slope

Find horizontal distance, x, using Pythagoras first.

$$\begin{aligned} x &= \sqrt{150^2 - 90^2} \\ &= 120 \end{aligned}$$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{90}{120} = \frac{3}{4}$$

Answer C

Question 8

1 cm on the map is equivalent to 1000 cm in real life

ie. 1 cm = 1000cm = 10 metres

To represent the horizontal distance of 120 metres (found in Question 7)

$$\begin{aligned} &\text{map(cm)} : \text{real(m)} \\ &\quad 1 : 10 \\ &\quad x : 120 \end{aligned}$$

x = 12 cm on the map

Answer D

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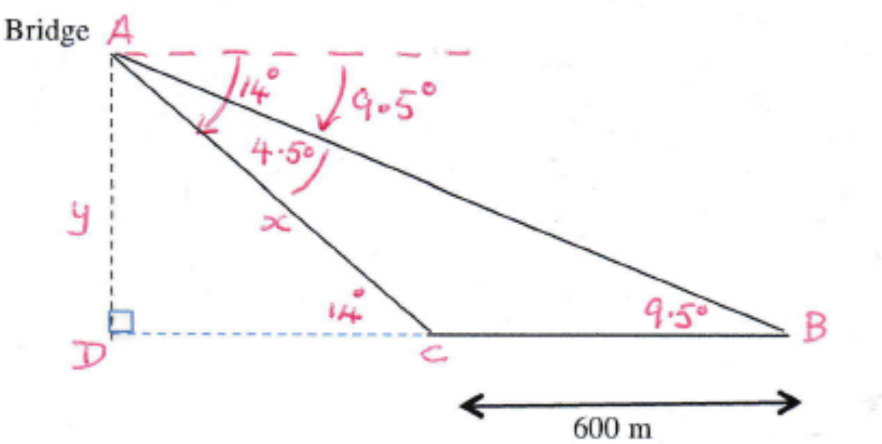
Question 9



First find x using the sine rule and the triangle labelled ABC

$$\frac{600}{\sin(4.5^\circ)} = \frac{x}{\sin(9.5^\circ)}$$

$$x = \frac{600 \times \sin(9.5^\circ)}{\sin(4.5^\circ)}$$

To find the height of the bridge, labelled y, use the right angle triangle ACD

And the trigonometric ratios SOH, CAH, TOA

$$\sin(14^\circ) = \frac{y}{x}$$

$$y = x \times \sin(14^\circ)$$

$$y = \frac{600 \times \sin(9.5^\circ) \times \sin(14^\circ)}{\sin(4.5^\circ)}$$

Answer B

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Module 3: Graphs and relations

Question 1

Using the points (0, 3) and (6, 0)

$$m = \frac{0-3}{6-0} = \frac{-3}{6} = -\frac{1}{2}$$

The y intercept is at 3 so the equation is

$$y = -\frac{1}{2}x + 3$$

Doubling the equation gives $2y = -x + 6$ and rearranging gives $2y + x = 6$

Answer A

Question 2

Let x = cost of one movie and y = cost of one CD

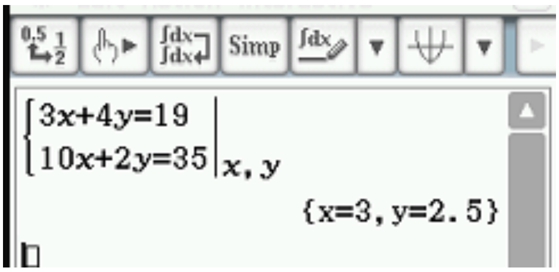
Equations are

$$3x + 4y = 19$$
$$10x + 2y = 35$$

Solving finds that

one movie = \$3

one CD = \$2.50



So Marks spends $2 \times 3 + 2 \times 2.5 = \$11$

Answer B

Question 3

Water added = 60-20=40 litres

Adds 4 litres per minute

Time = $\frac{40}{4} = 10\,\text{minutes}$

Answer B

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Question 4

Two points are (30, 60) and (45, 0)

$$m = \frac{0-60}{45-30} = -4$$

To find y-intercept sub m = -4 and point (30, 60) in

$$y = mx + c$$
$$60 = -4 \times 30 + c$$
$$c = 180$$

Equation is $V = 180 - 4t$

Answer D

Question 5

A. The highest petrol price is $1.55 per litre so A is false

B. Petrol price decreased in quarters 1, 2, 4, 7, 8, and 12. A total of six quarters, so B is false

C. The petrol prices changed as according to the following

Quarter	Change in cents per litre
1	-10
2	-5
3	+15
4	-10
5	+30
6	+15
7	-35
8	-15
9	+15
10	+5
11	+10
12	-20

The highest change occurred in the seventh quarter, so C is false

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D. Intial point (0, 120) and at the fifth month (5, 140)

The average change = $\frac{140-120}{5-0} = 4$ cents per litre , so D is correct

E. The lowest price was \$1.00 and the highest price is \$1.55. The difference is 55 cents, so E is incorrect.

Answer D

Question 6

$y = k \times \frac{1}{x^2}$ where k = gradient of the line

$k = \frac{9}{0.5} = 18$ so Equation is $y = \frac{18}{x^2}$ sub x = 3

$y = \frac{18}{3^2} = \frac{18}{9} = 2$

Answer A

Question 7

At least twice as many apples as bananas

Set up a table of values to help you

Apples (x)	Bananas (y)
2 or more	1
4 or more	2
6 or more	3
8 or more	4

This means that $x \geq 2y$

Answer B

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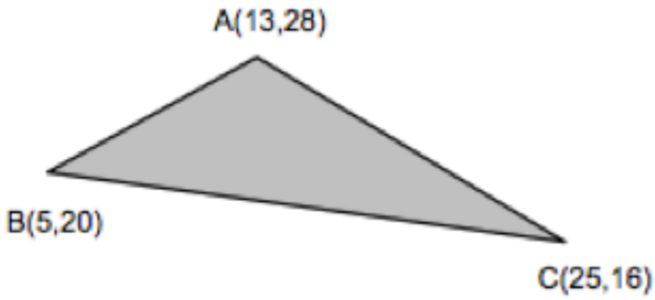
Question 8



Method 1: Using boundary points

The boundary line AB with gradient m = 1 contains the integer points
(5, 20), (6, 21), (7, 22), (8, 23), (9, 24), (10, 25), (11, 26), **(12, 27)** and (13, 28)

The boundary line BC with gradient m = -1/5 contains the integer points
(5, 20), **(10, 19),** (**15**, 18), (**20**, 17) and (25, 16)

The boundary line AC with gradient m = -1 contains the integer points
(13, 28) , (14, 27) , (**15**, 26), (16, 25), (17, 24), (18, 23), (19, 22), (**20**, 21) ,(21, 20), (22, 19), (23, 18) (24, 17) and (26, 16)

The points (10, 19) and (12, 27) are on boundary lines BC and AB respectively so they are in the feasible region.

The point (15, 21) lies between the two boundary points (15, 18) and (15, 26) so this is within the feasible region.

The point (16, 26) lies above the boundary point of (16, 25) at AC so this point is not in the feasible region

The point (20, 20) lies between the two boundary points (20, 17) and (20, 21) so this is also within the feasible region.

Method 2 : Using Inequalities

The feasible region is below the lines AC and AB and above the line BC

Inequality 1 : $x + y \le 41$

Inequality 2 : $x - y \ge -15$ $\qquad$ $16 + 26 \le 41$ ie. $42 \le 41$ is not true

Inequality 3 : $0.2x + y \ge 21$

The point (16, 26) does not satisfy inequality 1. All other points satisfy the three inequalities.

Answer D

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Question 9

The extreme points in the feasible region are at A (13, 28), B (5, 20) and C (25, 16)

Method 1: By substitution

Substitute each of the listed objective functions to determine where A is a maximum

Objective Function	A (13, 28)	B (5, 20)	C (25, 16)
A. $M = 10x + 5y$	130 + 140 = 270	50 + 100 = 150	250 + 80 = **330**
B. $M = 10x - 5y$	130 – 140 = -10	50 - 100 = - 50	250 – 80 = **170**
C. $M = 5x + 10y$	65 + 280= **345**	25 + 280 = 305	125 +160 = 285
D. $M = 5x + 5y$	65 +140 = **205**	25 + 100 = 125	125 + 80 = **205**
E. $M = 5y - 10x$	140 – 130 = 10	100 - 50 = **50**	80 – 250 = - 170

Method 2: Sliding line technique

The gradient of the objective function must be between gradient of line AB and the gradient of the line *AC ie.* $-1 < m < 1$

A. $M = 10x + 5y$ $\quad y = -\frac{10}{5} + \frac{M}{5} \quad \therefore m = -2$ **gradient is not in range**

B. $M = 10x - 5y$ $\quad y = \frac{10}{5} - \frac{M}{5} \quad \therefore m = 2$ **gradient is not in range**

C. $M = 5x + 10y$ $\quad y = -\frac{5}{10} + \frac{M}{10} \quad \therefore m = -\frac{1}{2}$

D. $M = 5x + 5y$ $\quad y = -\frac{5x}{5} + \frac{M}{5} \quad \therefore m = -1$ **gradient is not in range (on the line)**

E. $M = 5y - 10x$ $\quad y = \frac{10}{5} + \frac{M}{5} \quad \therefore m = 2$ **gradient is not in range**

Answer C

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Module 4: Business-related mathematics

Question 1

10% GST is added to the original price therefore it is multiplied by 1.1

$1.1 \times \text{price} = 4500$

$\Rightarrow \text{ price } = \dfrac{4500}{1.1}$

Answer A

Question 2

Finance Solver

$N = ?$

$I(\%) = 6.5$

$PV = 60000$

$Pmt = -700$

$FV = 0$

$PpY = 12$

$CpY = 12$

Solve for N

Number of monthly payments =115.54

$\dfrac{115.54}{12} = 9.63$ years therefore closest to 10 years.

Answer D

Question 3

Effective rate of interest $= \dfrac{2n}{n+1} \times$ flat rate of interest where n is the total number of payments.

$= \dfrac{2 \times 24}{24+1} \times 10$

$= \dfrac{480}{25}$

$= 19.2\%$

Answer D

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Question 4

$$I = \frac{PRT}{100}$$

$$3.37 = \frac{1685 \times R \times \frac{1}{12}}{100}$$

solving gives $R = 2.4\%$

Answer D

Question 5

$A = PR^n$

$R = 1 + \frac{r}{100}$ where r is the interest rate per quarter $= \frac{6.4}{4} = 1.6$

$R = 1 + \frac{1.6}{100} = 1.016$

3 years is equal to 12 quarters and 2 years is equal to 8 quarters

Interest in third year $= 5000 \times 1.016^{12} - 5000 \times 1.016^{8}$

Answer C

Question 6

With reducing balance loans, the amount of interest each repayment decreases therefore the amount paid off the principal increases with each payment.

Answer B

Question 7

Depreciation $= 32000 - 18500 = \$13500$

Rate of depreciation $= \frac{13500}{100000} = \$0.135 = 13.5$ cents per kilometre

Answer C

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Question 8

Method 1: Use Finance Solver

$\mathrm{N} = ?$

$\mathrm{I}(\%) = 6$

$\mathrm{PV} = -50000$

$\mathrm{Pmt} = 0$

$\mathrm{FV} = 100000$

$\mathrm{PpY} = 1$

$\mathrm{CpY} = 1$

Solve for N

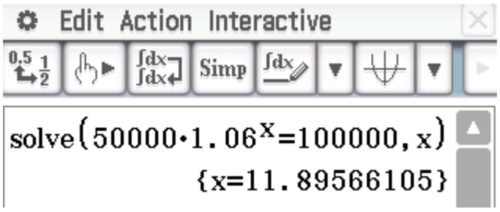
Number of compounding periods =11.89 (years)

Therefore closest to 12 years.

Method 2: Use Algebra

Solve

$$50\,000 \times 1.06^{x} = 100\,000$$



Answer D

Question 9

Finance Solver

$\mathrm{N} = ?$

$\mathrm{I}(\%) = 12$

$\mathrm{PV} = 5000$

$\mathrm{Pmt} = -250$

$\mathrm{FV} = 0$

$\mathrm{PpY} = 12$

$\mathrm{CpY} = 12$

Solve for N
N= 22.4257 months

Consider each statement

A. It takes 22.4257 months (less than two years) to repay the credit company – True

B. Amount owing plus 10% interest to Oscar's brother for two years is \$6 000
6 000 ÷ 250 = 24 months to repay Oscar's brother which is exactly two years - True

C. 22.4257 payments of \$250 therefore pays \$606.43 interest which is less than \$1000 - True

D. Oscar's brother has a total interest of \$1000 exactly therefore the statement is **incorrect**

E. Total payment to Credit company is \$5606.43 whereas total payment to Oscar is \$6 000 - True

Answer D

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Module 5: Networks and decision mathematics

Question 1

Maximum flow $=$ minimum cut

$= 3 + 4 = 7$

Answer B

Question 2

Using Euler's rule for planar graphs

$v - e + f = 2$

$4 - e + 3 = 2$

$e = 5$

Answer A

Question 3

The given graph with six vertices currently has 6 edges.

A complete graph with n vertices has $\frac{n(n-1)}{2}$ edges therefore $\frac{6 \times 5}{2} = 15$

9 additional edges must be added to the existing 6 to give 15.

