FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2

2010 Trial Examination 2

SOLUTIONS

Core

Question 1

a. Average = 78.77. Standard deviation = 1.82. A2

b. Female average = 82.78. Difference is 4.0 years. A1

c. **i.** 97.8% A1

ii. 97.8% of the variation in female life expectancy can be explained by calendar year. A1

Question 2

a. 10 hours A1

b. young A1

c. negatively A1

d. young A1

Question 3

a. Index for Spring is $4-(1+0.9+1.2)=0.9.$ A1

b. $155\times1.2=186.$ A1

c. Winter. It has the highest seasonal index and the highest de-seasonalised value and so must also have the highest actual sales volume. A1

Question 4

a. $\frac{25+40}{2}=32.5.$ A1

b. Old average = $\frac{25+75+101+40+0+12}{6}=42.17.$

New average = $\frac{25+75+101+40+100+12}{6}=58.83.$

Increase = 16.7. A1

c.

0	0
1	2
2	5
3	
4	0
5	
6	
7	5
8	
9	
10	1

A1

Module 1: Number Patterns

Question 1

a. $30 + 11 \times 2 = 52$ minutes A1

b. $30 + 2(n-1) > 60 \Rightarrow 2n > 32 \Rightarrow n > 16.$ In the 17^{th} week. A1

c.

$$7 \times \frac{12}{2}(30 + 52)$$
$$= 3{,}444 \text{ minutes.}$$

M1 A1

Question 2

a. $20000 = 0.95V_{2008} + 500 \Rightarrow V_{2008} = \$20{,}526.32.$ A1

b. 5% A1

c. Neither. The difference equation involves both a multiplicative factor and an additive factor thereby being a combination of arithmetic and geometric progression. Hence neither. M1

d.

$$V = 0.95V + 500$$
$$0.05V = 500$$
$$V = \$10{,}000.$$

M1

A1

Question 3

a. $B_n = 0.9B_{n-1}$ A1

b. $B_4 = 0.9^4 = 0.66$ metres. A1

c. $1 + 2\left(\frac{0.9}{1 - 0.9}\right) = 19$ metres. M1

A1

Question 4

a. $t_3 = 2, t_4 = 3$ and hence $t_4 - t_3 = 1$.

A1

b. 1,1,2,3,5,8,13,21,34,55,89,144. So $t_{12} = 144$.

A1

Module 2: Geometry and Trigonometry

Question 1

a. Label semi-diagonal on top face of cube as x. Label y as slant height on triangular prism.

$x = \frac{1}{2}\sqrt{4^2 + 4^2} = 2\sqrt{2}$

$y = \sqrt{\left(2\sqrt{2}\right)^2 + 8^2} = \sqrt{72}$ M1

Area of triangular face (Heron) is 16.49cm^2. Hence total surface area is

$4 \times 4 \times 5 + 16.49 \times 4 = 145.97\text{cm}^2$. A1

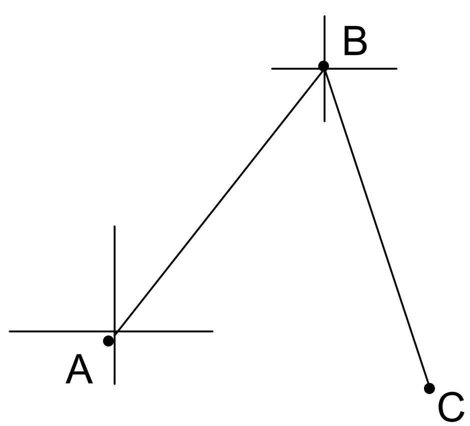
b. Volume is $4^3 + \frac{1}{3}4^2(8) = 106.67\text{cm}^3$. M1 A1

c. The revised surface area is $145.97 \times 25 = 3649.24\text{cm}^2$. A1

d. The revised volume is $106.67 \times 125 = 13{,}333.33\text{cm}^3$. A1

Question 2

a. AB = 10. BC = 10. First angle is 50 degrees from north. Second angle is 60 degrees from east.



A1

b.

$AC^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 80°$

$= 165.27$ A1

$\therefore AC = 12.9\text{km}.$

c.

$\frac{\sin\theta}{10} = \frac{\sin 80°}{12.9} \Rightarrow \theta = 43.9°.$

Bearing is $N73.9°W$. A1

d.

$s = \frac{20 + 12.9}{2} = 16.42$

$A = \sqrt{16.42(16.42 - 10)^2(16.42 - 12.9)} = 49.2\text{km}^2.$

M1

A1

e. $A = \frac{1}{2}(10)(10)\sin 80° = 49.2\text{km}^2$, as in (d). A1

Question 3

a.

