INSIGHT

YEAR 12 *Trial Exam Paper*

2011

FURTHER MATHEMATICS
UNIT 4

Written examination 2

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions.
- tips and guidelines.

SECTION A
Core: Data analysis

Question 1

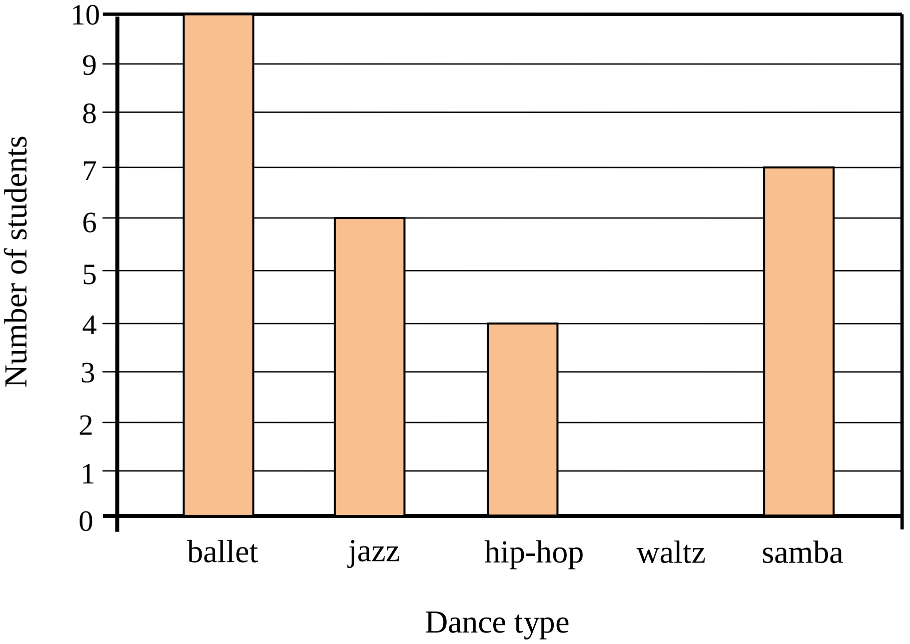
Shining Star Dance School teaches five different types of dance (ballet, jazz, hip-hop, waltz and samba) to four age groups (3–6 years, 7–10 years, 11–14 years, and 15 years and above). A survey of thirty-two Shining Star Dance School students who are in the 7–10 age group is conducted to find out what type of dance they enjoyed most.

Their responses are recorded below.

ballet	ballet	jazz	samba	jazz	hip-hop	ballet	samba
jazz	samba	waltz	waltz	hip-hop	samba	hip-hop	ballet
ballet	samba	jazz	waltz	ballet	jazz	ballet	samba
waltz	samba	ballet	jazz	hip-hop	waltz	ballet	ballet

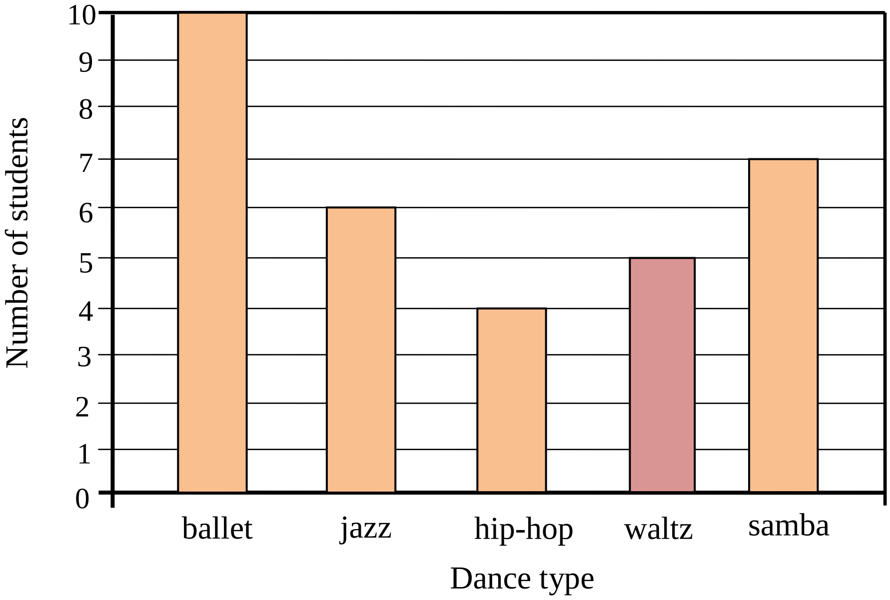
Use the data to:

a. complete the following bar chart by adding the missing column for waltz.



Worked solution

There are five students who enjoy waltz most.



1 mark

b. Determine the percentage of 7–10 year old Shining Star Dance School students who enjoyed jazz dancing most.

Worked solution

Six out of 32 students enjoyed jazz dancing most.

$\frac{6}{32} \times 100\% = 18.75\%$

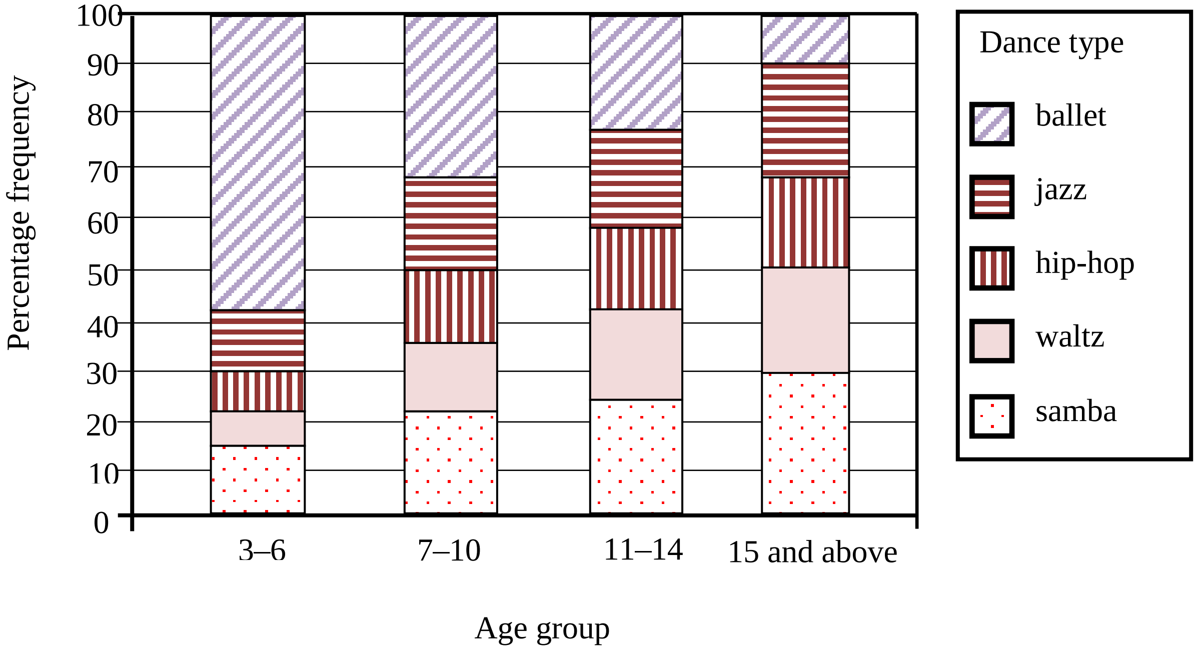
1 mark

Tip

- *Remind students that since the question does not indicate to round the answer, they will need to find the exact answer and should not round it.*

SECTION A – continued
TURN OVER

Question 2
The survey from Question 1 is conducted again, only this time all age groups of the dance school are surveyed. The results are displayed in the percentage segmented bar chart below.



Does the percentage segmented bar chart support the opinion that, for these students, the dance type that they enjoy most is associated with their age group? Justify your answer by quoting appropriate percentages.

Worked solution
Yes, the percentage segmented bar chart supports the opinion that, for these students, the dance type that they enjoy most is associated with their age group.
58% of students in 3-6 age group, 32% of students in 7-10 age group, 23% of students in11-14 age group and 10% of students in 15 and above age group enjoy ballet most. The percentage of students who enjoy ballet most decreases with age.

2 marks

Question 3
The birth years of 25 randomly chosen students of the Shining Star Dance School are given in the table below.

Name	Birth year
Abigail	2007
Timothy	1991
Bella	1994
Lara	1994
Mia	1996
Josh	1993
Jaden	1993
Sera	1993
Emma	1994
Craig	1998
Keith	1993
Ashley	1997
Stephanie	1993

Name	Birth year
Tyler	1992
Hattie	2001
Rebecca	1996
Jordan	1998
Courtney	1994
Matthew	1992
Joanne	1997
Vanessa	1992
Miguel	1991
Tiffany	1995
John	1993
Brennan	1999

a. What proportion of these 25 students are older than Tiffany?

Worked solution
Tiffany was born in 1995. The students who are older than Tiffany have birth years earlier than hers.

15 out of 25 students are older than Tiffany.

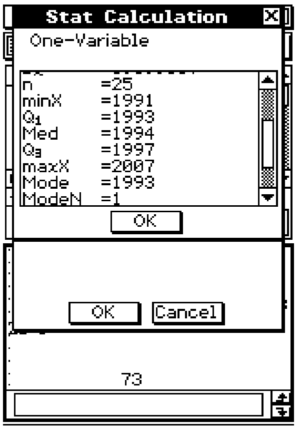
$\frac{15}{25} \times 100\% = 60\%$

1 mark

b. Complete the following sentence about the 25 Shining Star Dance School students:
The middle 50% of these students have birth years between ☐ and ☐ .

1 mark

Worked solution
Let’s calculate the five-figure summary of this data. The middle 50% of the data lies between Q_1 and Q_2. As you will see from the calculator screen below, Q_1 is 1993 and Q_2 is 1997.



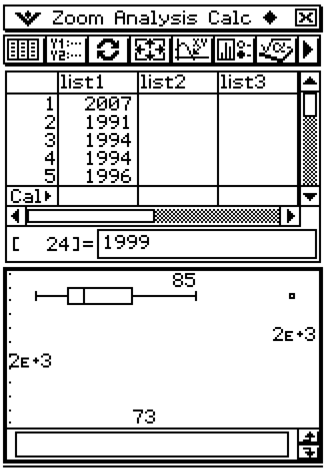
```
n      =25
minX   =1991
Q1     =1993
Med    =1994
Q3     =1997
maxX   =2007
Mode   =1993
ModeN  =1
```

The middle 50% of these students have birth years between 1993 and 1997.

1 mark

c. For this distribution, the median is a more appropriate measure of centre than the mean. Explain why.

Worked solution
Let’s use a calculator to determine the shape of the distribution.



	list1	list2	list3
1	2007		
2	1991		
3	1994		
4	1994		
5	1996		

```
[  24]=1999
```

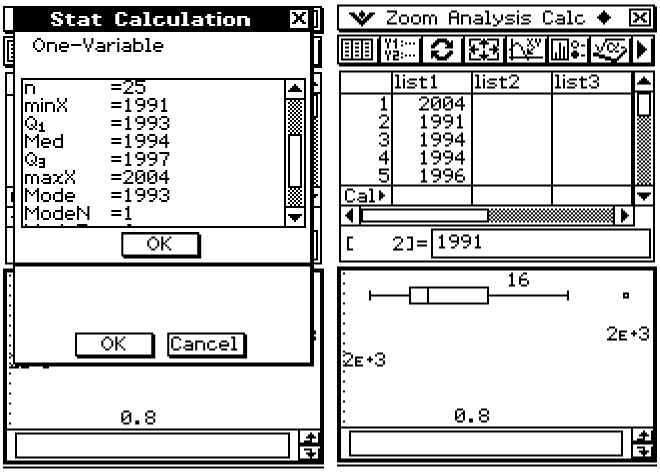
As seen from the boxplot, the distribution is positively skewed with an outlier. In this case the median is a more appropriate measure of centre than the mean because the median is not affected by extreme values, but the mean is.

1 mark

d. The birth year of Abigail is given as an outlier. This value is an error. Her real birth year is 2004. If the error is corrected, would Abigail's birth year still be an outlier? Give reasons for your answer showing an appropriate calculation.

Worked solution

Let's change Abigail's birth year from 2007 to 2004 and calculate the five-figure summary of this data once again.



```
Stat Calculation
One-Variable
n        =25
minX     =1991
Q1       =1993
Med      =1994
Q3       =1997
maxX     =2004
Mode     =1993
ModeN    =1
```

```
     list1   list2   list3
1    2004
2    1991
3    1994
4    1994
5    1996
[    2]= 1991
```

Now let's calculate the upper fence to see whether 2004 is still an outlier or not.

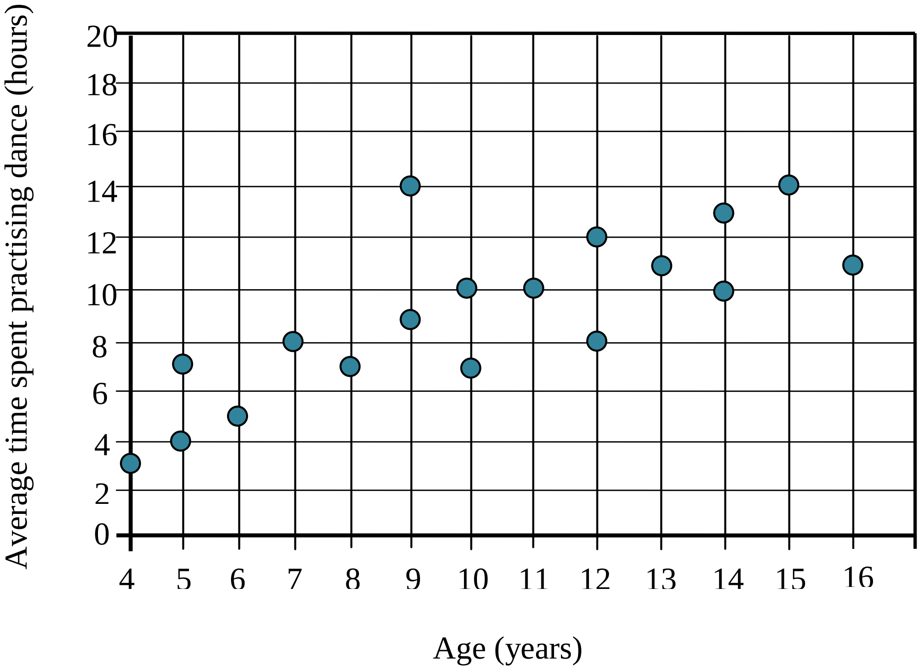
upper fence = Q_2 + 1.5 × IQR = 1997 + 1.5 × 4 = 2003

Since 2004 is a greater number than 2003, i.e., the upper fence, it is still an outlier.

1 mark

Question 4

In the scatter plot below, *average time spent practising dance each week*, in hours, is plotted against *age*, in years, of a random sample of 18 dance school students.

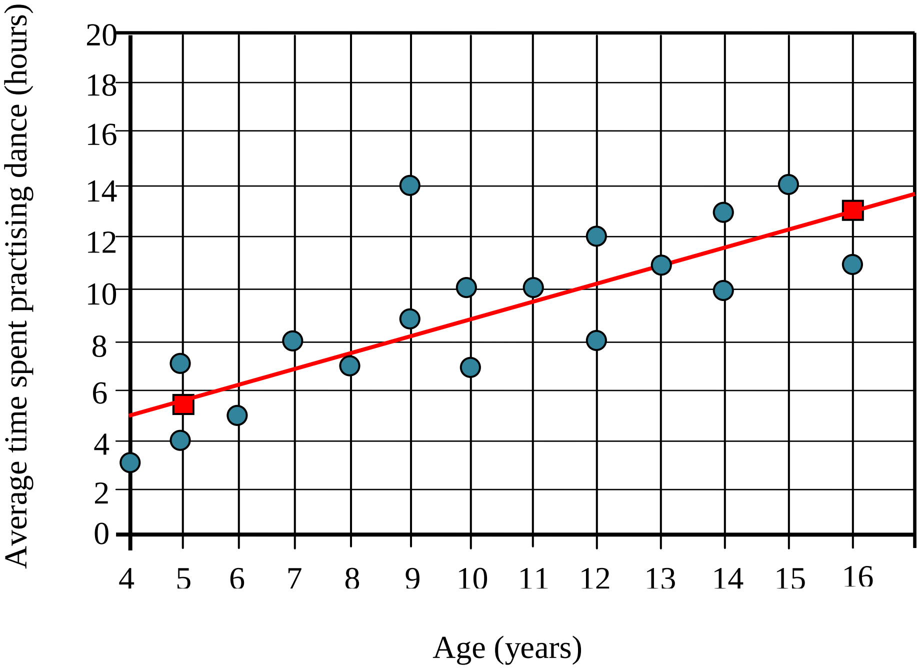


The equation of the least squares regression line for this data set is:
average time spent practising dance = 2.1696 + 0.6886 × age

a. Draw this least squares regression line on the scatterplot.

Worked solution

Let's choose two random points on the straight line.
When $x = 5$, $y = 2.1696 + 0.6886 \times 5 \cong 5.6$. So the line goes through the point (5, 5.6).
When $x = 16$, $y = 2.1696 + 0.6886 \times 16 \cong 13.2$. So the line goes through the point (16, 13.2).



1 mark

SECTION A – continued

Tip

- *Explain to students that they could also use the y-intercept as one of the points. But, since the x-axis starts at 4 instead of 0, this would not be appropriate for this question. Actually this is a mistake that students often make.*

b. One of the dance school students is 16 years old. The equation of the least squares regression line is used to predict the average time that this student spends practising dance each week.

Find the residual value for this prediction, in hours, correct to one decimal place and state whether this student's average weekly dance practice time is underestimated or overestimated by the least squares regression equation.

Worked solution

When $x = 16$, $y_{\text{predicted}} = 2.1696 + 0.6886 \times 16 = 13.1872$

$$\begin{aligned}\text{residual value} &= y_{\text{actual}} - y_{\text{predicted}}\\ &= 11 - 13.1872\\ &= -2.1872\\ &\cong -2.2\end{aligned}$$

So, this student's average weekly dance practice time is overestimated by the least squares regression equation.

1 mark

Tip

- *When the least squares regression line is above the actual point on the graph, the line overestimates the value. When it is below the point on the graph, it underestimates the value.*

c. The Pearson's product moment correlation coefficient for this data set is 0.7817.

i. Evaluate the coefficient of determination.
Write your answer, as a percentage, correct to two decimal places.

ii. Interpret the coefficient of determination in terms of the variables *average time spent practising dance each week* and *age.*

Worked solution

c. i.

$$\begin{aligned}r^2 &= 0.7817^2\\ &= 0.6111\\ &= 61.11\%\end{aligned}$$

1 mark

ii. 61.11% of the variation in *average time spent practising dance each week* can be explained by the variation in *age.*

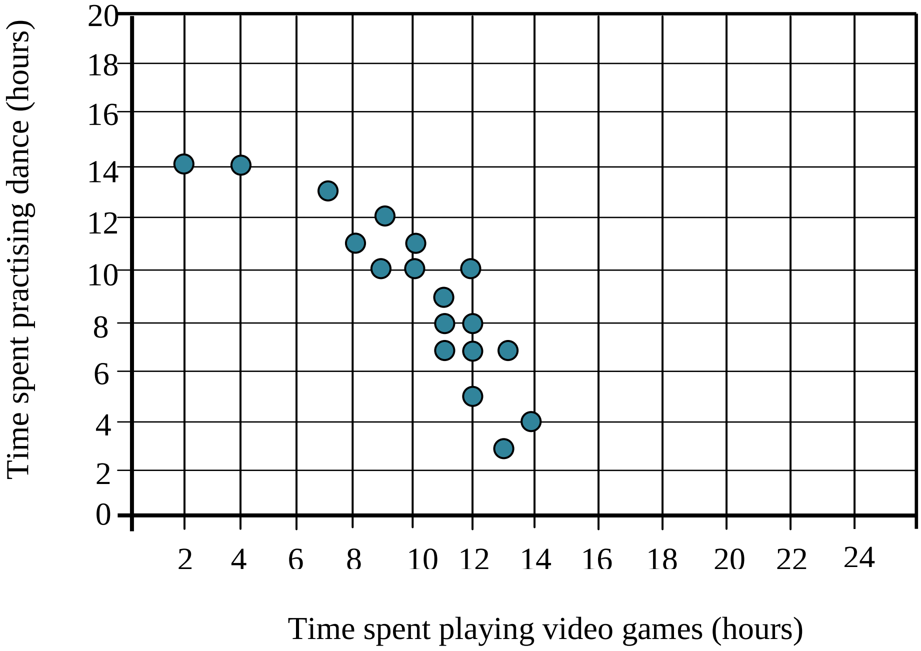
1 mark

1 + 1 = 2 marks

Question 5

Average time spent practising dance each week, in hours, and *average time spent playing video games each week*, in hours, were recorded for the same group of 18 dance school students.
The results are displayed in the table below and a scatterplot was constructed as shown.

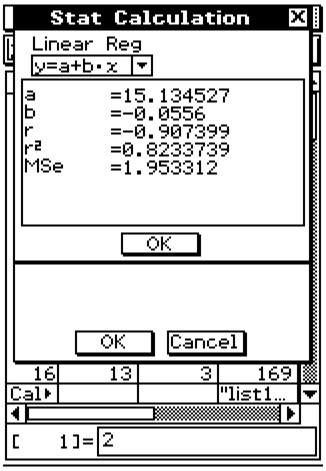
Time spent playing video games (hours)	**Time spent practising dance (hours)**
2	14
4	14
7	13
8	11
9	10
9	12
10	10
10	11
11	7
11	8
11	9
12	5
12	7
12	8
12	10
13	3
13	7
14	4



The relationship between *time spent practising dance* and *time spent playing video games* is clearly nonlinear.
Applying a **squared** transformation to the variable, *time spent playing video games*, will linearise this scatterplot.

a. Apply the squared transformation to the data and determine the equation of the least squares regression line that allows *time spent practising dance* to be predicted from the square of *time spent playing video games*.
Write the coefficients correct to three decimal places.

Worked solution



```
a       =15.134527
b       =-0.0556
r       =-0.907399
r²      =0.8233739
MSe     =1.953312
```

time spent practising dance = 15.135 – 0.056 × time spent playing video games^2

2 marks

Mark allocation

- Allocate 1 mark for each correct coefficients.

b. Use the least squares regression line to predict the number of hours that a dance school student who spends 6 hours per week playing video games spends practicing dance each week. Give your answer correct to one decimal place.

Worked solution

time spent practising dance = 15.135 – 0.056 × 6^2
= 13.1 hours

1 mark

Total 15 marks

END OF CORE
END OF SECTION A

END OF SECTION A
TURN OVER

SECTION B

Module 1: Number patterns

Question 1

Gemma owns a flower shop. She opens her shop at 6:00 am every morning when she has 319 bunches of fresh flowers delivered to be sold daily. She manages to sell 37 bunches of flowers each hour.

a. How many bunches of flowers will Gemma have in the shop at 9:00 am?

Worked solution

Gemma would sell $37 \times 3 = 111$ bunches of flowers in 3 hours.
She would have $319 - 111 = 208$ bunches of flowers left in the shop.

1 mark

b. The number of bunches of flowers, A_n, remaining in the shop n hours after it opened can be written as $A_n = a - 37(n - 1)$.
What is the value of a?

$a =$ ☐

Worked solution

Gemma would have $319 - 37 = 282$ bunches of flowers in the shop 1 hour after she opened the shop. Let's substitute this value into the arithmetic sequence rule:

$A_n = 282 - 37(n - 1)$

So: $a = 282$

1 mark

Tip

- *A common mistake that students usually make is to use* $t_0 = 319$ *in the arithmetic sequence rule* $A_n = a + (n - 1)d$ *instead of* $t_1 = 282$.

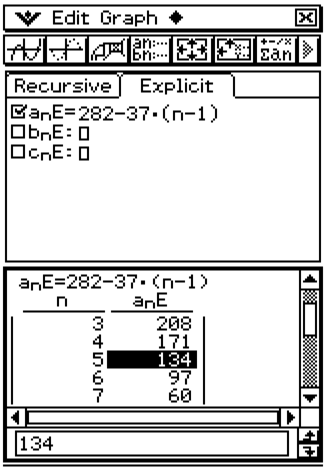
c. Gemma needs to sell 180 bunches of flowers to break even. After every hour she checks the number of flowers left in the shop. At what time will she first notice that she is making profit?

Worked solution

If Gemma sells 180 bunches of flowers, she will have $319 - 180 = 139$ bunches of flowers left in her shop.

SECTION B – continued

Let’s use a calculator to generate the sequence.



☑ a_nE=282-37·(n-1)
☐ b_nE: ▯
☐ c_nE: ▯

a_nE=282-37·(n-1)

n	a_nE
3	208
4	171
5	134
6	97
7	60

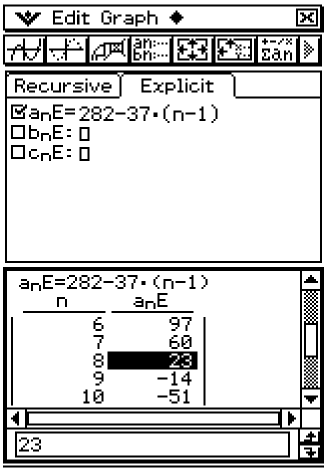
134

After 5 hours, 134 bunches of flowers are left in the shop which means that 185 bunches of flowers have been sold and a profit has been made. Gemma will first notice that she is making profit 5 hours after opening her shop (that is, at 11:00 am).

1 mark

d. For how many full hours can Gemma sell 37 bunches of flowers per hour?

Worked solution



☑ a_nE=282-37·(n-1)
☐ b_nE: ▯
☐ c_nE: ▯

a_nE=282-37·(n-1)

n	a_nE
6	97
7	60
8	23
9	-14
10	-51

23

As we see from the table on the calculator screen (previous page), there will be 23 bunches of flowers left 8 hours after Gemma opens her shop. Although she can continue selling in the ninth hour, she can only sell 37 bunches of flowers for exactly 8 full hours.

1 mark

e. The number of bunches of flowers, A_n, remaining in the shop n hours after opening the shop is given by the difference equation:

$$A_{n+1} = cA_n + d, \qquad A_0 = e$$

Write the values of c, d and e in the boxes below.

c = ☐ d = ☐ e = ☐

SECTION B – continued
TURN OVER

Worked solution

The difference equation representing this sequence is:

$A_{n+1} = A_n - 37, \qquad A_0 = 319$

So: $c = 1 \qquad d = -37 \qquad e = 319$

2 marks

Tip

- *The answer e = 282 is not accepted because the question clearly asks for A_0 , not A_1.*

Mark allocation

- Allocate 2 marks for two correct values. Allocate 1 mark if only two of the three values are correct. Allocate 0 marks for zero or one correct value.

Question 2

At the start of the day, Gemma sells each bunch of flowers for $30. Every hour, she reduces this price by 20% in order to attract more customers.

a. **i.** Brian wants to buy flowers for his daughter's engagement party. He goes to Gemma's flower shop at 9:00 am. How much is Brian going to pay for a bunch of flowers?

ii. Brian's budget for flowers is $245. How many bunches of flowers can he afford with his budget?

iii. How many more bunches of flowers would Brian afford with his budget of $245 if he bought them at 3:00 pm instead of at 9:00 am?

Worked solution

a. **i.** This is a geometric sequence with $r = 1 - \frac{20}{100} = 0.8$.

9:00 am is the start of the fourth hour after 6:00 am. The price of a bunch of flowers will be:

$30 \times 0.8^{4-1} = 30 \times 0.8^3$
$= \$15.36$

1 mark

ii. $245 ÷ $15.36 = 15.95

Brian can afford 15 bunches of flowers.

1 mark

iii. 3:00 pm is the start of the tenth hour after 6:00 am. The price of a bunch of flowers would be

$30 \times 0.8^{10-1} = 30 \times 0.8^9$
$= \$4.0265$

$245 ÷ $4.0265 = 60.85

Brian could afford to buy 60 bunches of flowers if he bought them at 3:00 pm.
He would afford 60 – 15 = 45 more bunches of flowers with his budget if he bought the flowers at 3:00 pm instead of at 9:00 am.

1 mark

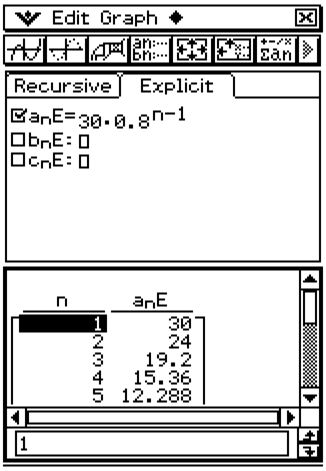
1 + 1 + 1 = 3 marks

Tip

- *Although mathematically, 15.95 would usually be rounded up to 16, Brian's money would only be enough for 15 bunches of flowers.*

b. How much money is Gemma going to earn from selling flowers in her shop's first 4 hours of operation?
Write your answer correct to the nearest cent.

Worked solution
We will first write the rule for this geometric sequence.
$t_n = 30 \times 0.8^{n-1}$
Now let's use a calculator to generate this sequence.



☑anE=$30 \cdot 0.8^{n-1}$
☐bnE:
☐cnE:

n	anE
1	30
2	24
3	19.2
4	15.36
5	12.288

1

The first hour, Gemma sells 37 bunches of flowers for $30 each, making revenue of:
37 × $30 = $1 110
The second hour, Gemma sells 37 bunches of flowers for $24 each, making revenue of:
37 × $24 = $888
The third hour, Gemma sells 37 bunches of flowers for $19.20 each, making revenue of:
37 × $19.20 = $710.40
The fourth hour, Gemma sells 37 bunches of flowers from $15.36 each, making revenue of:
37 × $15.36 = $568.32

In her shop's first four hours of operation, Gemma makes a total revenue of:
$1 110 + $888 + $710.40 + $568.32 = $3 276.72

1 mark

Tip

- *A common mistake made by students is to write the answer as $3276.70 because the question asks for the answer to be written correct to the nearest cent. $3276.72 is already written correct to the nearest cent.*

SECTION B – continued
TURN OVER

c. Determine Gemma’s total revenue from 10:00 am to 6:00 pm inclusive.
Write your answer correct to the nearest cent.

Worked solution

There are 12 hours from 6:00 am to 6:00 pm. We will use the geometric sum formula:

$$37 \times S_{12} = 37 \times 30 \times \frac{1 - 0.8^{12}}{1 - 0.8}$$

$$= 37 \times 139.69$$

$$= \$5168.6069$$

Now to find the revenue from **10:00 am to 6:00 pm**, we need to subtract from this the revenue for the first four hours:

$5168.6069 – $3276.72 = $1891.89

1 mark

Question 3

William also sells flowers. Just like Gemma, he opens his shop at 6:00 am every morning. The difference equation below provides a model for predicting the price of a bunch of flowers in William’s shop.

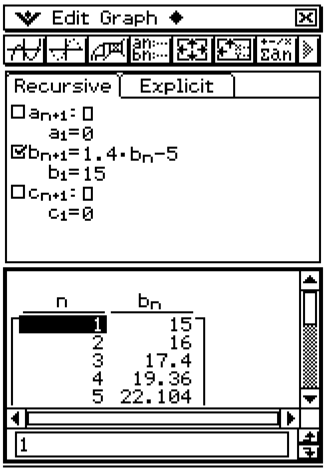
$B_{n+1} = 1.4B_n - 5, \quad B_1 = 15$

where B_n is the price of a bunch of flowers at the start of the nth hour after opening the shop.

a. Show that the sequence generated by this difference equation is neither arithmetic nor geometric.

Worked solution

First, we need to use a calculator to generate the sequence.



☐a_{n+1}: ☐
$a_1=0$
☑$b_{n+1}=1.4\cdot b_n-5$
$b_1=15$
☐c_{n+1}: ☐
$c_1=0$

n	b_n
1	15
2	16
3	17.4
4	19.36
5	22.104

1

$B_2 - B_1 = 16 - 15 = 1$

$B_3 - B_2 = 17.4 - 16 = 1.4$

Since $B_2 - B_1 \neq B_3 - B_2$, this sequence is not arithmetic.

$$\frac{B_2}{B_1} = \frac{16}{15} = 1.0667$$

$$\frac{B_3}{B_2} = \frac{17.4}{16} = 1.0875$$

Since $\frac{B_2}{B_1} \neq \frac{B_3}{B_2}$, this sequence is not geometric.

So, this sequence is neither arithmetic nor geometric.

1 mark

b. How much will a bunch of flowers cost at William’s shop at 11:00 am?
Write your answer correct to the nearest cent.

Worked solution

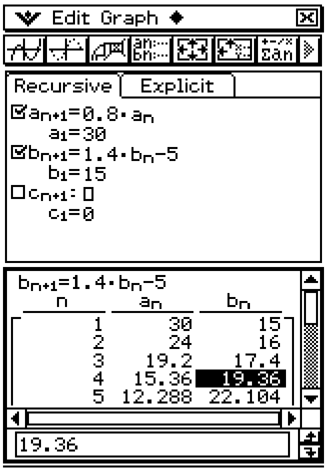
If the hour starting at 6:00 am is the first hour, 11:00 am is the start of the sixth hour after 6:00 am.
The price of a bunch of flowers at 11:00 am will be:
$25.946 ≅ $25.95

1 mark

c. Determine how many hours after opening it will be before the price of a bunch of flowers in William’s shop first exceeds the price of a bunch of flowers in Gemma’s shop. At what time will this happen?

Worked solution

Let’s use a calculator to generate both sequences.



☑ $a_{n+1}=0.8\cdot a_n$
$a_1=30$
☑ $b_{n+1}=1.4\cdot b_n-5$
$b_1=15$
☐ c_{n+1}: ☐
$c_1=0$

$b_{n+1}=1.4\cdot b_n-5$

n	a_n	b_n
1	30	15
2	24	16
3	19.2	17.4
4	15.36	19.36
5	12.288	22.104

19.36

At the start of the fourth hour, i.e., at 9:00 am, the price of a bunch of flowers in William’s shop first exceeds the price of a bunch of flowers in Gemma’s shop.

2 marks

Total 15 marks

Mark allocation

- You can allocate 1 mark for making the table of values for both sequences.

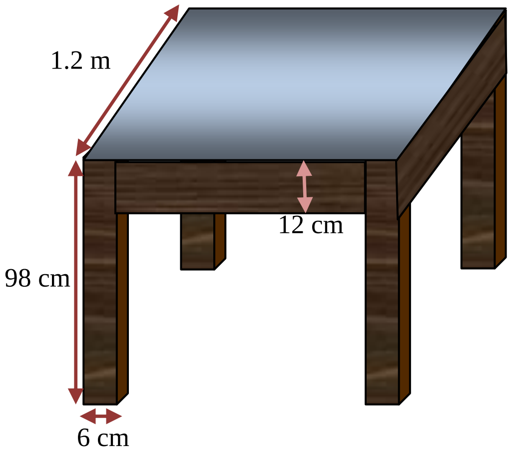
END OF MODULE 1

SECTION B – continued
TURN OVER

Module 2: Geometry and trigonometry

Question 1

Ryan bought a table shaped like a square with side lengths of 1.2 metres.
The table has a height of 98 centimetres and the table top is 12 centimetres thick.
The legs of the table are in the shape of four identical cuboids with square bases.
The side lengths of the bases of the legs are 6 centimetres each.



a. On a model of the table, a 3 cm^2 area is used to represent 12 m^2.

i. What scale factor is used to make this model?

ii. What is the height of the model of the table?

iii. Find the volume of the table.
Write your answer in m^3, correct to two decimal places.

Worked solution

a. **i.** A ratio must have all measurements expressed in the same units so, to find the scale ratio of the model table to the real table, we must change the 3 cm^2 and 12 m^2 to the same units before comparing them. We change the 12 m^2 to cm^2:

1 m^2 = 1 m × 1 m
= 100 cm × 100 cm
= 10 000 cm^2

12 m^2 = 12 × 10 000 cm^2
= 120 000 cm^2

So the area ratio of the model table to the real table is:
3 : 120 000 = 1 : 40 000 (Dividing both sides by 3).

$$k^2 = \frac{1}{40\ 000}$$

So: $k = \frac{1}{200}$

The scale factor is 1 : 200.

1 mark

ii. Since the height of the table is 98 cm, the model of the table has a height of:
98 ÷ 200 = 0.49 cm

1 mark

iii. volume of the table = volume of the table top + 4 × volume of a table leg

$= 1.2 \times 1.2 \times 0.12 + 4 \times 0.06 \times 0.06 \times (0.98 - 0.12)$

$= 0.1728 + 0.012384$

$= 0.185184 \text{ m}^3$

$\cong 0.19 \text{ m}^3$

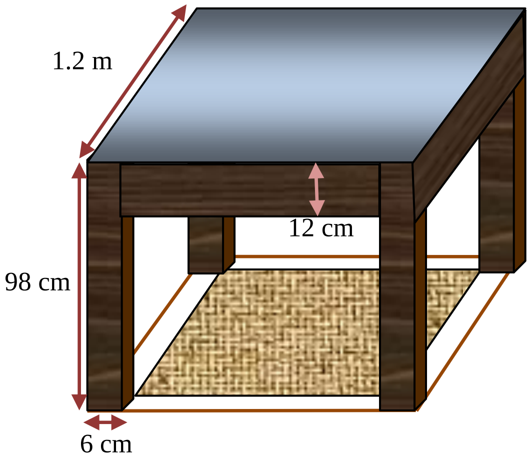
2 marks

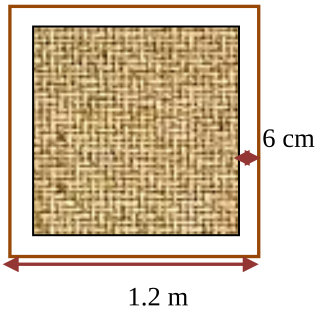
1+1+2=4 marks

Mark allocation

- Allocate 2 marks for the correct volume. Allocate 1 mark for finding the correct volume of only the top of the table or the table leg.

b. Ryan first lays a plastic floor mat under the table.
He then buys a square of carpet and places it on top of the floor mat.
The corners of the square of carpet touch the inside corners of the table legs.



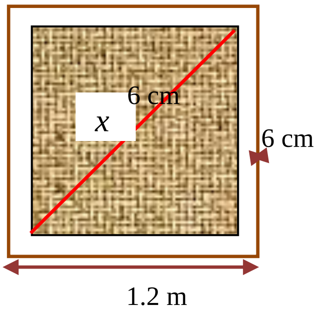


i. Find the length of the diagonal of the square of carpet.
Give your answer in metres, correct to two decimal places.

ii. What is the area of the plastic floor mat that is not covered by the carpet (including the part that is covered by the base of the table legs)?

Worked solution

b. i.



side length of the carpet square = 1.2 – 2 × 0.06 = 1.08 m

Let's use Pythagoras' theorem to calculate the length of the diagonal of the carpet:

$x^2 = 1.08^2 + 1.08^2$

$x = 1.53$ m

1 mark

SECTION B – continued

TURN OVER

ii. Area of the plastic floor mat $= 1.2^2 - 1.08^2$

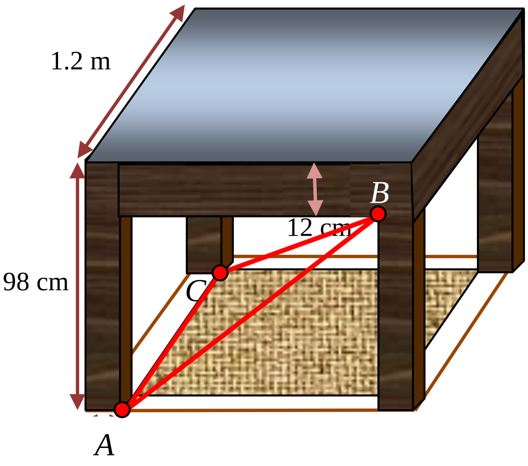
$= 1.44 - 1.1664$

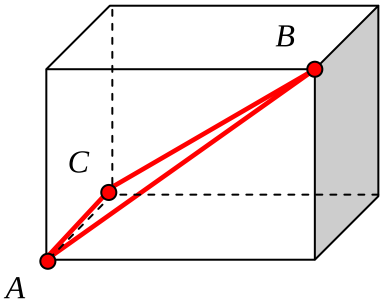
$= 0.2736 \text{ m}^2$

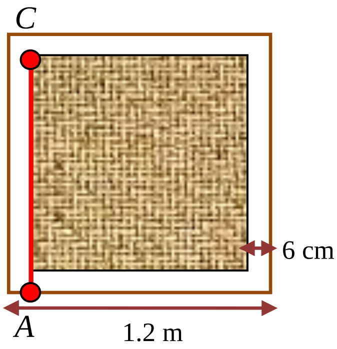
1 mark

1+1=2 marks

c. Find the area of the triangle that is formed by connecting the points *A*, *B* and *C* shown below.
Write your answer in cm^2, correct to one decimal place.





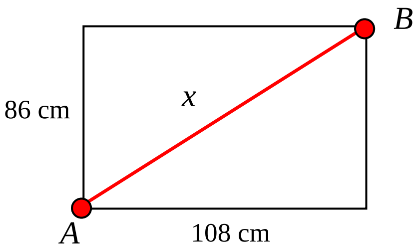


Worked solution
We will first use Pythagoras' theorem to find the length of the diagonal *AB*.

$x^2 = 86^2 + 108^2$

$x = \sqrt{19060}$

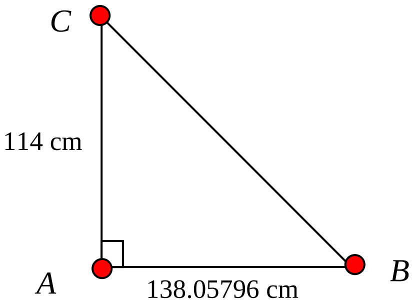
$= 138.05796 \text{ cm}$



length of side $AC = 120 - 6 = 114$ cm
Sides *AB* and *AC* are perpendicular to each other.

area of triangle $= \frac{1}{2} \times 114 \times 138.05796$

$= 7869.3 \text{ cm}^2$



2 marks

SECTION B – continued

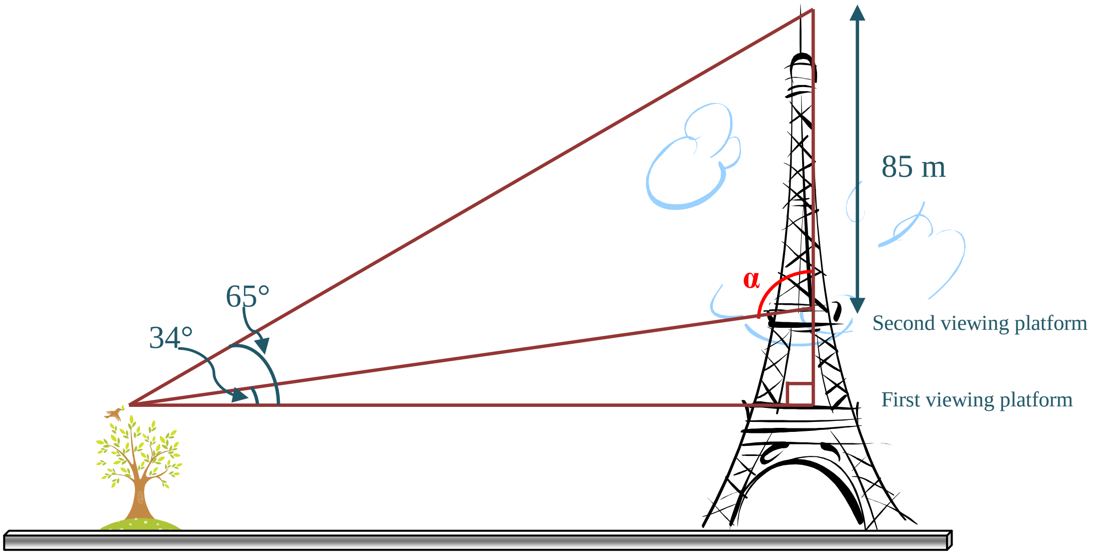
Question 2

From a bird in a tree the angle of elevation of the top of a tower is 65° and that of the second viewing platform of the tower is 34°.

The vertical distance between the second viewing platform and the top of the tower is 85 metres.

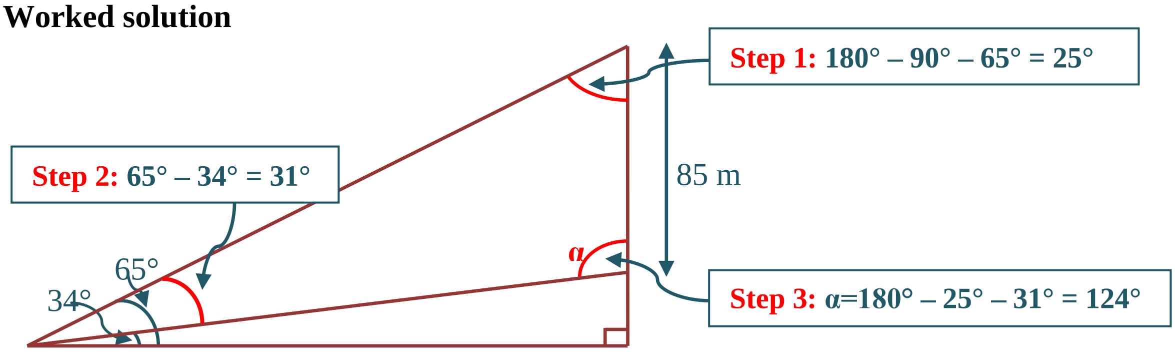
The bird is horizontally at the same level as the first viewing platform of the tower.

The diagram below shows the positions of the bird and the tower.



a. Show that the size of angle α in the diagram above is 124°.

Worked solution



Step 1: 180° – 90° – 65° = 25°

Step 2: 65° – 34° = 31°

Step 3: α=180° – 25° – 31° = 124°

1 mark

b. Calculate the area of the triangle that is formed between the bird, the top of the tower and the second viewing platform of the tower. Write your answer in m^2, correct to one decimal place.

Worked solution

Let's call the side between the bird and the second viewing platform *b***,** then use the sine rule.

$$\frac{85}{\sin\ 31°} = \frac{b}{\sin\ 25°}$$

$b = 69.74737$ m

Now let's find the area of the triangle by using the rule:

$$\begin{aligned} \text{area} &= \frac{1}{2} \times b \times c \times \sin A \\ &= \frac{1}{2} \times 69.74737 \times 85 \times \sin 124° \\ &= 2457.5\ \text{m}^2 \end{aligned}$$

1 mark

c. Calculate the vertical distance between the first and the second viewing platforms of the tower. Write your answer in metres, correct to one decimal place.

Worked solution

Let the vertical distance between the first and the second viewing platforms of the tower be *y*.

$$\sin 34° = \frac{y}{69.74737}$$

$$y = 39.0\ \text{m}$$

1 mark

Question 3

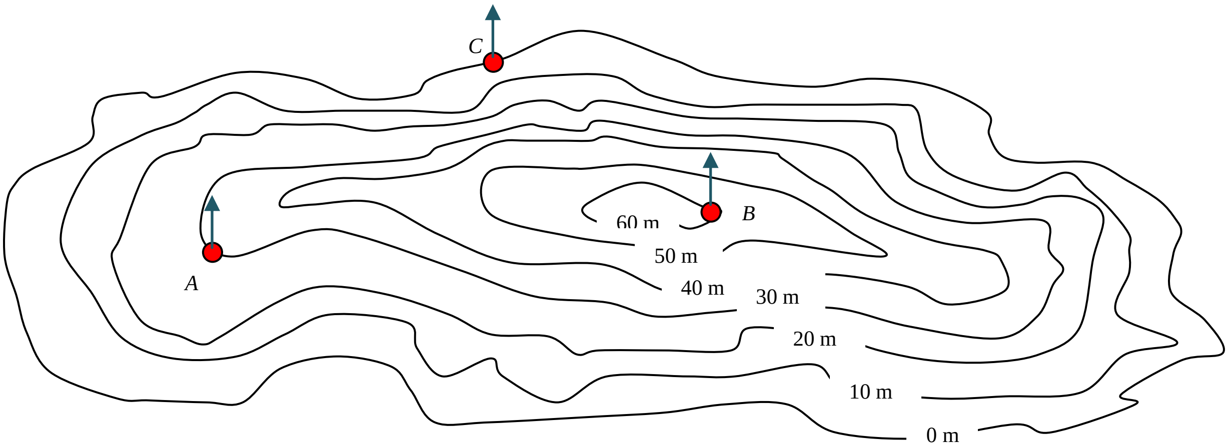
A reduced image of a contour map of a botanic garden is shown below. It has contours drawn at intervals of 10 metres.

The map shows three observation cafés, in three different spots: Café Alexander (*A*), Café Benjamin (*B*) and Café Chloe (*C*).

Eva measures the distance between Café Alexander and Café Chloe on the actual contour map as 15 centimetres and the distance between Café Chloe and Café Benjamin as 10 centimetres.

The bearing of Café Chloe from Cafe Alexander is 070°T and the bearing of Café Benjamin from Cafe Chloe is 135°T.

The arrows on the points are pointing in the north direction.



a. On the contour map, 1 centimetre represents 3 metres on the horizontal level.

i. Find the horizontal distance between Café Alexander and Café Benjamin. Write your answer in metres, correct to three decimal places.

ii. By using your answer from **part i**, find the bearing of Café Alexander from Cafe Benjamin. Give your answer correct to the nearest degree.

Worked solution

a. **i.** scale of the actual contour map is: 1 cm : 300 cm

So, the horizontal distance between points *A* and *C* is:

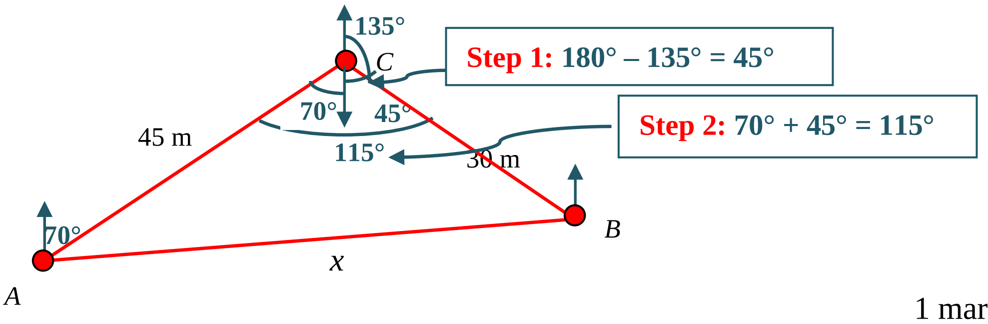
15 × 300 = 4500 cm

= 45 m

The horizontal distance between points *B* and *C* is:

10 × 300 = 3000 cm

= 30 m



Step 1: 180° – 135° = 45°

Step 2: 70° + 45° = 115°

1 mark

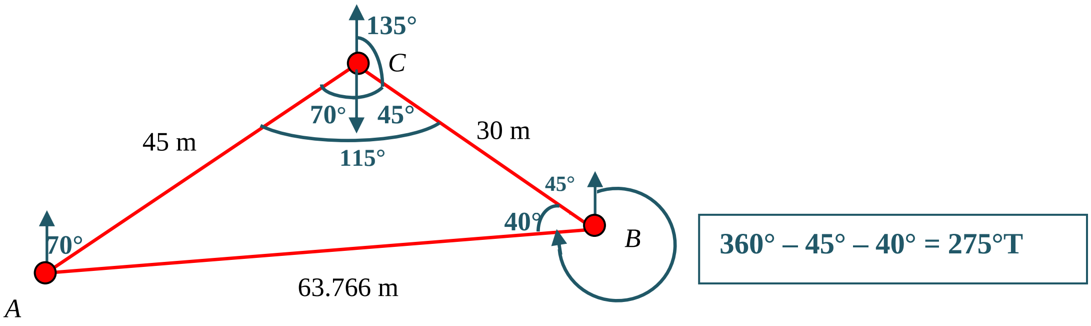
Now let's use the cosine rule to determine x.

$x^2 = 45^2 + 30^2 - 2 \times 45 \times 30 \times \cos 115°$
$x = 63.766$ m

ii. We will use the sine rule to calculate the size of angle ABC:

$$\frac{63.766}{\sin 115°} = \frac{45}{\sin ABC}$$

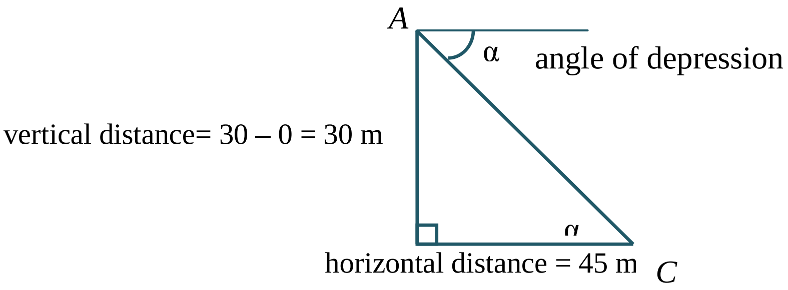
size of angle $ABC = 39.76°$
$\cong 40°$



2 marks

b. Determine the angle of depression of a person looking at Café Chloe from Café Alexander. Give your answer correct to the nearest degree.

Worked solution



$$\tan \alpha = \frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{30}{45}$$

$$\alpha = \tan^{-1}\frac{30}{45} = 33.69$$

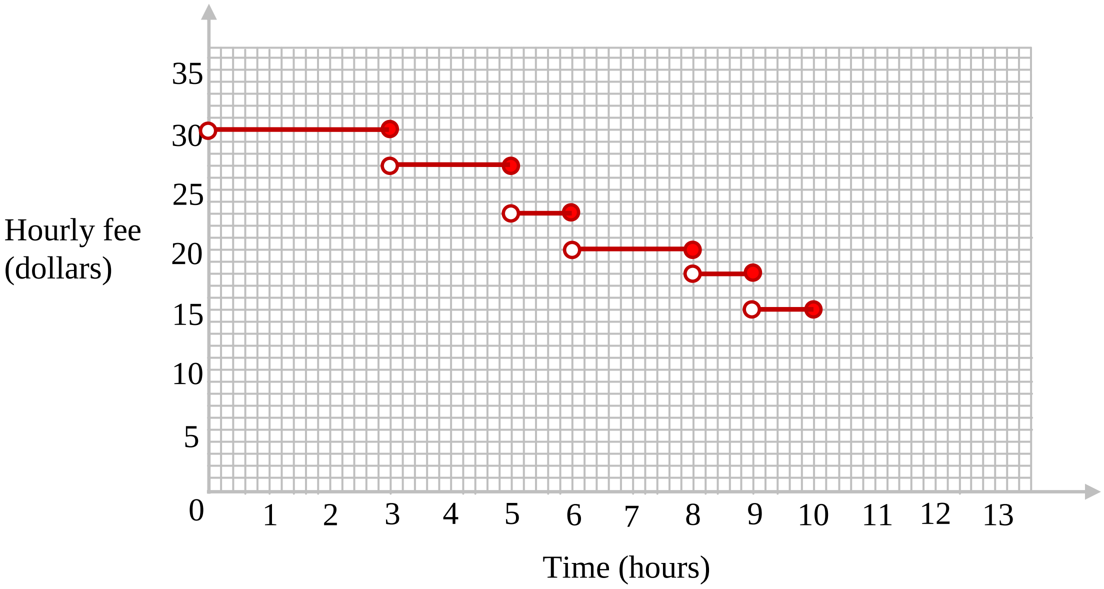
1 mark
Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1

Sarah operates a house-cleaning service. The hourly fee she charges depends on the time it takes to clean the house. The hourly fees that Sarah's house-cleaning service charges for times up to 10 hours are shown on the graph below.



a. How much would it cost if Sarah cleaned a house in six hours?

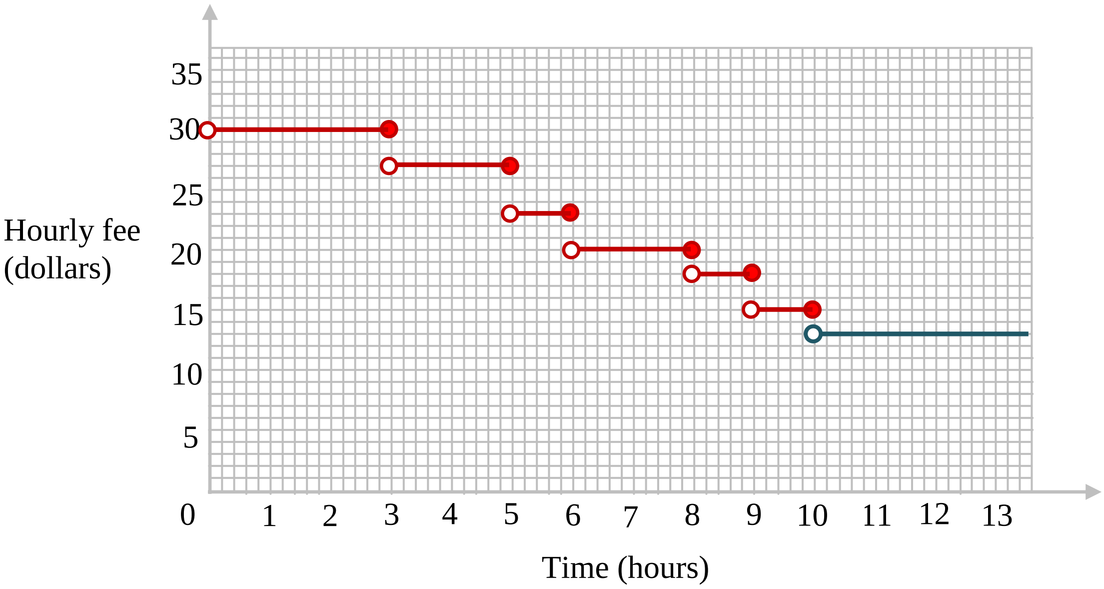
Worked solution

Cleaning a house in 6 hours would cost 6 × \$23 = \$138.

1 mark

b. Sarah's house cleaning service charges \$13 per hour when customers require more than 10 hours of cleaning. Draw this information on the graph.

Worked solution



1 mark

SECTION B – continued
TURN OVER

Question 2
Sarah's brother Nathan manufactures and sells vacuum cleaners. When he first opened his business he spent $5500 on the necessary equipment.
The cost of manufacturing one vacuum cleaner is $300.

a. Write an equation that gives the total amount of money, M dollars, that it costs Nathan to manufacture n vacuum cleaners.

Worked solution
$M = 5500 + 300n$

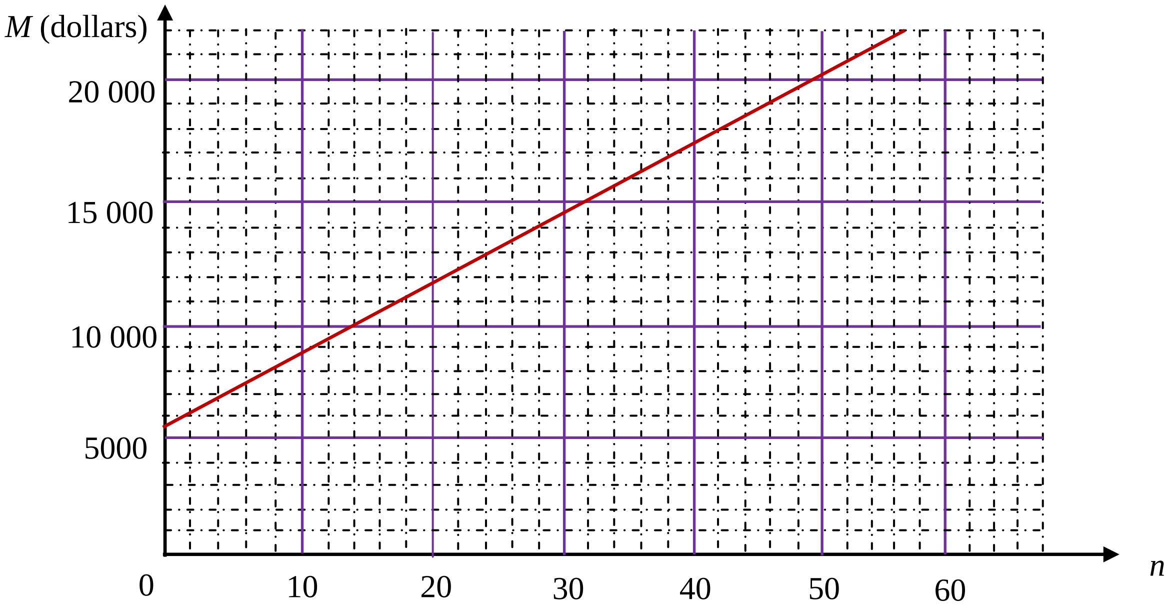
1 mark

b. Nathan sells the vacuum cleaners for $520 each. Write an equation for the revenue, R dollars, that Nathan receives from the sale of n vacuum cleaners.

Worked solution
$R = 520n$

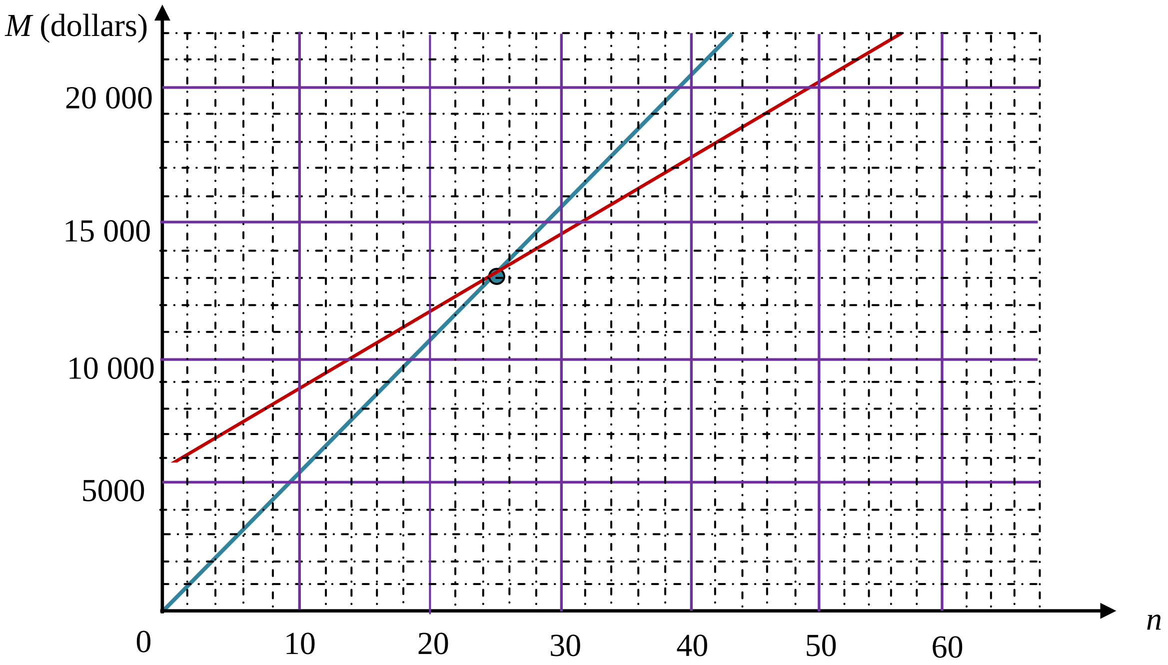
1 mark

The total amount of money, M, that Nathan pays to manufacture n vacuum cleaners is graphed below.



c. Draw the graph of the revenue, R, that Nathan receives from the sale of n vacuum cleaners. From the graph, or using another method, find the smallest number of vacuum cleaners that Nathan needs to sell in order to make profit.

Worked solution



From the graph we can conclude that Nathan needs to sell 26 vacuum cleaners in order to make a profit.

Alternatively, we can find the break-even point using the two equations we found in **part a** and **part b**:
$M = 5500 + 300n$
$R = 520n$

The break-even point occurs when $M = R$.

$$\begin{aligned} M &= R \\ 5500 + 300n &= 520n \\ 5500 &= 220n \\ n &= 25 \end{aligned}$$

So, Nathan needs to sell 26 vacuum cleaners (one more than the break-even number) in order to make a profit.

1 mark

d. How many vacuum cleaners did Nathan sell if he made $1100 profit?

Worked solution
profit = revenue – cost
$1100 = 520n – (5500 + 300n)$
$n = 30$

1 mark

Question 3

Nathan's wife, Sandra is a hairdresser. She specialises in two areas, cutting hair and styling hair.

Let x be the number of customers who get a haircut from Sandra
and let y be the number of customers who get a hairstyle from Sandra.

It takes 40 minutes to cut hair and half an hour to style hair.
Sandra has 10 hours available each day to cut and style the customers' hair.

a. Write an inequality to describe this information in terms of x and y.

Worked solution

Inequality 1: $40x + 30y \le 600$ (in terms of minutes)

Or: $\frac{2}{3}x + \frac{1}{2}y \le 10$ (in terms of hours)

1 mark

b. A constraint is given by Inequality 2:
$x \ge 2$
Explain the meaning of Inequality 2 in terms of the context of this problem.

Worked solution

At least two people must get a haircut from Sandra each day.

1 mark

c. Inequalities 3 to 5 below represent the remaining constraints.

Inequality 3: $x \ge 0$
Inequality 4: $y \ge 0$
Inequality 5: $y \ge \frac{x}{3}$

By using the constraints given, find the maximum number of haircuts Sandra can perform on a day when she styles three customers' hair.

Worked solution

By substitution, let's find the possible x values when $y = 3$.

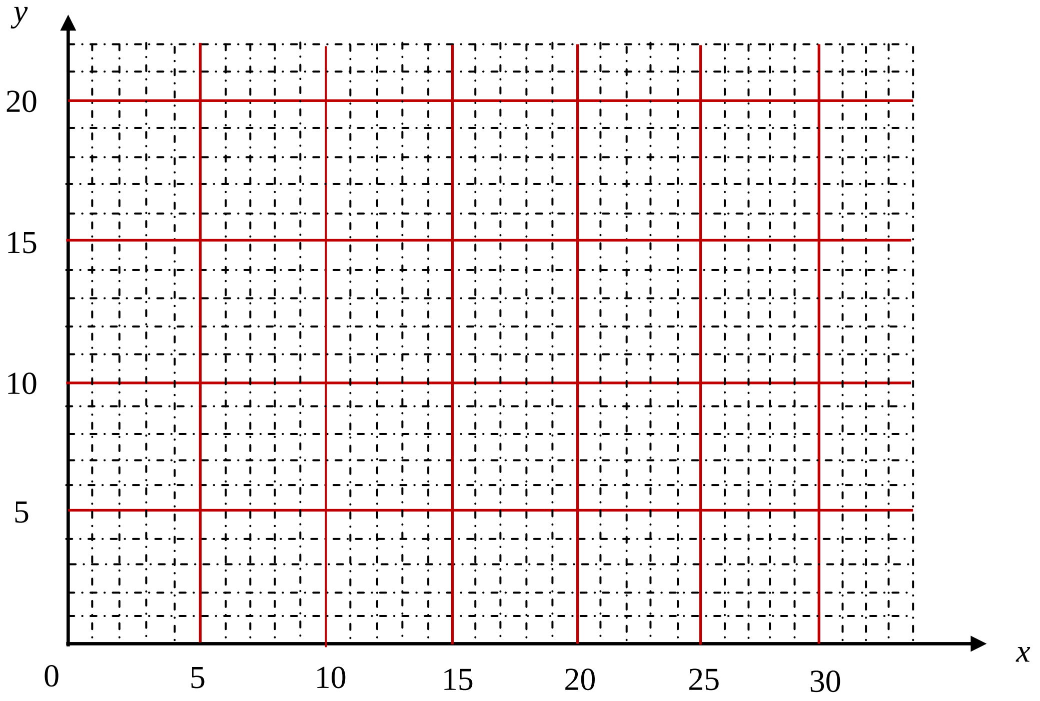
Inequality 1: $40x + 30y \le 600 \rightarrow 40x + 30 \times 3 \le 600 \rightarrow x \le 12.75$

Inequality 5: $y \ge \frac{x}{3} \rightarrow 3 \ge \frac{x}{3} \rightarrow x \le 9$

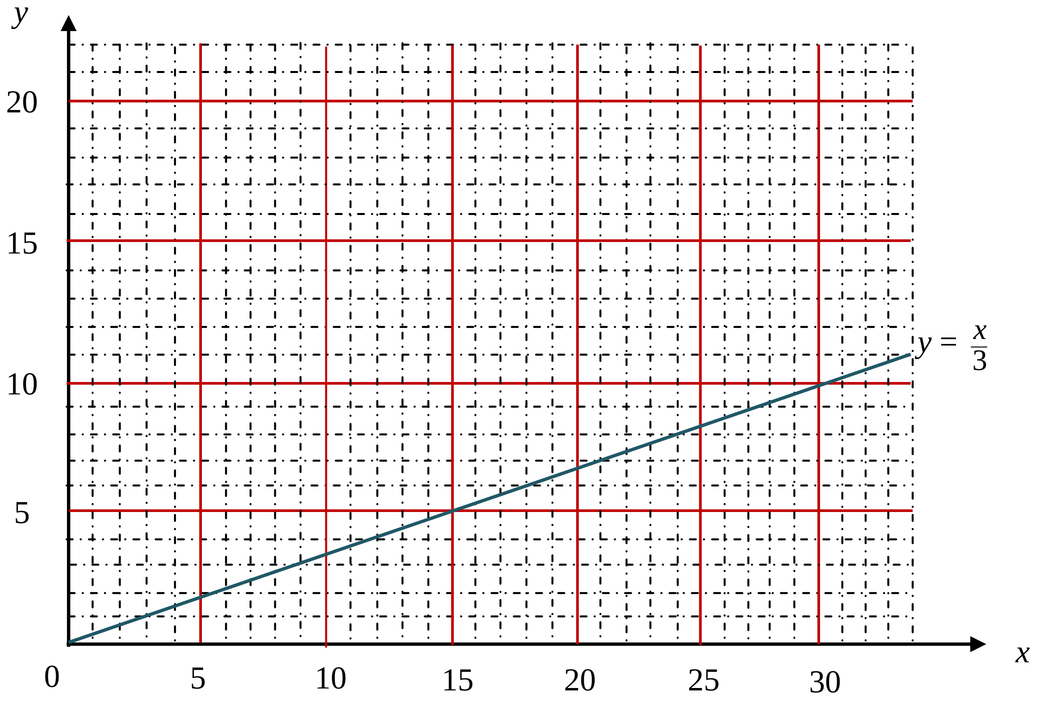
Sandra can perform at most 9 haircuts on a day when she styles three customers' hair.

1 mark

d. Draw a graph that represents $y = \frac{x}{3}$.



Worked solution

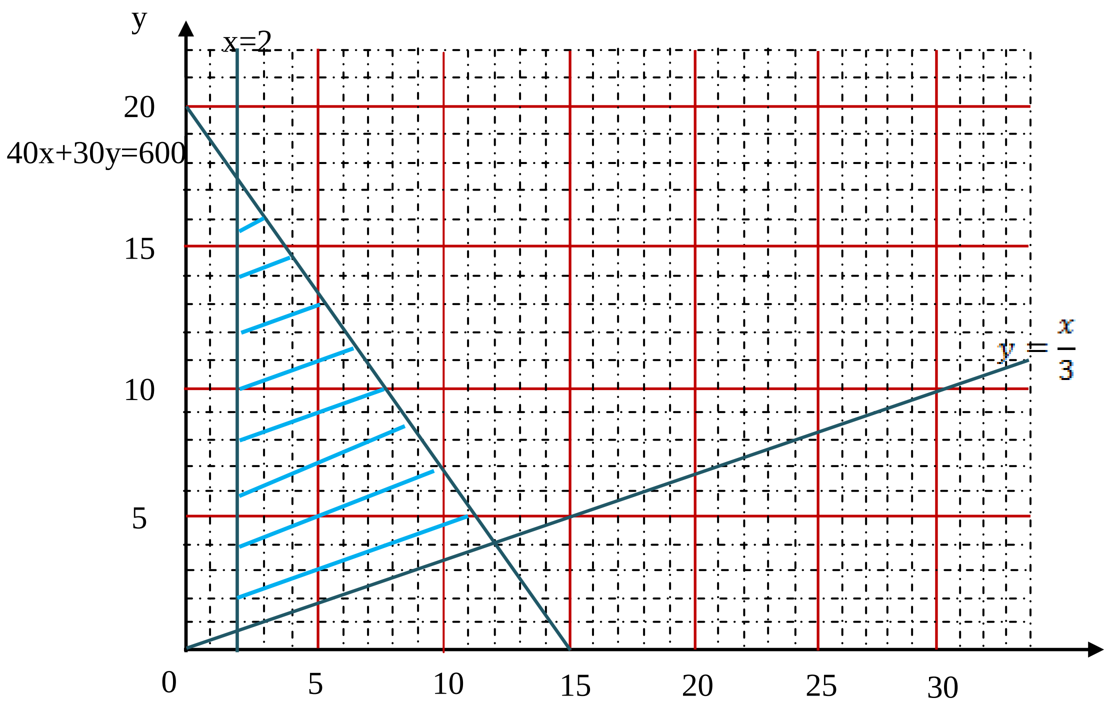


1 mark

SECTION B – continued
TURN OVER

e. On the graph above, draw the boundaries of the region represent ted by Inequalities 1 to 4. Clearly shade the feasible region represented by these inequalities.

Worked solution



2 marks

Mark allocation

- Allocate 1 mark for drawing three lines representing three main constrains.

The profit that Sandra receives from each haircut is \$65 and the profit that she receives from each hairstyle is \$50.

f. Write an equation for the total profit, *P*, that Sandra receives in terms of *x* and *y*, when *x* is the number of haircuts and *y* is the number of hairstyles.

Worked solution

$P = 65x + 50y$

1 mark

g. Determine the number of customers that Sandra needs to have their hair cut and the number of customers that she needs to have their hair styled for her to maximise her profit.

Worked solution

Let’s substitute the discrete points in the region into the profit equation.

$P = 65x + 50y$

(2, 17) → $P = 65 \times 2 + 50 \times 17 = 980$

(3, 16) → $\boldsymbol{P = 65 \times 3 + 50 \times 16 = 995}$ **(maximum profit)**

(6, 12) → $P = 65 \times 6 + 50 \times 12 = 990$

(9, 8) → $P = 65 \times 9 + 50 \times 8 = 985$

(12, 4) → $P = 65 \times 12 + 50 \times 4 = 980$

Sandra needs to cut three customers’ hair and style 16 customers’ hair in order to maximise her profit.

2 marks

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

Gabrielle wants to buy a laptop for her daughter's birthday. She visits three different stores and compares their prices.

The first store has the laptop marked at $2 500 but will give 22% discount if the payment is made in cash at the time of sale.

a. If Gabrielle decides to buy the laptop from the first store, what would she pay for it after the discount is applied, provided that she makes the payment in cash.

Worked solution

$$\text{discounted price} = \$2\,500 - \$2\,500 \times \frac{22}{100} = \$1\,950$$

1 mark

b. A second store offers the same $2500 laptop under a hire-purchase agreement with $350 deposit and 18 monthly instalments of $152.

i. Determine the total amount that Gabrielle would pay if she decides to buy the laptop from the second store.

ii. Find the total interest Gabrielle would pay over 18 months.

iii. Determine the annual flat interest rate that would be applied to this hire-purchase agreement. Write your answer as a percentage, correct to one decimal place.

Worked solution

b. **i.**

$$\begin{aligned}\text{total amount} &= \$350 + 18 \times \$152 \\ &= \$350 + \$2736 \\ &= \$3086\end{aligned}$$

1 mark

ii. $\text{total interest} = \$3086 - \$2500 = \$586$

1 mark

iii.

$$\begin{aligned}\text{annual flat interest rate} &= \left(\frac{586}{2736} \div 1.5\right) \times 100\% \\ &= 14.3\%\end{aligned}$$

1 mark

1+1+1=3 marks

SECTION B – continued

c. The third store requires 35% deposit for the \$2 500 laptop. The balance is to be paid in 12 equal monthly instalments. No interest is charged.

i. Determine the amount of deposit that Gabrielle would pay if she decides to buy the laptop from the third store.

ii. Determine the amount of each of the 12 instalments Gabrielle will pay. Write your answer correct to the nearest cent.

Worked solution

c. **i.** deposit = \$2 500 × $\frac{35}{100}$ = \$875

1 mark

ii. amount of each instalment = (\$2 500 – \$875) ÷ 12
= \$135.42

1 mark

Tip

- *Students should keep in mind that, when rounding the value correct to the nearest cent, they should round it correctly to two decimal places. A very common error is to give the answer as \$135.40.*

Question 2

The bank statement below shows the transactions on Joshua’s account for the month of April. Interest for this account is calculated on the minimum monthly balance at a rate of 5.3% per annum.

Date	Description of transaction	Withdrawals	Deposits	Balance
01 April	Opening balance			\$53 467
09 April	Withdrawal–internet transfer	\$4 538		\$48 929
12 April	Withdrawal–cash	\$12 890		\$36 039
17 April	Deposit–cash		\$13 485	\$49 524
20 April	Withdrawal–internet transfer			\$48 279
24 April	Deposit–cheque			\$51 365
28 April	Deposit–internet transfer		\$525	\$51 890
30 April	Closing balance			\$51 890

a. Find the amount that was withdrawn from the account on 20 April and the amount that was deposited to the account on 24 April.

Worked solution

amount withdrawn on 20 April = \$49 524 – \$48 279
= \$1245

amount deposited on 24 April = \$51 365 – \$48 279
= \$3086

1 mark

SECTION B – continued
TURN OVER

b. Calculate the interest for April, correct to the nearest cent.

Worked solution

The interest is calculated on the minimum monthly balance. From the table, we can clearly see that the minimum monthly balance for April is \$36 039.

interest $= \$36\,039 \times \frac{5.3}{12 \times 100} = \$159.17$

1 mark

Question 3

Lucas has \$6 800 invested in an account which pays interest at the rate of 5.6% per annum compounding quarterly.

a. Show that the interest rate per quarter is 1.4%.

Worked solution

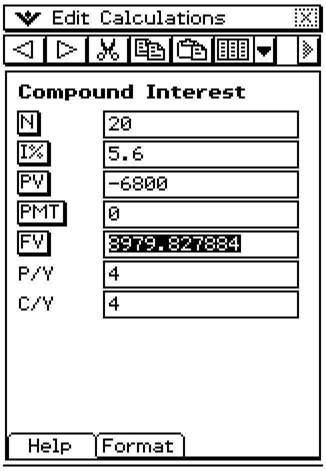
$$\text{interest rate per quarter} = \frac{\text{interest rate per annum}}{4}$$
$$= \frac{5.6\%}{4}$$
$$= 1.4\%$$

1 mark

b. Determine the value of a \$6 800 investment after five years.
Write your answer correct to the nearest cent.

Worked solution

Let's enter the given values into the calculator and solve this using the TVM solver.



Compound Interest

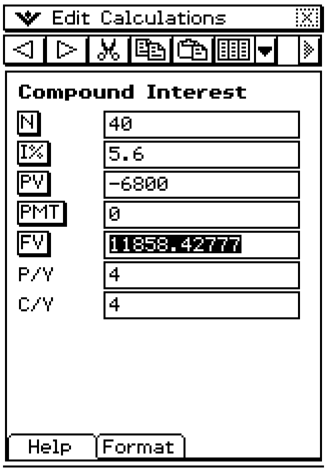
N 20
I% 5.6
PV -6800
PMT 0
FV 8979.827884
P/Y 4
C/Y 4

The value of \$6800 investment after 5 years is \$8979.83.

1 mark

c. Calculate the interest a \$6 800 investment will earn over 10 years.
Write your answer correct to the nearest cent.

Worked solution
First, we'll first use the TVM solver to find the value of a \$6 800 investment after 10 years.



Compound Interest

N 40
I% 5.6
PV -6800
PMT 0
FV 11858.42777
P/Y 4
C/Y 4

So: interest = \$11 858.43 – \$6 800
= \$5058.43

1 mark

Question 4
Jessica takes out a reducing balance loan to purchase a house.
Interest on the loan will be calculated and paid monthly at the rate of 7.652% per annum.

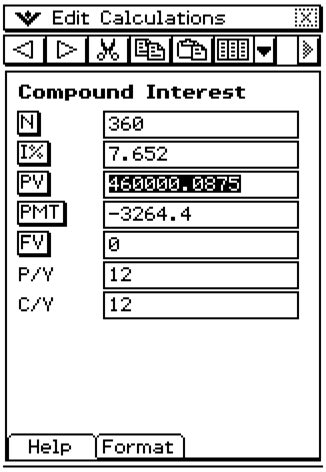
a. Jessica's loan will be fully repaid in equal monthly instalments of \$3 264.40 over 30 years.

i. Find the amount Jessica borrowed from the bank.
Write your answer correct to the nearest dollar.

ii. By using your answer from **part i**, calculate the total interest that will be paid over the 30-year term of Jessica's loan.

Worked solution

a. **i.** We'll use the calculator's TVM solver to find the amount Jessica borrowed from the bank.



Compound Interest

N 360
I% 7.652
PV 460000.0875
PMT -3264.4
FV 0
P/Y 12
C/Y 12

Jessica borrowed $460 000 from the bank.

1 mark

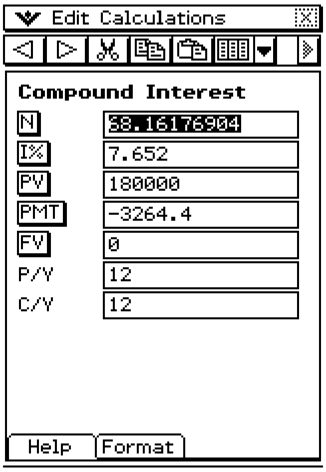
ii. total interest = $3264.40 × 360 – $460 000 = $715 184

1 mark

b. Jessica reduces the principal of her loan to $180 000 by making a lump sum payment after fifteen years. Her monthly repayments and the interest rate remain the same. How many months, in total, will Jessica take to fully repay her home loan?

Worked solution

We'll use the calculator's TVM solver to find the number of months it will take to repay $180 000.



Compound Interest

N 68.16176904
I% 7.652
PV 180000
PMT -3264.4
FV 0
P/Y 12
C/Y 12

It will take 69 months to repay $180 000.

number of months to fully repay the loan = 15 × 12 + 69
= 249 months

2 marks

Tips

- *Although the calculator shows the number of months as 68.16 176 904, this requires 69 full months, not 68.*
- *The 69 months calculated above is the time it will take Jessica to repay the remaining money from when the amount owing was reduced to $180 000. This amount of time still had to be added to the number of monthly payments she made in the 15 years before that time.*

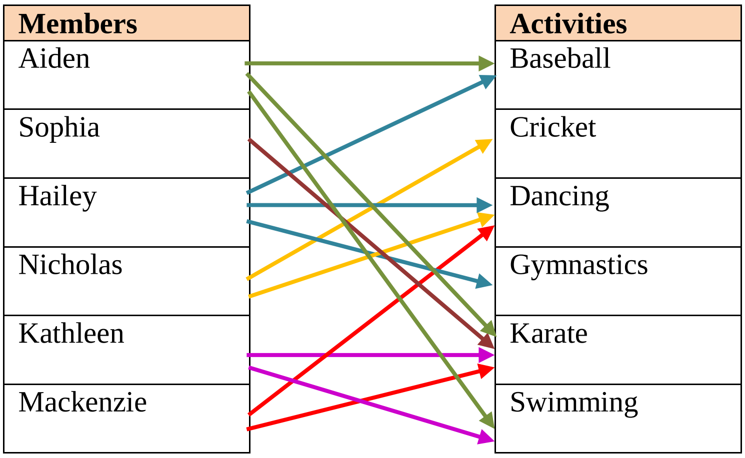
Total 15 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1

Aiden, Sophia, Hailey, Nicholas, Kathleen and Mackenzie are members of a local youth club.
The youth club has six activities running at the same time.
Although the students make their preferences, each member can only do one activity and each activity must be done by one participant.
The following bipartite graph illustrates the activity preferences of the six youth club members.



a. Who must do the dancing activity?

Worked solution

- Sophia has only selected karate, so she must do karate.
- Kathleen has only selected karate and swimming. She must do swimming because Sophia has already been allocated karate.
- Mackenzie selected dancing and karate. Sophia has already been allocated karate, so Mackenzie must do dancing.

Mackenzie must do the dancing activity.

1 mark

b. Complete the table below showing the names of the members who must do the following activities.

Members	**Activities**
	Baseball
	Karate
	Gymnastics
	Cricket
	Swimming

Worked solution

- Sophia has only selected karate, so she must do karate.
- Kathleen has only selected karate and swimming. She must do swimming because Sophia has already been allocated karate.
- Mackenzie selected dancing and karate Sophia has already been allocated karate, so Mackenzie must do dancing.
- Nicholas selected dancing and cricket. Mackenzie has been allocated dancing, so Nicholas must do cricket.
- Aiden selected baseball, karate and swimming, Karate and swimming have already been allocated to Sophia and Kathleen, so Aiden must do baseball.
- Hailey selected baseball, dancing and gymnastics. Baseball and dancing have already been allocated to Aiden and Mackenzie, so Hailey must do gymnastics.

Members	**Activities**
Aiden	Baseball
Sophia	Karate
Hailey	Gymnastics
Nicholas	Cricket
Kathleen	Swimming

2 marks

Mark allocation

- 2 marks awarded for all five names attributed to the correct activities. 1 mark is awarded if at least three of the five names were attributed to the correct activities.

Question 2

The six youth club members from Question 1 decided to arrange a round-robin table tennis tournament among themselves.

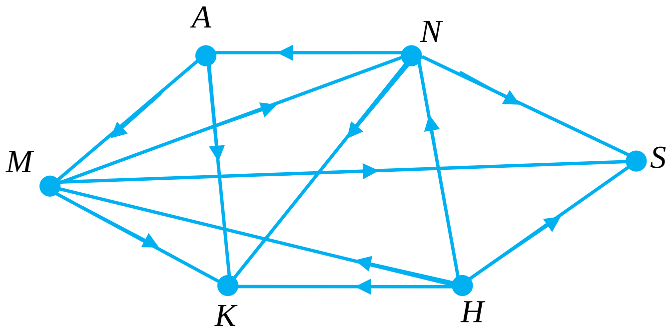
a. If each club member is to play table tennis with every other member once only, how many games will be played in the tournament?

Worked solution

$$\text{number of games played} = \frac{n(n-1)}{2}$$
$$= \frac{6 \times 5}{2}$$
$$= 15$$

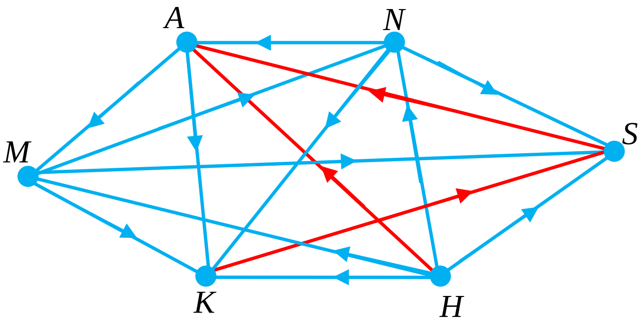
1 mark

b. The network graph below has been partially constructed to represent the results of the tournament. ($A \rightarrow K$ means Aiden defeated Kathleen.)



Given that Hailey and Sophia both defeated Aiden, and Kathleen defeated Sophia, complete the network graph.

Worked solution



1 mark

c. By representing 'defeated' by the element '1' and 'did not defeat' by the element '0', construct a dominance matrix, **M**, for the results of the tournament.

Worked solution

$$\mathbf{M} = \begin{array}{c c} & \begin{array}{cccccc} A & S & H & N & K & M \end{array} \\ \begin{array}{c} A \\ S \\ H \\ N \\ K \\ M \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

1 mark

d. Determine and write the dominance vector associated with the matrix $\mathbf{M} + \mathbf{M}^2$.

Worked solution

$$\mathbf{M} + \mathbf{M}^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \right)^2$$

$$= \begin{bmatrix} 0 & 2 & 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 3 & 4 & 0 & 2 & 4 & 2 \\ 2 & 2 & 0 & 0 & 2 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 1 & 2 & 0 \end{bmatrix}$$

2 marks

Adding the elements in each row gives the dominance vector, i.e., the sum of the level 1 and level 2 wins in the table tennis tournament.

$$\text{dominance vector} = \begin{bmatrix} 6 \\ 3 \\ 15 \\ 7 \\ 2 \\ 8 \end{bmatrix}$$

e. Rank the club members in order, from first to last, for the table tennis tournament.

Worked solution

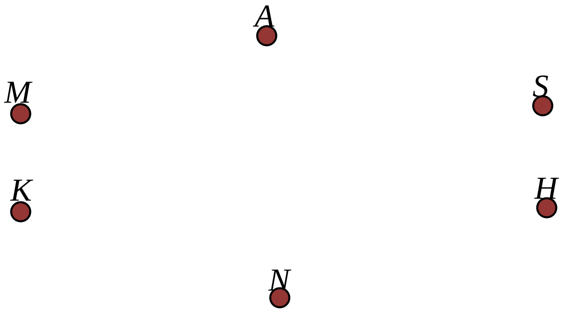
The dominance vector could be labelled as shown, to relate its content to the six participants.

$$\begin{array}{cc} \begin{bmatrix} 6 \\ 3 \\ 15 \\ 7 \\ 2 \\ 8 \end{bmatrix} & \begin{array}{c} A \\ S \\ H \\ N \\ K \\ M \end{array} \end{array}$$

The ranking of the club members in order, from first to last, is:
Hailey, Mackenzie, Nicholas, Aiden, Sophia and Kathleen.

1 mark

Question 3
Aiden, Sophia, Hailey, Nicholas, Kathleen and Mackenzie are also neighbours.
Their houses are labelled *A*, *S*, *H*, *N*, *K* and *M* below.



To be able to communicate with each other comfortably, they want their houses to be connected by private telephone cables.

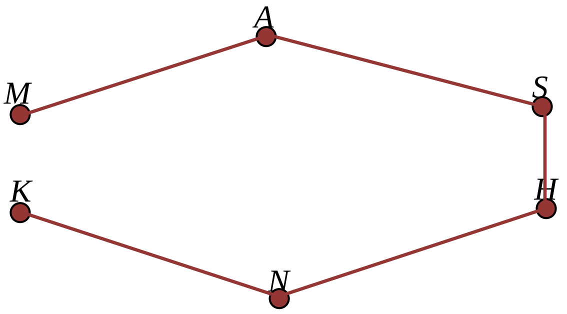
a. What is the minimum number of edges needed to connect the six houses.

Worked solution
The minimum number of edges needed to connect the six houses is five.

1 mark

b. On the diagram above, draw a connected graph with the number of edges you answered for **part a**.

Worked solution

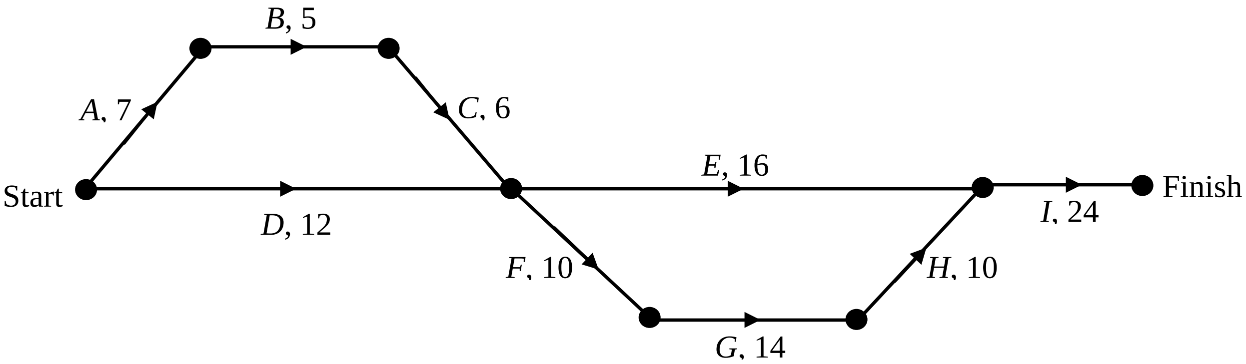


1 mark

Tip

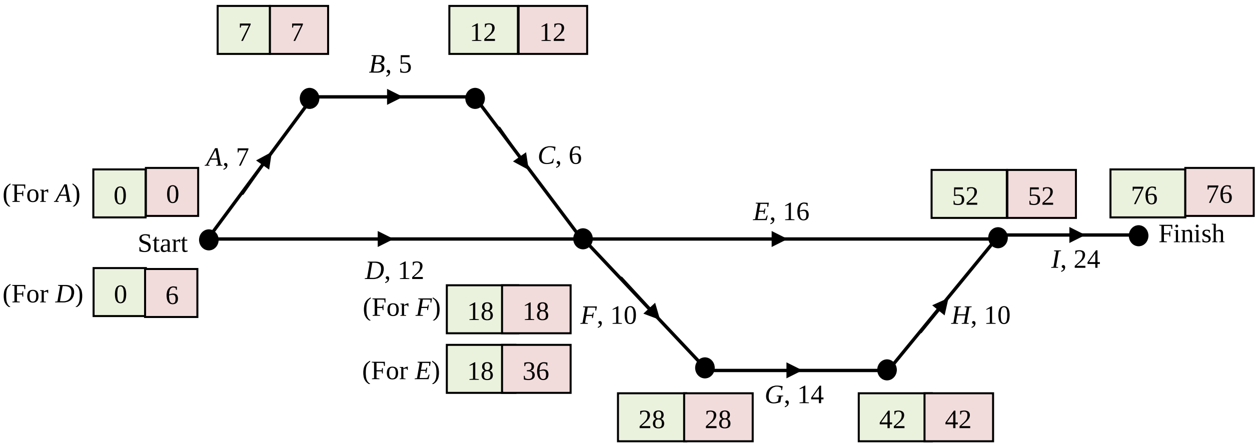
- *This is only one of the possible answers. Any graph with five edges that connected all six points with each other, whether directly or indirectly, is acceptable.*

Question 4
The supervisor of the telephone cable connection project described in Question 3 has identified nine activities that must be performed in order to complete the connection job. Those activities together with the times that they take to complete, in hours, are shown on the directed network below.



a. Identify the critical path for this project.

Worked solution
The numerals in each of the two joined boxes indicate the earliest start time (EST) and the latest start time (LST) possible for each of the activities.



The critical path for this project is $A \to B \to C \to F \to G \to H \to I$

1 mark

b. What is the latest start time possible for activity D without delay resulting to the overall project?

Worked solution
The latest start time for activity D is 6 hours.

1 mark

c. The project supervisor correctly records the float time for each activity that can be delayed and makes a list of these times.

Determine the longest float time in hours.

Worked solution
float time = LST – EST

Let's determine this for each activity:

float time for activity $A = 0 - 0 = 0$
float time for activity $B = 7 - 7 = 0$
float time for activity $C = 12 - 12 = 0$
float time for activity $D = 6 - 0 = 6$
float time for activity $E = 36 - 18 = 18$
float time for activity $F = 18 - 18 = 0$
float time for activity $G = 28 - 28 = 0$
float time for activity $H = 42 - 42 = 0$
float time for activity $I = 52 - 52 = 0$

1 mark

d. What is the minimum time in which the project can be completed?

Worked solution
The minimum time in which the project can be completed is 76 hours.

1 mark
Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

Brook goes to the Sunday market to buy some vegetables. She needs to buy avocado, cucumber, pumpkin and cauliflower.

Z is a row matrix that represents the price (in dollars) of each vegetable.

$\mathbf{Z} = [4 \;\; 2 \;\; 5 \;\; 6]$

Brook buys 2 avocados, 3 cucumbers, 1 pumpkin and 4 cauliflowers.

a. Use a column matrix, **Y**, to show the number of each vegetable that Brook bought from the Sunday market.

Worked solution

$$\mathbf{Y} = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 4 \end{bmatrix}$$

1 mark

Tip

- *Students should note that they must write a 4 × 1 column matrix, not a row matrix.*

b. Matrix **X** is found by multiplying matrix **Z** with matrix **Y** so that:
X = **Z** × **Y**
Evaluate matrix **X**.

Worked solution

$$\mathbf{X} = [4 \;\; 2 \;\; 5 \;\; 6] \times \begin{bmatrix} 2 \\ 3 \\ 1 \\ 4 \end{bmatrix} = [43]$$

1 mark

c. In this context, what does the information in matrix **X** provide?

Worked solution

In this context, the matrix **X** represents the total money (in dollars) that Brook spent buying 2 avocados, 3 cucumbers, 1 pumpkin and 4 cauliflowers from the Sunday market.

1 mark

d. Another matrix, **W**, is found by multiplying matrix **Y** with matrix **Z** so that:
W = **Y** × **Z**
What is the order of matrix **W**?

Worked solution

$$\begin{array}{ccccc} \mathbf{Y} & \times & \mathbf{Z} & = & \mathbf{W} \\ \downarrow & & \downarrow & & \downarrow \\ (4\times 1) & \times & (1\times 4) & \rightarrow & (4\times 4) \end{array}$$

So, matrix **W** must be a 4 × 4 matrix.

1 mark

Question 2

The student representative council of a high school is selling doughnuts to raise money for the Queensland flood appeal. They formed themselves into four groups and each group sold four different types of doughnuts.

The table below shows the number of doughnuts sold by each group and the total value of sales.

Types of doughnuts sold	**Group 1**	**Group 2**	**Group 3**	**Group 4**
Original glazed doughnut	51	56	35	14
Glazed cinnamon doughnut	27	18	37	41
Chocolate iced custard-filled doughnut	15	29	42	42
Cinnamon apple-filled doughnut	22	19	17	24
Total sales	$284.90	$304.80	$342.90	$333.30

An original glazed doughnut costs $$x$.
A glazed cinnamon doughnut costs $$y$.
A chocolate iced custard-filled doughnut costs $$z$.
A cinnamon apple-filled doughnut costs $$w$.

a. The situation described above is represented in the matrix equation below.
Complete the missing information.

$$\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 284.90 \\ 304.80 \\ 342.90 \\ 333.30 \end{bmatrix}$$

Worked solution

$$\begin{array}{c} \begin{array}{cccc} \text{Number of original glazed doughnuts} & \text{Number of glazed cinnamon doughnuts} & \text{Number of chocolate iced custard-filled doughnuts} & \text{Number of cinnamon apple-filled doughnuts} \\ \downarrow & \downarrow & \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 51 & 27 & 15 & 22 \\ 56 & 18 & 29 & 19 \\ 35 & 37 & 42 & 17 \\ 14 & 41 & 42 & 24 \end{bmatrix} \end{array} \times \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 284.90 \\ 304.80 \\ 342.90 \\ 333.30 \end{bmatrix}$$

1 mark

b. Do the linear equations that are represented by the matrix equation in **part a** have a unique solution?
Provide an explanation to justify your response.

Worked solution

The linear equations that are represented by the above matrix equation have a unique solution

$$\text{since:}\quad \det \begin{bmatrix} 51 & 27 & 15 & 22 \\ 56 & 18 & 29 & 19 \\ 35 & 37 & 42 & 17 \\ 14 & 41 & 42 & 24 \end{bmatrix} = -370\ 570 \neq 0$$

1 mark

Tip

- *Incorrect answers may result from students justifying their responses by quoting the actual solutions to the equations.*

c. Use the matrix equation to find the cost of a chocolate iced custard-filled doughnut.

Worked solution

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 51 & 27 & 15 & 22 \\ 56 & 18 & 29 & 19 \\ 35 & 37 & 42 & 17 \\ 14 & 41 & 42 & 24 \end{bmatrix}^{-1} \times \begin{bmatrix} 284.90 \\ 304.80 \\ 342.90 \\ 333.30 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 2.5 \\ 3 \\ 3.2 \end{bmatrix}$$

So, the cost of a chocolate iced custard-filled doughnut is \$3.

2 marks

Mark allocation

- 2 marks awarded for the correct cost.1 mark awarded for finding the resulting matrix only.

Tip

- *Solving the matrix equation and finding the resulting matrix is not enough to get full marks. The cost of a chocolate iced custard-filled doughnut needs to be clearly written.*

Question 3

The bakery that makes the doughnuts sells 1 200 original glazed doughnuts (O), 900 glazed cinnamon doughnuts (G) and 1 900 chocolate iced custard-filled doughnuts (C) in the first week of its operation. The bakery starts making cinnamon apple filled doughnuts (A) in the second week.

The initial state matrix, $\mathbf{S}_1$, shows the number of each type of doughnut sold by the bakery in the first week.

$$\mathbf{S}_1 = \begin{array}{c} \begin{bmatrix} 1200 \\ 900 \\ 1900 \\ x \end{bmatrix} \end{array}\begin{array}{c} O \\ G \\ C \\ A \end{array}$$

a. What is the value of x in the initial state matrix?

Worked solution

Since the bakery starts making cinnamon apple-filled doughnuts (A) in the second week:
$x = 0$

1 mark

SECTION B – continued

Each week some customers change their doughnut preferences and so the number of each type of doughnut sold changes accordingly. The transition matrix, **T**, reflecting the changes in the doughnuts sales from one week to the next is shown below.

$$\mathbf{T} = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{cccc} O & G & C & A \\ \end{array}$$

$$\mathbf{T} = \begin{bmatrix} 0.65 & 0.45 & 0.30 & d \\ 0.20 & b & 0.25 & 0.10 \\ a & 0.15 & 0.20 & 0.05 \\ 0.05 & 0.20 & c & 0.35 \end{bmatrix}\begin{array}{c} O \\ G \\ C \\ A \end{array}$$

b. Find the sum of the values a, b, c and d in the transition matrix, **T**.

Worked solution

$a = 1 - (0.65 + 0.2 + 0.05) = 0.1$
$b = 1 - (0.45 + 0.15 + 0.2) = 0.2$
$c = 1 - (0.3 + 0.25 + 0.2) = 0.25$
$d = 1 - (0.1 + 0.05 + 0.35) = 0.5$

$a + b + c + d = 0.1 + 0.2 + 0.25 + 0.5$
$= 1.05$

1 mark

c. The store manager notices that each customer buys only one doughnut every week.

i. Determine how many customers are not expected to have changed their doughnut preference in the second week.

ii. How many customers are expected to buy cinnamon apple-filled doughnuts in the fifth week?

Worked solution

c. i. $1200 \times 0.65 + 900 \times 0.2 + 1900 \times 0.20 + 0 \times 0.35 = 1340$
1340 customers are not expected to have changed their doughnut preference in the second week.

1 mark

ii. $\mathbf{S}_5 = \mathbf{T}^4 \times \mathbf{S}_1$

$$\mathbf{S}_5 = \begin{bmatrix} 0.65 & 0.45 & 0.30 & 0.5 \\ 0.20 & 0.2 & 0.25 & 0.10 \\ 0.1 & 0.15 & 0.20 & 0.05 \\ 0.05 & 0.20 & 0.25 & 0.35 \end{bmatrix}^4 \times \begin{bmatrix} 1200 \\ 900 \\ 1900 \\ 0 \end{bmatrix} = \begin{bmatrix} 2199.084375 \\ 761.4475 \\ 452.386875 \\ 587.08125 \end{bmatrix}$$

$$\begin{bmatrix} 2199.084375 \\ 761.4475 \\ 452.386875 \\ 587.08125 \end{bmatrix}\begin{array}{c} O \\ G \\ C \\ A \end{array}$$

So, 587 customers are expected to buy cinnamon apple filled doughnuts in the fifth week.

1 mark
1 + 1 = 2 marks

SECTION B – continued
TURN OVER

d. Determine how many more cinnamon apple-filled doughnuts are expected to be sold in week five than in week six.
Write your answer correct to the nearest whole number.

Worked solution

$\mathbf{S}_6 = \mathbf{T}^5 \times \mathbf{S}_1$

$$\mathbf{S}_6 = \begin{bmatrix} 0.65 & 0.45 & 0.30 & 0.5 \\ 0.20 & 0.2 & 0.25 & 0.10 \\ 0.1 & 0.15 & 0.20 & 0.05 \\ 0.05 & 0.20 & 0.25 & 0.35 \end{bmatrix}^5 \times \begin{bmatrix} 1200 \\ 900 \\ 1900 \\ 0 \end{bmatrix} = \begin{bmatrix} 2201.312906 \\ 763.9112188 \\ 453.957 \\ 580.818875 \end{bmatrix}$$

The number of cinnamon apple-filled doughnuts expected to be sold in week six is 580.818875. Subtract this from the number expected to be sold in week five to determine how many more are expected to be sold in week five than in week six.

$$587.08125 - 580.818875 = 6.272375$$
$$\cong 6$$

Approximately six more cinnamon apple-filled doughnuts are expected to be sold in week five than in week six.

1 mark

e. In week 10, every person buying an original glazed doughnut or a chocolate iced custard-filled doughnut will be given a free cup of coffee (the numbers of customers are written correct to the nearest whole number).
How many free cups of coffee, in total, are expected to be given away?

Worked solution

$$\mathbf{S}_{10} = \mathbf{T}^9 \times \mathbf{S}_1 = \begin{bmatrix} 2200.960374 \\ 764.7864547 \\ 454.7376092 \\ 579.5155618 \end{bmatrix} \cong \begin{bmatrix} 2201 \\ 765 \\ 455 \\ 580 \end{bmatrix}$$

To calculate the number of free cups of coffee, we add the number of original glazed donuts and the number of chocolate iced custard-filled donuts from the matrix above:
2 201 + 455 = 2 656

In total, 2 656 free cups of coffee are expected to be given away.

1 mark

In the first week, 1 200 original glazed doughnuts and 900 glazed cinnamon doughnuts were sold.

Suppose, instead, that 2 000 original glazed doughnuts and 1 000 glazed cinnamon doughnuts were sold in the first week.

f. Describe the way in which the number of original glazed doughnuts sold is expected to change over the next 60 or so weeks.

Worked solution

$$\mathbf{S}_{60} = \begin{bmatrix} 0.65 & 0.45 & 0.30 & 0.5 \\ 0.20 & 0.2 & 0.25 & 0.10 \\ 0.1 & 0.15 & 0.20 & 0.05 \\ 0.05 & 0.20 & 0.25 & 0.35 \end{bmatrix}^{59} \times \begin{bmatrix} 2000 \\ 1000 \\ 1900 \\ 0 \end{bmatrix} = \begin{bmatrix} 2696.172249 \\ 936.861244 \\ 557.0526316 \\ 709.9138756 \end{bmatrix}$$

$$\mathbf{S}_{61} = \begin{bmatrix} 0.65 & 0.45 & 0.30 & 0.5 \\ 0.20 & 0.2 & 0.25 & 0.10 \\ 0.1 & 0.15 & 0.20 & 0.05 \\ 0.05 & 0.20 & 0.25 & 0.35 \end{bmatrix}^{60} \times \begin{bmatrix} 2000 \\ 1000 \\ 1900 \\ 0 \end{bmatrix} = \begin{bmatrix} 2696.172249 \\ 936.861244 \\ 557.0526316 \\ 709.9138756 \end{bmatrix}$$

The number of original glazed doughnuts sold is expected to change from 2 000 to 2 696 over the next 60 or so weeks.

1 mark

Total 15 marks

Tip

- *The answer, 'The number of original glazed doughnuts sold is expected to increase by 696,' is also acceptable.*

END OF MODULE 6
END OF SOLUTIONS BOOK