Answer C

Question 4

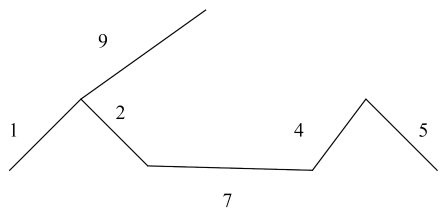

Diagram indicates the edges used for the minimal spanning tree

The length of the tree is $1 + 2 + 4 + 5 + 7 + 9 = 28$

Answer D

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Question 5

Melanie is unable to speak about sport.

Answer D

Question 6

The earliest start time for activity K is represented by the longest path from start to K

Path is $A \to C \to F \to I \to K$ which takes 16 hours.

Answer E

Question 7

The minimum time for completion of the project is the time of the critical path.

Critical path is $A \to C \to F \to H \to J \to L$ which takes 22 hours

Answer E

Question 8

The float time for non-critical activities is latest start time $-$ earliest start time

For the activities listed, float times are

B: $9-2=7$

E: $13-2=11$

G: $9-7=2$

I: $14-13=1$

K: $17-16=1$

Activity E has the greatest float time.

Answer B

Question 9

Since Bob does task B, cross out the values in the row for Bob and the column for task B.

Cedric could do task A or D (9 mins) but other times for task D are less than remaining times for task A therefore Cedric should do task A.

This leaves Andi and Dimitar for tasks C and D. Both could do task D (12 mins) however Andi takes less time on C. So Andi should do task C and Dimitar D.

$9+$ Bob's time $+20+12=51$ therefore Bob takes 10 minutes.

Answer C

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Module 6: Matrices

Question 1

$AB = \begin{bmatrix} a+d+g & b+e+h & c+f+i \end{bmatrix}$

therefore adds the columns of matrix B

Answer D

Question 2

By multiplication

$$\begin{bmatrix} x & 2 \\ 3 & y \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} x+4 & 2x+2 \\ 3+2y & 6+y=8 \end{bmatrix} = \begin{bmatrix} 8 & 10 \\ 7 & 8 \end{bmatrix}$$

$x+4=8 \Rightarrow x=4$

$6+y=8 \Rightarrow y=2$

Therefore $x=2y$

Answer B

Question 3

Since E can be added to C, E must be a 3×2 matrix.

Option A : $A\times D\times B$ dimensions $(3\times 4)\times(4\times 3)\times(2\times 3)$ is undefined

Option B: $C\times B\times A$ dimensions $(3\times 2)\times(2\times 3)\times(3\times 4)$ produces a 3×4 matrix

Option C: $B\times A\times C$ dimensions $(2\times 3)\times(3\times 4)\times(3\times 2)$ is undefined

Option D: $B\times A\times D$ dimensions $(2\times 3)\times(3\times 4)\times(4\times 3)$ produces a 2×3 matrix

Option E: $A\times D\times C$ dimensions $(3\times 4)\times(4\times 3)\times(3\times 2)$ produces a 3×2 matrix

Answer E

Question 4

Since $A\times\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ which is the identity matrix, $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

Answer B

Question 5

A is of order 2×2, B is of order 3×1, C is of order 2×3, D is of order 1×2

Product matrices that are defined (with order in brackets) are

$AC(2\times 3)$, $BD(3\times 2)$, $CB(2\times 1)$, $DA(1\times 2)$, $DC(1\times 3)$

Answer B

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Question 6

Let $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$AB = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}B^{-1}$

$A = 5B^{-1}$

$B^{-1} = \frac{A}{5} = \frac{1}{5}A$

Answer A

Question 7

The transition matrix $T = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}$

For a steady state raise T to a high power e.g. $T^{50} = \begin{bmatrix} 0.25 & 0.25 \\ 0.75 & 0.75 \end{bmatrix}$

Therefore a 75% chance of travelling by bus

Answer D

Question 8

A, B and C can be seen to be correct from inspection of the transition matrix.

Test D $T^4\begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 138 \\ 63 \\ 99 \end{bmatrix}$ rounded off so D is correct

Test E $T\begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 111 \\ 87 \\ 102 \end{bmatrix}$ therefore 111 go by bus not 90

Answer E

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Question 9

$$R_2 = \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} \times \begin{bmatrix} 65 \\ 10 \end{bmatrix} - \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 52 \\ 11 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} \times \begin{bmatrix} 52 \\ 11 \end{bmatrix} - \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$= \begin{bmatrix} 42 \\ 9 \end{bmatrix}$ so 42 attend extra dance rehearsals in week 3

Answer D

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