The Mathematical Association of Victoria

Trial Exam 2013

FURTHER MATHEMATICS

Written Examination 2

STUDENT NAME: ________________________________

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of Book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 26 pages, with a detachable sheet of miscellaneous formulas at the back.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the back of this book during reading time.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

This examination consists of core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer all questions with the modules selected.
You need not give numerical answer as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.
Diagrams are not to scale unless specified otherwise.

Core

Question 1

Hunter owns a bike shop and has noted that there is seasonal variation apparent when considering the number of bikes sold each season. Over a period of three years from 2010 to 2012 inclusive the seasonal indices for bike sales are calculated and given in the following (incomplete) table.

Season	**Summer**	**Autumn**	**Winter**	**Spring**
Seasonal index	1.31		0.72	1.09

a. Complete the table with the seasonal index for Autumn.

1 mark

b. Interpret the seasonal index for Winter in the context of the problem

1 mark

c. During Spring 2012, Hunter sold 654 bikes. Determine the deseasonalised sales figure for this particular season.

1 mark

Using Summer 2010 as time period 1, Autumn 2010 as time period 2 etc, the following least squares regression equation was calculated

$$\text{Deseasonalised sales } = 540.1 + 4.25 \times \text{ time period}$$

d. Calculate the residual value of deseasonalised sales for Spring 2012.

2 marks

e. Use the regression equation and other available information to forecast the actual number of bikes Hunter will sell during Winter 2014. Give your answer to the nearest whole number.

2 marks

Core – continued
TURN OVER

Question 2

Hunter sells two popular models of bike, the Mountain Climber and the Easy Rider.
Over a period of a week Hunter recorded the ages of the customers who purchased each model of bike.
This data is to be displayed using parallel boxplots.

a. Complete the following sentence

Parallel boxplots are appropriate to use here as we are investigating the relationship

between a ______ variable

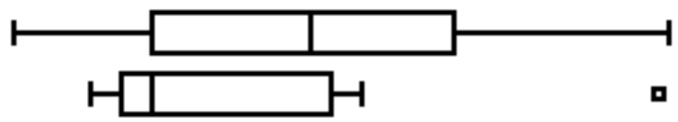
and a two-level ______ variable.

1 mark

b. The two boxplots are shown in the following diagram.

Mountain Climber

Easy Rider



Compare the two boxplots in terms of shape, centre, spread and outliers.

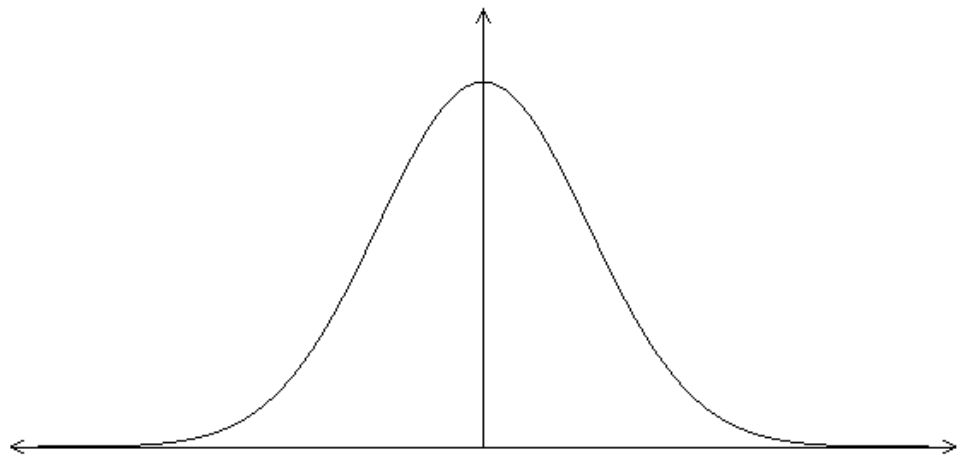
2 marks

Core – continued

Question 3

Hunter competes in the tour de Oz and for stage one it was found that the times of the competitors to complete the course formed a normal (bell-shaped) distribution with mean of 150 minutes and standard deviation of 3 minutes.

a. On the diagram below label the values that represent $\bar{x}$, $\bar{x} \pm s$, $\bar{x} \pm 2s$ and $\bar{x} \pm 3s$



1 mark

b. Hunter finished the stage in a time of 153 minutes. What percentage of riders finished ahead of Hunter?

