Neap:

Trial Examination 2011

VCE Further Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

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SECTION A – Core: Data analysis

Question 1 **E**

3 + 7 + 12 + 8 + 4 + 6 + 8 + 9 + 2 + 7 = 66

Question 2 **D**

S_x for Nick is 3.56 and S_x for Jacob is 2.70. The larger the value of S_x the bigger the spread.

Question 3 **A**

12 is the number of hours spent on the game, and not the mode.

Question 4 **A**

The difference between Q_3 and the maximum is smaller than Q_1 and the minimum. There is also a slight negative skew so the mean is closer to the right hand end.

Question 5 **A**

$$\frac{13.1 + 16.4 + 17.3 + 14.7 + 18.9}{5} = 16.08$$

Question 6 **E**

The data is positively skewed and the mode and IQR are correct.

Question 7 **A**

In this case the replacement of 35 with 25 will only impact upon the mean and standard deviation because 25 remains the extreme value. The mean and standard deviation are impacted as the actual value is used in calculation.

Question 8 **A**

Using $b = \frac{rs_y}{s_x}$, $b = 6.83$, the mean for x and y are calculated using $\frac{\Sigma_x}{n}$ and $\frac{\Sigma_y}{n}$ before using $a = \bar{y} - b\bar{x}$.

Question 9 **B**

Using the formula to calculate the predicted value of absorption gives 47.8.

The residual is actual – predicted = 48 – 47.8 = 0.2

Question 10 **B**

min = 10, Q_1 = 20, med = 20, Q_3 = 30, max = 40

Question 11 **C**

The seasonal indices must add to 4 so the missing seasonal index is 4 – (0.65 + 0.65 + 1.1) = 1.6. The deseasonalised figure is $\frac{4750}{1.6} = 2968.75$.

Question 12 A

Service speed versus shoe size would be expected to have a weak positive correlation (bigger shoe size = taller person = faster serve). Prize money versus matches won is a strong positive correlation.

Question 13 C

81.5% is one standard deviation either side of the mean (68%) and another standard deviation on one side (13.5%).

SECTION B

Module 1: Number patterns

Question 1 **E**

It is an arithmetic sequence so the second term is the mean of the first and third terms. Thus it is 10. Alternatively, an algebraic approach can be used.

$t_1 = a = 18$

$t_3 = a + 2d = 2$

$\therefore 2d = -16$

$d = -8$

$t_2 = a + d = 10$

Question 2 **D**

The sequence is geometric. The common ratio is 0.8. The first term is 15.

$t_4 = ar^3 = 15(0.8)^3 = 7.68$ cm

Question 3 **C**

The second term of an arithmetic sequence is the mean of the first 3 terms. Thus it is $\frac{27}{3} = 9$.

$$\text{mean} = \frac{t_1 + t_2 + t_3}{3}$$

$$= \frac{a + a + d + a + 2d}{3}$$

$$= \frac{3a + 3d}{3} = a + d = t_2$$

Question 4 **D**

Choosing a value of term 1 and thus obtaining a few terms shows that the sequence is neither geometric nor arithmetic. This shows option **D** to be true. All of the other options except **A** are also clearly untrue.

Question 5 **D**

For a sequence to be arithmetic, the points must be equally spaced (as points on a straight line).

For a sequence to be geometric, the points must either move closer to the horizontal (n) axis or increase with increasing rapidity. It is also impossible to have a value of zero in a geometric sequence unless all terms are zero.

Options **C** to **E** are similar but with some important differences. All are a combination of arithmetic and geometric sequences terms. If $a > 1$ or $a < -1$, the gap between terms will increase instead of decrease. Students can see this by trying values in their calculator. The graph clearly shows decreasing separation and thus neither **C** nor **E** could be true.

Question 6 **B**

$$\frac{a}{1-r}(1-r^3) = 57$$

$$\frac{a}{1-r} = 81$$

$$\therefore 81(1-r^3) = 57$$

$$r^3 = \frac{8}{27}$$

$$r = \frac{2}{3}$$

Alternatively the calculator and some logic may be used. The common ratio must be less than 1 and thus only **A** and **B** are credible. Each option can be tried and only **B** will be found to be credible.

Question 7 **B**

$$S_4 = 14$$

$$S_5 - t_1 = 18$$

$$t_5 + 14 - t_1 = 18$$

$$a + 4d - a = 4$$

$$d = 1$$

Question 8 **A**

In order to calculate each term, the existing frog population is multiplied by 0.8 and then 15 is added. It is clear that the 15 new frogs are added after the death/survival calculation.

Question 9 **E**

This is a geometric sequence of improvements but it must be noted that alternate terms (including term 1) are negative values as they involve time reductions.

$$a = -4$$

$$r = 0.75$$

$$S_\infty = \frac{a}{1-r} = \frac{-4}{1-(-0.75)} = \frac{-16}{7}$$

$$\text{time} = 27 - \frac{16}{7} = 24.714$$

Module 2: Geometry and trigonometry

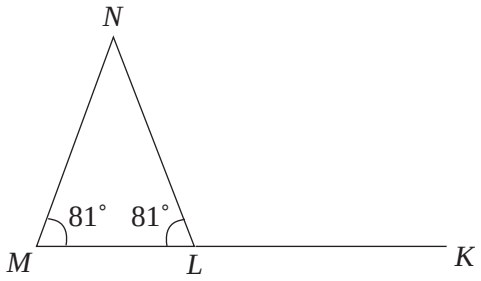
Question 1 **B**

$(YZ)^2 = 39^2 - 36^2$

$(YZ)^2 = 225$

$YZ = 15$ cm

Question 2 **B**



$81° + 81° = 162°$

$180° - 162° = 18°$

Question 3 **A**

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{83}{\sin 48°} = \frac{BK}{\sin 39°}$$

$$\frac{83 \times \sin 39°}{\sin 48°} = BK$$

$70.287\ldots = BK$

Question 4 **D**

The cosine rule calculates a distance when two distances and the included angle are known.

$a^2 = b^2 + c^2 - 2bc \cos A$

Question 5 **C**

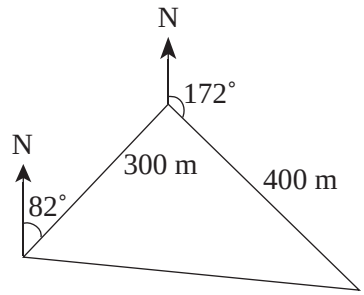
$a + b + c = 91.5$, $s = \frac{95.1}{2} = 47.55$, $a = b = c = \frac{95.1}{3} = 31.7$

so $A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{47.55(47.55 - 31.7)(47.55 - 31.7)(47.55 - 31.7)}$

$A = 435.1301$

Question 6 **D**



The third jog completed by Hal can be calculated via the cosine rule.

A simple method is to recognise that the triangle contains a right angle.

$h^2 = 300^2 + 400^2$

$h^2 = 25000$

$h = 500 \text{ m}$

Total $= 300 + 400 + 500 = 1200 \text{ m}$.

Question 7 **C**

$\text{Area} = \pi d \times 2.5$

$= \pi \times 22 \times 2.5$

$= 172.783$

Question 8 **E**

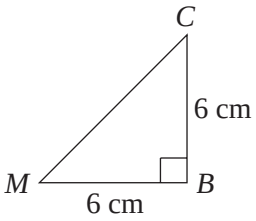
$$\frac{\text{area building}}{\text{area drawing}} = k^2$$

$$\frac{36000000}{36} = k^2$$

$$1000000 = k^2$$

$$1000 = k$$

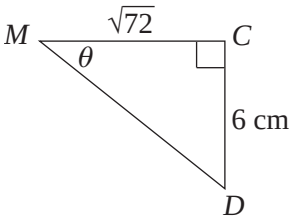
Question 9 **C**



$CM^2 = 6^2 + 6^2$

$CM^2 = 72$

$CM = \sqrt{72}$



$$\frac{6}{\sqrt{72}} = \tan\theta$$

$35.2644° = \theta$

$35.3° = \theta$

Module 3: Graphs and relations

Question 1 D

The fastest change does not require an increase. The rate of change was greatest from November to December – a rapid decrease.

Question 2 C

Students can perform a process of trial and error. Each of the two points given must work in the correct equation. Thus it may have been necessary to perform 10 such calculations before the correct option is detected.

Alternatively, students can determine the gradient between the points and eliminate other equations on that basis. Only options **C** and **D** have the correct gradient. Either point can be chosen to determine which of these two options is valid.

Question 3 D

Cost: $C = 100 + 3n$

Revenue: $R = 5n$

Profit: $P = R - C$

$= 2n - 100$

Question 4 B

It is vital to note that it is the unshaded region that is necessary to describe, not the shaded region. Two clear boundaries appear on the graph. The first is $y = 2x$. The region required is above this boundary line (larger y-value) so inequation IV is valid.

The second boundary line has x-intercept 3 and y-intercept 2. This is consistent with inequation I. Inequations II and III are alternatives to I and IV and are thus invalid and incorrect. Finally, it can be noted that inequation V is also true for the required region so it must be included in the correct response.

Question 5 A

The total protein for five geese is $5 \times 15 = 75$. The total protein gained from x pili nuts is $2x$. That gained from y peanuts is $3y$. Thus the total protein is $2x + 3y$ and this figure must be at least 75.

Question 6 E

There are many ways to determine the correct answer to this question. If students graph the two lines it will become obvious that they are parallel and thus never meet. Students can achieve the same result by checking the gradient of each line.

Students who use any algebraic method to find the intersection of the lines will not arrive at a solution.

Question 7 E

The relationship is clearly not linear. The values of y are decreasing toward zero so quadratic and cubic relationships are also impossible. Thus options **A**, **B** and **C** are ruled out. Students can test option **D** by plotting a few points on axes y against $\frac{1}{x}$. The result is not a straight line and thus is clearly not the correct response. Plotting y against $\frac{1}{x^2}$, however, does produce a straight line so that is correct.

Question 8 B

One approach to this question would be to calculate the value of C at each of the critical points on the graph:

At (0, 9), $C = 45$

At (1.5, 4.5), $C = 31.5$

At (2, 4), $C = 32$

At (4, 2), $C = 34$

At (8, 0), $C = 48$

The least value is at (1.5, 4.5).

Question 9 C

The cost equation can be obtained from the graph: $C = 5 + \frac{3n}{4}$

The revenue is $3n$ and thus we need to solve $3n = 5 + \frac{3n}{4}$. The result is $n = 20$.

An alternative approach would be graphical using the same equations.

Module 4: Business-related mathematics

Question 1 **E**

$87 \times 0.8 \times 0.95 = \$66.12$

Question 2 **D**

$1500 \times 1.0009^{30} = 1541.03$, so the interest is closest to \$41.

Question 3 **B**

Repayment is $1500 \times 0.05 = 75$ which is greater than \$40, thus the \$75 charge applies.

Question 4 **D**

Transposing Q and substituting in $A = \frac{Q(R^n - 1)}{R - 1}$ gives $Q = \frac{A(R - 1)}{R^n - 1}$

$$R = 1 + \frac{r}{100} = 1 + \frac{5}{\frac{12}{100}} = 1.00417$$

$n = 24$ $A = 35\ 000$

Question 5 **D**

Year 1 $25\ 000 \times 0.75 = \$18\ 750$

For years 2 to 4, 85% of the value is retained each year. Therefore:

Year 4 $18\ 750 \times 0.85^3 = \$11\ 514.84$

Question 6 **D**

$\frac{4}{12}$% on the minimum balance of \$3100 is \$10.33. Add this amount to the balance:

\$5800 + \$10.33 = \$5810.33.

Question 7 **D**

This question requires two applications of the TVM-solver or equivalent. Using the calculator to firstly find the repayments for a \$200 000 loan to be repaid in 10 years we find repayments of \$2348. To find the balance owing after 3 years change *n* to 36 and look for the amount still owing.

Question 8 **C**

The total amount repaid is $(36 \times 138 + 0.2 \times 4560) = \$5880$. Subtracting the cash price of \$4560 gives a saving of \$1320.

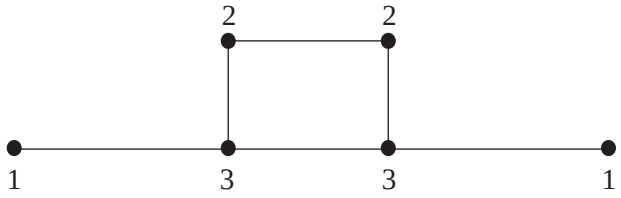
Question 9 **C**

$r = \frac{1320}{3648 \times 3} = 12.06\%$

Effective interest is calculated using $r_e = \frac{2 \times 36 \times 12.06}{36 + 1} = 23.47\%$.

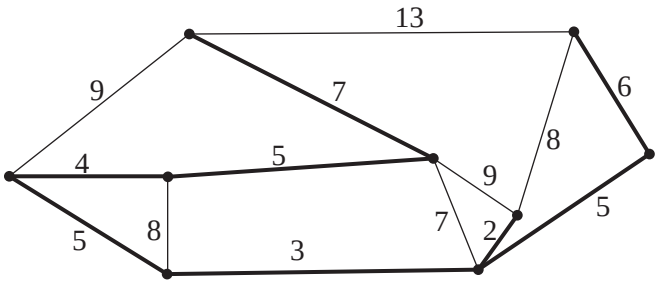
Module 5: Networks and decision mathematics

Question 1 **D**



As shown above, there are four vertices with an odd degree.

Question 2 **E**



To solve this question, mark select edges as shown above and total their values.

$2 + 3 + 5 + 4 + 5 + 7 + 5 + 6 = 37$

Question 3 **C**

$U \to Y \to X \to Z \to V \to W \to U$ is the only Hamiltonian circuit as it visits each vertex only once and returns to the starting vertex.

Question 4 **D**

D to *E* is the only option that maintains exactly two vertices of odd degree, as necessary for an Euler path.

Question 5 **A**

A Hamiltonian circuit visits every vertex only once and starts and finishes at the same vertex.

Question 6 **E**

$v + f = e - 2$

$9 + 5 = e - 2$

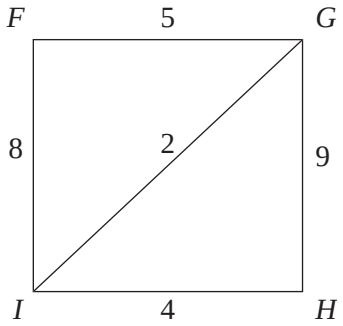
$14 = e - 2$

$16 = e$

This shows that 16 edges and 9 vertices exist.

Question 7 **A**

The network generated from the matrix is:



The shortest path from F to H is $F \to G \to I \to H$, so $5 + 2 + 4 = 11$.

Question 8 **B**

The critical path is $13 + 12 + 10 + 6 = 41$. Another path is $17 + 6 + 10 + 6 = 39$. Activity P can therefore be crashed for 2 minutes; crashing for any more than two minutes is of no value.

Question 9 **E**

n	$k_n = \frac{n(n-1)}{2}$
12	$\frac{12 \times 11}{2} = 66$
13	$\frac{13 \times 12}{2} = 78$
14	$\frac{14 \times 13}{2} = 91$
15	$\frac{15 \times 14}{2} = 105$

The table above shows the only possible combination of vertices and edges. Of the combinations shown, only $v = 15$ and $e = 105$ is allowed.

Module 6: Matrices

Question 1 **D**

Singular means that the determinant of the matrix is zero, thus $3a - 2 = 0$.

Question 2 **D**

$$\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$$

$\Delta = 2(2) - 3(-3) = 13$

$$A^{-1} = \frac{1}{13} \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \end{bmatrix}$$

Question 3 **A**

$AB = C$

Firstly we know that the number of columns of A must match the rows of B in order for multiplication to be possible. Thus B has 3 rows.

Secondly, it is true that the columns of B must match the columns of the product matrix C. Thus B must have 4 columns.

Question 4 **C**

$$AC = \begin{matrix} \begin{bmatrix} J & F \end{bmatrix} \\ \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix} \\ \begin{matrix} B \\ S \end{matrix} \end{matrix} \begin{matrix} \begin{bmatrix} K & L \end{bmatrix} \\ \begin{bmatrix} 7 & 8 \\ 9 & 9 \end{bmatrix} \\ \begin{matrix} J \\ F \end{matrix} \end{matrix} = \begin{matrix} \begin{bmatrix} K & L \end{bmatrix} \\ \begin{bmatrix} 57 & 60 \\ 73 & 77 \end{bmatrix} \\ \begin{matrix} B \\ S \end{matrix} \end{matrix}$$

The multiplication above shows that the columns represent factories and the rows represent the bulb types.

Question 5 **C**

$A_{2,2}$ is a matrix element. It is not a matrix of any order.

Question 6 **C**

Since A has 3 columns, BC must have 3 rows and thus B must have 3 rows. This is simply a definition of order of matrix multiplication. Also, since a transition matrix is square, B is 3×3. It also follows that C must have 3 rows. It is also true that P and C must have the same number of columns and that P must have 5 rows.

We cannot uniquely determine the order of P and C but some combinations are impossible.

Options **A** and **B** are already contradicted by information already determined.

For option **D** to be true, C must have order 3×3. However, this gives order 5×3 when multiplied by A which contradicts the statement that P and C are the same order.

Option **E** has a problem in that P must have 5 rows and C must have 3 rows. They are not the same order. This fits option **C** quite well, however.

Question 7 D

In order to multiply/allocate correctly, we require a 3×3 diagonal matrix. 2% reduction indicated 98% retention and thus the first element in the first row is 0.98. The other 2 diagonal elements are thus 0.95 and 0.92 for the same reason.

Question 8 D

$$T^{50} = \begin{bmatrix} 0.269 & 0.269 & 0.269 \\ 0.346 & 0.346 & 0.346 \\ 0.384 & 0.384 & 0.384 \end{bmatrix}$$

$$S_{50} = T^{50}S_0 = \begin{bmatrix} 0.269(a+b+c) \\ 0.346(a+b+c) \\ 0.384(a+b+c) \end{bmatrix}$$

Question 9 A

$$F = \begin{bmatrix} 0.90 & 0 & 0 \\ 0 & 0.85 & 0 \\ 0 & 0.05 & 0.80 \end{bmatrix} \begin{bmatrix} 1.10 & 0 & 0.1 \\ 0 & 1.06 & 0 \\ 0.1 & 0 & 1.05 \end{bmatrix}$$

$$= \begin{bmatrix} 0.99 & 0.005 & 0.08 \\ 0 & 0.901 & 0 \\ 0.09 & 0.0525 & 0.84 \end{bmatrix}$$

The second row relates to the new hardware prices. As can be seen, it is only column 2 that is non-zero. This column corresponds to the old hardware prices and thus only the old hardware price influences the new hardware price. This shows option **B** to be false. The new price is 0.901 times the old price. This represents a reduction of 9.9%.