$\frac{BC}{4} = \frac{10}{6} \Rightarrow BC = \frac{20}{3}\text{cm}.$ A1

b.

$\frac{DF}{9} = \frac{6}{10} \Rightarrow DF = 5.4\,\text{cm}.$ A1

c.

$V = 15\sqrt{12.83(12.83 - 10)(12.83 - 9)\left(12.83 - \frac{20}{3}\right)} = 29.3\text{cm}^3.$

M1 A1

Module 3: Graphs and Relations

Question 1

a. $0.15(25000-15000)=\$15,000.$ A1

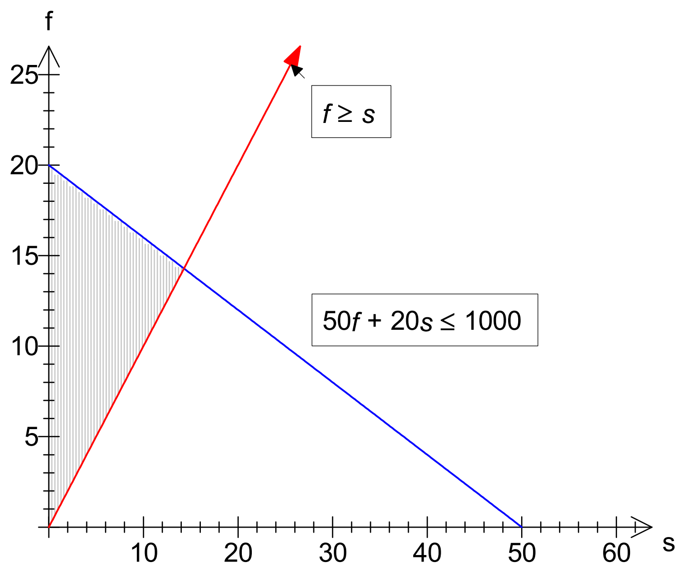
b. $2,250+0.3(48,000-30,000)=\$7,650.$ A1

c. $T=11,250+0.4(S-60,000)$ A2

Question 2

a. $50f+20s\le 1000$ and $f-s\ge 0.$ A2

b. Point of intersection is $\left(\frac{100}{7},\frac{100}{7}\right)$.



A2

c. The maximum profit occurs at the point $\left(\frac{100}{7},\frac{100}{7}\right)$. This is $\frac{100}{7}(1000+600)=\$22,857.14.$

Note that partial computers have been allowed for in this calculation. If students round the number of computers to 14 this is equally acceptable.

A1

Question 3

a. Revenue = $20b$. A1

b. Costs = $2000 + 5b$. A1

c. Need to solve

$20b - 2000 - 5b = 0$
$b = 133.3$.
Hence, must make 134 bunches. A1

d. Loss = Costs – Revenue M1
$= 2000 + 5(x + 10) - 20x$ A1
$= 2050 - 15x$.

Module 4: Business-Related Mathematics
Question 1

a. $2250 + 0.3 \times 10{,}000 = \$5{,}550.$ A1

b. $20{,}550 = 17{,}750 + 0.4(S - 80{,}000) \Rightarrow S = \$87{,}000$ M1 A1

Question 2
a. $2000 \times 0.7 = \$1{,}400.$ A1

b. $2000 \times 0.92^5 = \$1{,}318.16.$ A1

c. After 8 years, the difference is minimum at \$13.56. Use calculator to find sequence using a difference equation.

M1 A1

Question 3
a. \$100 A1

b. $100 = 5000\frac{r}{12} \Rightarrow r = 24\%.$ A1

c. $4{,}300 + 4{,}300\frac{0.24}{12} = \$4{,}386.$ M1 A1

Question 4
a. \$2,997.75 using TVM solver. A1

b. \$418,429 using TVM solver. A1

c. $30 \times 12 \times 2{,}997.75 - 500{,}000 = \$579{,}190.$ M1 A1

Module 5: Networks and Decision Mathematics

Question 1

a. Applying the Hungarian algorithm, we have

$$\begin{pmatrix} 15 & 45 & 0 & 25 \\ 30 & 70 & 0 & 30 \\ 0 & 70 & 20 & 50 \\ 0 & 25 & 0 & 10 \end{pmatrix} \Rightarrow \begin{pmatrix} 15 & 20 & 0 & 15 \\ 30 & 45 & 0 & 20 \\ 0 & 45 & 20 & 40 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 15 & 20 & 15 & 15 \\ 30 & 45 & 15 & 20 \\ 15 & 60 & 35 & 55 \\ 15 & 15 & 15 & 15 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 5 & 0 & 0 \\ 15 & 30 & 0 & 5 \\ 0 & 45 & 20 & 40 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Hence, Adrian: Type 4, Bill: Type 3, Charlene: Type 1, Danielle: Type 2.

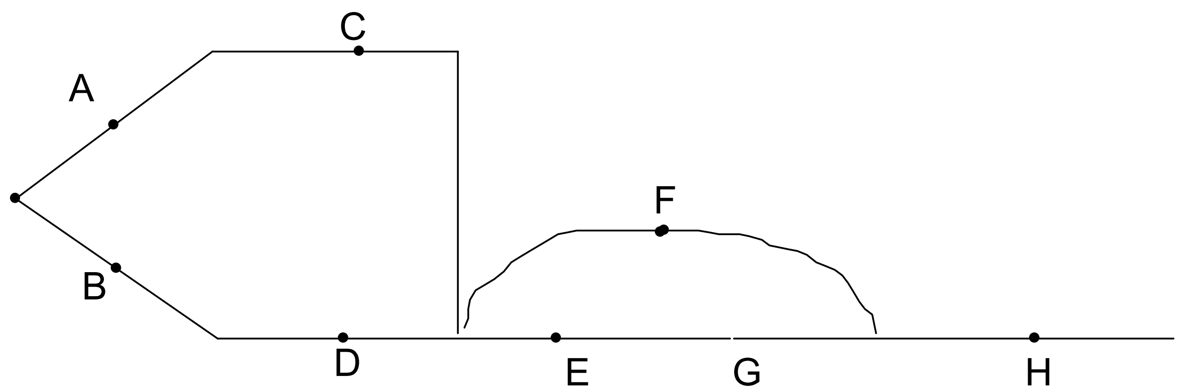
Minimum time is 115 hours. A4

b. Write a matrix with values equal to 100 minus the values in the given time matrix in the question. Note that any value greater than or equal to the maximum time in the given table could be used in place of 100. Then apply the Hungarian algorithm to this new matrix. The result will be the minimum allocation of 100 minus times or equivalently the maximum time allocation.

M2

Question 2

a.



A3

b. A:0, B:0, C:4, D:7, E:8, F:8, G:13, H:21. A2

c. A:3, B:0, C:7, D:7, E:8, F:15, G:13, H:21. A2

d. B-D-E-G-H A1

e. 23 hours A1

Module 6: Matrices

Question 1

a. The total number of sales of beer, red wine, white wine and soft drink in one year. A1

b. The price for 1 beer, 1 red wine, 1 white wine and 1 soft drink. A1

c. The sales revenue for Autumn. A1

d. The sales revenue from soft drinks in a one-year period. A1

e.

$$N=\begin{pmatrix}220 & 50 & 150 & 300\\ 165 & 75 & 120 & 250\\ 132 & 100 & 100 & 150\\ 176 & 75 & 125 & 200\end{pmatrix} \text{ and } P=\begin{pmatrix}4.2\\ 8.4\\ 7.35\\ 3.15\end{pmatrix} \Rightarrow NP=\begin{pmatrix}3{,}391.5\\ 2{,}992.5\\ 2{,}601.9\\ 2{,}917.95\end{pmatrix}.$$

Hence, total is \$11,904. M1 A1

Question 2

a.

$$2x+6y+7z=52$$
$$x+4y+8z=39$$
$$3x+5y+z=43$$

A1

b.

$$\begin{pmatrix}2 & 6 & 7\\ 1 & 4 & 8\\ 3 & 5 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}52\\ 39\\ 43\end{pmatrix} \Rightarrow \begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}7\\ 4\\ 2\end{pmatrix}.$$

M1 A1

Question 3

a. $S_{2011} = T^2 S_{2009} = \begin{pmatrix} 509 \\ 269 \\ 126.5 \\ 95.5 \end{pmatrix}$. Hence 509. A1

b. $S_{2014} = T^5 S_{2009} \Rightarrow \begin{pmatrix} 537.7 \\ 264.4 \\ 111.7 \\ 86.2 \end{pmatrix}$. Hence 86. A1

c. $S_n = T^{n-2009} S_{2009}$. A1

d. $\begin{pmatrix} 541 \\ 264 \\ 110 \\ 85 \end{pmatrix}$. A1

e. $\begin{pmatrix} 0.944 & 0.6 \\ 0.056 & 0.4 \end{pmatrix}$. Note that $\Pr(\text{Cash} \mid \text{Risky}) = \dfrac{(541 + 264).05 + 110(.1)}{541 + 264 + 110} = 0.056$. M1 A1