TSSM™
Creating VCE Success

2010 Trial Examination

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER

Figures										Letter
Words										

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2

Reading time: 15 minutes
Writing time: 1 hour and 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	15
Module		
Number of Modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator and a scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 37 pages.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer all questions within the modules selected.

You need no give numerical answers as decimals unless instructed to do so. Alterative forms may involve, for example, π, surds or fractions.

Diagrams are not so scale unless specified otherwise.

TURN OVER

Core

Question 1

The number of years that a newborn baby is expected to live is called the life expectancy. The life expectancy for males and females has increased over time. The male and female life expectancies for males and females born from 1996 to 2006 for a particular country are given in the table below.

Year	Life Expectancy (Males)	Life Expectancy (Females)
1996	76.2	80.2
1997	76.7	80.7
1998	77.1	81.6
1999	77.2	81.7
2000	78.0	81.9
2001	79.1	82.6
2002	79.4	83.8
2003	80.1	83.9
2004	80.3	84.1
2005	81.1	84.9
2006	81.3	85.2

a. Calculate the mean and standard deviation for the life expectancy for **males** during this eleven year period. Give your answers correct to two decimal places.

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2 marks

Core - continued

b. Calculate the difference in the mean life expectancies between females and males over this eleven year period. Give your answer correct to one decimal place.

1 mark

c. **i.** Find, giving your answer as a percentage correct to two decimal places, the coefficient of determination, based on a regression of female life expectancy against year from 1996 to 2006.

ii. Explain briefly the meaning of the coefficient of determination from part **c (i)**.

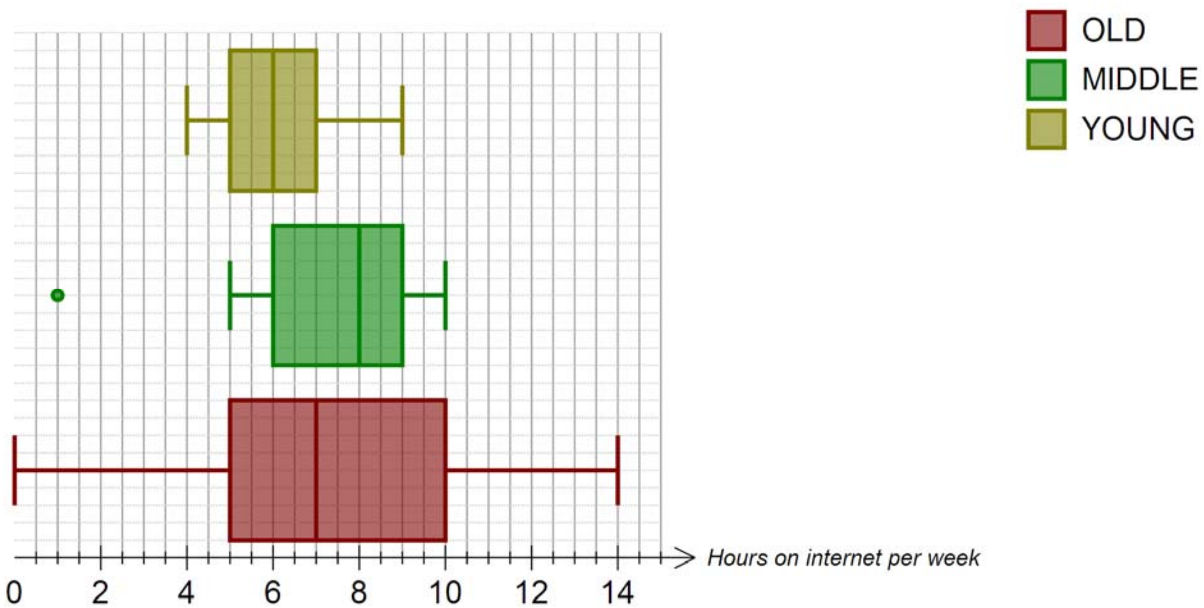
1 + 1 = 2 marks

Core – continued

TURN OVER

Question 2

Consider the following parallel boxplots which show the distribution of the number of hours spent on the internet each week by young, middle and old aged people. Note that young are at the top, middle-aged in the middle and old-aged at the bottom.



Complete the following sentences:

a. 25% of old people spend more than _______ hours on the internet each week.

b. The inter-quartile range is the smallest for _____________ people.

c. The distribution of hours spent each week on the internet for middle-aged people is __________________ skewed.

d. The range of the number of hours spent on the internet each week is lowest for ____________-aged people

4 marks

Core - continued

Question 3

De-seasonalised quarterly sales for a retail shop are given in the table below.

Season	**Summer**	**Autumn**	**Winter**	**Spring**
Year 1	150	180	200	170
Year 2	180	190	170	190
Year 3	145	165	155	165

The seasonal indices, calculated using many years data, are given in the table below.

Summer	**Autumn**	**Winter**	**Spring**
1.0	0.9	1.2	x

a. Calculate the seasonal index for Spring.

1 mark

b. Calculate the actual number of sales during Winter for Year 3.

1 mark

c. Which season has the highest actual number of sales in Year 1? Justify your answer.

1 mark

Core – continued

TURN OVER

Question 4

The number of runs scored by a cricketer in a recent season are

25, 75, 101, 40, 0, 12.

a. What is the cricketer's median run score?

1 mark

b. If the score of 0 were changed to a score of 100, calculate the resultant increase in the cricketer's average score. Give your answer correct to one decimal place.

1 mark

c. What score should be added to 25, 75, 101, 40, 0, 12 to obtain the mean score of 50?

1 mark

Total: 15 marks

END OF CORE

Module 1: Number patterns

Question 1

Robert begins a fitness program. During the first week of the program he jogs for 30 minutes each day (on each of the seven days during the week) making a total of 3.5 hours of exercise during the first week.

In each subsequent week, Robert increases his daily jogging time by 2 minutes. The amount of time he spends jogging in the first four weeks of his program is given in the table below.

Week Number	1	2	3	4	…
Daily Jogging Time	30	32	34	36	…

a. How many minutes does Robert jog per day during the 12^{th} week of his fitness program?

1 mark

b. Find which week Robert's daily exercise jogging time **exceeds 1 hour** for the first time.

1 mark

c. Find the total amount of time, in minutes, Robert jogs for during the first 12 weeks of his fitness program.

2 marks

Module 1: Number patterns – continued
TURN OVER

Question 2

The value of a car in year n is V_n.

The following difference equation shows how the value of the car changes over time

$$V_n = 0.95V_{n-1} + 500.$$

The 0.95 factor in the difference equation above represents depreciation while the addition of 500 represents improvements made to the car each year.

The value of the car at the start of 2009 was \$20 000.

a. What was the value of the car at the start of 2008? Give your answer correct to the nearest cent.

1 mark

b. What is the percentage depreciation of the car each year without taking into account the improvements?

1 mark

c. Does the value of the car at the start of each year follow an arithmetic progression, a geometric progression or neither? Explain your reasoning briefly.

1 mark

Module 1: Number patterns – continued

d. What is the smallest value the car can reach?

2 marks

Question 3

A ball is dropped from a height of 1 metre. On each bounce the ball rises to a height of 90% of that reached on the previous bounce.

a. Write down a difference equation relating B_n and B_{n-1}, the heights of the ball reached on the nth and $(n\text{-}1)^{\text{th}}$ bounces.

1 mark

b. Find the height reached by the ball on the 4^{th} bounce in metres, correct to two decimal places.

1 mark

Module 1: Number patterns – continued

TURN OVER

c. Find the total distance travelled by the ball if it is left bouncing indefinitely.

2 marks

Question 4

You are given that $t_n = t_{n-1} + t_{n-2}$ and that $t_1 = t_2 = 1$.

a. Find $t_4 - t_3$.

1 mark

b. Find t_{12}.

1 mark

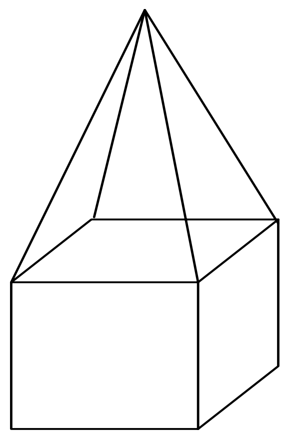
Total: 15 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

Question 1

Consider the composite solid below. It consists of a cube of side length 4cm and a square pyramid of height 8cm above it.



a. Calculate the surface area of the solid. Give your answer in square centimetres correct to two decimal places.

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2 marks

b. Calculate the volume of the solid. Give your answer in cubic centimetres correct to two decimal places.

____

____

____

2 marks

Module 2: Geometry and trigonometry – continued

TURN OVER

Suppose now that each of the side lengths of the solid are multiplied by 5.

c. Find the surface area of the new solid. Give your answer in square centimetres correct to two decimal places.

1 mark

d. Find the volume of the new solid. Give your answer in cubic centimetres correct to two decimal places.

1 mark

Question 2

A hiker begins at point A. She walks for 10km in a direction $N50°E$ to a point B. She then rests at point B. After her rest, she walks 10km in a direction $E60°S$ to the point C.

a. Draw a diagram showing clearly the given information

1 mark

Module 2: Geometry and trigonometry - continued

b. Find the distance from A to C. Give your answer in kilometres correct to one decimal place.

1 mark

c. Find the compass bearing of A from C. Quote the angle in your answer in degrees correct to one decimal place?

1 mark

d. Use Heron's formula to show that the area of triangle ABC is 49.2km^2, correct to one decimal place.

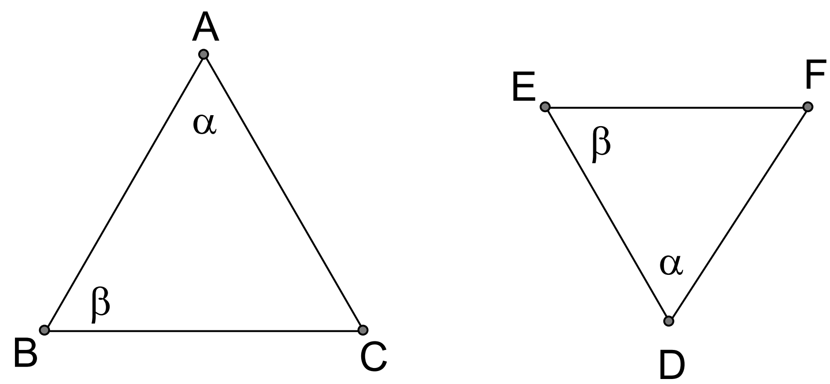
1 mark

e. Confirm the given value for the area of triangle ABC using a different method.

1 mark

Question 3

Triangles ABC and DEF, shown below, are similar.



AB is 10 centimetres. AC is 9 centimetres. EF is 4 centimetres. DE is 6 centimetres.

a. Find the length of side BC.

1 mark

b. Find the length of side DF.

1 mark

Module 2: Geometry and trigonometry - continued

Suppose now that triangle ABC forms the base of a triangular prism of height 15cm.

c. Find the volume of this triangular prism. Give your answer correct to one decimal place.

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2 marks

Total 15 marks

Module 2: Geometry and trigonometry – continued

TURN OVER

Working space

END OF MODULE 2

Module 3: Graphs and Relations

Question 1

The income tax payable in a certain country depends on the level of personal income. The income tax required to be paid is given in the following table.

Income Range	**Tax Payable**
\$0 - \$15,000	0
\$15 001 - \$30 000	\$0.15 for every dollar of income over \$15 000
\$30 001 - \$60 000	\$2 250 plus \$0.30 for every dollar of income over \$30 000
Over \$60 000	\$11 250 plus \$0.40 for every dollar of income over \$60 000

a. Find the amount of income tax payable by a person who earns \$25,000.

1 mark

b. Find the amount of income tax payable by a person who earns \$48 000.

1 mark

Module 3: Graphs and relations – continued
TURN OVER

c. Determine a formula for the amount of tax payable, T, for a person earning S dollars per annum, where $S > \$60\,000$.

2 marks

Question 2

A shop produces and sells two types of computer, F (fast) and S (slow).

The time required to produce a fast computer is 50 hours. The time required to produce a slow computer is 20 hours.

The profit made on selling a fast computer is $1 000. The profit made on selling a slow computer is $600.

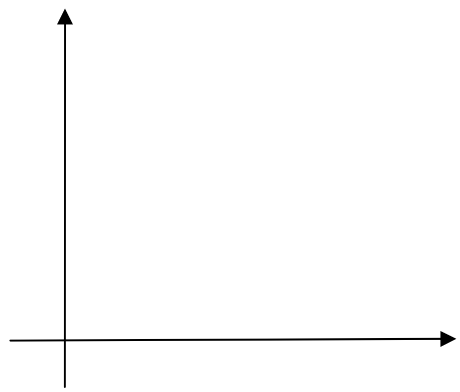
The total time available for production is 1000 hours. The shop must sell at least as many fast computers as it does slow computers.

a. Suppose a shop produces f fast computers and s slow computers. Clearly we have that $f \geq 0$ and $s \geq 0$. Write down two other constraints on f and s based on the information above.

2 marks

Module 3: Graphs and relations – continued

b. Graph all four constraints on a single diagram and shade the feasible region for the problem. Place the number of slow computers, s, on the horizontal axis.



2 marks

c. Find the maximum amount of profit that can be made. Give your answer correct to the nearest cent.

1 mark

Question 3

Joanne sells flowers. For each bunch of flowers sold, she receives \$20. This is her sales revenue. The costs associated with running her business are \$2 000 per week plus \$5 per bunch of flowers produced.

a. Suppose Joanne sells b bunches of flowers in a week. Write down an expression for her sales revenue from that week.

1 mark

Module 3: Graphs and relations – continued

TURN OVER

b. Write down an expression for the costs incurred during a week where Joanne produces b bunches of flowers.

1 mark

c. If Joanne sells all the flower bunches that she produces, find the number of bunches of flowers she must sell in order to make a profit.

2 marks

d. Suppose that in one week, Joanne sells x bunches of flowers and produces $(x+10)$ bunches of flowers. Find an expression for the **loss** she incurs during that week.

2 marks

Total: 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

The income tax payable in a particular country is determined from the following table.

Income Range	**Tax Payable**
$0 - $20,000	0
$20 001 - $35 000	$0.15 for every dollar of income over $20,000
$35 001 - $80 000	$2 250 plus $0.30 for every dollar of income over $35 000
Over $80 000	$15 750 plus $0.40 for every dollar of income over $80 000

a. Calculate the tax payable by an individual earning $45 000.

1 mark

b. An individual paid $20 550 in tax. Find this individual's income level.

2 marks

Module 4: Business-related mathematics – continued
TURN OVER

Question 2

I purchased a computer for $2 000. For the purposes of depreciating the value of the computer, I have two options:

Option A: Flat rate depreciation at 6% per annum.

Option B: Reducing balance depreciation at 8% per annum.

a. Find the book value of the computer after five years under Option A.

1 mark

b. Find the book value of the computer after five years under Option B. Give your answer correct to the nearest cent.

1 mark

c. Excluding time zero, after how many whole years are the book values under Options A and B **closest**?

2 marks

Module 4: Business-related mathematics – continued

Question 3

Consider the following extract from a bank account.

Date	Deposit ($)	Withdrawal ($)	Balance ($)
1 July 2009			5000.00
15 July 2009	500.00		5500.00
28 July 2009		200.00	5300.00
31 July 2009	Interest (a):		5400.00
12 August 2009		600.00	4800.00
13 August 2009		500.00	4300.00
31 August 2009	Interest:		

Interest is payable on the minimum monthly balance. The rate of interest payable is the same in July and August 2009.

a. Find the amount of interest deposited in July.

____

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1 mark

b. What is the rate of interest payable per annum?

____

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1 mark

Module 4: Business-related mathematics – continued
TURN OVER

c. Find the closing balance, after interest is paid, on 31 August 2009.

2 marks

Question 4

Michelle and David borrow $500,000 to purchase a new house. Interest is charged at 6% per annum, compounded monthly. Monthly repayments are made over 30 years in order to repay the loan.

a. Find the monthly loan repayment. Give your answer correct to the nearest cent.

1 mark

b. Find the amount that David and Michelle still owe the bank after ten years. Give your answer correct to the nearest dollar.

1 mark

Module 4: Business-related mathematics – continued

c. Find the total interest paid by David and Michelle over the thirty year term of the loan. Give your answer correct to the nearest 10 dollars.

2 marks

Total: 15 marks

END OF MODULE 4
TURN OVER

Module 5: Networks and decision mathematics

Question 1

The times taken, in hours, for each of four computer technicians to build each of four different computer types are given in the table below. No more than one technician can work on production at any time.

	Type 1	Type 2	Type 3	Type 4
Adrian	30	60	15	40
Bill	50	90	20	50
Charlene	10	80	30	60
Danielle	20	45	20	30

a. Using the Hungarian algorithm and showing full working, find which technician should be given the task of building which computer in order to minimise the total time required to build all four computer types. Assume each technician must build exactly one computer. Also, find the minimum time taken to build all four machines in this way.

4 marks

Module 5: Networks and decision mathematics – continued

b. Explain, without performing any calculations, how you would find the allocation of technicians to computer types in order to maximise the total time to build all four machines. Again assume that each technician must build exactly one computer.

___

___

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2 marks

Question 2

The following tasks are required to complete a project.

Task	Predecessors	Time to complete task (hours)
A	-	4
B	-	7
C	A	1
D	B	1
E	D,C	5
F	D,C	6
G	E	8
H	F, G	2

a. Create a weighted di-graph showing each of the tasks and the time required to complete them.

3 marks

Module 5: Networks and decision mathematics – continued

TURN OVER

b. Find the earliest start times for each of tasks A to H.

2 marks

c. Find the latest start times for each of tasks A to H.

2 marks

d. Identify the critical path.

1 mark

e. Find the total time required for completion of the project.

1 mark

Total: 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

The sales of four types of drink vary by season. A large retailer has recorded his sales of beer, red wine, white wine and soft drink for each season during 2008. These are given in the matrix N below.

$$N = \begin{array}{c} \\ \text{Summer} \\ \text{Autumn} \\ \text{Winter} \\ \text{Spring} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Beer} & \text{Red Wine} & \text{White Wine} & \text{Soft Drink} \end{array} \\ \begin{bmatrix} 200 & 50 & 150 & 300 \\ 150 & 75 & 120 & 250 \\ 120 & 100 & 100 & 150 \\ 160 & 75 & 125 & 200 \end{bmatrix} \end{array}.$$

The prices, in dollars of each drink are given in the column vector, P, below.

$$P = \begin{array}{c} \text{Beer} \\ \text{Red Wine} \\ \text{White Wine} \\ \text{Soft Drink} \end{array} \begin{bmatrix} 4.00 \\ 8.00 \\ 7.00 \\ 3.00 \end{bmatrix}.$$

a. Explain what the sum of all elements in N represents.

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1 mark

b. Explain what the sum of all elements in P represents.

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1 mark

Module 6: Matrices – continued

TURN OVER

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Module 6: Matrices – continued

c. Explain what the sum of all elements in the second row of the matrix product *NP* represents.

1 mark

d. Explain what the sum of all elements in the fourth column of the matrix product *NP* represents.

1 mark

e. Suppose that during 2009 beer sales increase uniformly across all seasons by 10%. All other sales volumes remain at 2008 levels. Suppose also that all prices increase by 5% in 2009 from their 2008 levels. Calculate the total amount received by the retailer for sales of all drinks during 2009. Give your answer correct to the nearest dollar.

2 marks

Module 6: Matrices – continued

TURN OVER

Question 2

In a new game of football, known as 'Tassie Rules', players score points as follows:

Type A Goal: x points
Type B Goal: y points
Type C Goal: z points

John, a keen Tassie Rules player, scored points and goals in three games as given in the following table.

Game Number	Type A Goals	Type B Goals	Type C Goals	Points
1	2	6	7	52
2	1	4	8	39
3	3	5	1	43

a. This information can be used to find the number of points scored for each of a Type A, Type B and Type C goal. Set up a system of simultaneous equations which can be solved to find the values of x, y and z.

1 mark

b. Write your system of simultaneous equations from **part a** in matrix form and **hence** find the values for x, y and z.

2 marks

Module 6: Matrices - continued

Question 3

Investors with Bank XYZ have a choice of four different funds.

Fund A: Aggressive
Fund G: Growth
Fund B: Balanced
Fund C: Cash

At the end of each year investors may elect to change fund. The relevant transition matrix, where this year is given by the columns (in order A, G, B and then C) and next year is given by the rows (in order A, G, B and then C) is given below.

$$T = \begin{bmatrix} 0.7 & 0.5 & 0.2 & 0.1 \\ 0.2 & 0.4 & 0.3 & 0.2 \\ 0.05 & 0.05 & 0.4 & 0.3 \\ 0.05 & 0.05 & 0.1 & 0.4 \end{bmatrix}.$$

During 2009 there were 400 investors in Fund A, 300 in Fund G, 150 in Fund B and 150 in Fund C. This information can be shown as the column matrix below.

$$S_{2009} = \begin{array}{c} A \\ G \\ B \\ C \end{array}\begin{bmatrix} 400 \\ 300 \\ 150 \\ 150 \end{bmatrix}.$$

a. Predict the number of aggressive investors in 2011. Give your answer to the nearest whole number.

1 mark

Module 6: Matrices – continued
TURN OVER

b. Predict the number of cash investors in 2014. Give your answer to the nearest whole number.

1 mark

c. Write an expression for S_n, the column vector giving the number of investors in each of category A, G, B and C (where n is an integer greater than 2009) in terms of S_{2009} and T.

1 mark

d. Find the number of investors in each category in the long run. Given each element in your answer correct to the nearest whole number.

1 mark

Module 6: Matrices – continued

e. Suppose we have reached the state in part **d**, with the number of investors in each category rounded to the nearest whole number. Suppose now that funds A, G and B are combined to form a new fund R (risky). Fund C remains unchanged.

Find the 2×2 transition matrix for transitions between funds R and C (between two consecutive years) given that we have reached the state identified in **d**. Give your matrix values correct to 3 decimal places.

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK