**Victorian Certificate of Education
2010**

FURTHER MATHEMATICS
Written examination 1

Monday 1 November 2010

**Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)**

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
A	13	13			13
B	54	27	6	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 42 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Core: Data analysis

The following information relates to Questions 1, 2 and 3.

To test the temperature control on an oven, the control is set to 180 °C and the oven is heated for 15 minutes. The temperature of the oven is then measured. Three hundred ovens were tested in this way. Their temperatures were recorded and are displayed below using both a histogram and a boxplot.
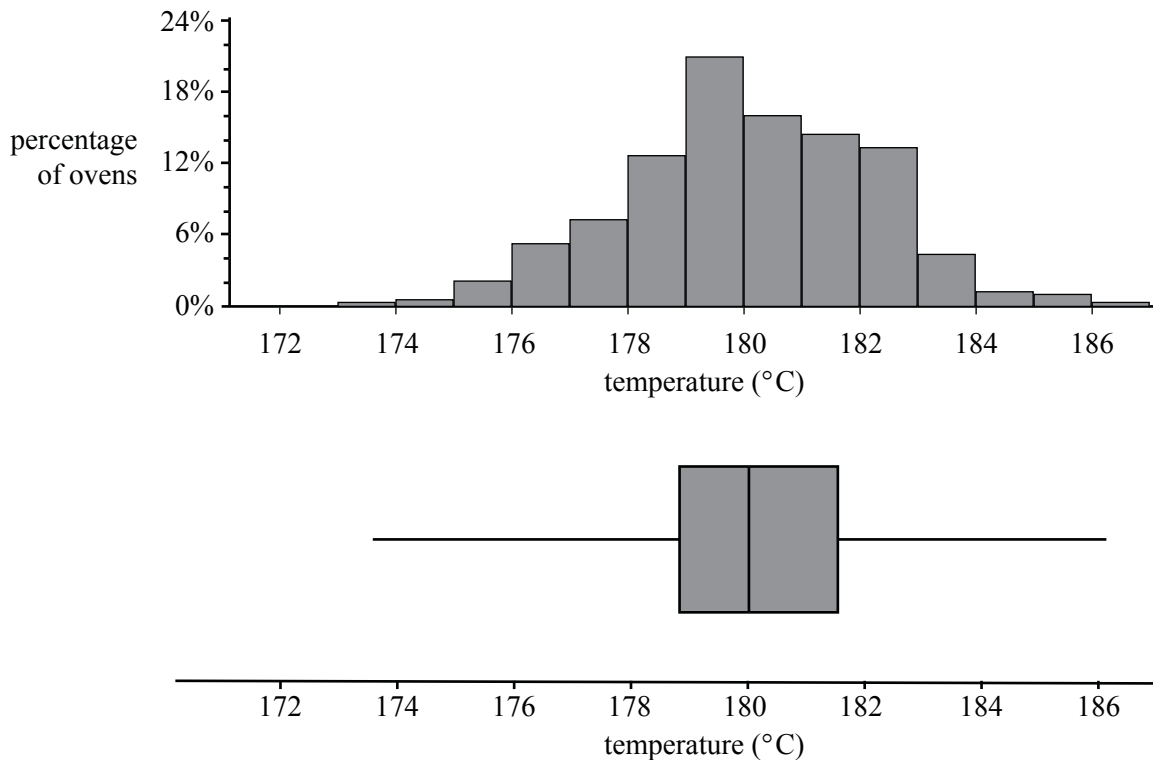
Question 1

A total of 300 ovens were tested and their temperatures were recorded.

The number of these temperatures that lie between 179 °C and 181 °C is closest to

A.   40

B.   50

C.   70

D.   110

E.   150

Question 2

The interquartile range for temperature is closest to

A.   1.3 °C

B.   1.5 °C

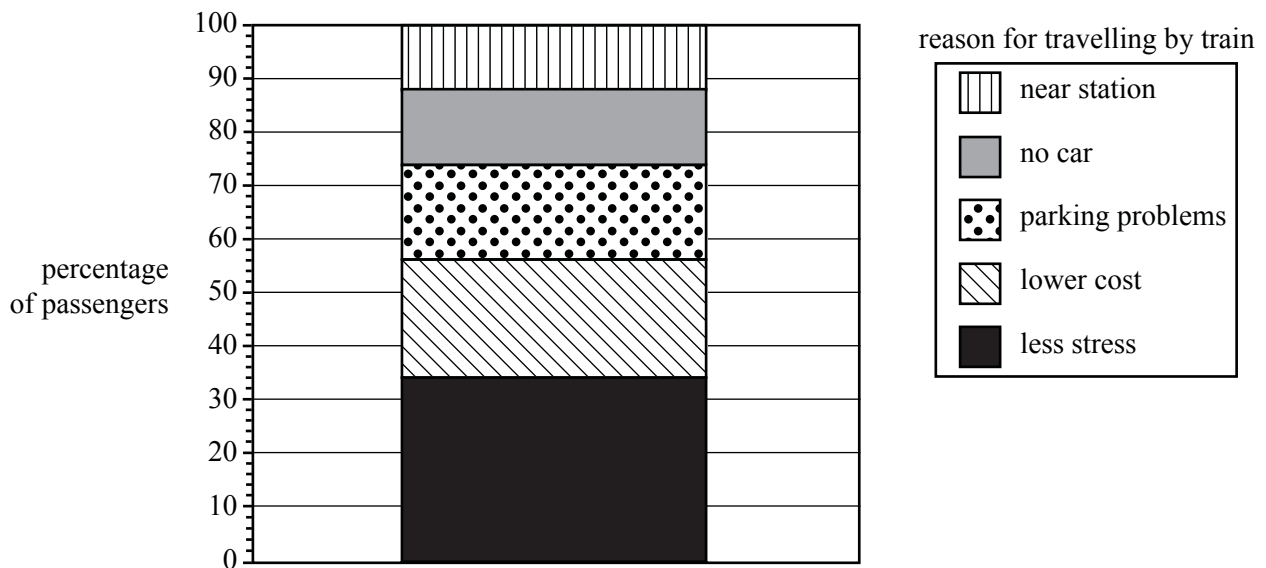C.   2.0 °C

D.   2.7 °C

E.   4.0 °C

Question 3

Using the 68–95–99.7% rule, the standard deviation for temperature is closest to

A.   1 °C

B.   2 °C

C.   3 °C

D.   4 °C

E.   6 °C

Question 4

The passengers on a train were asked why they travelled by train. Each reason, along with the percentage of passengers who gave that reason, is displayed in the segmented bar chart below.

The percentage of passengers who gave the reason 'no car' is closest to

A.   14%

B.   18%

C.   26%

D.   74%

E.   88%

The following information relates to Questions 5 and 6.

The lengths of the left feet of a large sample of Year 12 students were measured and recorded. These foot lengths are approximately normally distributed with a mean of 24.2 cm and a standard deviation of 4.2 cm.

Question 5

A Year 12 student has a foot length of 23 cm.

The student's standardised foot length (standard z score) is closest to

A.   −1.2

B.   −0.9

C.   −0.3

D.   0.3

E.   1.2

Question 6

The percentage of students with foot lengths between 20.0 and 24.2 cm is closest to

A.   16%

B.   32%

C.   34%

D.   52%

E.   68%

The following information relates to Questions 7, 8 and 9.

The *height* (in cm) and *foot length* (in cm) for each of eight Year 12 students were recorded and displayed in the scatterplot below.

A least squares regression line has been fitted to the data as shown.
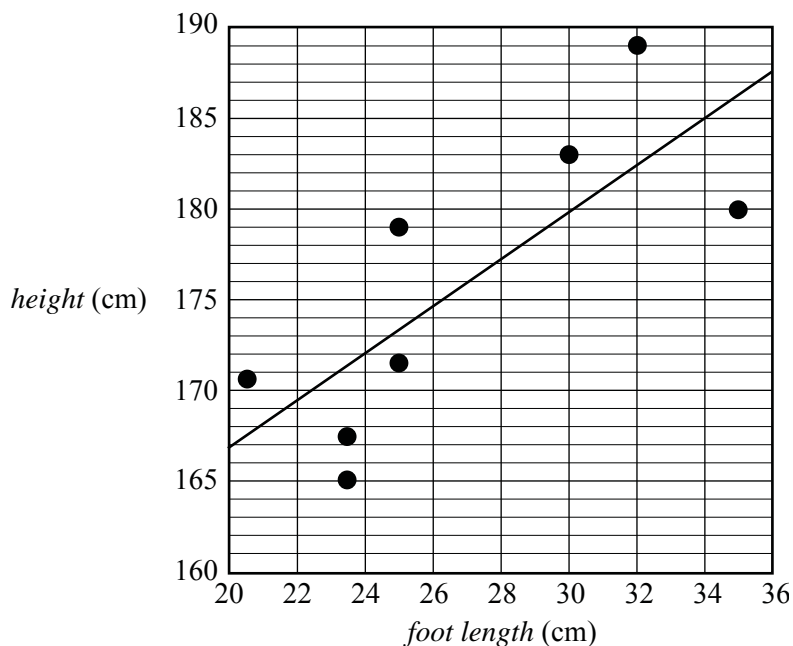
Question 7

By inspection, the value of the product-moment correlation coefficient (*r*) for this data is closest to

A.   0.98

B.   0.78

C.   0.23

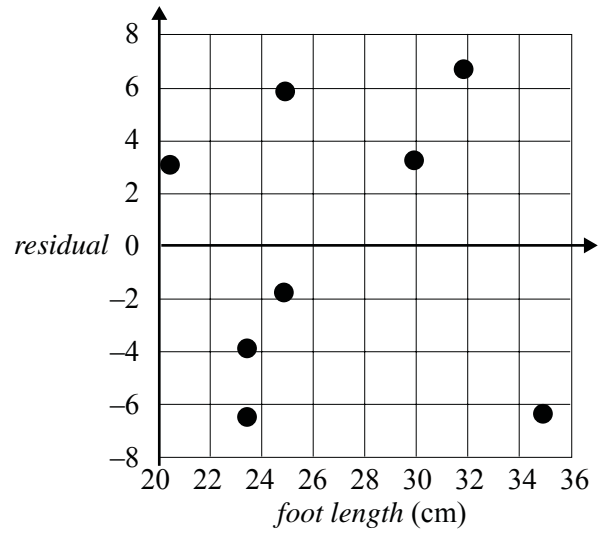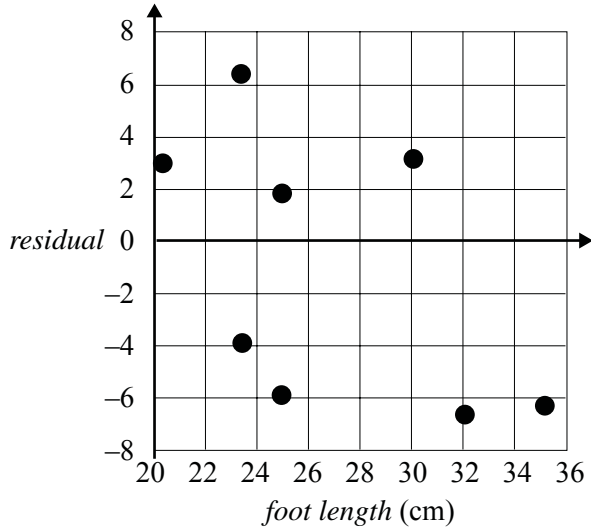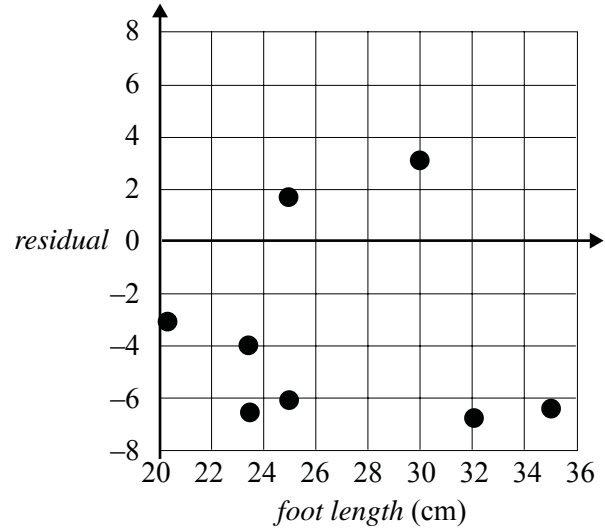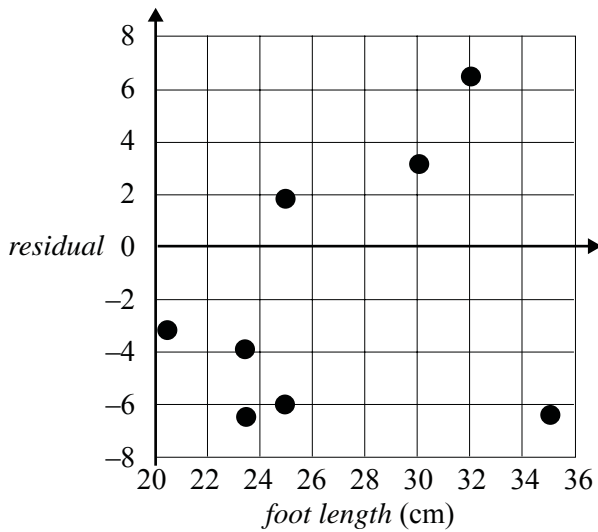**D.**   –0.44

E.   –0.67

Question 8

The independent variable is *foot length*.

The equation of the least squares regression line is closest to

A.   *height* = –110 + 0.78 × *foot length*.

B.   *height* = 141 + 1.3 × *foot length*.

C.   *height* = 167 + 1.3 × *foot length*.

D.   *height* = 167 + 0.67 × *foot length*.

E.   *foot length* = 167 + 1.3 × *height*.

Question 9

The plot of the *residuals* against *foot length* is closest to

A.
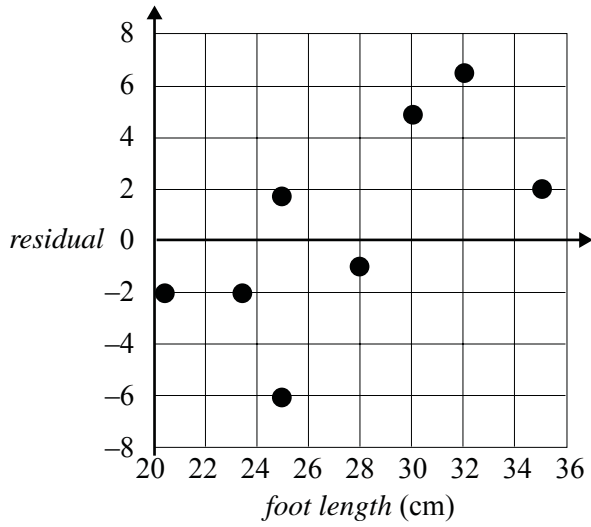
B.

C.

D.

E.

Question 10

For a set of bivariate data that involves the variables x and y, with y as the dependent variable

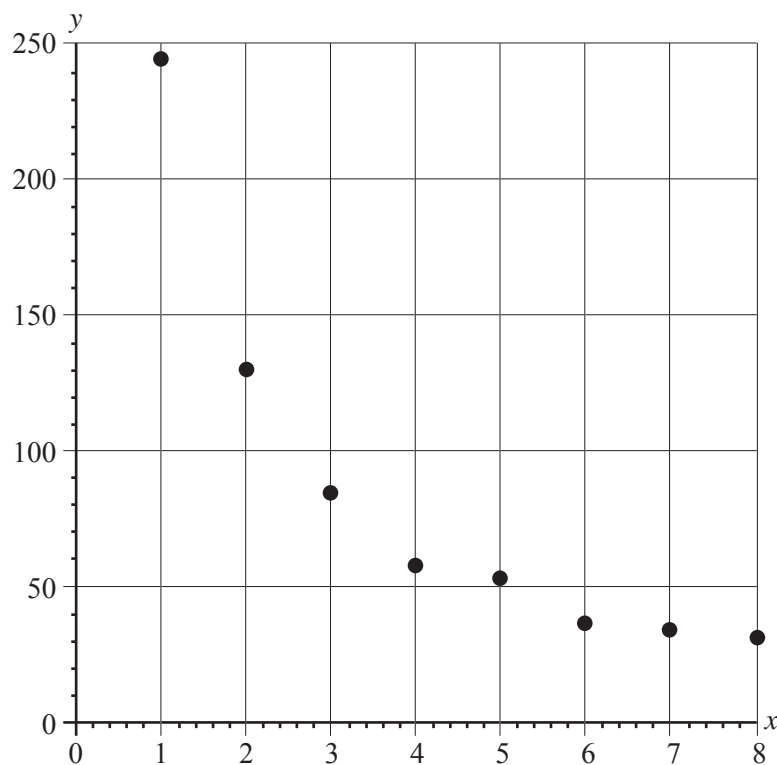$$r = -0.644, \quad \bar{x} = 5.30, \quad \bar{y} = 5.60, \quad s_x = 3.06, \quad s_y = 3.20$$

The equation of the least squares regression line is closest to

A. $y = 9.2 - 0.7x$
B. $y = 9.2 + 0.7x$
C. $y = 2.0 - 0.6x$
D. $y = 2.0 - 0.7x$
E. $y = 2.0 + 0.7x$

Question 11

A student uses the following data to construct the scatterplot shown below.

| x	1	2	3	4	5	6	7	8
y	245	130	84	58	52	36	33	30

A reciprocal transformation is applied to the x-axis and is used to linearise the scatterplot.

With y as the dependent variable, the slope of the least squares regression line that is fitted to the **linearised** plot is closest to

A.   –249
B.   –25
C.   0.004
D.   25
E.   249

Question 12

The time series plot below shows the number of calls each month to a call centre over a twelve-month period.
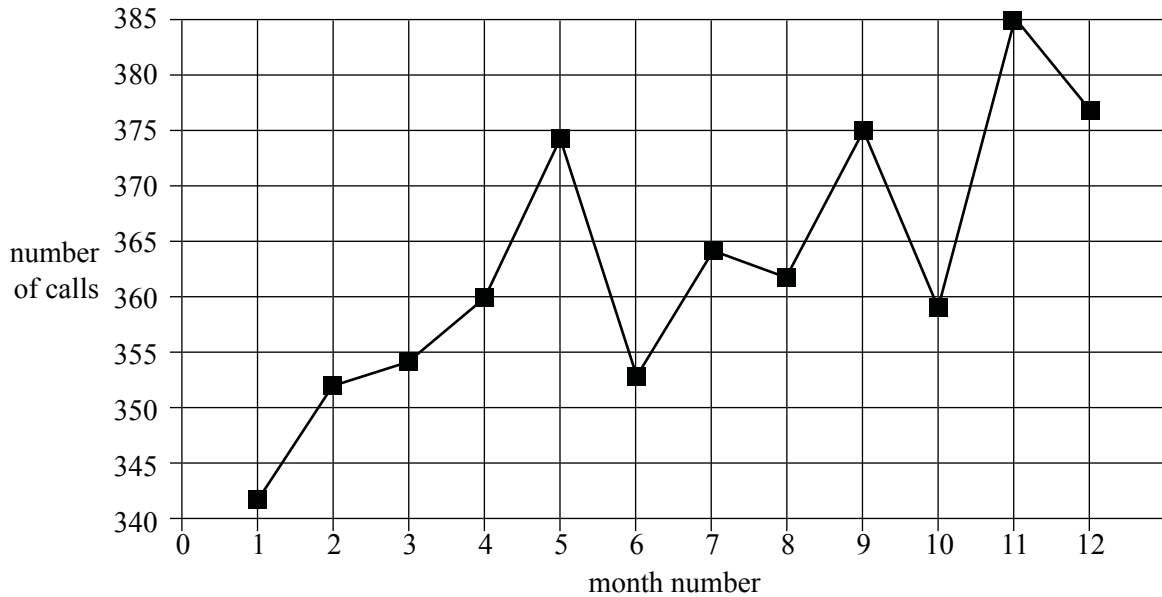
The plot is to be smoothed using five-point **median** smoothing.

The smoothed number of calls for month number 10 is closest to

A.   358

B.   364

C.   371

D.   375

E.   377

Question 13

A garden supplies outlet sells water tanks. The monthly seasonal indices for the revenue from the sale of water tanks are given below.

The seasonal index for September is missing.

Seasonal index											
Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1.26	0.96	0.74	0.48	0.31	0.39	0.75	1.55		1.32	1.25	1.58

The revenue from the sale of water tanks in September 2009 was $104 500.

The deseasonalised revenue for September 2009 is closest to

A. $42 800

B. $74 100

C. $104 500

D. $141 000

E. $147 300

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B
Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.
Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
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Module 1: Number patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for
multiple-choice questions and write the name of the module in the box provided.

Question 1

The sequence

$$3, 6, 9, 12 \ldots$$

could be

A. Fibonacci.
B. arithmetic.
C. geometric.
D. alternating.
E. decreasing.

Question 2

The first three terms of a geometric sequence are

$$0.125, \ 0.25, \ 0.5$$

The fourth term in this sequence would be

A. 0.625
B. 0.75
C. 0.875
D. 1
E. 1.25

Question 3

The prizes in a lottery form the terms of a geometric sequence with a common ratio of 0.95.

If the first prize is $20 000, the value of the eighth prize will be closest to

A. $7 000
B. $8 000
C. $12 000
D. $13 000
E. $14 000

Question 4

The first four terms of a geometric sequence are

$$4, -8, 16, -32$$

The sum of the first ten terms of this sequence is

A.   −2048

B.   −1364

C.   684

D.   1367

E.   4096

Question 5

A team of swimmers was training.

Claire was the first swimmer for the team and she swam 100 metres.

Every other swimmer in the team swam 50 metres further than the previous swimmer.

Jane was the last swimmer for the team and she swam 800 metres.

The total number of swimmers in this team was

A.   9

B.   13

C.   14

D.   15

E.   18

Question 6

$t_1 = 10$, $t_2 = k$ and $t_3 = 90$ are the first three terms of a difference equation with the rule $t_n = t_{n-1} + t_{n-2}$.
The value of k is

A.   30

B.   40

C.   50

D.   60

E.   80

Question 7

Each trading day, a share trader buys and sells shares according to the rule

$$T_{n+1} = 0.6\, T_n + 50\,000$$

where T_n is the number of shares the trader owns at the start of the nth trading day.

From this rule, it can be concluded that each day

A. the trader sells 60% of the shares that she owned at the start of the day and then buys another 50 000 shares.

B. the trader sells 40% of the shares that she owned at the start of the day and then buys another 50 000 shares.

C. the trader sells 50 000 of the shares that she owned at the start of the day.

D. the trader sells 60% of the 50 000 shares that she owned at the start of the day.

E. the trader sells 40% of the 50 000 shares that she owned at the start of the day.

TURN OVER

Question 8

The nth term in a geometric sequence is t_n.

The common ratio is greater than one.

A graph that could be used to display the terms of this sequence is

A.
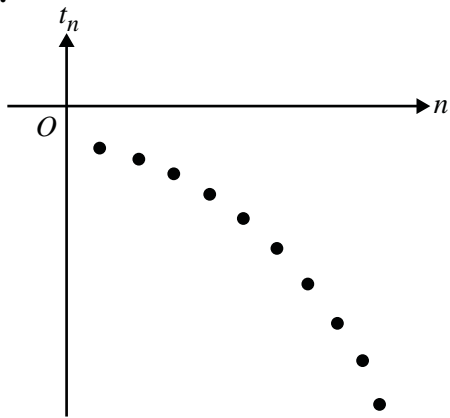
B.
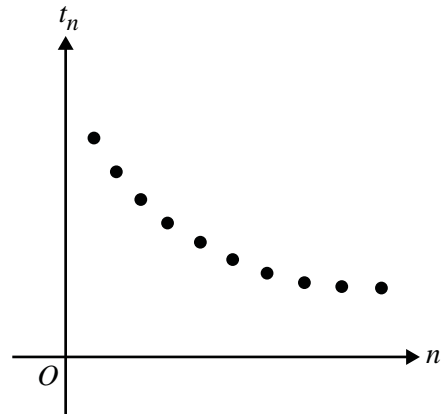
C.
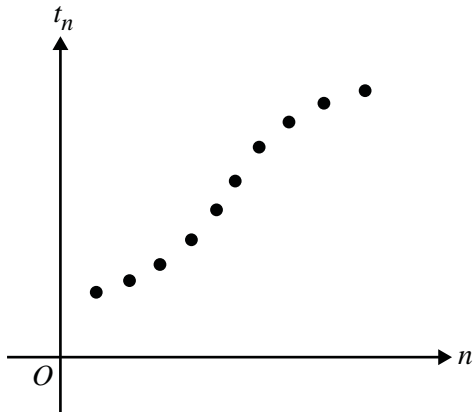
D.
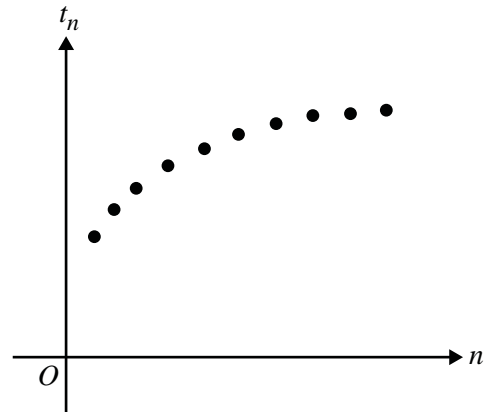
E.
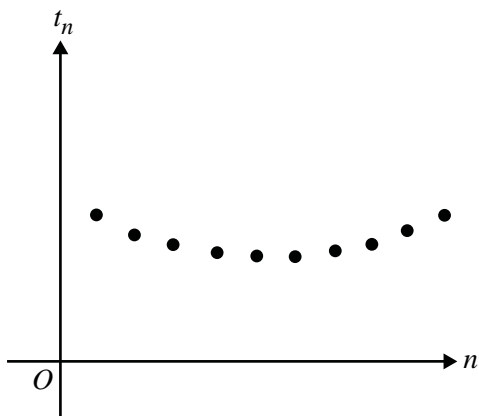
Question 9

Before he began training, Jethro's longest jump was 5.80 metres.

After the first month of training, his longest jump had increased by 0.32 metres.

After the second month of training, his longest jump had increased by another 0.24 metres.

After the third month of training, his longest jump had increased by another 0.18 metres.

If this pattern of improvement continues, Jethro's longest jump, correct to two decimal places, will be closest to

A.   6.54 metres.
B.   6.68 metres.
C.   7.00 metres.
D.   7.08 metres.
E.   7.25 metres.

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.
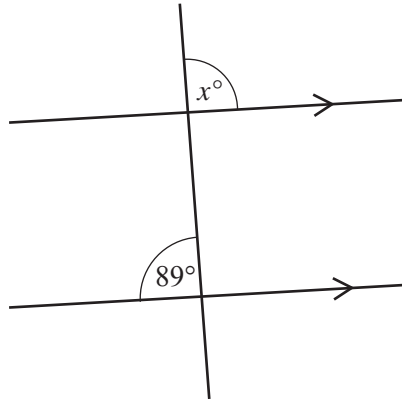
Question 1

The value of x in the diagram above is

A.　89
B.　90
C.　91
D.　101
E.　180

Question 2

A circle has a circumference of 10 cm.

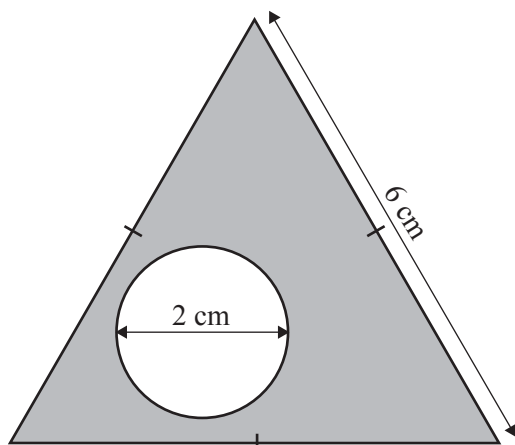The radius of this circle is closest to

A.　1.3 cm
B.　1.6 cm
C.　1.8 cm
D.　3.2 cm
E.　5.0 cm

Question 3

An equilateral triangle of side length 6 cm is cut from a sheet of cardboard.

A circle is then cut out of the triangle, leaving a hole of diameter 2 cm as shown below.

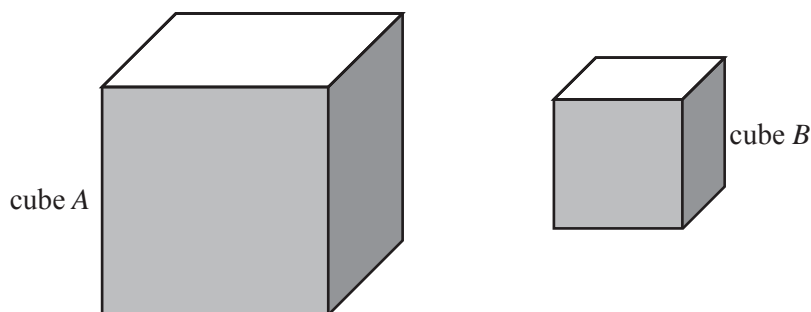The area of cardboard **remaining**, as shown by the shaded region in the diagram above, is closest to

A.   3 cm^2

B.   9 cm^2

C.   12 cm^2

D.   15 cm^2

E.   16 cm^2

Question 4

Cube *A* and cube *B* are shown below.

The side length of cube *A* is 1.5 times the side length of cube *B*.

The **surface** area of cube *B* is 256 cm^2.

The **surface** area of cube *A* is

A.   114 cm^2

B.   256 cm^2

C.   384 cm^2

D.   576 cm^2

E.   864 cm^2

SECTION B – Module 2: Geometry and trigonometry – continued
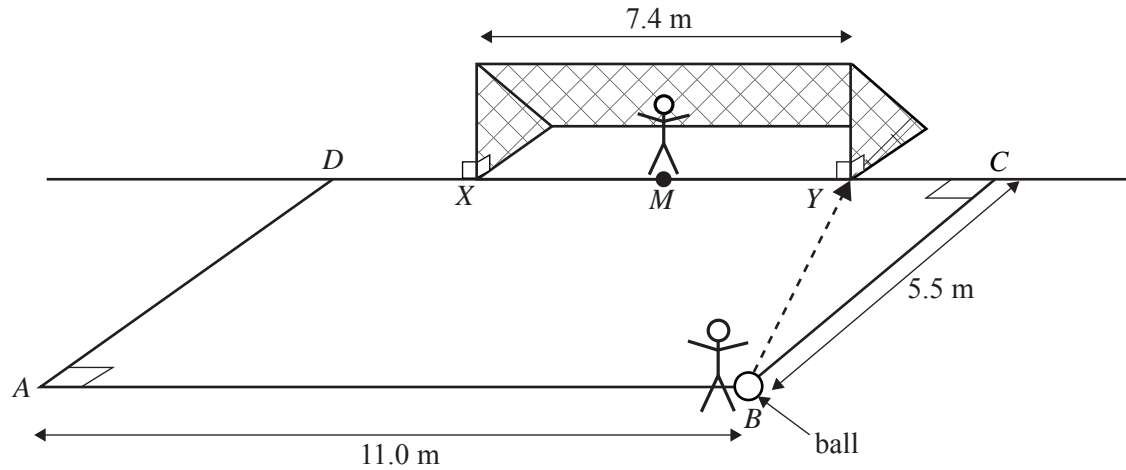**TURN OVER**

The following information relates to Questions 5 and 6.

A soccer goal is 7.4 metres wide.

A rectangular region *ABCD* is marked out directly in front of the goal.

In this rectangular region, *AB* = *DC* = 11.0 metres and *AD* = *BC* = 5.5 metres.

The goal line *XY* lies on *DC* and *M* is the midpoint of both *DC* and *XY*.
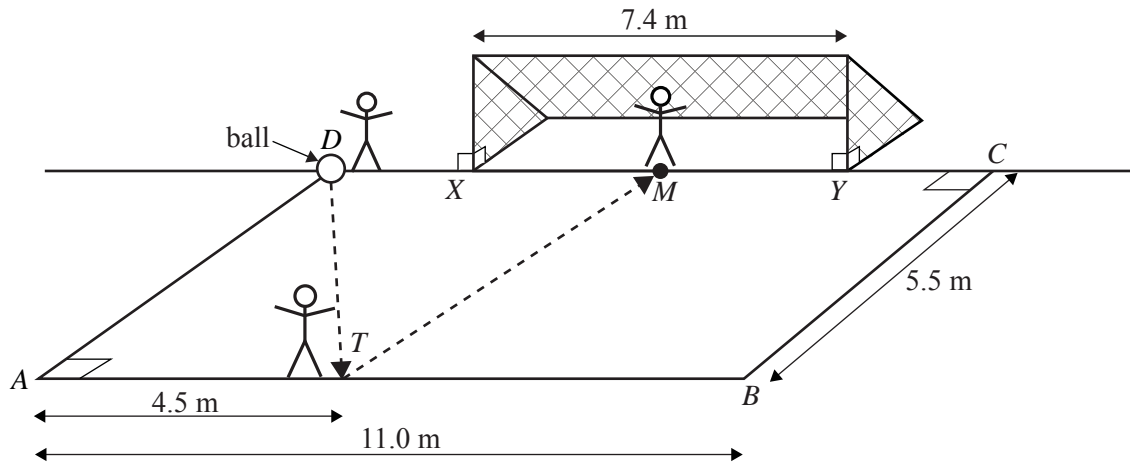

Question 5

Ben kicks the ball from point *B*. It travels in a straight line to the base of the goal post at point *Y* on the goal line.

Angle *CBY*, the angle that the path of the ball makes with the line *BC*, is closest to

A.   18°

B.   33°

C.   45°

D.   67°

E.   72°

Question 6

David kicks the ball from point *D* in a straight line to Tara. Tara is standing near point *T* on the line *AB*, a distance of 4.5 metres from point *A*. Tara then kicks the ball from point *T* in a straight line to the midpoint of the goal line at *M*.
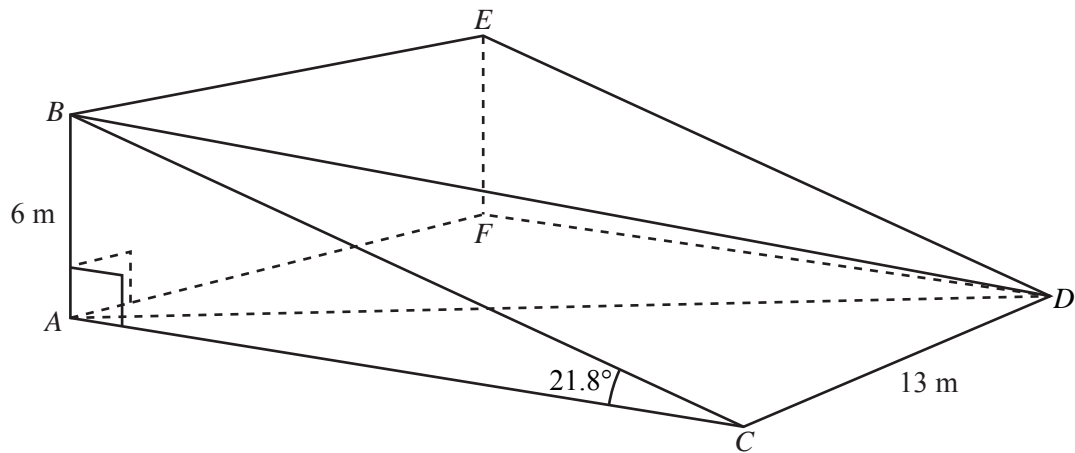

The total distance that the ball will travel in moving from point *D* to *T* to *M* is closest to

A.   5.5 m

B.   12.1 m

C.   12.5 m

D.   12.7 m

E.   12.9 m

Question 7

The diagram below shows a right-triangular prism *ABCDEF*.

In this prism, $AB = 6$ m, angle $ACB = 21.8°$ and $CD = 13$ m.

The size of the angle *CBD* is closest to

A. 21.6°

B. 26.7°

C. 38.8°

D. 40.9°

E. 51.2°

Question 8

Dan takes his new aircraft on a test flight.

He starts from his local airport and flies 10 km on a bearing of 045° until he reaches his brother's farm.

From here he flies 18 km on a bearing of 300° until he reaches his parents' farm.

Finally he flies back directly from his parents' farm to his local airport.

The total distance (in km) that he flies is closest to

A. 37

B. 42

C. 46

D. 59

E. 61

Question 9

A conical water filter has a diameter of 60 cm and a depth of 24 cm. It is filled to the top with water.

The water filter sits above an empty cylindrical container which has a diameter of 40 cm.

The water is allowed to flow from the water filter into the cylindrical container.
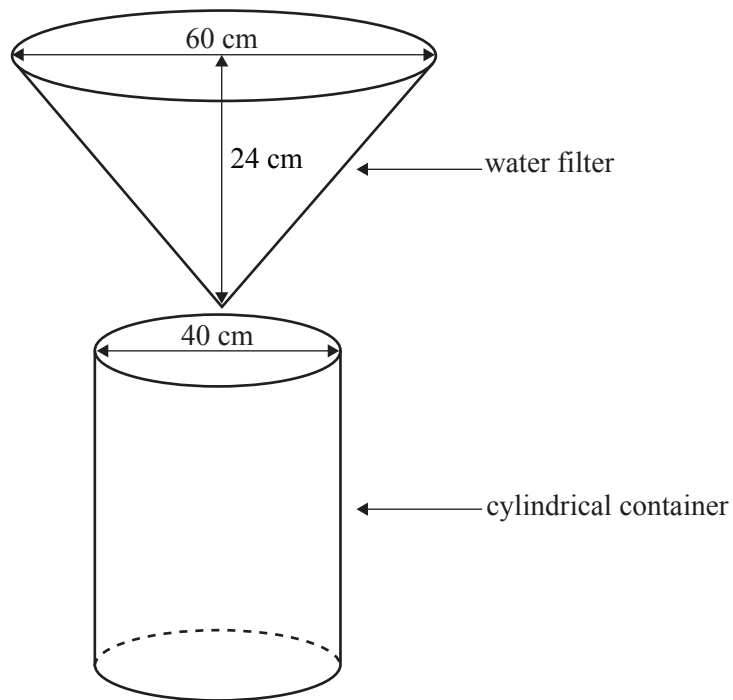
When the water filter is empty, the depth of water in the cylindrical container will be

A.   8 cm
B.   18 cm
C.   24 cm
D.   32 cm
E.   96 cm

Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

The following information refers to Questions 1 and 2.

The volume of water that is stored in a tank over a 24-hour period is shown in the graph below.
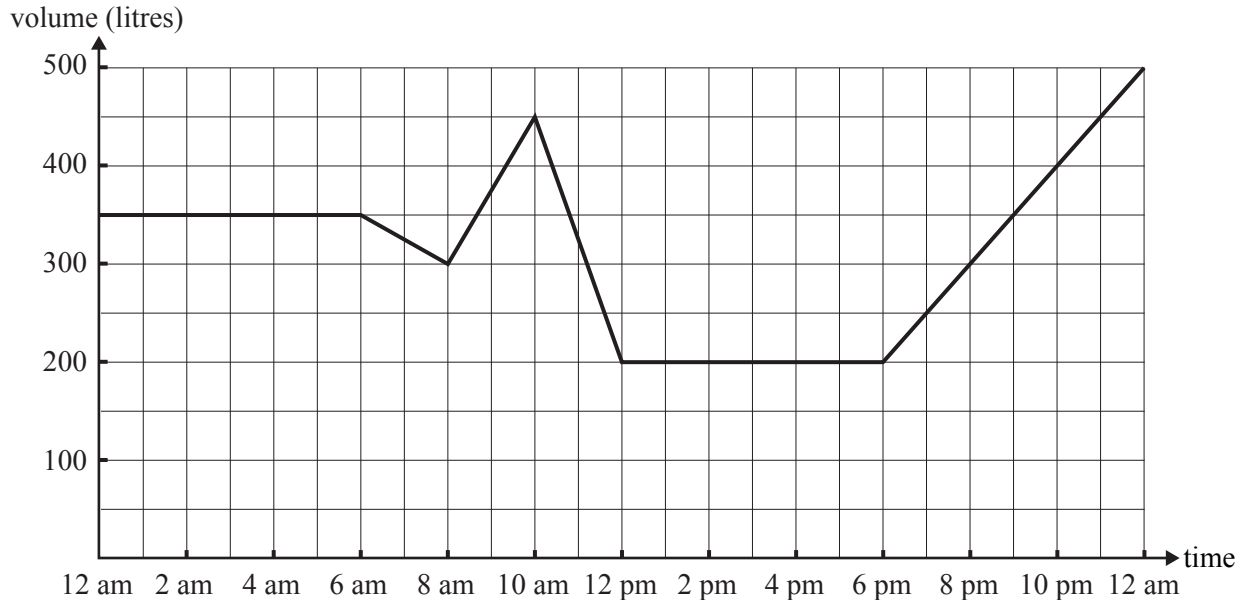
Question 1

What is the difference in the volume of water (in litres) in the tank between 8 am and 6 pm?

A.   50
B.   100
C.   120
D.   200
E.   400

Question 2

The rate of increase in the volume of water in the tank (in litres/hour) between 8 am and 10 am is

A.   37.5
B.   50
C.   75
D.   125
E.   150

Question 3

An equation for the straight line that passes through the points (10, 1) and (4, –2) is

A. $x + 2y = 12$

B. $2x + y = 6$

C. $4x + y = 14$

D. $x – 4y = 14$

E. $x – 2y = 8$

Question 4

The manager of an office is ordering finger food for an office party.

Hot items cost $2.15 each and cold items cost $1.50 each.

Let x be the number of hot items ordered.

Let y be the number of cold items ordered.

The manager can spend no more than $5 for each of the 200 employees.

An inequality that can be used to represent this constraint is

A. $1.5x + 2.15y \leq 5$

B. $1.5x + 2.15y \leq 200$

C. $1.5x + 2.15y \leq 1000$

D. $2.15x + 1.5y \leq 200$

E. $2.15x + 1.5y \leq 1000$

Question 5

The cost in dollars, C, of making n pottery mugs is given by the equation

$$C = 150 + 6n$$
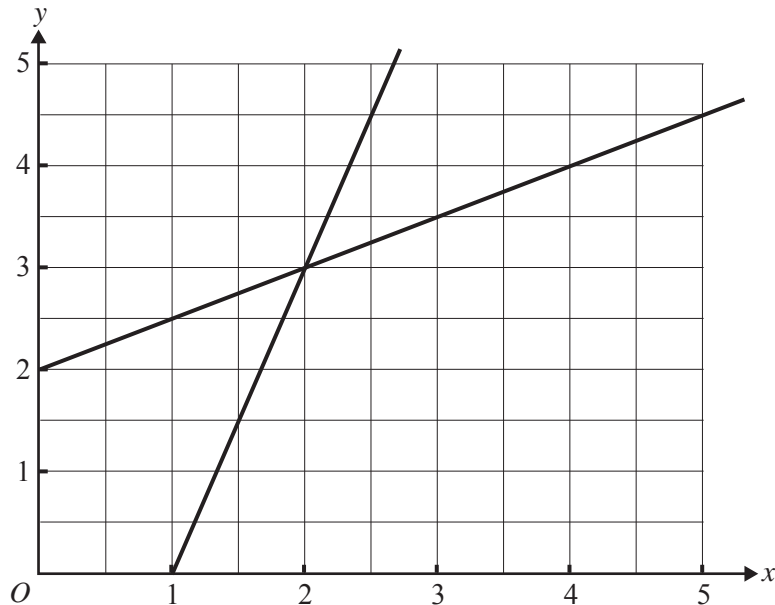
A loss will result from selling

A.   60 mugs at $9.00 each.

B.   70 mugs at $8.50 each.

C.   80 mugs at $7.50 each.

D.   90 mugs at $8.00 each.

E.   100 mugs at $9.50 each.

SECTION B – Module 3: Graphs and relations – continued
TURN OVER

Question 6

The graphs of the linear relations

$$x - 2y = -4 \quad \text{and} \quad 3x - y = 3$$

are shown below.

A point that satisfies both the inequalities

$$x - 2y \geq -4 \quad \text{and} \quad 3x - y \geq 3$$

is

A.   (1, 2)

B.   (1, 2.5)

C.   (2, 4)

D.   (3, 2)

E.   (3, 4)

Question 7

Bruce and John both work in a factory. They assemble bicycles and scooters.

It takes 45 minutes to assemble a bicycle and 15 minutes to assemble a scooter.

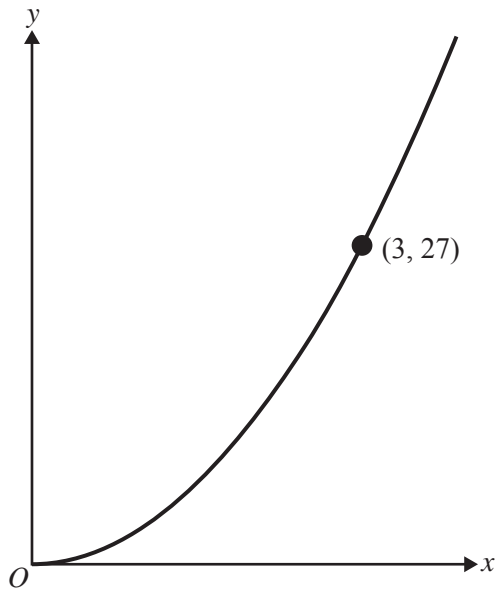Bruce assembled 7 bicycles and 8 scooters.

In the same time, John assembled 6 bicycles and a number of scooters.
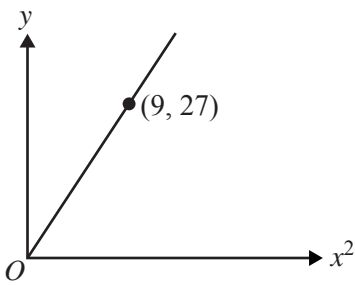
The number of scooters that John assembled is

A.   3

B.   6

C.   11

D.   13
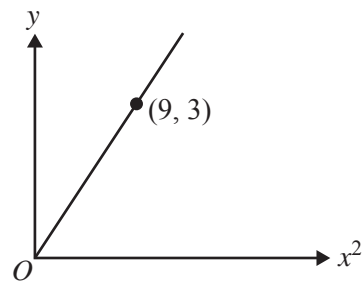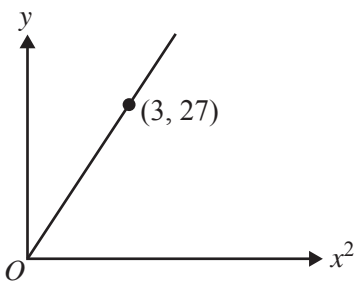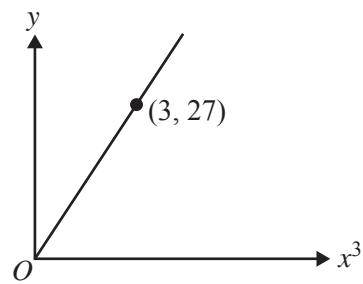
E.   31

SECTION B – Module 3: Graphs and relations – continued

Question 8

The graph of $y = 3x^2$ is shown below.

Another graph that represents this relationship between x and y is

A.

B.

C.

D.

E.
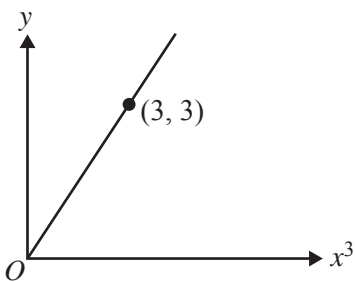
Question 9

Two roof sections of a building are separated by a vertical window.
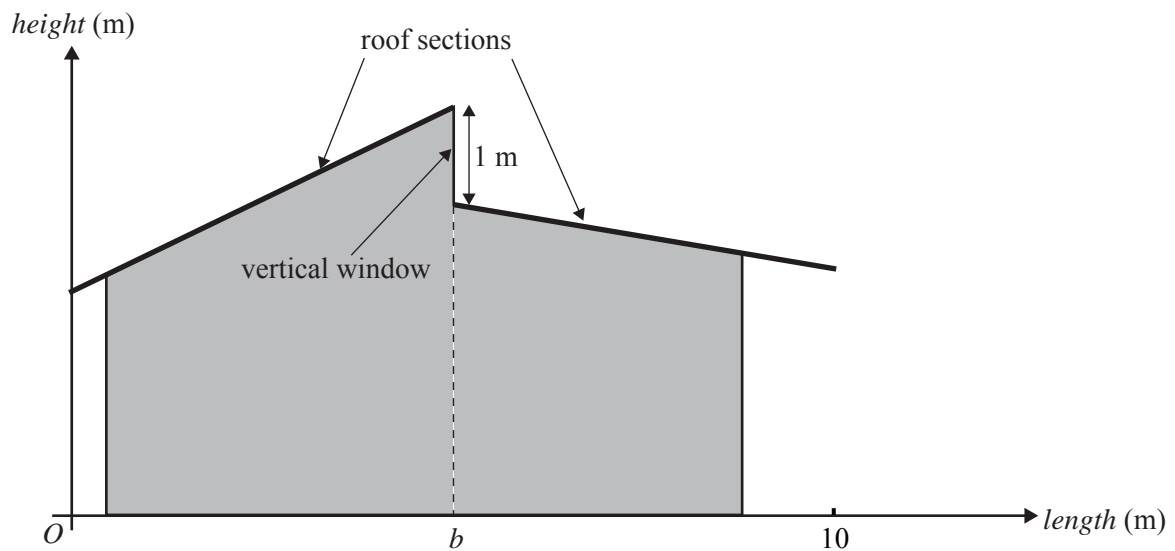
The roof sections are modelled by two straight line graphs as follows.

$$height = \begin{cases} \dfrac{1}{2} \times length + 3 & 0 \leq length < b \\[2mm] -\dfrac{1}{6} \times length + 5 & b \leq length \leq 10 \end{cases}$$

The vertical window has height 1 m.

The value of b is

A.   3.0

B.   4.5

C.   4.8

D.   5.0

E.   5.3

Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Sally purchased an electronic game machine on hire purchase. She paid $140 deposit and then $25.50 per month for two years.

The total amount that Sally paid is

A. $191

B. $446

C. $612

D. $740

E. $752

Question 2

$6000 is invested in an account that earns simple interest at the rate of 3.5% per annum.

The total interest earned in the first four years is

A. $70

B. $84

C. $210

D. $840

E. $885

Question 3

Peter received a quote from the Artificial Grass Company for his new front lawn.

The quote is for $1880 plus a Goods and Services Tax (GST) of 10%.

The final amount that Peter pays for the new front lawn is

A. $188

B. $1880

C. $1890

D. $1899

E. $2068

Question 4

The price of a one-way airfare between two cities varies each day according to demand.

- On Monday the price is $160.
- The price on Tuesday is 25% greater than the price on Monday.
- The price on Wednesday is 10% less than the price on Tuesday.
- The price on Thursday is 25% less than the price on Wednesday.
- The price on Friday is 20% greater than the price on Thursday.
- The Saturday price is the same as the Friday price.

The price on Saturday is

A. $150

B. $162

C. $176

D. $288

E. $330

Question 5

A file server costs $30 000.

The file server depreciates by 20% of its value each year.

After three years its value is

A. $6 000

B. $12 000

C. $15 360

D. $19 200

E. $24 000

Question 6

The transaction details for an online savings account, for the month of March 2010, are shown below.

Date	Transaction detail	Debit $	Credit $	Balance $
1 March 2010	Balance			1650.00
16 March 2010	Withdrawal	200.00		1450.00
31 March 2010	Deposit		350.00	1800.00

The bank pays a simple interest rate of 3.6% per annum on the minimum monthly balance.

The amount of interest that is credited to the account, for the month of March 2010, would be closest to

A. $4.35

B. $4.95

C. $5.20

D. $43.50

E. $52.20

SECTION B – Module 4: Business-related mathematics – continued

Question 7

A loan of $300 000 is to be repaid over a period of 20 years. Interest is charged at the rate of 7.25% per annum compounding quarterly.

The quarterly repayment to the nearest cent is

A. $2371.13

B. $5511.46

C. $7113.39

D. $7132.42

E. $7156.45

Question 8

Rae paid $40 000 for new office equipment at the start of the 2007 financial year.

At the start of each following financial year, she used flat rate depreciation to revalue her equipment.

At the start of the 2010 financial year she revalued her equipment at $22 000.

The annual flat rate of depreciation she used, as a percentage of the purchase price, was

A. 11.25%

B. 15%

C. 17.5%

D. 35%

E. 45%

Question 9

Rebecca invested $4000 at 5.0% per annum with interest compounding quarterly.

After interest is paid at the end of each quarter, Rebecca adds $800 to her investment.

The value of her investment at the end of the second quarter, after the $800 has been added, is closest to

A. $4101

B. $4901

C. $4911

D. $5711

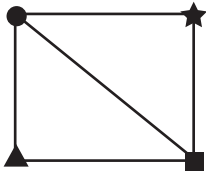E. $6060

Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.
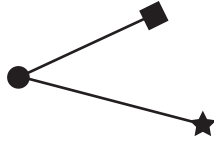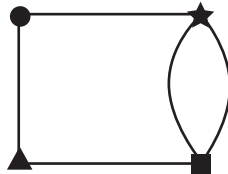
Question 1

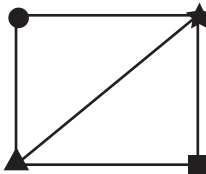The graph above is a subgraph of which one of the following graphs?

A.

B.

C.

D.
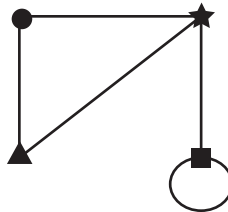
E.
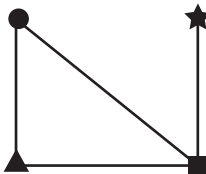
Question 2
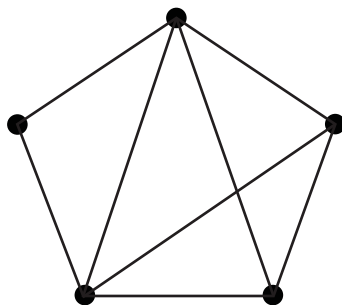
The number of edges in the graph above is

A.   5
B.   7
C.   8
D.   10
E   11

Question 3

$$
\begin{array}{ccccc}
A & B & C & D & E \\
\end{array}
$$

$$
\begin{bmatrix}
0 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 2 \\
0 & 0 & 1 & 0 & 1 \\
1 & 1 & 2 & 1 & 0
\end{bmatrix}
\begin{array}{l}
A \\
B \\
C \\
D \\
E
\end{array}
$$

A graph that can be drawn from the adjacency matrix above is

A.

B.

C.

D.

E.

SECTION B – Module 5: Networks and decision mathematics – continued
TURN OVER
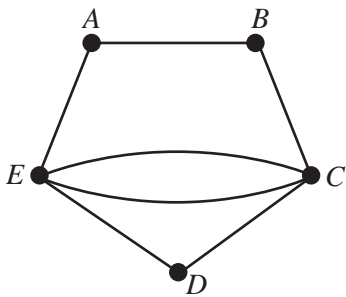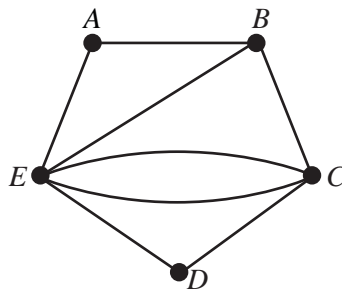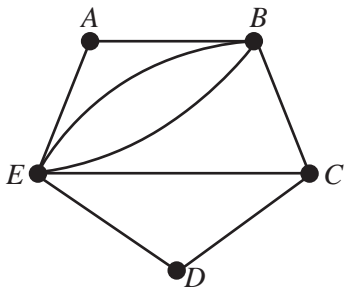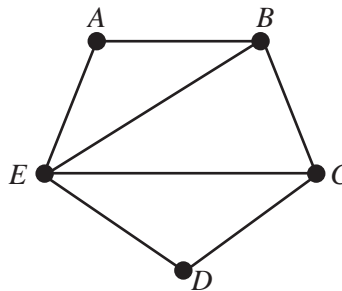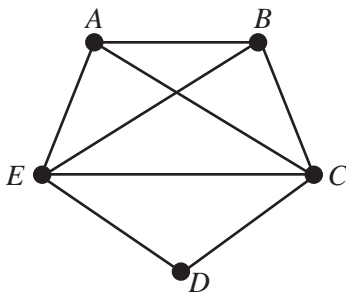
Question 4

A board game consists of nine labelled squares as shown.

A player must start at square J and, moving one square at a time, aim to finish at square R.

Each move may only be to the **right one square** or **down one square**.

A player who lands on square N must stay there and cannot move again.

A player can only stop moving when they reach N or R.

J	K	L
M	N	O
P	Q	R

A digraph that shows all the possible moves that a player could make to reach N or R from J is

A.
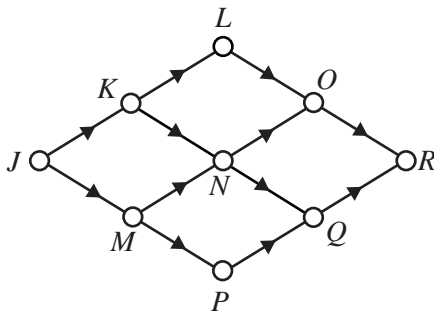
B.
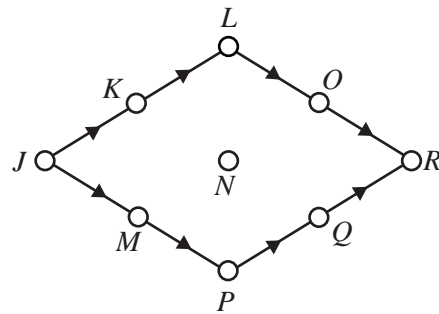
C.
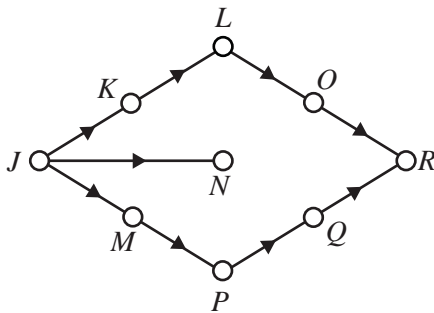
D.
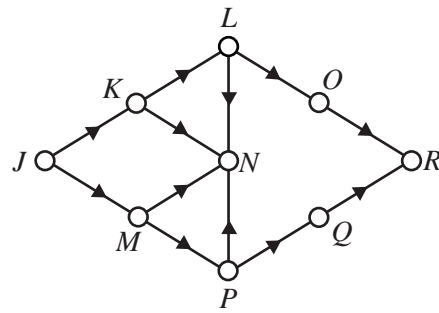
E.
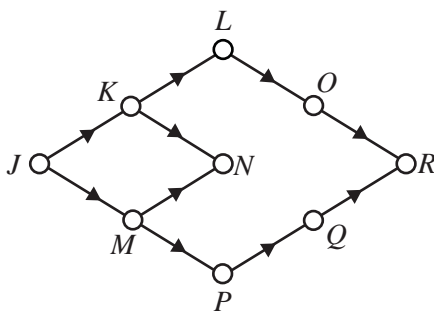
Question 5
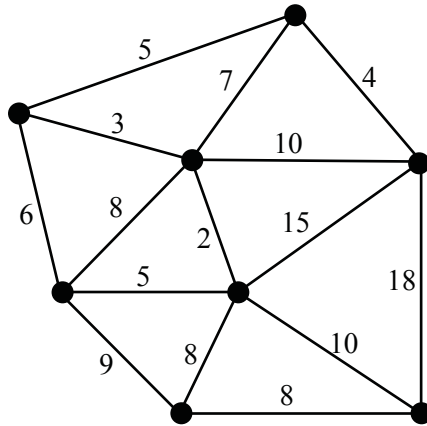
For the network above, the length of the minimal spanning tree is

A. 30
B. 31
C. 35
D. 39
E. 45

The following information refers to Questions 6 and 7.

In the network below, the values on the edges give the maximum flow possible between each pair of vertices. The arrows show the direction of flow. A cut that separates the source from the sink in the network is also shown.
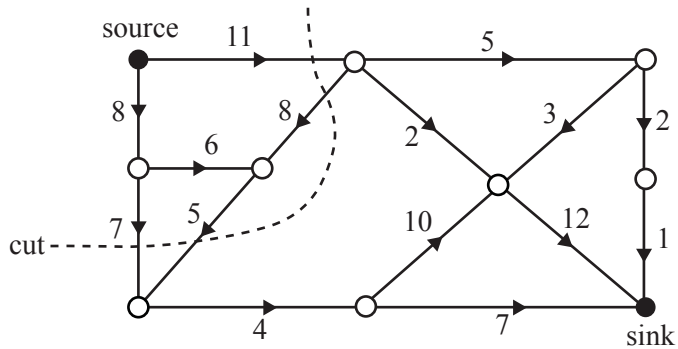
Question 6

The capacity of this cut is

A.   14

B.   18

C.   23

D.   31

E.   40

Question 7

The maximum flow between source and sink through the network is

A.   7

B.   10

C.   11

D.   12

E.   20

SECTION B – Module 5: Networks and decision mathematics – continued

Question 8

A project has 12 activities. The network below gives the time (in hours) that it takes to complete each activity.
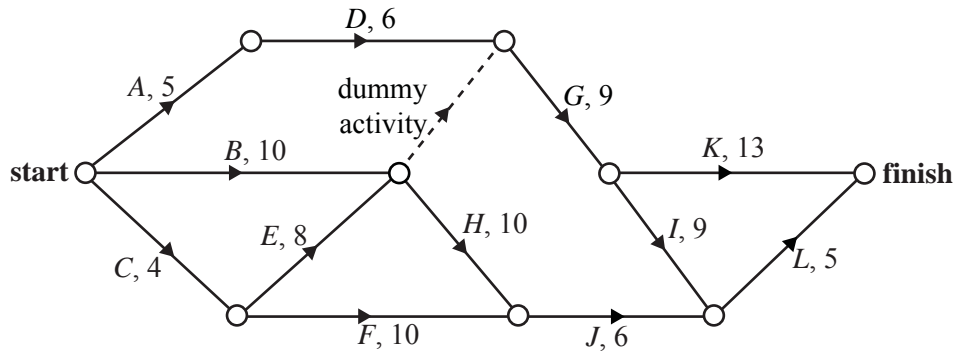
The critical path for this project is

A. *ADGK*

B. *ADGIL*

C. *BHJL*

D. *CEGIL*

E. *CEHJL*

Question 9

The table below shows the time (in minutes) that each of four people, Aiden, Bing, Callum and Dee, would take to complete each of the tasks *U*, *V*, *W* and *X*.

	Task			
	U	*V*	*W*	*X*
Aiden	3	2	9	9
Bing	5	6	12	12
Callum	9	6	12	14
Dee	8	3	8	12

If each person is allocated one task only, the minimum total time for this group of people to complete all four tasks is

A. 22 minutes.

B. 28 minutes.

C. 29 minutes.

D. 30 minutes.

E. 32 minutes.

Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The order of the matrix $\begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{bmatrix}$ is

A. 2×2

B. 2×3

C. 3×2

D. 4

E. 6

Question 2

Peter bought only apples and bananas from his local fruit shop.

The matrix

$$N = \begin{matrix} A & B \\ [3 & 4] \end{matrix}$$

lists the number of apples (A) and bananas (B) that Peter bought.

The matrix

$$C = \begin{bmatrix} 0.37 \\ 0.43 \end{bmatrix} \begin{matrix} A \\ B \end{matrix}$$

lists the cost (in dollars) of one apple and one banana respectively.

The matrix product, NC, gives

A. the total amount spent by Peter on the fruit that he bought.

B. the total number of pieces of fruit that Peter bought.

C. the individual amounts that Peter spent on apples and bananas respectively.

D. the total number of pieces of fruit that Peter bought and the total amount that he spent.

E. the individual number of apples and bananas that Peter bought and the individual amounts that Peter spent on these apples and bananas respectively.

Question 3

The total cost of one ice cream and three soft drinks at Catherine's shop is $9.

The total cost of two ice creams and five soft drinks is $16.

Let x be the cost of an ice cream and y be the cost of a soft drink.

The matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ is equal to

A. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

B. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

D. $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

E. $\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

Question 4

Which matrix expression results in a matrix that contains the sum of the numbers 2, 5, 4, 1 and 8?

A. $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2 & 5 & 4 & 1 & 8 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 5 & 4 & 1 & 8 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ 4 \\ 1 \\ 8 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

Question 5

A system of three simultaneous linear equations is written in matrix form as follows.

$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & 3 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 11 \\ -5 \end{bmatrix}$$

One of the three linear equations is

A. $x - 2y + z = 4$

B. $x + y + 3z = 11$

C. $2x - y = -5$

D $x + 3z = 11$

E. $3y - z = -5$

Question 6

Vince, Nev and Rani all service office equipment.

The matrix T shows the time that it takes (in minutes) for each of Vince (V), Nev (N) and Rani (R) to service a photocopier (P), a fax machine (F) and a scanner (S).

$$T = \begin{array}{c} \\ \\ \end{array} \begin{array}{ccc} V & N & R \\ \left[\begin{array}{ccc} 12 & 15 & 14 \\ 8 & 7 & 8 \\ 20 & 19 & 17 \end{array}\right] & & \end{array} \begin{array}{c} P \\ F \\ S \end{array}$$

The matrix U below displays the number of photocopiers, fax machines and scanners to be serviced in three schools, Alton (A), Borton (B) and Carlon (C).

$$U = \begin{array}{ccc} P & F & S \\ \left[\begin{array}{ccc} 5 & 3 & 2 \\ 4 & 4 & 3 \\ 6 & 1 & 2 \end{array}\right] \end{array} \begin{array}{c} A \\ B \\ C \end{array}$$

A matrix that displays the time that it would take each of Vince, Nev and Rani, working alone, to service the photocopiers, fax machines and scanners in each of the three schools is

A.

$$\begin{bmatrix} 17 & 18 & 16 \\ 12 & 11 & 11 \\ 26 & 20 & 19 \end{bmatrix}$$

B.

$$\begin{bmatrix} 204 & 110 & 97 \\ 116 & 60 & 53 \\ 278 & 153 & 131 \end{bmatrix}$$

C.

$$\begin{bmatrix} 124 & 134 & 128 \\ 140 & 145 & 139 \\ 120 & 135 & 126 \end{bmatrix}$$

D.

$$\begin{bmatrix} 7 & 12 & 12 \\ 4 & 3 & 5 \\ 14 & 18 & 15 \end{bmatrix}$$

E.

$$\begin{bmatrix} 60 & 15 & 28 \\ 32 & 35 & 24 \\ 120 & 19 & 34 \end{bmatrix}$$

Question 7

A new colony of several hundred birds is established on a remote island. The birds can feed at two locations, *A* and *B*. The birds are expected to change feeding locations each day according to the transition matrix

$$T = \begin{array}{cc} & \textit{this day} \\ & \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} & \begin{array}{c} A \\ B \end{array} \end{array} \quad \textit{next day}$$

In the beginning, approximately equal numbers of birds feed at each site each day.

Which of the following statements is **not** true?

A.   70% of the birds that feed at *B* on a given day will feed at *B* the next day.

B.   60% of the birds that feed at *A* on a given day will feed at *B* the next day.

C.   In the long term, more birds will feed at *B* than at *A*.

D.   The number of birds that change feeding locations each day will decrease over time to zero.

E.   In the long term, some birds will always be found feeding at each location.

Question 8

m and *n* are positive whole numbers.

Matrix *P* is of order *m* × *n*.

Matrix *Q* is of order *n* × *m*.

The matrix products *PQ* and *QP* are both defined

A.   for no values of *m* and *n*.

B.   when *m* is equal to *n* only.

C.   when *m* is greater than *n* only.

D.   when *m* is less than *n* only.

E.   for all values of *m* and *n*.

Question 9

Robbie completed a test of four multiple-choice questions.

Each question had four alternatives, *A*, *B*, *C* or *D*.

Robbie randomly guessed the answer to the first question.

He then determined his answers to the remaining three questions by following the transition matrix

$$
\begin{array}{c}
\textit{this question} \\
\begin{array}{cccc} A & B & C & D \end{array}
\end{array}
$$

$$
T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{array}{l} A \\ B \\ C \\ D \end{array} \quad \textit{next question}
$$

Which of the following statements is **true**?

A.   It is impossible for Robbie to give the same answer to all four questions.

B.   Robbie would always give the same answer to the first and fourth questions.

C.   Robbie would always give the same answer to the second and third questions.

D.   If Robbie answered *A* for question one, he would have answered *B* for question two.

E.   It is possible that Robbie gave the same answer to exactly three of the four questions.

END OF MULTIPLE-CHOICE QUESTION BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: $\text{seasonal index} = \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n-1)d) = \dfrac{n}{2}\big[2a + (n-1)d\big] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, \; r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, \; |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc \sin A$

Heron's formula: $A = \sqrt{s(s - a)(s - b)(s - c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $\qquad c^2 = a^2 + b^2$

sine rule: $\qquad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $\qquad c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $\qquad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $\qquad y = mx + c$

Module 4: Business-related mathematics

simple interest: $\qquad I = \dfrac{PrT}{100}$

compound interest: $\qquad A = PR^n \quad$ where $\; R = 1 + \dfrac{r}{100}$

hire purchase: $\qquad$ effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $\qquad v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $\qquad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $\qquad A^{-1} = \dfrac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET