Neap:

Trial Examination 2011

VCE Further Mathematics Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
Core	2	2	15
Section	**Number of modules**	**Number of modules to be answered**	**Number of marks**
Modules	6	3	45

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white-out liquid/tape.

Materials supplied

Question booklet of 25 pages with a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.

Please ensure that you write your **name** and your **teacher's name** in the space provided on this page.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2011 VCE Further Mathematics Units 3 & 4 Written Examination 2.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Diagrams are not to scale unless specified otherwise.

Page

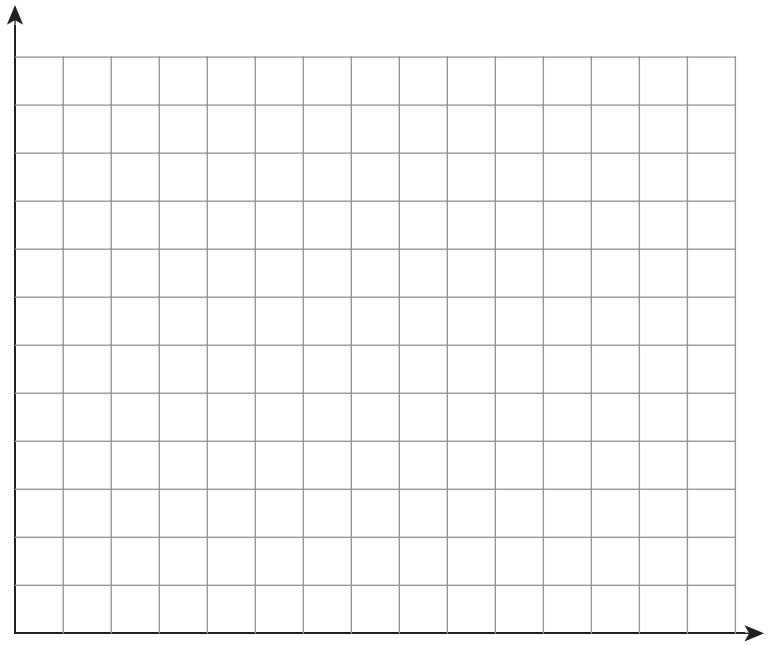
Core

Question 1

"Surf Mart" sales records have been kept for the last 12 quarters and results are shown in the table below.

Quarter	1	2	3	4	5	6	7	8	9	10	11	12
Total sales in $1000s	14	18	25	24	40	27	34	38	28	40	34	47

a. Use the axes shown to draw and label a time series graph to show the data.



2 marks

b. Comment on any relationship you see.

_____

1 mark

c. Use the 3 median method to plot a line of best fit on the axes for **part a**. All working must be shown directly on the graph, including the 3 points used to generate the line. Use the graph to find the gradient (to 1 decimal place) and approximate intercept to complete this statement:

Sales = _____ + _____ × Quarter

3 marks

Question 2

Statistics are gathered on the length of putts Cam has during a round of golf and their success (i.e. if they go in the hole). The results for one particular round are shown in the following table.

length of putt / Result	**0 – < 1 m**	**1 – < 2 m**	**2 – < 3 m**	**3 – < 4 m**	**4 – < 5 m**	**> 5 m**
Success	14	2	1	0	1	0
Miss	3	4	4	5	2	2

a. Find the mean length of Cam's successful putts in this round (to 2 decimal places).

_____

_____

2 marks

b. Find the mode of the length of Cam's missed putts.

_____

1 mark

c. A golfing research team investigates the relationship between the length of putt and the chance of that putt going in. To do this they set up 100 putts at 1 m in distance, another 100 at 2 m in distance and so on up to 8 m in length and record how many putts go in. The results are shown in the table below.

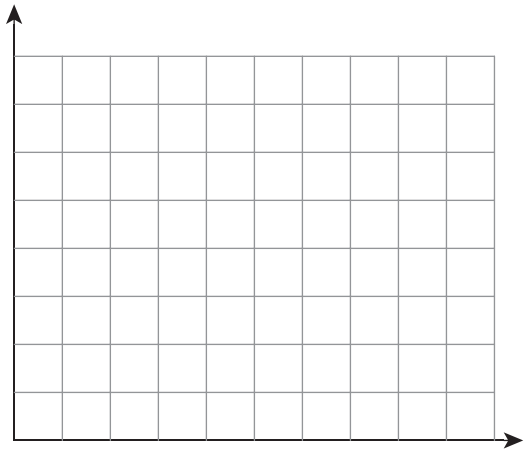
Length of putt (m)	1	2	3	4	5	6	7	8
Number of successes per 100 putts	90	50	25	16	8	4	1	0

i. Which of the variables is the independent variable?

_____

_____

ii. Draw a scatterplot on the following axes.



iii. Briefly explain why it is not appropriate to calculate pearson's product–moment correlation coefficient for this dataset.

_____

_____

iv. Perform a $\frac{1}{x}$ transformation and complete the following (to 2 decimal places):

Pearson's product moment correlation coefficient = _____

Number of successful putts per 100 putts = _____ + _____ × $\frac{1}{\text{Length of putt (m)}}$

1 + 2 + 1 + 2 = 6 marks

END OF CORE

MODULES

Module 1: Number patterns

Question 1

Shouad is investigating likely future increases in house insurance premiums. The annual premium in 2010 for a standard house (value $400 000) is $600 but this will increase every year by $20.
Unless otherwise stated, all answers for this question should be expressed to the nearest dollar.

a. State the type of sequence involved in premiums.

_____

1 mark

b. Find the cost of insuring a house in 2030 and also the total cost of insuring a house between 2010 and 2030.

_____

_____

_____

_____

_____

_____

2 marks

c. During which calendar year would the total of all premiums paid first exceed the actual value of the house?

_____

_____

_____

_____

2 marks

Shouad's colleague, Jill, forecasts an annual 2% increase in premiums instead of the $20 rise. Prices and premiums are the same in both models for 2010.

d. Find the premium that Jill would predict for 2030.

_____

_____

_____

1 mark

e. Find the year in which Jill's estimate first exceeds that of Shouad.

2 marks

Jill and Shouad decide to work together to produce a prediction model that involves some features of each model. A difference equation of the form $P_{n+1} = aP_n + b$ is to be used. They each agree that each year's premium should be calculated by increasing the previous year's by 1% and then adding $10.

f. Find the correct values of *a* and *b.*

2 marks

g. Find the 2015 premium to the nearest dollar.

2 mark

Shouad notices that it is uncommon for large rises in premiums in consecutive years. They decide to try a second-order difference equation with the following characteristics: the premium for year $(n + 2)$ is found by increasing that premium for year $(n + 1)$ by 2.5% but reduced by 40% of the premium rise from year n to year $(n + 1)$. The difference equation is of the form $P_{n+2} = a.P_{n+1} + b.P_n$.

h. Find values of *a* and *b.*

1 mark

i. If premiums were \$600 and \$650 in 2010 and 2011 respectively, find the premium in 2013 to the nearest cent.

_____

_____

_____

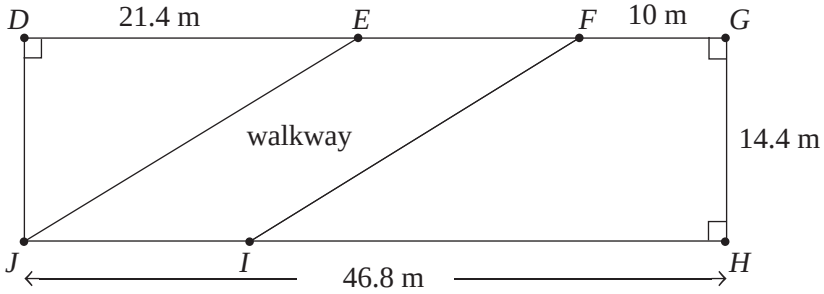
2 marks

Total 15 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1



A flat rectangular block of land is broken up into two parts that are separated by a walkway, as shown in the diagram above. The walkway has a constant width.

a. Calculate the size of angle *DEJ*. Write your answer correct to the nearest degree.

1 mark

b. Calculate the length *EF*.

1 mark

c. Calculate the length *EJ*. Write your answer correct to one decimal place.

1 mark

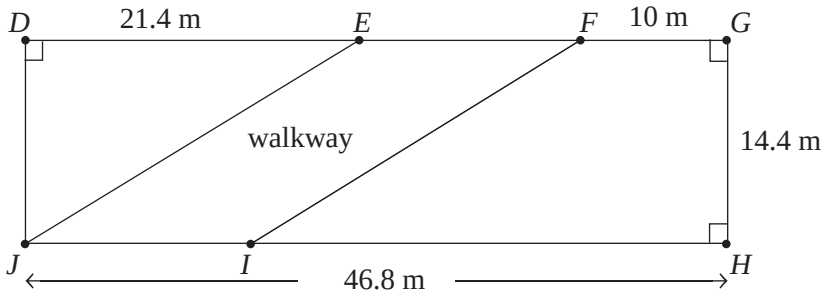
d. Triangle *DEJ* and trapezium *FGHI* are to be tiled.
Calculate the total area of the tiled sections. Write your answer correct to one decimal place.

2 marks

e. Calculate the perpendicular width of the walkway. Write your answer correct to one decimal place.

2 marks

Question 2



Point *G* is due north of point *H*.

a. Calculate the three figure bearing of point *F* from point *H*. Write your answer correct to one decimal place.

_____

_____

_____

2 marks

A 13.5 metre tall vertical flagpole is built at point *F*.

Point *F* and point *H* are both 64 metres above sea level.

b. Calculate the angle of depression of point *H* from the top of the flagpole. Write your answer correct to one decimal place.

_____

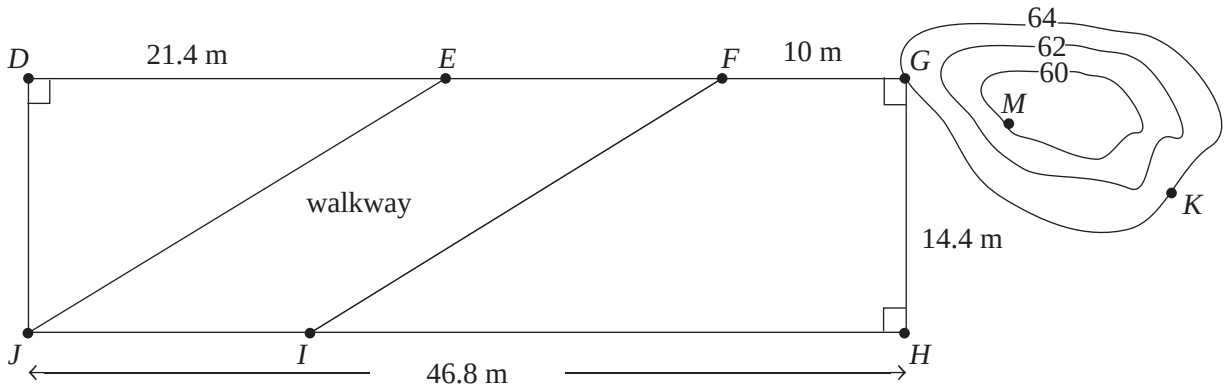
_____

_____

_____

2 marks

Question 3



The contour map above shows a region of land adjacent to the block *DGHJ*. It has contours drawn at two metre intervals. The average slope between point *M* and point *G* is 0.25.

a. Calculate the horizontal distance between point *M* and point *G*.

_____

_____

1 mark

The scale used on the contour map is 1:300.

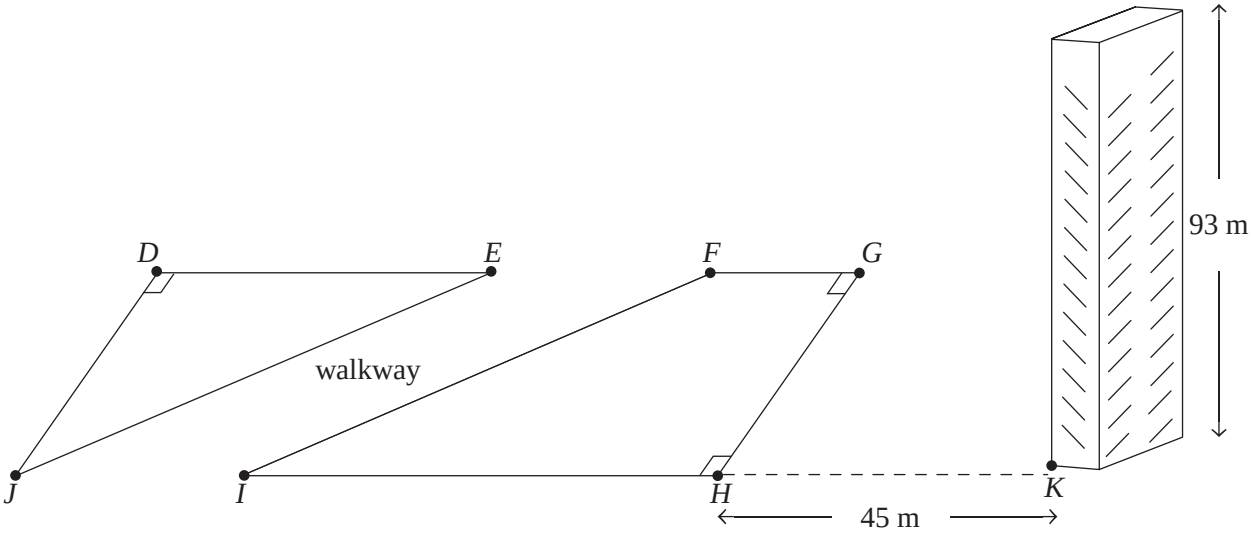
b. Calculate the length of a line on the contour map that represents an actual distance of 135 metres. Write your answer in cm.

_____

_____

1 mark

Question 4



A building close to the rectangular block *DGHJ* is 93 metres tall. The closest point of the building to the block is at point *K* which lies on the straight line *IHK*. The distance between point *H* and point *K* is 45 metres.

Is it possible for the building to land on point *D* if it falls? Explain your answer with relevant calculations.

_____

_____

_____

_____

_____

_____

2 marks

Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1

Bridget operates a parking lot. She charges by the hour or part thereof. Thus if someone parks for 2.3 hours, they will be charged for 3 hours. Charges consist of a $2 fee and a further $1 per hour in addition to that.

a. Frank parks for 3 hours and 37 mins.

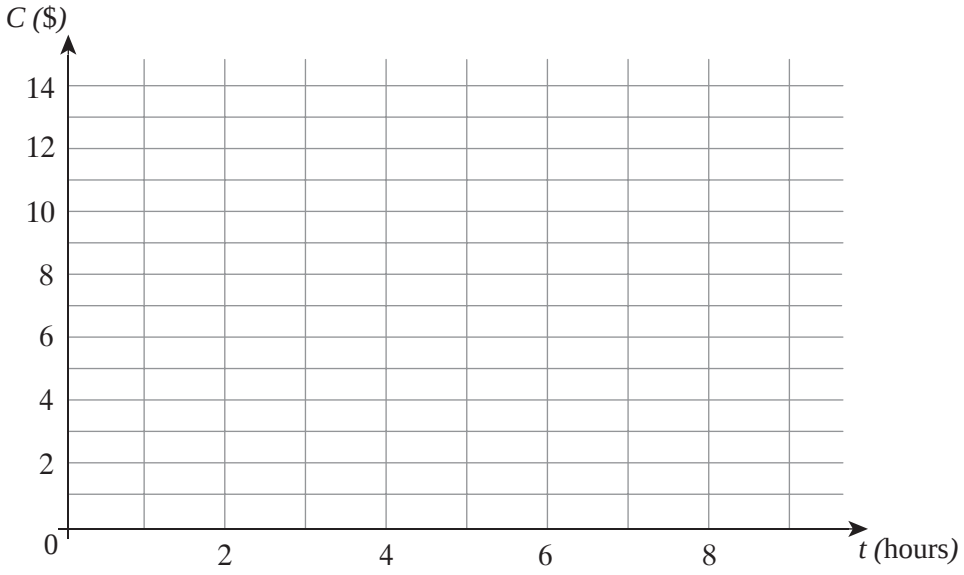
Find how much he must pay.

_____

_____

1 mark

b. Sketch a graph on the axes below of cost against time parked.



2 marks

Bridget pays a set rate for use of the car park land and facilities. It costs her $500 per day to be able to operate the car park. In addition to this, there is a cost of 50 c per car in insurance premiums that she must pay. On a certain day, customers are charged for parking for an average of 9 hours.

c. Find an equation for the cost, C, of operating the car park for one day with x number of cars.

_____

_____

1 mark

d. Find an equation for the total revenue (money received), R, for operating the car park with x number of cars.

_____

_____

1 mark

e. Find the minimum number of cars that need to be parked each day for Bridget to make a profit.

_____

_____

_____

_____

1 mark

Question 2

Bridget is able to buy a new piece of land that can be used as a parking lot. The total land area is 720 square metres. It can be divided into large and small car spaces. A small car space occupies 5 square metres while a large one is 8 square metres. The space allocated for small cars must be at least double that for large cars. The number of large car places must be at least 10.

a. One inequation that applies is $5x + 8y \leq 720$.

Explain why this inequation applies.

_____

_____

_____

1 mark

b. Given $x \geq 0$, write two inequations that describe the other conditions.

_____

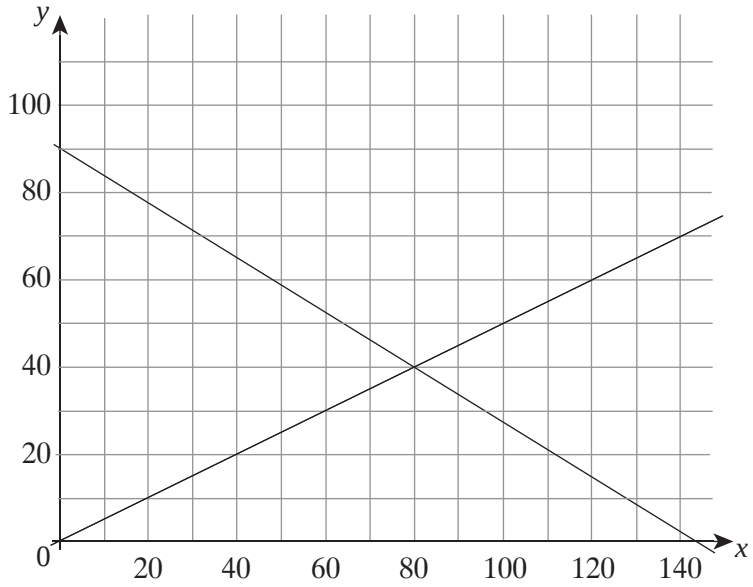
_____

_____

_____

2 marks

c. In the graph below, two of the boundaries are shown.



Mark in any other boundaries that exist and clearly indicate the region that satisfies all of the conditions represented by the inequations above.

2 marks

Bridget can make a profit of $20 per day from a small space and $30 from a large one.

d. Write an equation that gives the profit, *P*, if she allocates *x* small and *y* large parking spaces.

_____

_____

1 mark

e. Hence determine the maximum profit that Bridget can make.

_____

_____

_____

_____

2 marks

f. In order to maximise profit, how many of each size car places should be allocated?

_____

_____

1 mark

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

Book sales have increased over the last few years and nile.com sell both hardcopy and electronic books online.

a. A current vampire novel is priced at $31.50 for the hard cover version and $15 for the eReader version.

Assuming you already own an eReader, what % saving (to one decimal place) can be made by buying the eReader version rather than the hardcover version?

_____

_____

_____

1 mark

b. To use the eReader version you must purchase an e-reader first. It is available for a cash price of $199 or on terms of 20% deposit and 18 monthly payments of $12.

i. Calculate the cost over the 18 months of the e-reader when buying on terms.

_____

_____

_____

1 mark

ii. Calculate the flat interest rate charged to 1 decimal place.

_____

_____

1 mark

iii. Calculate the effective interest rate to 1 decimal place.

_____

_____

1 mark

c. Having purchased the e-reader you find the resale value on eBay has depreciated from the purchase price of $199 to only $45 in 24 months.

Calculate the flat rate of depreciation (to one decimal place).

_____

_____

_____

1 mark

Question 2

a. Rajid is planning to buy his first home in twelve months' time. He has looked into it with the bank and discovers he can afford to make the repayments on a loan of about $250 000 but he must demonstrate a good savings history by saving an amount of $40 000 as a deposit.

How much must Rajid deposit each month to achieve his goal if the bank pays 8% per annum compounding monthly on his savings? (Give your answer to the nearest dollar.)

_____

_____

_____

1 mark

b. Having saved the deposit of $40 000, Rajid finds a studio apartment priced at $320 000 and due to his good savings record he has now qualified for a loan of up to $300 000. The terms are a 25 year loan at an interest rate of 7.75% per annum compounding monthly. Rajid borrows all $300 000, allowing him to purchase furniture with the extra.

What will his monthly repayments be? (Give your answer to the nearest dollar.)

_____

_____

_____

1 mark

c. Another possibility is to take a loan at the same interest rate but for a 20 year period only with increased payments.

How much money is saved under this new plan in comparison to the 25 year plan?

_____

_____

2 marks

d. Rajid decides to stick with the original 25 year period but in order to save interest he pays an extra $500 per month off the loan.

Assuming interest rates stay constant throughout the loan period, how long to the nearest month will it take to pay out the loan?

_____

_____

1 mark

Question 3

The 'Bottletops R Us' company spends $85 000 on a new machine to put caps on bottles of drinking water. It is anticipated that the machine will put tops on 20 million bottles before being scrapped for $4000.

a. Calculate the depreciation rate per bottle capped.

_____

_____

_____

1 mark

b. Calculate the book value after capping on 3 million bottles.

_____

_____

_____

2 marks

c. If the average number of bottles capped in year is 3.5 million, how long to the nearest month until the machine is scrapped?

_____

_____

_____

1 mark

d. The company is offered an alternative to purchasing the machine by a finance company who will lease the machine to them for $1220 a month until the 20 million bottles are topped.

Show by calculation which offer ends up costing the least amount over the life of the machine.

_____

_____

_____

1 mark

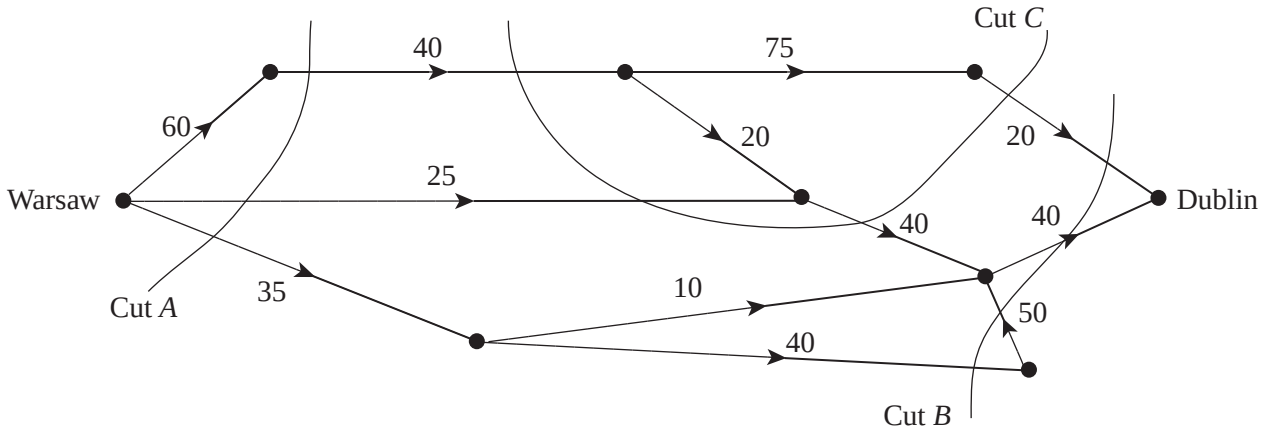
Total 15 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1

Slam Airlines offer low-cost flights throughout Europe. A selection of their flights from Warsaw to Dublin is shown in the network graph below. The number of available seats for each flight is shown as the weight of each edge.



a. Write down the capacity of Cut *A* and the capacity of Cut *B*.

_____

_____

_____

2 marks

b. Cut *C* is not a valid cut when trying to find the maximum flow between Warsaw and Dublin.
Explain why.

_____

_____

1 mark

A large organisation wishes to use Slam Airlines to fly as many people as possible from Warsaw to Dublin.

c. Calculate the maximum number of available seats that exist between Warsaw and Dublin.

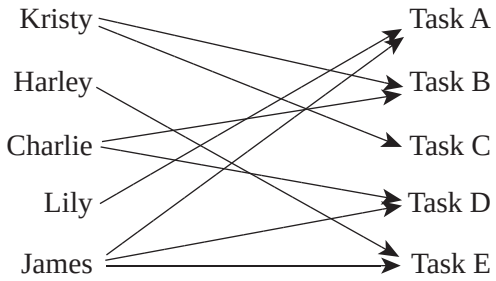
_____

_____

1 mark

Question 2

Slam Airlines employ five people to complete various tasks required to create an online booking facility. The following bipartite graph shows the tasks that each employee is qualified to perform.



a. Which task must James complete?

_____

_____

_____

_____

1 mark

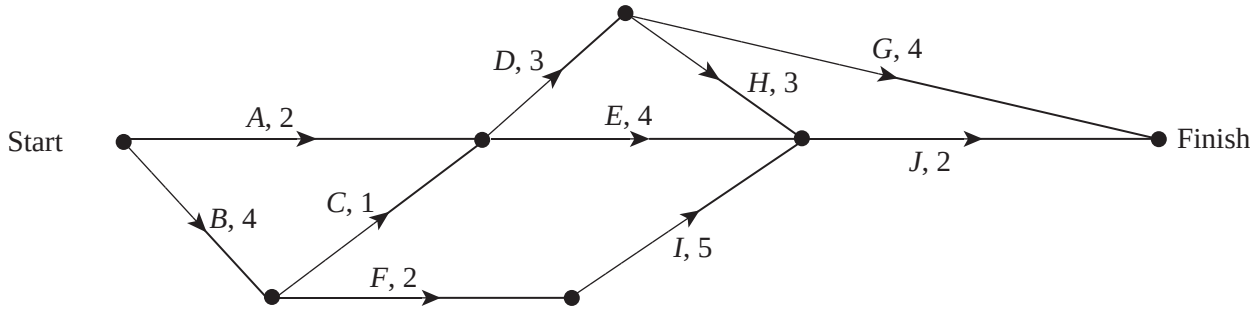
b. Fill in the table below to show the tasks that each employee must complete:

Employee	Task
Kristy	
Harley	
Charlie	
Lily	

2 marks

Question 3

The air traffic controllers at Dublin International Airport are required to complete a project involving safety of incoming flights. The directed graph below shows the activities, their immediate predecessors, and the duration of each activity in minutes.



a. Use the information above to complete the table below filling in the shaded cells.

Activity	Immediate Predecessor	Earliest Start Time	Latest Start Time
A	–	0	3
B	–	0	0
C	*B*	4	4
D	*A, C*	5	5
E	*A, C*	5	
F	*B*	4	4
G	*D*	8	9
H		8	8
I	*F*	6	6
J	*E, H, I*		11

3 marks

b. Write down the number of critical paths that exist

_____

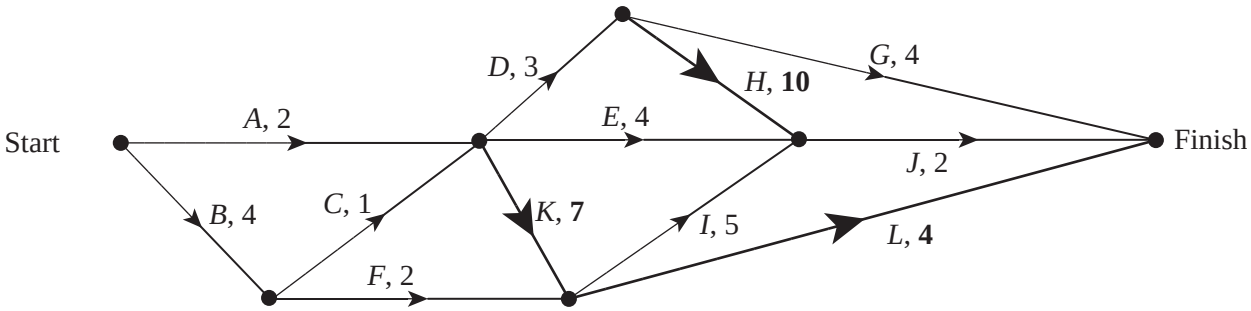
_____

_____

1 mark

Question 4

Due to poor weather at Dublin Airport, new activities, *Activity K* and *Activity L*, are required to complete the project. Also, the duration of *Activity H* has increased. These variations are shown below.



For the **revised** project, write down:

a. all non-critical activities.

_____

_____

1 mark

b. the earliest start time for *Activity H*.

_____

_____

1 mark

c. the difference between the original slack (float) time for *Activity E* and the revised slack (float) time for *Activity E*.

_____

_____

_____

_____

_____

_____

2 marks

Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

Sam operates a shirt factory. They have three styles of shirt and prices are as given in the matrix:

$P = \begin{bmatrix} 20 \\ 23 \\ 28 \end{bmatrix}$ for the standard, business and formal shirts respectively.

The numbers of each sold last week were $N = \begin{bmatrix} 54 & 21 & 18 \end{bmatrix}$ for those three types in the same order.

a. Calculate *PN* and *NP* and state in each case whether the product concerned has any physical meaning and use and, if so, what such meaning and use is.

_____

_____

_____

_____

_____

3 marks

Sam decides to increase prices on all shirts as follows: *x*% increase on standard, *y*% on business and *z*% on formal shirts.

He decides to use a matrix of the form $R = \begin{bmatrix} 1.06 & 0 & 0 \\ 0 & 1.08 & 0 \\ 0 & 0 & 1.10 \end{bmatrix}$.

b. Find the values of *x*, *y* and *z*.

3 marks

Sam has a set of deals available to customers:

1. If you buy 1 business and 2 standard shirts, you get a 20% discount off the total cost.
2. If you buy any formal shirt, you get a standard shirt ½ price
3. If you buy 2 formal shirts, you can get a free business shirt.

$D = \begin{bmatrix} deal\ 1 \\ deal\ 2 \\ deal\ 3 \end{bmatrix}$ is the price of these 3 deals.

For example, the *deal* 1 price is the cost of 1 business and 2 standard shirts after the 20% discount is applied. It may be assumed that those using any deal will buy the minimum amount of merchandise to activate each deal and that comparisons to be made are against the cost of the same set of items if no deal existed. For example, a customer using *deal* 2 buys 1 formal and 1 standard shirt.

c. If $D = CP$, find matrix C.

_____

_____

_____

_____

2 marks

d. Hence find the price of *deal* 2 after the price change.

_____

_____

_____

_____

1 mark

Question 2

Sam knows that the proportions of sales at two retail outlets were 40% at Manton's and 60% at Greatfit in 2009. In 2010, those proportions were 52% Manton's and 48% Greatfit.

A transition matrix of the form $T = \begin{bmatrix} a & 1-a \\ 1-a & a \end{bmatrix}$ applies to sales state data of the form.

a. Show that $a = 0.4$

2 marks

b. Hence determine the proportions for each retailer in 2011.

1 mark

c. Determine the long-term proportions for each retailer.

1 mark

d. If it is assumed that the transition matrix applies to years prior to 2009, use it to find what the proportions would have been in 2008.

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOKLET