



**2011 Further Mathematics GA 3: Written examination 2**

**GENERAL COMMENTS**

There were 30 732 students who sat the Further Mathematics examination 2 in 2011. The selection of modules by the students is shown in the table below.

| MODULE                                | %<br>2011 |
|---------------------------------------|-----------|
| 1 – Number patterns                   | 32        |
| 2 – Geometry and trigonometry         | 75        |
| 3 – Graphs and relations              | 44        |
| 4 – Business-related mathematics      | 37        |
| 5 – Networks and decision mathematics | 43        |
| 6 – Matrices                          | 68        |

Most students were able to complete their three selected modules. In general, students were able to answer the first question in each module very well but were challenged by the more complex questions they encountered later in the module.

It was concerning that some students did not read questions carefully. For example, in Question 1ai. in the Core section, it seemed that many students did not read the key words in the introductory sentence stating that the stemplot showed ‘the distribution of average age of women ....’. The question was really only asking, ‘What is the smallest number in the stemplot?’. ‘The average of all the numbers in the stemplot’ was a common incorrect answer. Students are reminded to read questions carefully before answering and to ensure they answer the question asked.

Students are expected to recognise an inappropriate answer, yet many examples of inappropriate answers were given. This was most evident in Question 4b. in the Core section, where unrealistic answers for ‘average age at first marriage’ were given.

Some students rounded off answers involving money to the nearest five cents despite the instruction to give the answer correct to the nearest cent. Unless directed otherwise, all answers involving currency should be treated as electronic payments, such as by credit card or direct debit where no rounding to the nearest five cents occurs.

Students should be prepared to explain their reasoning when answering questions. Such explanations should be concise and relate clearly to the context of the question.

All students are encouraged to bring a ruler to the examination. Where lines are drawn freehand, on graphs in particular, these are often insufficiently accurate to gain marks or to be useful for reading values from the graph.

Many students who wrote answers without showing working missed out on method marks or consequential marks. Method marks often apply in questions worth two marks or more, while consequential marks may apply to some single-mark questions. To qualify for a possible consequential mark, students must show a mathematical calculation or statement that clearly shows how a previously wrong result has been used correctly to obtain a consequentially wrong answer.

Work that is crossed out will not be assessed unless there is written notation by the student that indicates the crossing out should be ignored.



**SPECIFIC INFORMATION**

**Core**

**Question 1ai.–2b.**

| Marks | 0 | 1 | 2 | 3 | 4  | 5  | 6  | 7  | 8  | Average |
|-------|---|---|---|---|----|----|----|----|----|---------|
| %     | 1 | 2 | 5 | 8 | 12 | 13 | 18 | 16 | 26 | 5.7     |

**1ai.**

25.0 years

Many students did not read this question clearly and found the mean of all the numbers in the stemplot. As a result, a common incorrect answer was 28.04.

**1aii.**

28.2 years

**1b.**

1.1 years

**1c.**

$$1.5 \times IQR = 1.5 \times 1.1 = 1.65$$

$$\text{and } Q_1 - 1.65 = 29.9 - 1.65 = 28.25$$

Since  $26.0 < 28.25$ , the age of 26.0 is an outlier

Many students did not answer this question fully. Most calculated the 28.25 but then did not discuss how 26.0 related to this. Mathematical symbols were sometimes used incorrectly; for example,  $28.25 \leq 26.0 \leq 32.65$ .

**2a.**

65.5 %

**2b.**

The data for each of the four age groups in the table does support the opinion that age at first marriage is associated with the year of marriage. For example, of all first marriages, the percentage of women aged 25–29 years increased from 23.4% (1986) to 31.7% (1996) to 34.5% (2006).

Many students tried to work down the table despite the direction to work across a row.

**Question 3a.–bii.**

| Marks | 0  | 1  | 2  | 3  | 4  | Average |
|-------|----|----|----|----|----|---------|
| %     | 22 | 23 | 18 | 20 | 17 | 1.9     |

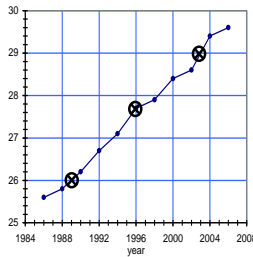
**3a.**

The average age at first marriage remained relatively constant from 1915 to around 1935, and then decreased during the period 1935 to 1970.

This question was generally answered very poorly. Many students noted a ‘decreasing trend from 1915 to 1970’, without considering the two sections of the plot. Some gave very specific answers rather than answering in general terms. Many students said the data was either ‘positively skewed’ or ‘negatively skewed’, neither of which is an applicable expression for a time-series plot.



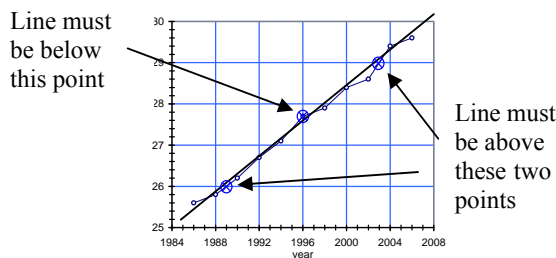
3bi.



3bii.

Most students were able to find the three points. However, a significant number divided the 11 points in a 3:5:3 ratio rather than 4:3:4.

3bii.



Many students performed three-median smoothing instead of finding the 3-median line. Some students only connected the two outer points and did not slide this line. A significant number of answers that did involve moving an initial line seemed to have been pivoted about either the top or the bottom point and consequently passed through it. This was not accepted. A number of students who found the correct three points then incorrectly joined these with two-line segments.

**Question 4a.–b.**

| Marks | 0  | 1 | 2 | 3  | Average |
|-------|----|---|---|----|---------|
| %     | 59 | 7 | 7 | 27 | 1       |

4a.

2.39, 5.89

Few students found the two answers correctly by correct data entry and log transformation. A common mistake was to ignore the log transformation requirement. For some students it did not register that the dependent variable was in the first column of the table and hence calculated the regression line incorrectly. These students could still access one of the two marks if they then correctly applied a log transformation to the wrong variable.

4b.

27.7 years

$$\text{average age} = 0.09 \times \log(20\,000) \approx 27.2$$

Very few students were able to answer this question correctly as most of those who attempted it did not find  $\log(20\,000)$  in their calculation. Very commonly, answers were inappropriate in the context of the question. For example, many answers were in the thousands, millions or were written as dollars. Answers such as these could not apply to a question about the average age at first marriage.

## Module 1 – Number patterns

**Question 1**

Many students did not answer this question.

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## Question 1a.–c.

| Marks | 0 | 1 | 2  | 3  | Average |
|-------|---|---|----|----|---------|
| %     | 2 | 6 | 16 | 75 | 2.7     |

**1a.**

A point plotted at (7, 600)

A large number of students plotted all the points for months 4 to 12.

**1b.**

50

**1c.**

150

## Question 2a.–e.

| Marks | 0  | 1  | 2  | 3  | 4  | 5  | 6 | Average |
|-------|----|----|----|----|----|----|---|---------|
| %     | 14 | 17 | 20 | 16 | 14 | 13 | 6 | 2.6     |

**2a.**

\$3.20

**2b.**

\$26.40

$$\frac{8}{2}(2 \times 4 + 7 \times -0.2) = 26.4$$

A common incorrect answer was  $t_8 = 2.60$ .

**2c.**

20 GB

First free GB when  $t_n = 0$

$$4 - 0.2(n-1) = 0 \quad \therefore n = 21$$

Minimum number of GB = 20 for the same cost as 21 or more GB

A common incorrect answer was 21.

**2di.**

11 GB

**2dii.**

\$33

$$\frac{11}{2}(2 \times 4 + (11-1) \times -0.2) = 33$$

**2e.**

Month 9

### Month Data

|   |       |
|---|-------|
| 7 | 47.6  |
| 8 | 70.3  |
| 9 | 104.5 |

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## Question 3a–4c.

| Marks | 0  | 1  | 2  | 3  | 4  | 5  | 6 | Average |
|-------|----|----|----|----|----|----|---|---------|
| %     | 26 | 13 | 13 | 13 | 13 | 14 | 8 | 2.5     |

### 3a.

\$47.50

### 3b.

\$11.31

$$\$50 - \$50 \times (0.95)^5 = 11.31$$

Many students seemed to have rounded down this result to the nearest 5 cents, ignoring the instruction to give the answer to the nearest cent. Quite a few of these students wrote \$11.30 but did not show any working and therefore were not eligible for any consideration of a rounding error allowance.

### 3c.

\$460

$$S_{12} = \frac{50(1-0.95^{12})}{1-0.95} \approx 459.639\dots$$

### 4a.

198 100

$$200\,000 \times 0.99 + 100 = 198\,100$$

### 4b.

$$C_{n+1} = 0.99 C_n + 100, C_1 = 200\,000$$

The initial condition was required but was left off by many students. Some had trouble converting 1% to a decimal

$$(1\% = \frac{1}{100} = 0.01).$$

A common incorrect answer was  $C_{n+1} = 0.01 C_n + 100, C_1 = 198\,100$ .

### 4c.

3781

Very few students answered this question correctly.

## Module 2 – Geometry and trigonometry

### Question 1a–2c.

| Marks | 0  | 1  | 2 | 3 | 4  | 5  | 6  | Average |
|-------|----|----|---|---|----|----|----|---------|
| %     | 14 | 22 | 8 | 9 | 15 | 16 | 14 | 3       |

### 1a.

150m

$$AB = 30 \times 5 = 150$$

This question was not answered well. Most incorrect answers resulted from poor understanding of the reference to ‘the horizontal level’. All lengths on a map are measured as if the land is perfectly flat and will always be perpendicular to a vertical line that measures the altitude of a point.

Many students determined the length  $AB$  on the hill was 150 m, but then stated that this was measured along the slope from  $A$  to  $B$  (rather than horizontal). They then used it as the hypotenuse in a Pythagoras calculation to find  $AB$ .

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**1b.**  
0.2

$$\text{Average slope} = \frac{\text{rise}}{\text{run}} = \frac{15}{75} = \frac{1}{5} = 0.2$$

This question was not answered successfully by many students as many did not seem to understand *average slope*. Many students incorrectly found the angle of elevation in degrees. Another common incorrect interpretation of the question resulted in some students finding the *length* of the *distance* along the slope from *B* to *L*.

**2ai.**  
152°

**2aii.**  
78°

**2b.**  
208°

**2c.**  
079°

$\triangle ABL$  is an isosceles triangle with base angles =  $t$

$$t = \frac{180 - (28 + 50)}{2} = \frac{102}{2} = 51$$

Therefore angle from North to  $LB$  is  $28 + 51 = 79^\circ$ .

True (three-figure) bearings were required so that the bearing was  $079^\circ$ , rather than just  $79^\circ$ .

### Question 3a–d.

| Marks | 0  | 1  | 2  | 3  | 4  | 5  | Average |
|-------|----|----|----|----|----|----|---------|
| %     | 16 | 17 | 13 | 10 | 15 | 29 | 2.8     |

**3a.**

$$\frac{360}{8} = 45^\circ$$

A 'show that' question requires a mathematical calculation or explanation that produces the required result. A number of students inappropriately began with, and used, the  $45^\circ$  to 'show that' there are  $360^\circ$  in a circle.

**3b.**  
1.4 m<sup>2</sup>

$$A = \frac{1}{2} \times 2 \times 2 \times \sin(45^\circ) = 1.414\dots$$

Some students found the area of the octagon instead of the triangle  $POQ$  as required.

**3c.**  
1.2 m

A common incorrect answer was 2.4 m.

**3d.**  
21 m<sup>2</sup>

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$$\begin{aligned} \text{Area} &= \text{Area of circle} - \text{Area of octagon} \\ &= \pi \times 3.2^2 - 8 \times 1.4 = 20.969\dots \end{aligned}$$

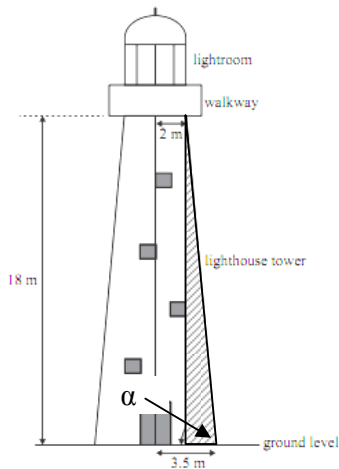
Some students multiplied the area of triangle  $POQ$  by 6 rather than 8. Others used  $A = \pi r^2$  with  $r = 2$  for the area of the octagon.

### Question 4a–bii.

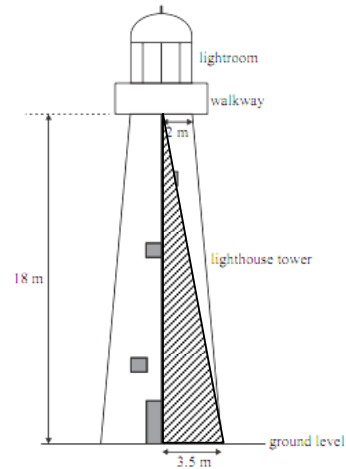
| Marks | 0  | 1  | 2 | 3 | 4 | Average |
|-------|----|----|---|---|---|---------|
| %     | 70 | 11 | 4 | 8 | 6 | 0.7     |

4a.  
85.2°

$$\tan^{-1}\left(\frac{18}{1.5}\right) = 85.236\dots$$



**Correct**



**Incorrect**

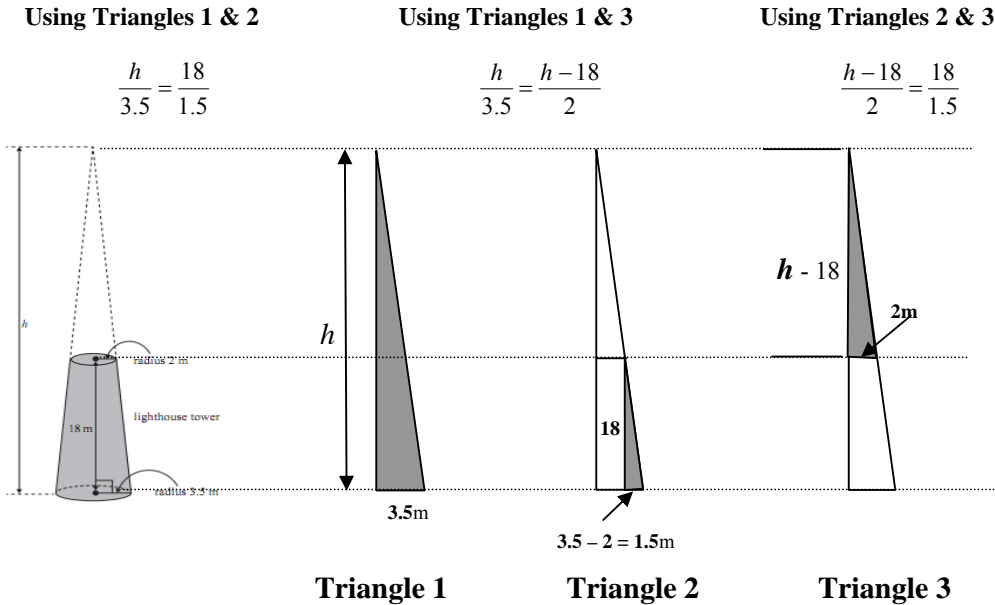
This question was generally answered poorly. Many students drew an inappropriate right triangle with base = 3.5m and perpendicular height = 18 m and then found the angle formed at the corresponding corner by using

$$\tan^{-1}\left(\frac{18}{3.5}\right) = 78.996\dots$$



4bi.  
42m

Various similar triangles (such as those labelled 1, 2 and 3 in the diagrams below) could be used to form the relevant equation.



Many students struggled with this question, as most were unable to identify the required similar triangles.

A common error was to equate the ratios  $\frac{h}{18} = \frac{3.5}{2}$ . This attempts to equate a triangle with a trapezium.

4bii.  
438 m<sup>2</sup>

Large cone – small cone =

$$\left(\frac{1}{3} \times \pi \times 3.5^2 \times 42\right) - \left(\frac{1}{3} \times \pi \times 2^2 \times 24\right) = 438.469\dots$$

Of those who did attempt this question, many students gave incomplete/incorrect answers consisting of only the volume of a large cone with height = 42 m or height = 18 m.

### Module 3 – Graphs and relations

#### Question 1a–f.

| Marks | 0 | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8 | 9 | Average |
|-------|---|---|---|----|----|----|----|----|---|---|---------|
| %     | 5 | 8 | 7 | 12 | 15 | 12 | 13 | 14 | 5 | 8 | 4.7     |

1a.  
236 g

1b.  
200 g

$$320 \times 0.2 + 0.04 \times r = 72$$

$$\therefore r = 200$$



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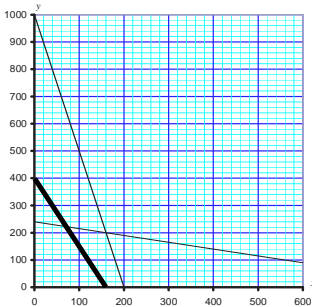


A common incorrect answer was 8 g, which is the number of grams of protein needed from the raisins. If the calculation method could be followed, this answer could have gained a method mark. However, this was not often the case as many students chose to write only their final answers.

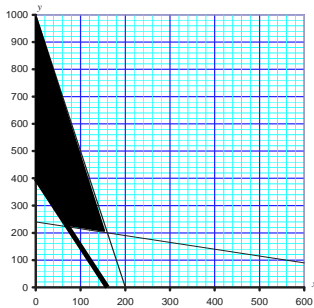
**1c.**

$$0.1x + 0.04y \geq 16$$

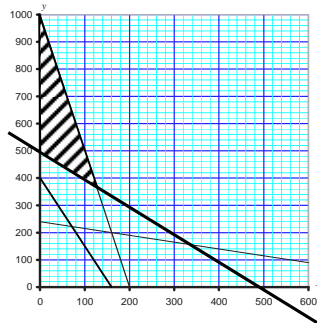
**1di.**



**1dii.**



125 g



Draw in the new constraint  $x + y \geq 500$

This gives a new feasible region shaded above.

The line  $x + y = 500$  crosses  $0.2x + 0.04y = 40$  at (125, 375)

This gives the maximum  $x = 125$

This question was generally answered poorly, with the majority of students ignoring the new constraint  $x + y \leq 500$ . This usually produced an incorrect answer of 160 g.

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## Question 2a–d.

| Marks | 0  | 1  | 2  | 3  | 4 | 5 | 6 | Average |
|-------|----|----|----|----|---|---|---|---------|
| %     | 27 | 23 | 15 | 22 | 9 | 2 | 3 | 1.8     |

### 2a.

2.3 km/h

$$\frac{16}{7} = 2.285\dots$$

### 2b.

$a = 1.5, b = 5.5$

Solve

$$10 = 3a + b \quad \dots\dots\dots \textcircled{1}$$

$$16 = 7a + b \quad \dots\dots\dots \textcircled{2}$$

The correct value for  $a$  was common, but the value of  $b$  evaded many.

### 2c.

2 hours

Solve

$$d = -3t + 16 \quad \dots\dots\dots \textcircled{1}$$

$$d = 5t \quad \dots\dots\dots \textcircled{2}$$

While not required by the question, very few students chose to add the line for Katie's hike on the graph as part of their working. Consequently, most equated Katie's equation with the equation  $d = 1.5t + 5.5$  (from Question 2b.) for the wrong section of Michael's graph. This usually led to an incorrect answer of 2.3 hours.

### 2d.

1.38 hours

$$3 \text{ km apart at } (-3t+16) - 5t = 3$$

$$\therefore t = \frac{13}{8} = 1.625$$

Also 3 km apart at  $t = 3$

$$\therefore \text{can talk for } 3 - 1.625 = 1.375 \text{ hrs}$$

As most students had not drawn in Katie's graph, few were able to complete this question correctly.

## Module 4 – Business-related mathematics

### Question 1a–2d.

| Marks | 0 | 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8 | Average |
|-------|---|---|----|----|----|----|----|----|---|---------|
| %     | 1 | 7 | 11 | 13 | 16 | 16 | 15 | 13 | 9 | 4.6     |

### 1a.

\$330

### 1b.

\$462

### 1c.

8%

A common incorrect answer was 92%.

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**2a.**  
\$5833.60

**2b.**  
0.072

**2c.**  
\$78.42

**Tom:** Using TVM will have \$8497.58  
**Patty:** Using formula will have \$8576.00  
 $\$8576.00 - \$8497.58 = 78.42$

Many students were able to find the value of Patty's investment correctly but then used their answer from Question 2a. (value after first month) instead of calculating Tom's investment after 12 months.

**2d.**  
12.5 %

$$1000 = \frac{8000 \times r \times 1}{100}$$

**Question 3a–4b.**

| Marks | 0  | 1  | 2  | 3  | 4  | 5 | 6 | 7 | Average |
|-------|----|----|----|----|----|---|---|---|---------|
| %     | 33 | 17 | 10 | 10 | 10 | 7 | 7 | 5 | 2.2     |

**3a.**  
\$13 070

$$2870 + 6\% \text{ of } (300\,000 - 130\,000)$$

This question was answered poorly. The majority of students appeared to have difficulty understanding how to use the table. Consequently, a common incorrect answer was  $2870 + 0.06 \times 300,000 = \$20\,870$ .

**3b.**  
\$351 000

$$300\,000 \times 1.0317^5 = 350\,661.76 \dots$$

**3c.**  
2024

$$300\,000 \times 1.0317^n = 450\,000$$

$\therefore n = 12.992\dots$  or  $n \approx 13$   
 $\therefore$  Year is  $2011 + 13 = 2024$

Many students calculated the year number only and then did not complete the answer.

**4ai.**  
300 months

**N** = 299.573...  
**I** = 7.62  
**PV** = 265000  
**PMT** = -1980  
**FV** = 0  
**P/Y** = 12  
**C/Y** = 12

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**4a.ii.**

\$3 694.25

$N = 12$

$I = 7.62$

$PV = 265000$

$PMT = -1980$

$FV = -261305.747\dots$

$P/Y = 12$

$C/Y = 12$

and  $265\ 000 - 261\ 305.75 = 3694.25$

**4b.**

\$22 65.04

$N = 12 \times 19$

$I = 8.2$

$PV = 261305.75$

$PMT = -2265.04 \dots$

$FV = 0$

$P/Y = 12$

$C/Y = 12$

## Module 5 – Networks

**Question 1a.–2b.**

| Marks | 0 | 1 | 2  | 3  | 4  | 5  | 6  | Average |
|-------|---|---|----|----|----|----|----|---------|
| %     | 1 | 4 | 10 | 17 | 27 | 28 | 13 | 4       |

**1a.**

200 km

**1b.**

6

*FDC, FEBC, FDEBC, FEDC, FEABC, FDEABC*

**1c.**

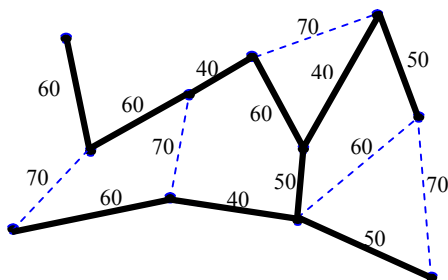
Bredon

**1d.**

240 km

Many students calculated the distance to be travelled by the engineer rather than the assistant.

**2a.**



A number of students included circuits in their graphs or missed one or more vertices.

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**2b.**  
510 m

A consequential mark was available for the correct total length of any spanning tree drawn in Question 2a. A common error was for students to omit the length of one of the edges from their diagram.

**Question 3a.–e.**

| Marks | 0  | 1  | 2  | 3  | 4 | 5  | Average |
|-------|----|----|----|----|---|----|---------|
| %     | 22 | 27 | 15 | 10 | 9 | 17 | 2.1     |

**3a.**  
2 hours

A common incorrect answer was one hour.

**3b.**  
3 hours

**3c.**  
F and H

A common incorrect answer included activity G, either by itself or with other activities. This activity was not on the critical path (it was not a predecessor for activity I) and could be delayed by one hour.

**3d.**  
13 hours

While not required by the question, an activity diagram would have been helpful when answering this question.

**3e.**  
14 hours

**Question 4a.–c.**

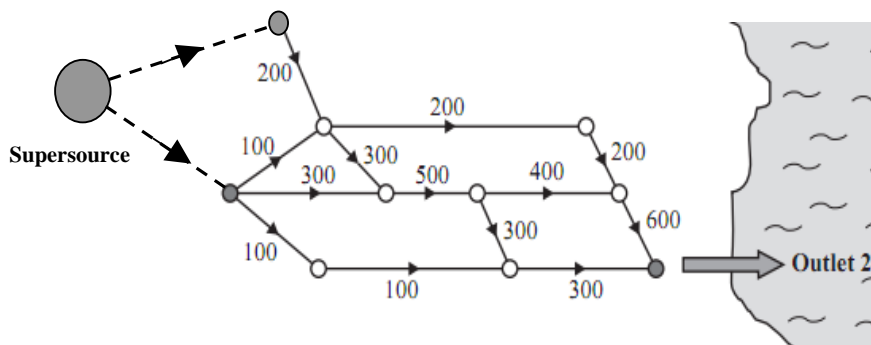
| Marks | 0  | 1  | 2  | 3  | 4 | Average |
|-------|----|----|----|----|---|---------|
| %     | 10 | 35 | 34 | 14 | 7 | 1.7     |

**4a.**  
2, 1

**4bi.**  
700

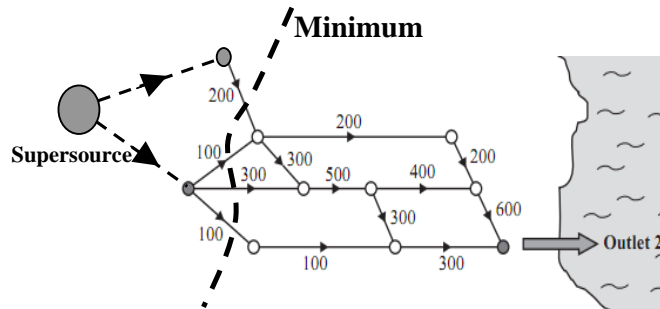
**4bii.**  
700

Water that eventually flowed through Outlet 2 came from Source 2 and also along the pipe labelled 200 from Source 1. To address this, a single Supersource can be considered, as shown in the diagram below.





It is now clearer that the required minimum cut must separate the Supersource from Outlet 2 and, in this case, includes the 200 pipe coming down from Source 1.



4c.  
300

Question 4bii. gave a minimum cut of 700 that included the damaged pipe. The next smallest cut in the lower pipe system is 800. Therefore the original 200 damaged pipe can be lengthened by 100 so that the cut from Question 4bii. is now also 800 and so the replacement pipe should allow 300 kilolitres per minute.

## Module 6 – Matrices

### Question 1ai–2d.

| Marks | 0 | 1 | 2 | 3 | 4  | 5  | 6  | 7  | Average |
|-------|---|---|---|---|----|----|----|----|---------|
| %     | 1 | 2 | 5 | 6 | 13 | 26 | 30 | 18 | 5.2     |

1ai.

Birds eat lizards.

A common inaccurate interpretation was ‘one bird eats one lizard’.

1aii.

No birds, lizards or insects eat birds.

Many students had difficulty with this question. Common incorrect answers included:

- no lizards or insects eat birds (this omitted one of the zeros)
- insects do not eat insects, birds or lizards (confused a column with a row)
- none of these animals eat their own kind (confused a diagonal with a row).

1b.

$$Z = \begin{matrix} & \begin{matrix} I & B & L & F \end{matrix} \\ \begin{matrix} I \\ B \\ L \\ F \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

2a.

$$[99\ 500\ 20\ 25\ 240]$$

The correct answer shown is a  $1 \times 4$  matrix with the four elements clearly separated by spaces that are wider than the space that groups digits in large or small numbers, as in 99 500.

Some students wrote their four elements separated by commas or dots; however, this is not correct mathematical notation.

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**2b.**  
20

A common wrong answer was 380.

**2c.**  
[ 285 ]

**2d.**  
Total number of birds, lizards and frogs that were killed

A variety of incorrect answers suggested that many students may benefit from increased practice interpreting the result of a matrix product within a context. For this question, manual calculation rather than using the calculator may have provided more understanding of the result. Incorrect answers often included the number of insects killed or suggested that the answer referred to the number of birds, lizards and frogs that survived.

**Question 3a–cii.**

| Marks | 0 | 1  | 2  | 3  | 4  | 5  | Average |
|-------|---|----|----|----|----|----|---------|
| %     | 6 | 12 | 19 | 10 | 19 | 34 | 3.3     |

**3a.**  
96

**3b.**  
[ 128 ]  
[ 40 ]

**3ci.**  
Two points plotted at (3, 46) and (3, 104)

Some very inaccurate attempts at plotting points on the grid were evident.

**3cii**  
144

The total number of female ducks = the sum of 96 juveniles and 48 adults.

**Question 3d–e.**

| Marks | 0  | 1  | 2  | 3 | Average |
|-------|----|----|----|---|---------|
| %     | 64 | 19 | 10 | 7 | 0.6     |

**3d.**  
Year 5

At the start, there were 96 female ducks  
Must find when we get 48 or fewer  
End of year 4 = 51 female ducks  
End of year 5 = 41.5 female ducks

**3e.**  
400 juvenile ducks and 0 adult ducks

End year 1 =

$$W_1 = \begin{bmatrix} 0 & 1 \\ 0.25 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \end{bmatrix}$$

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and

*End year 0 =*

$$W_0 = \begin{bmatrix} 0 & 1 \\ 0.25 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 100 \end{bmatrix} = \begin{bmatrix} 400 \\ 0 \end{bmatrix}$$

A common error was to only calculate  $W_1$ .