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1 mark

c. Another competitor Nathan had a standardised score of $z = -1.2$. What time did Nathan record for this stage? Give your answer in minutes correct to one decimal place.

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1 mark

When Hunter competed in stage two of the tour it was found that the times also formed a normal distribution with mean of 120 minutes.

d. Hunter's time for this stage was 116.4 minutes. Only 2.5% of competitors finished before him. Determine the standard deviation for this distribution.

---

2 marks

Total 15 marks

END OF CORE
TURN OVER

Module 1: Number Patterns

Question 1

Tickets to the U2 concert went on sale at 9am in the morning. During the first minute 3000 tickets were sold, during the second minute 2700 tickets were sold and during the third minute 2430 tickets were sold. Suppose that this decreasing pattern continues.

a. Show that the sequence is geometric.

1 mark

b. Find the number of tickets sold in the fourth minute.

1 mark

c How many tickets were sold during the first 10 minutes of ticket sales?

1 mark

d. The venue can only hold a capacity crowd of 28 000. During which minute would the last ticket be sold?

1 mark

Question 2

Before the concert began a select group of 200 people were initially let in to the stadium. Several doors were then opened to the general public so that 400 people entered the stadium per minute.

a. Calculate the number of people in the stadium 1 minute after the doors were opened.

1 mark

b. Find the number of people in the stadium 10 minutes after the doors were opened.

1 mark

c. The number of people in the stadium *n* minutes after the doors were opened is given by the general expression $P_n = bn + c$.

Find the values of *b* and *c*.

2 marks

d. Determine how long it will take for the stadium to reach a capacity crowd of 28 000 people. Give your answer in minutes correct to one decimal place.

1 mark

Question 3

A large car park with several thousand parking spaces is situated near the stadium. The number of parking spaces available at 5 pm was 750. Every minute after 5 pm the number of car spaces available increases by 2% as cars leave while 28 other cars arrive to fill the empty parking spaces.

a. Find the number of car spaces available 1 minute after 5 pm.

I mark

b. P_n represents the number of parking spaces available n minutes after 5 pm. The value of P can be specified by the difference equation $P_{n+1} = bP_n + c, \quad P_0 = a$.

State the values of a, b and c

3 marks

c. At what time to the nearest minute will the car park be full?

1 mark

Once the car park is full, the number of cars, C_n, in the car park follows a sequence that is specified by the difference equation $C_{n+1} = 0.97C_n + 18$, $C_0 = 2800$ where n is the number of minutes after 6 pm.

d. Find the number of cars in the car park at 6.30 pm.

1 mark

e. Explain, with a calculation, why the number of cars parked will never fall below 600.

1 mark

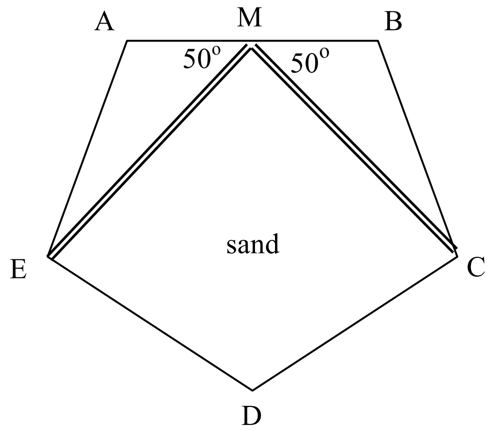
Total 15 marks

END OF MODULE 1

Module 2: Geometry and Trigonometry

Question 1

The local childcare centre has a sandpit in the shape of a regular pentagon ABCDE with equal side lengths of 2 metres shown in the diagram below.



The sandpit is symmetrical about MD where M is the midpoint of AB. The angles AME and BMC are both 50°. The triangular sections AME and BMC are made of timber that lift up to reveal storage areas for sandpit toys. The remaining section MCDE contains the sand.

a. Find the interior angle of a pentagon and hence show that the angle BCM is 22°

1 mark

b. Determine the length of MC in metres correct to 1 decimal place.

1 mark

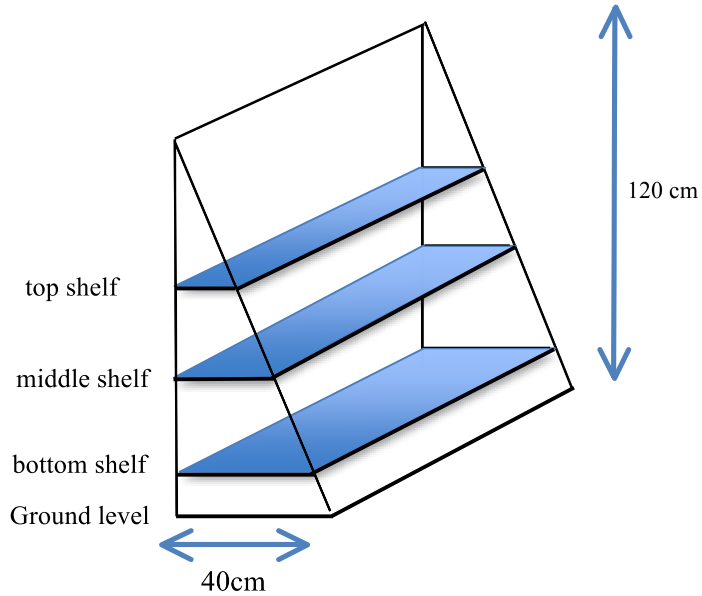
c. i. Find the area of the quadrilateral MCDE to the nearest square metre.

ii. Hence find the volume of sand required if the sandpit is 45 cm deep. Give your answer in cubic metres correct to 2 decimal places.

2+1=3 marks

Question 2

During rest time the children can select a picture book from a book-stand displaying three shelves of books parallel to each other. The height of the book-stand is 120 cm and the depth of the book-stand at ground level is 40 cm as shown on the diagram below.



The top shelf is 96 cm above the ground.

a. Determine the depth of the top shelf.

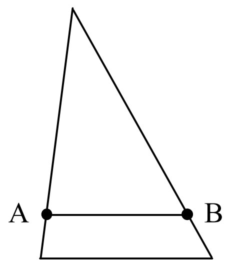
1 mark

b. How high above the ground is the middle shelf if it has a depth of 18 cm?

1 mark

c. The book-stand contains two triangular sides to support the shelf ends. The bottom shelf, AB, is placed at a height so that the ratio of the area underneath it to the area above it is 23 : 121.

Determine the height of the bottom shelf.



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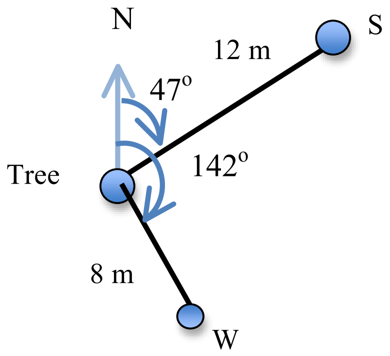
2 marks

Question 3

The sandpit and the swings are positioned near a tree that casts the maximum area of shade during the afternoon.

The sandpit, S, is situated 12m from the tree on a bearing of 047^{o} and the swings, W, are 8 metres from the tree on a bearing of 142^{o}.



a. Show that the direct distance, to the nearest metre, between the sandpit and the swings is 15 metres.

2 marks

b. Find the bearing of the sandpit from the swings to the nearest degree.

2 marks

The childcare centre decided to maximise the use of the shade by building a long bench seat directly from the sandpit to the swings. The tree is 6 metres high and there is concern that it may hit the bench seat if it falls.

c. Determine whether the tree will hit the bench seat if it does fall. Justify your answer.

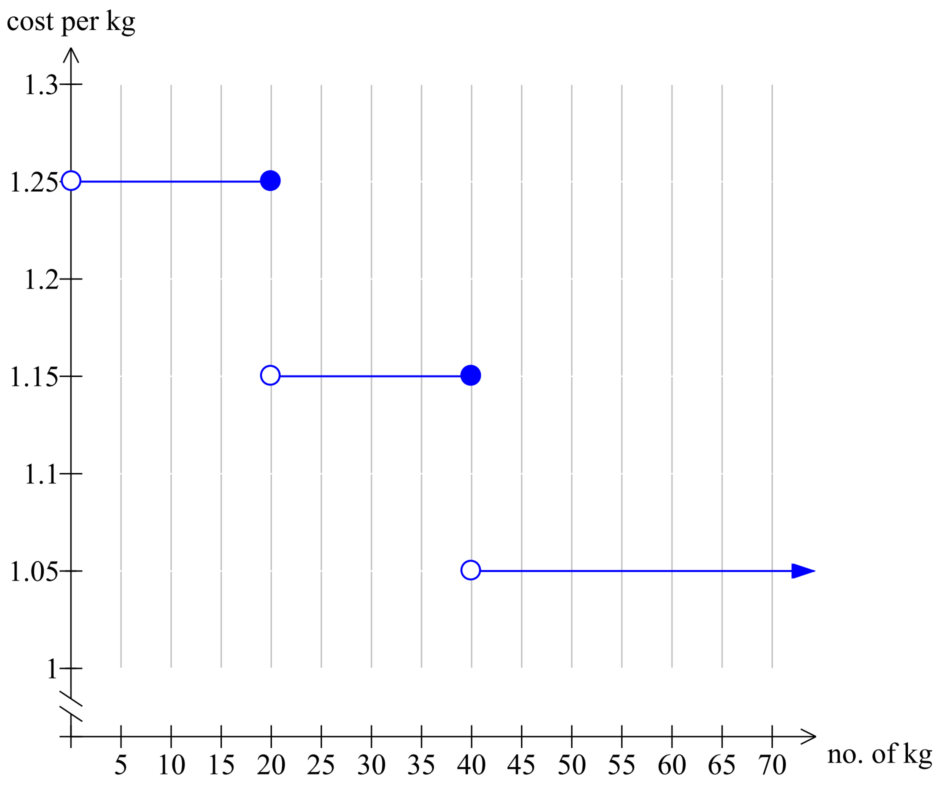
2 marks
Total 15 marks

END OF MODULE 2
TURN OVER

Module 3: Graphs and relations

Question 1

As incentive for the quick sale of produce, Con, the fruit farmer sells larger quantities of tomatoes to his distributers at a cheaper rate. The prices per kg are shown in the step graph below.



a. Con sold 30 kg of the tomatoes to one supplier and 18 kg to another. How much money did he receive?

1 mark

b. Determine two possible weights of tomatoes, to the nearest kg, that a distributor can purchase with $45.

1 mark

Question 2

The profit made from selling apples is constant for each kilogram. A profit of \$32.80 is obtained from selling 260 kg of apples and a profit of \$75.70 is obtained from selling 315 kg of apples.

a. Find the profit obtained from selling one kilogram of apples.

1 mark

b. What is the loss made from selling no apples?

1 mark

c. Determine, to the nearest whole kilogram, the amount of apples that must be sold to break even.

1 mark

Question 3

Con grows strawberries and apples on his farm.

Let x be the number of hectares of strawberries grown and
y be the number of hectares of apples grown.

It takes one day to trim a hectare of strawberries and 2 days to trim a hectare of apples.
There are at least 140 days available to trim the strawberries and apples.

This information can be written as Inequalities 1 to 3.

Inequality 1: $x \geq 0$

Inequality 2: $y \geq 0$

Inequality 3: $x + 2y \geq 140$

a. If it takes 38 days to trim the strawberries, what is the minimum number of hectares of apples that may be trimmed?

1 mark

It takes 0.3 days to harvest a hectare of strawberries and 0.1 days to harvest a hectare of apples. There are at most 30 days available for harvesting.

b. Write an inequality to describe this information in terms of x and y.

Inequality 4 :

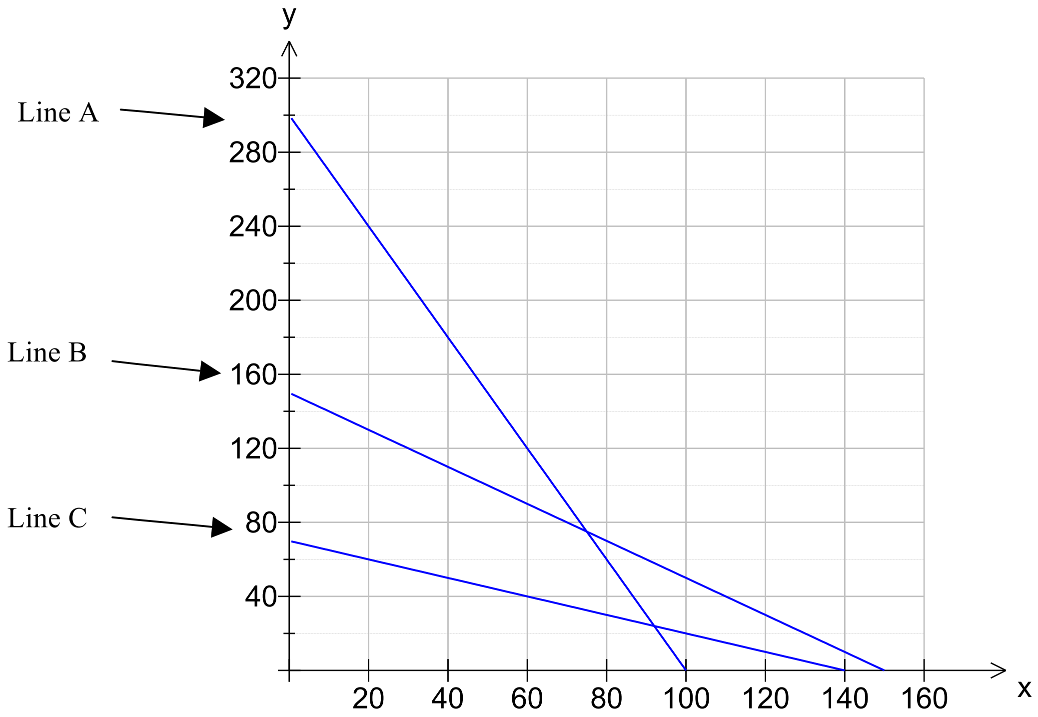
1 mark

Inequlity 5: $x + y \leq 150$

c. Explain what is meant by inequality 5 in terms of the land available.

1 mark

The boundary lines for inequalities 3, 4 and 5 are shown in the graph below.



d. In the spaces provided below, write the equations of these lines.

Line A:

Line B:

Line C:

2 marks

e. On the graph clearly indicate the feasible region that satisfies all the inequalities.

1 mark

The profit is \$345 per hectare of strawberries and \$115 per hectare of apples.

f. Find the number of hectares of each fruit that can be planted to maximise the profit.

2 marks

Due to distributor demand it was necessary to include a sixth inequality.
Con needed to use at least twice as many hectares of strawberries as apples.

g. i. Write the sixth inequality associated with the production of strawberries and apples.

1 mark

ii. Find the minimum number of hectares of strawberries that will produce a maximum profit.
Give your answer correct to one decimal place.

1 mark
Total 15 marks

END OF MODULE 3
TURN OVER

Module 4 : Business-related mathematics

Question 1

Diesel has a savings account for which simple interest is calculated at 2.4% per annum based on the minimum monthly balance. Interest is paid quarterly on the first day of each new quarter.
Diesel receives quarterly statements on his savings account.
The (incomplete) statement below is for the first quarter of 2013.

Date	Transaction detail	Debit	Credit	Balance
1/1/13	Balance brought forward			1525.50
24/1/13	Withdrawal			
5/2/13	Deposit		500.00	
19/2/13	Bill payment - gas	350.00		
4/3/13	Withdrawal	400.00		
22/3/13	Deposit		1000.00	

a. Determine the interest rate per month for Diesel's savings account.

1 mark

b. The interest paid for the month of January was $2.70
Calculate the amount withdrawn by Diesel on January 24.

2 marks

c. Determine the missing balances in the statement above.

1 mark

d. Calculate the interest earned by Diesel for the first quarter of 2013.

2 marks

Module 4 – Business-related mathematics –continued
TURN OVER

Question 2

At some later time Diesel finds that the balance of his account is $2500.
He re-invests this money in a new account that offers 2.4% per annum interest compounding monthly.

a. Determine the interest Diesel earns in the first year. Give your answer correct to the nearest cent.

1 mark

b. If Diesel correctly used the compound interest formula $A = PR^n$, determine the value of R and the value of n that he would use.

2 marks

c. Determine the balance of Diesel's account after one year if he added an additional $100 into the account each month immediately after interest was added.
Give your answer correct to the nearest cent.

1 mark

Question 3

Diesel has purchased a new computer for $5000.
He intends to depreciate the computer over five years.
The depreciated (scrap) value of the computer at the end of the five-year period is expected to be $400.

a. If Diesel uses straight-line depreciation, determine the annual percentage rate of depreciation that he is using.

1 mark

b. What percentage of the computer's value has been written off by Diesel after 5 years?

1 mark

c. Diesel believes that the computer's value depreciates, on average, by \$0.50 for each hour of use. How many hours, on average, does Diesel use his computer each year?

1 mark

d. Diesel now considers depreciating the value of the computer to the same scrap value after 5 years using the reducing balance method.
What annual depreciation rate is he using?
Give your answer correct to 1 decimal place.

2 marks
Total 15 marks

END OF MODULE 4
TURN OVER

Module 5 : Networks and decision mathematics

Question 1

Pickles, Snoopy, Mo and Bobby are four dogs who like to play and walk together.
A dominance hierarchy exists amongst the dogs as follows

Pickles has dominance over Snoopy only
Snoopy has dominance over Mo and Bobby
Mo has dominance over Pickles only
Bobby has dominance over Pickles and Mo

a. Draw a directed graph below that displays this information using arrows from the dominant dog to the other dog to indicate a dominance relationship.

1 mark

b. Explain the 2-step dominance that Bobby has over Snoopy.

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1 mark

c. The dominance order of the four dogs from most dominant to least dominant is determined by adding each dog's total for one-step and two-step dominance. This order is given in the table below. Complete the table with the total of one-step and two-step dominances for each dog.

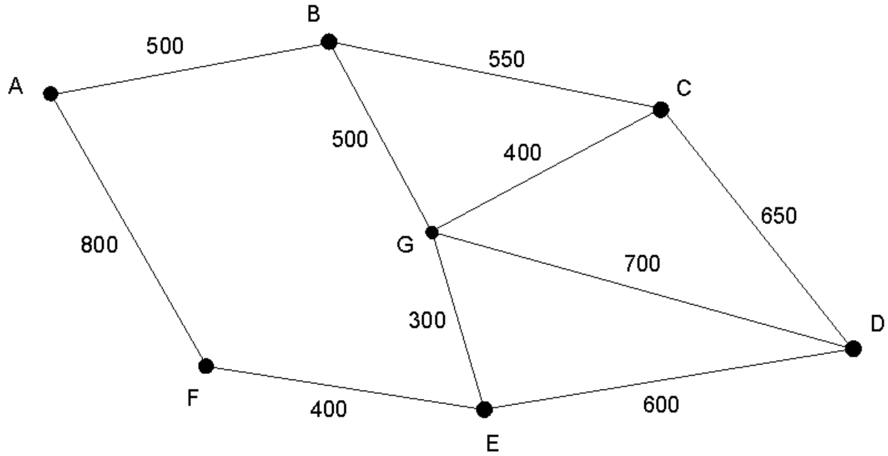
Dominance order	Total
Snoopy	
Bobby	
Pickles	
Mo	

2 marks

Module 5 – Networks and decision mathematics –- continued

Question 2

The dogs are often walked at Ripley Park. On the network diagram below the available walking paths are represented by edges and the weightings on the edges represent distances in metres.



a. The owners want to walk the dogs along every path just once and return to the starting point. Explain why this is not possible for this network.

1 mark

b. An extra walking path between two existing vertices would need to be created in order for an Euler path to exist. This can be done in several ways.

i. Write down two vertices, which if joined, would create an Euler path in this network.

ii. Hence write down a possible Euler path that would result from the addition of this edge.

1+ 1 = 2 marks

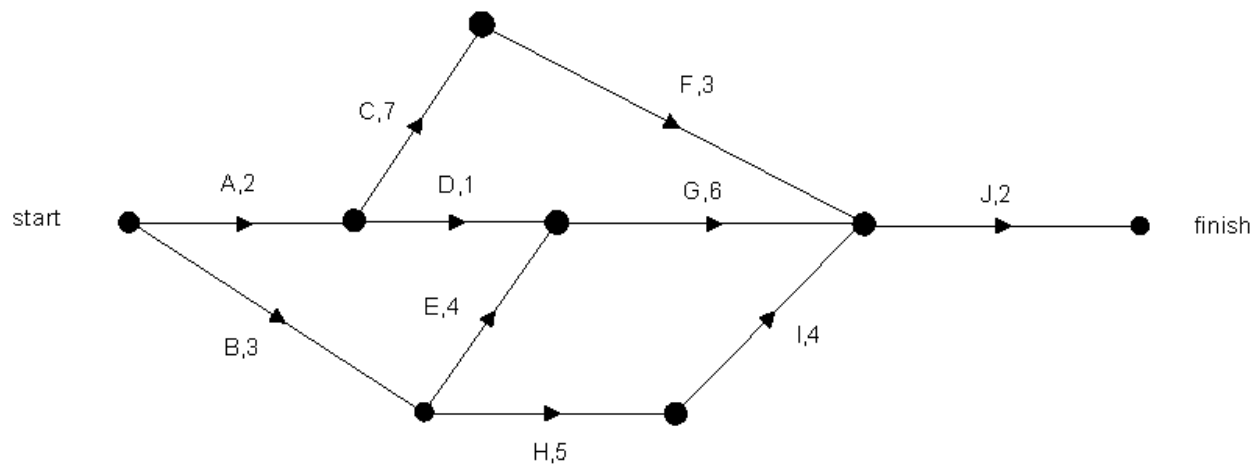
c. Water is available for the dogs at each of the vertices in the network.
Pickles starts at vertex C and his owner walks him to each other point once for a drink.
If Pickles does not return to C, determine the longest possible distance he can walk.

1 mark

Module 5 – Networks and decision mathematics - continued
TURN OVER

Question 3

The four dogs are to take part in a dog show organised by the local council.
There are ten activities that must be completed for this project.
The directed network below shows the activities, A-J, and their completion times in hours.



a. Two of the possible paths from start to finish take the same length of time. Write down these two paths.

1 mark

b. Determine the minimum time, in hours, for the completion of the project.

1 mark

c. Write down each activity that could be delayed individually, without increasing the minimum completion time of the project.

1 mark

d. Write down a calculation that shows that activity F has a float time of 1 hour.

1 mark

e. Of all the activities that can be delayed without increasing the minimum completion time of the project, how many have a float time of 1 hour?

1 mark

It is decided that one additional activity, K, should be added to the project.
Activity K has an earliest start time of 3 hours and a latest start time of 7 hours.
The duration of this new activity is 6 hours.

f. Given that activity K does not affect the critical path, draw in this activity on the network diagram above.

1 mark

g. If activity K actually takes 5 hours longer than originally intended, what will be the minimum time for the project's completion?

1 mark
Total 15 marks

END OF MODULE 5
TURN OVER

Module 6 : Matrices

Question 1

For concerts at the Mountain winery in Catville, the venue usually requires the following employees

20 security staff
15 car parking attendants
10 ushers
12 catering staff

This information can be represented in the matrix $\begin{bmatrix} 20 \\ 15 \\ 10 \\ 12 \end{bmatrix}$

For the forthcoming concert by Young Neil, the venue organiser has decided that the usual number in each group is to be doubled and one additional team leader should be added to each group.

a. Write a matrix expression that will produce a 4×1 column matrix that gives the totals of the four groups of employees required.

1 mark

b. Hence determine the total number of staff to be employed for this concert.

1 mark

Question 2

Tickets for the concert come in two categories, Platinum, costing \$250, and Standard, costing \$150.
The total number of tickets sold was 5500.
The total value of all tickets sold was \$975 000.

a. Write a pair of simultaneous linear equations, **in matrix form**, where P represents Platinum and S represents Standard.

2 marks

Module 6 – Matices – Question 2 - continued

b. These simultaneous equations have a unique solution. Explain why with reference to the determinant of the 2×2 matrix used.

1 mark

c. Insert the five missing values in the following matrix product that gives the solution to the equations.

$$\begin{bmatrix} P \\ S \end{bmatrix} = \begin{bmatrix} -1.5 & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

1 mark

d. Write down the number of each type of ticket sold.

1 mark

Question 3

When Young Neil performs a concert he always opens the show with either of two songs, "Fingerpowder" (F) or "Like a Cyclone" (L). The following transition matrix, T, can be used to predict the opening song from one concert to the next.

$$\begin{array}{cc} & \textit{this concert} \\ & \begin{array}{cc} F & L \end{array} \\ T = & \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} \begin{array}{l} F \\ L \end{array} \; \textit{next concert} \end{array}$$

a. The information contained in the transition matrix can also be represented by an equivalent transition diagram. Draw this diagram below.

1 mark

Module 6 – Matices – Question 3 - continued

b. Explain what the figure of 0.8 in the matrix tells us in the context of the problem.

1 mark

c. For the first concert of his new "infinite" tour, Young Neil opened the show with Fingerpowder. Using F for the first row and L for the second row, write down a 2×1 initial state matrix, S_1, that represents this information.

1 mark

d. Determine the chance that Young Neil opens the fourth show of this tour with Like a Cyclone.

1 mark

e. Show as the "infinite" tour draws to a close, there is a 40% chance that Young Neil will open a show with Fingerpowder.

1 mark

f. If Young Neil had opened the first show with Like a Cyclone, what effect would this have on the answer of 40% from part **e.**? Explain your answer briefly.

1 mark

g. For his next "infinite" tour, in the long run, Young Neil would prefer to open the show with Fingerpowder 75% of the time. Fill in the transition matrix with values that would allow this to happen.

$$\begin{array}{cc} & \textit{this concert} \\ & \begin{array}{cc} F & L \end{array} \\ T = & \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{array}{l} F \\ L \end{array} \quad \textit{next concert} \end{array}$$

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOKLET