TSSM ™
Creating VCE Success

2010 Trial Examination

STUDENT NUMBER

Letter

Figures

Words

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 1

Reading time: 15 minutes
Writing time: 1 hour and 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
A	13	13			13
B	54	27	6	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator and a scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 38 pages.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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SECTION A

Instructions for Section A
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Core: Data analysis

The following information relates to Questions 1 and 2

The IQ of students in a particular town is normally distributed with a mean of 100 and standard deviation of 15.

Question 1

The proportion of students in this town with IQ between 85 and 130 is closest to

A. 95%

B. 81.5%

C. 64.5%

D. 68%

E. 70%

Question 2

The proportion of students in this town with IQ between the upper quartile of IQ scores for this town and 115 is closest to

A. 9%

B. 17%

C. 25%

D. 34%

E. 59%

The following information relates to Questions 3,4 and 5

The number of hours spent playing sport each week by a group of students are shown in the dot plot below. Note that 2 students play no sport each week, 4 students play sport for 1 hour each week, etc.
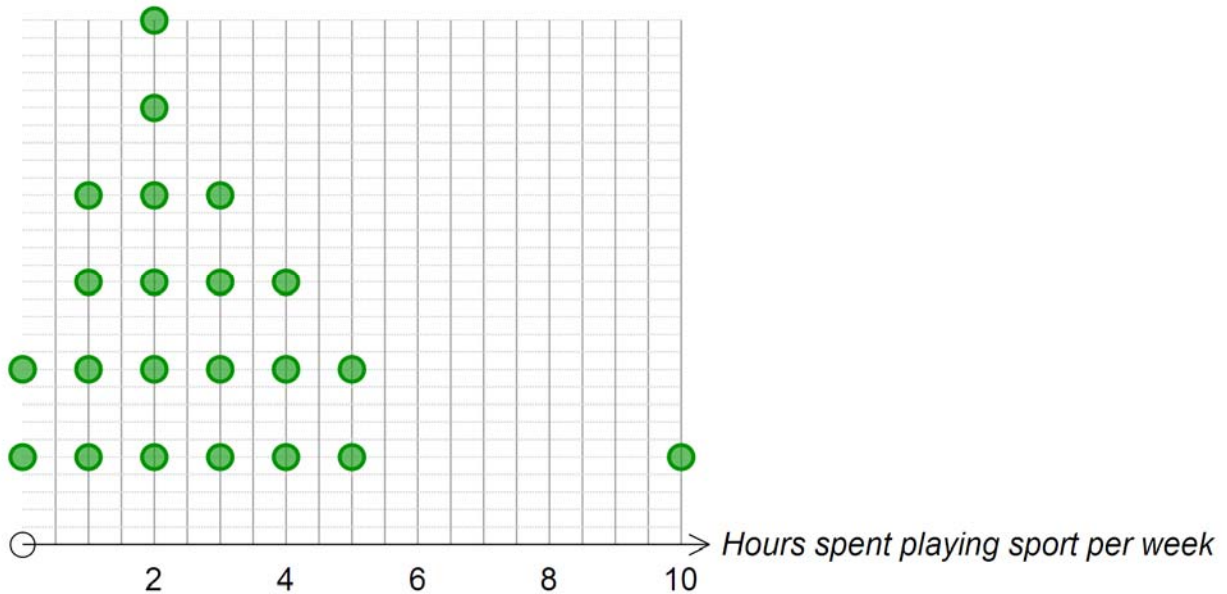
Question 3

The mode and median for these data are

A.   2 and 2 respectively

B.   2 and 2.5 respectively

C.   2 and 3 respectively

D.   3 and 2 respectively

E.   3 and 2.5 respectively

Question 4

The mean and standard deviation of the **number of minutes** spent playing sport per week, based on these data, are closest to

A.   163.6 and 7791.3 respectively

B.   3540 and 120.6 respectively

C.   2.68 and 242.2 respectively

D.   2.68 and 2.01 respectively

E.   163.6 and 129.9 respectively

Question 5

The shape of this distribution is best described as

A. Negatively skewed with outlier

B. almost symmetric with outlier

C. positively skewed with outlier

D. bi-modal

E. uniform

Question 6

In a recent election, the votes cast by a sample of individuals are summarised in the following two-way table:

	TYPE OF VOTE CAST		
	Labor Party	**Liberal Party**	**Other**
Males	251	236	17
Females	280	212	25

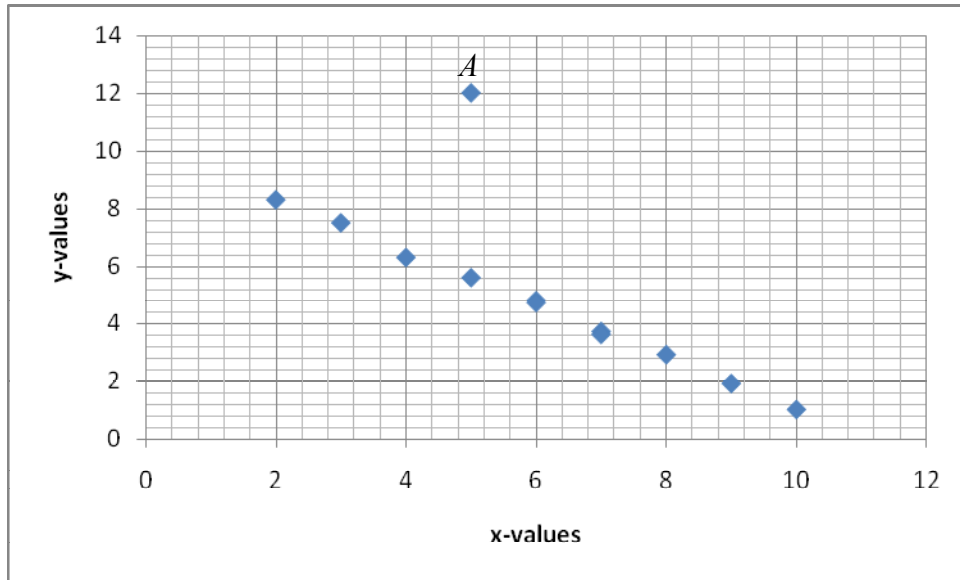The Labor party and the Liberal party are considered the two major parties.

The percentage of males who did not vote for a major party is closest to

A. 4.8%

B. 96.6%

C. 95.2%

D. 51.3%

E. 3.4%

Question 7

Consider the following scatterplot and the point A with co-ordinates (5,12).

Removal of point A would cause

A.   an increase in Pearson's correlation coefficient

B.   a decrease in Pearson's correlation coefficient

C.   an increase in the least squares regression coefficient for slope

D.   a decrease in the range of x-values

E.   none of the above

Question 8

A least squares regression line is estimated for data relating to ENTER score (Y) and hours spent studying each week (X). The regression line estimated was

$$Y = 30.0 + 2.0X.$$

The average ENTER score from the sample was 60.5. The average number of hours spent studying per week from the sample is

A.   14

B.   15

C.   15.25

D.   15.5

E.   cannot be determined from the given information
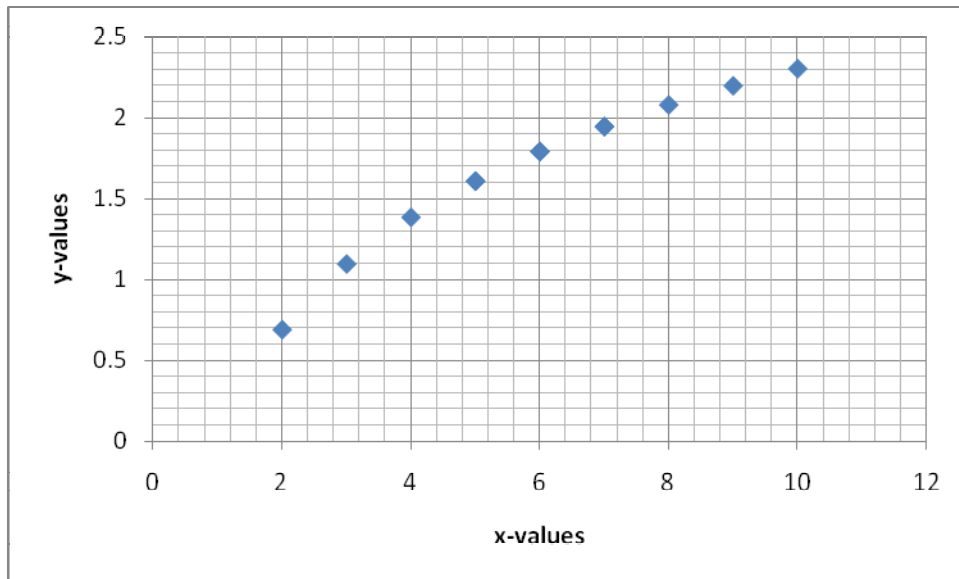
SECTION A - continued

Question 9

When the least squares regression line is used for prediction, a negative residual means

A. a predicted value is below the mean of the independent variable values

B. a predicted is value below the mean of the dependent variable values

C. a predicted value for the dependent variable is less than the actual value of the dependent variable

D. a predicted value of the dependent variable is greater than the actual value of the dependent variable

E. none of the above

Question 10

Consider the graph below.

A transformation that could be used to linearise the above data is

A. $x \rightarrow \log x$

B. $x \rightarrow x^2$

C. $y \rightarrow \log y$

D. all of the above

E. none of A, B or C

The following information relates to Questions 11 and 12

Time series data, over a twelve-month period, are given in the table below.

Time (Month)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Red Wine Sales (Litres)	120	125	132	148	198	212	260	258	155	165	127	111

Question 11

Suppose these data are smoothed using the five-moving median method. The final smoothed value is

A. 132

B. 155

C. 165

D. 127

E. none of the above

Question 12

Suppose there are four seasons in the year where the seasons relate to the months as given in the table below.

SEASON	MONTHS
Summer	Dec, Jan, Feb
Autumn	Mar, Apr, May
Winter	Jun, Jul, Aug
Spring	Sep, Oct, Nov

Based on these data, the seasonal index for Winter is closest to

A. 0.708

B. 0.951

C. 1.452

D. 0.889

E. 0.731

Question 13

Suppose you estimate a trend line based on monthly time series data from January 2007 (month 1) to December 2009 (month 36). The estimated trend line is

$$y_t = 2.0 + 1.1t.$$

The difference in estimated values of y at January 2007 and December 2008 is

A. 25.3
B. 26.4
C. 27.3
D. 28.4
E. 46.0

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Module 1: Number patterns

> Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

In week one, a beginning typist can accurately type 20 words per minute. Each week, she increases her speed by 2 words per minute. How many words can she type per minute in week 10?

A. 36
B. 38
C. 40
D. 42
E. 44

Question 2

Let $t_n = 0.7t_{n-1}$ be the nth term of a sequence. If $t_0 = 70,$ the value of t_{10} correct to 3 decimal places, is

A. 1.977
B. 3.297
C. 1190.000
D. 63.700
E. 2.825

Question 3

The temperature in a room n minutes after turning on a heater is given by $T_n = 1.05T_{n-1}$. Suppose $T_0 = 15°C$. The heater is turned off when the temperature first exceeds $21°C$. How long will the heater be on, to the nearest minute, before it is switched off?

A. 4
B. 5
C. 6
D. 7
E. 8

TURN OVER

Question 4

The number of oranges in the bottom (1st) row of a huge fruit display is 131. Each successive row contains two fewer oranges than the row below. The top row has a single orange. How many oranges are used in the entire display?

A. 7 336

B. 5 148

C. 8 646

D. 6 419

E. 4 356

The following information relates to Questions 5 and 6

During 2010 there are 10 000 kangaroos in a parkland. If left alone, this kangaroo population will increase by 7% each year. In order to control this population, 1000 kangaroos are shot each year. Let K_n be the number of kangaroos in this parkland n years after 2010.

Question 5

A difference equation, along with an initial condition, that can be used to predict the number of kangaroos in this parkland into the future is

A. $K_n = 0.93K_{n-1} + 1000, \quad K_0 = 10\,000$

B. $K_n = 1.07K_{n-1}, \quad K_0 = 2010$

C. $K_n = K_{n-1} - 300, \quad K_0 = 10\,000$

D. $K_n = 1.07K_{n-1} - 1000, \quad K_0 = 10\,000$

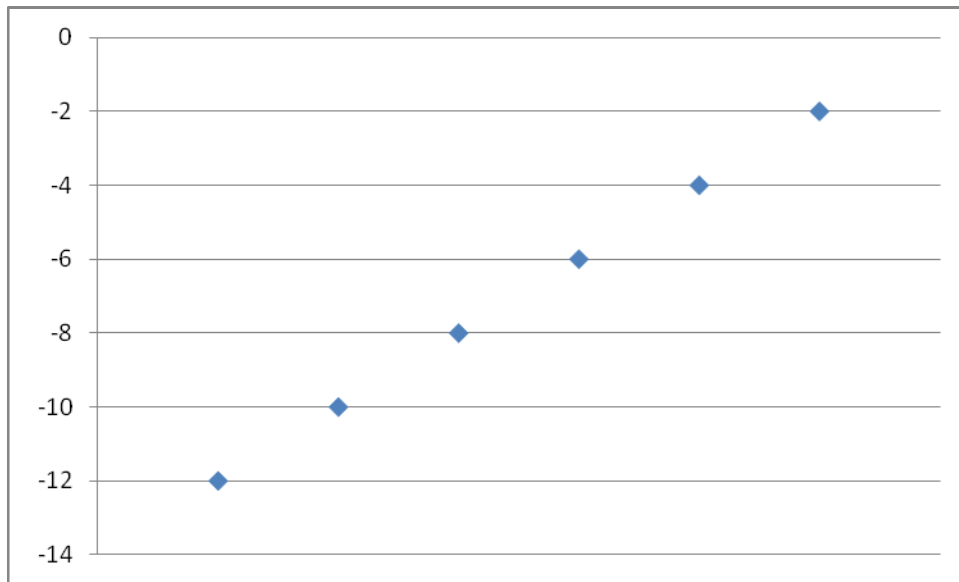E. $K_n = 0.93K_{n-1}, \quad K_0 = 2010$

Question 6

The number of kangaroos predicted to be in this parkland in 2025 is, to the nearest whole number,

A. 3 235

B. 3 958

C. 2 461

D. 1 634

E. 2 025

SECTION B – Module 1: Number patterns - continued

Question 7

The successive terms in a sequence are graphed below.

The sequence is

A. arithmetic with $d > 0$

B. arithmetic with $d < 0$

C. geometric with $r > 0$

D. geometric with $r < 0$

E. Fibonacci-related

Question 8

For a Fibonacci sequence, $t_8 = 100, t_{10} = 230$ and $t_{12} = 590$. The value of t_9 is

A. 130

B. 180

C. 360

D. 400

E. 440

Question 9

The first term of a geometric sequence is 1. If its sum to infinity is 2, the common ratio of this sequence is

A. 1

B. $\dfrac{1}{3}$
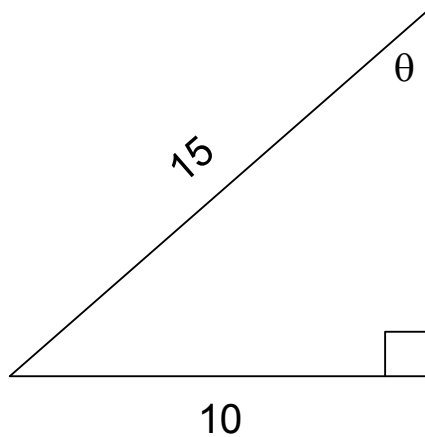
C. 2

D. $\dfrac{1}{2}$

E. not possible to determine

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

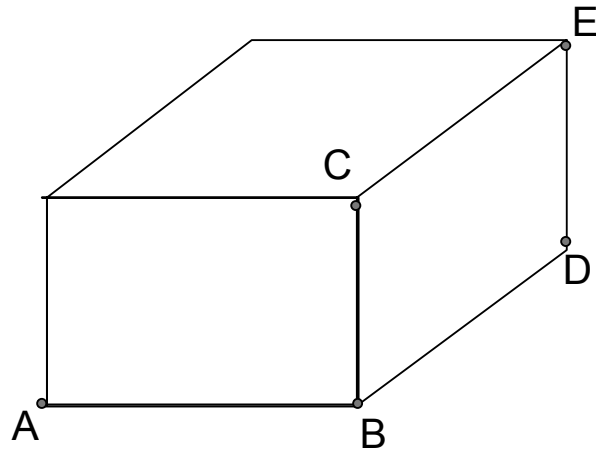The size of the angle marked θ, correct to the nearest degree, is

A. 48°
B. 45°
C. 34°
D. 43°
E. 42°

SECTION B – Module 2: Geometry and trigonometry – continued
TURN OVER

Question 2

A rectangular prism is shown below. AB is 10cm, BC is 9cm and BD is 12cm. The length AE, to the nearest centimetre, is
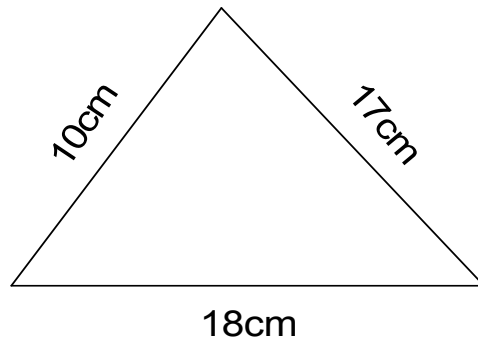

A. 17
B. 18
C. 19
D. 20
E. 21

Question 3

On a map, where the scale is 1:10000, the area of a particular land enclosure is 5cm^2. The actual area of the land enclosure, correct to the nearest hectare, where 1 hectare equals $10000m^2$, is
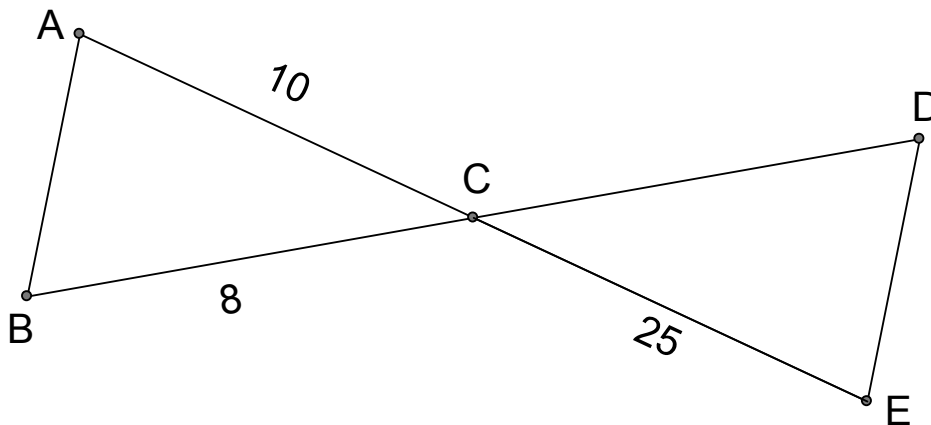
A. 5
B. 50
C. 500
D. 5000
E. 50000

SECTION B – Module 2: Geometry and trigonometry – continued

Question 4

The area of the triangle shown below is closest to

A. 153.0cm^2

B. 83.4cm^2

C. 1091.2cm^2

D. 52.0cm^2

E. 251.4cm^2

Question 5

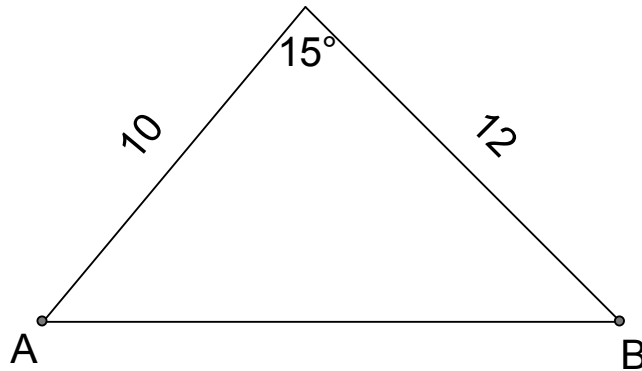In the diagram below, where sides AB and DE are parallel, the length of the side CD is closest to

A. 20

B. 0.31

C. 25

D. 15

E. 22

SECTION B – Module 2: Geometry and trigonometry – continued

TURN OVER

Question 6

The length of the side marked AB in the triangle below is closest to

A. 12.1
B. 2.6
C. 3.9
D. 9.4
E. 3.5

Question 7

Suppose that the points A and B are on contour lines at heights 200 metres and 100 metres respectively. The horizontal distance between the points A and B is 500 metres. The distance AB, in metres, is closest to

A. 500
B. 441
C. 600
D. 483
E. 510

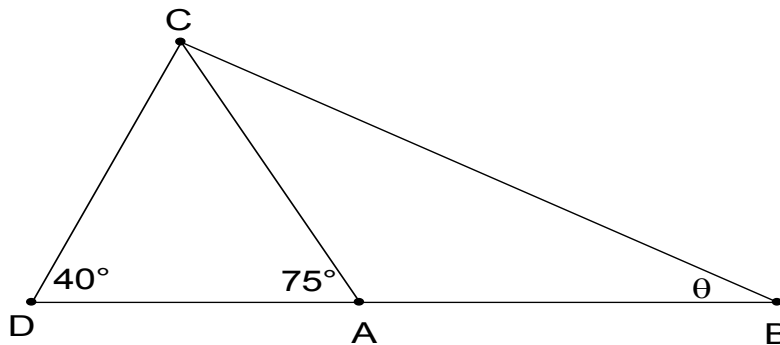**SECTION B – Module 2: Geometry and trigonometry** – continued

Question 8

Given that $\angle ABD = 5°$, $\angle BCA = 90°$, $BC = 10\text{m}$ and $CD = 3\text{m}$, as shown in the diagram below, find AD, in metres, correct to two decimal places.
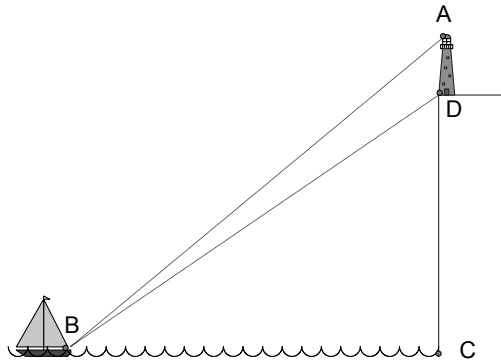
A. 0.98
B. 0.99
C. 1.00
D. 1.01
E. 1.02

Question 9

In the diagram below, where AB is of length 10 and CD is of length 12, the size of the angle marked θ, correct to the nearest degree, is

A. 32°
B. 33°
C. 34°
D. 35°
E. 36°

Working space

Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The cost of posting a letter is $0.50 plus $0.25 for every 500 grams that the mass of the letter exceeds 1 kilogram. The cost of posting a letter with mass 2.5 kilograms is

A. $1.75
B. $1.50
C. $1.25
D. $1.00
E. $0.50

Question 2

The amount of water in a pool begins the day at 500,000 litres. Rain falls so that the amount of water in the pool increases by 500 litres per hour. Which of the following is an expression for the amount of water, in litres, in the pool t hours after the rain begins?

A. 500,000
B. $500,000 + 500t$
C. $500,000 - 500t$
D. $499,500 + 500t$
E. $499,500 - 500t$

Question 3

The life expectancy for a newborn male has increased over time so that in 1950 males had a life expectancy of 72 years while in 1990, newborn males had a life expectancy of 77 years. The average rate of change in the life expectancy of newborn males over the period 1950 to 1990 is

A. 0.125 per year
B. 1.925 per year
C. 0.250 per year
D. 0.500 per year
E. 1.250 per year

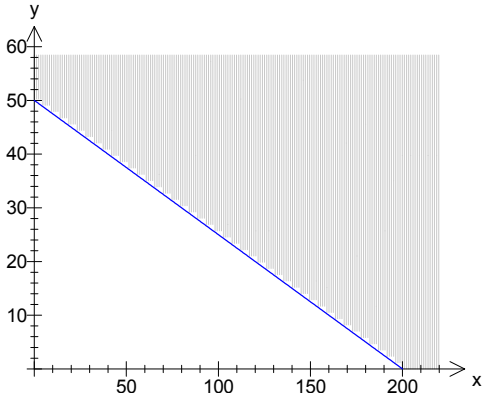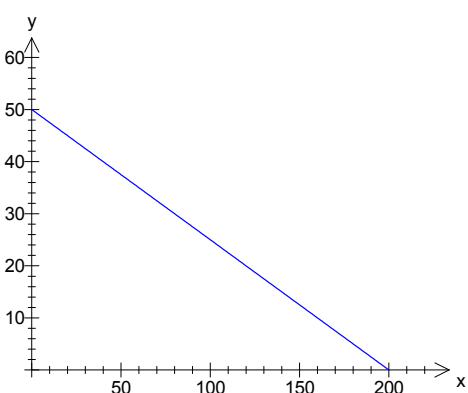**TURN OVER**

Question 4

Suppose that scientific calculators can be bought for $20 and that graphics calculators can be bought for $80. Suppose a school purchases x scientific calculators and y graphics calculators.

Which shaded region in the graphs below shows the possible numbers of scientific and of graphics calculators that the school can purchase if it is willing to spend no more than $4000 on calculators.
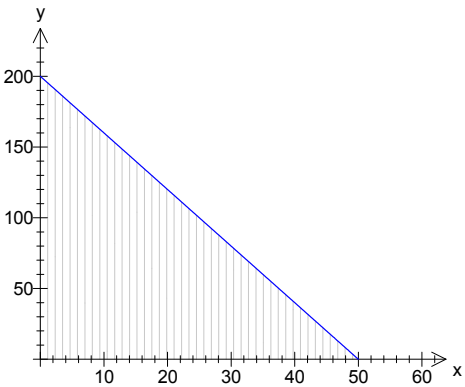
A.
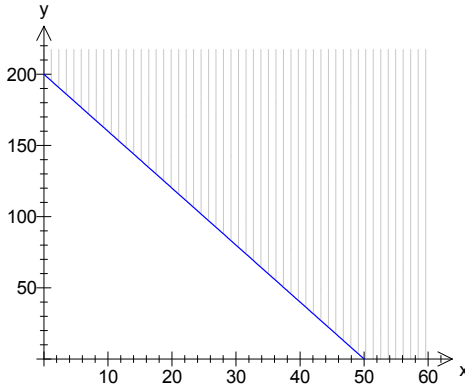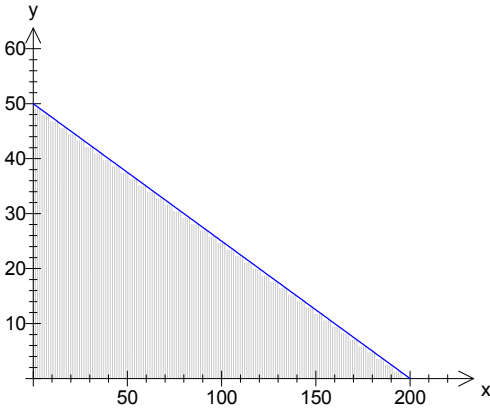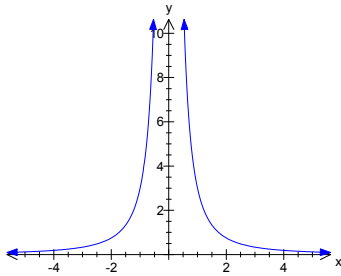

D.

B.

E.

C.

SECTION B – Module 3: Graphs and relations – continued

Question 5

Which of the following could be a graph of $y = 3x^{-2}$?

A.

B.
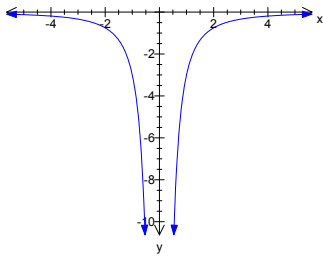
C.
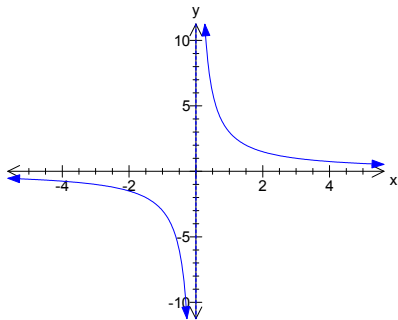
D.
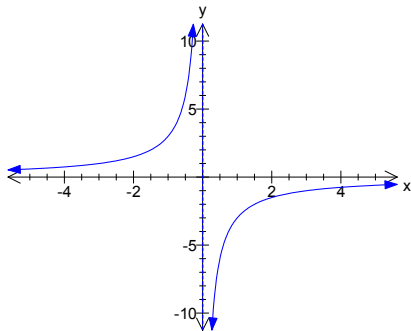
E.   None of the above

Question 6

The total of Adam's and Bill's scores on a test is 145. Adam scored 15 marks more than Bill.

By setting Adam's score equal to a and Bill's score equal to b, which system of equations could be solved to find a and b?

A. $a + b = 145, a - b = 15$

B. $a + b = 145, b - a = 15$

C. $2a + b = 145, a - b = 15$

D. $2a - b = 145, b - a = 15$

E. None of the above

Question 7

Consider the diagram below.
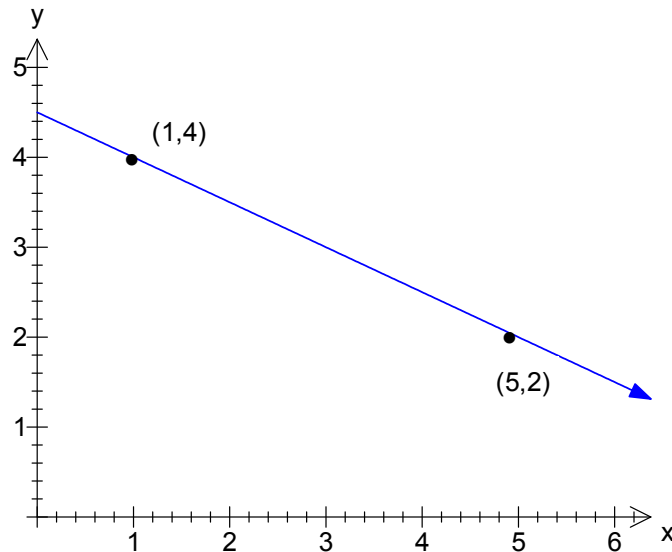
The equation of the line joining the points (1,4) and (5,2) is

A. $-x + 2y = 9$

B. $x - 2y = 9$

C. $-x - 2y = 9$

D. $x + 2y = 9$

E. None of the above

SECTION B – Module 3: Graphs and relations – continued

Question 8
Consider the diagram below.
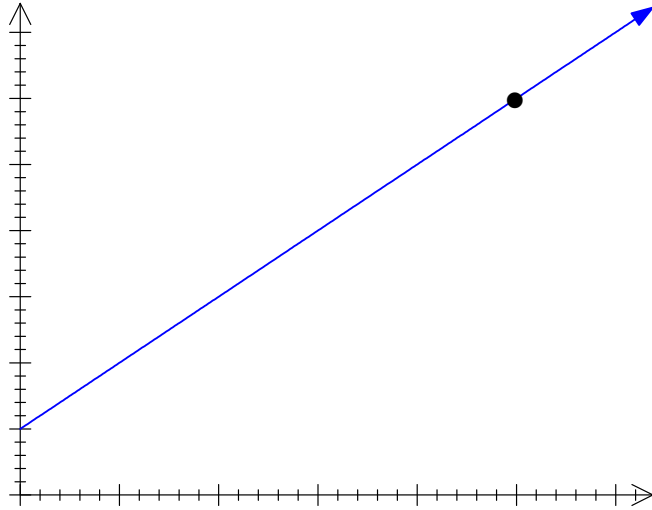
The equation of the line represented above, where the labelled point has co-ordinates $x = 5$ and $y = 6$, is

A. $y = 1 + x$

B. $y = 1 + \dfrac{6}{5}x$

C. $y = 1 - x$

D. $y = 1 - \dfrac{1}{x}$

E. None of the above

SECTION B – Module 3: Graphs and relations – continued
TURN OVER

Question 9

The feasible region for a linear programming problem is shaded in the diagram below.
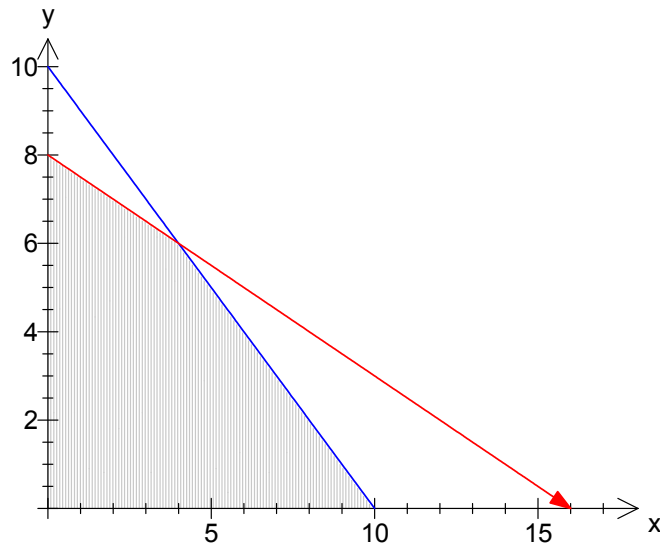
If the profit is given by $P = 100y - 3x$, the maximum profit occurs at

A. (0,8)

B. (0,0)

C. (10,0)

D. (4,6)

E. None of the above

Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Suppose the price of an item, including GST at 10%, is $880. The price of the item, excluding GST, is

A. $792

B. $800

C. $880

D. $968

E. None of the above

Question 2

Simon earned simple interest at the rate of 9% per annum for a nine-month period. The final balance in his account was $640.50. The balance of his account at the start of the nine-month period, assuming no other transactions apart from the simple interest, was

A. $590

B. $600

C. $610

D. $620

E. $630

TURN OVER

Question 3

Natalie earns compound interest at 12% per annum compounded m times per year. An expression which gives the accumulated value of $1000 invested for three years is

A. $1000\left(1+\dfrac{12}{100}\right)^{3}$

B. $1000\left(1+\dfrac{12}{100m}\right)^{3m}$

C. $1000\left(1+\dfrac{12}{100m}\right)^{3}$

D. $1000\left(1+\dfrac{12}{100}\right)^{3m}$

E. $1000\left(1+\dfrac{12}{1200}\right)^{36}$

Question 4

A scholarship is to be paid annually in perpetuity from a fund of F dollars. The fund is invested to earn interest at 10% per annum and the scholarship is to provide an annual payment of $5000. The value of F is closest to

A. $4 545

B. $500 000

C. $5 500

D. $50 000

E. $5 000

Question 5

Mark borrows $100 000 with an interest only loan. He only begins to repay principal ten years after the loan commences. The loan will be repaid 30 years after the $100 000 was lent. Interest is charged at 8% per annum. The amount of the loan outstanding five years after the money was first lent is closest to

A. $100 000

B. $94 821.30

C. $146 932.80

D. $60 000

E. $54 386.21

SECTION B – Module 4: Business-related mathematics – continued

The following information relates to Questions 6 and 7

Sally purchases a tractor by a hire-purchase agreement. The price of the tractor is $15 000. She pays a deposit equal to 10% of the price of the tractor. She agrees to make monthly repayments of $1 300 for a one-year period.

Question 6

What is the yearly flat rate of interest, to the nearest percentage, payable by Sally?

A. 16%

B. 17%

C. 18%

D. 19%

E. 20%

Question 7

What is the yearly effective rate of interest, to the nearest percentage, payable by Sally?

A. 16%

B. 20%

C. 25%

D. 29%

E. 33%

Question 8

Anne earns compound interest at 10% per annum compounded twice each year.
Bill earns compound interest at 10% per annum compounded quarterly.
If each of Anne and Bill invest $2,000 for four years

A. Anne has $14.10 more than Bill after four years

B. Anne has $14.10 less than Bill after four years

C. Anne has $549.01 more than Bill after four years

D. Anne has $549.01 less than Bill after four years

E. Anne and Bill have equal amounts after four years

SECTION B – Module 4: Business-related mathematics – continued
TURN OVER

Question 9

Simone earns compound interest on $1 000 for two years at 5% per annum compounded annually. She then takes her resulting accumulated value and deposits it into an account earning 8% per annum simple interest for one year. The final accumulated value of her investment is closest to

A. $1 015

B. $1 024

C. $1 191

D. $1 158

E. $1 260

Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

A planar graph has 6 vertices and 2 faces. How many edges does it have?

A.   12
B.   11
C.   10
D.   8
E.   6

Question 2

A complete graph has 12 vertices. It contains how many edges?

A.   12
B.   24
C.   66
D.   72
E.   132

Question 3

A Hamiltonian path for a graph always

A.   passes through all the vertices more than once
B.   passes through all the vertices exactly once
C.   passes over all the edges more than once
D.   passes over all the edges exactly once
E.   none of the above

The following information relates to Questions 4, 5 and 6

Consider the following network diagram.
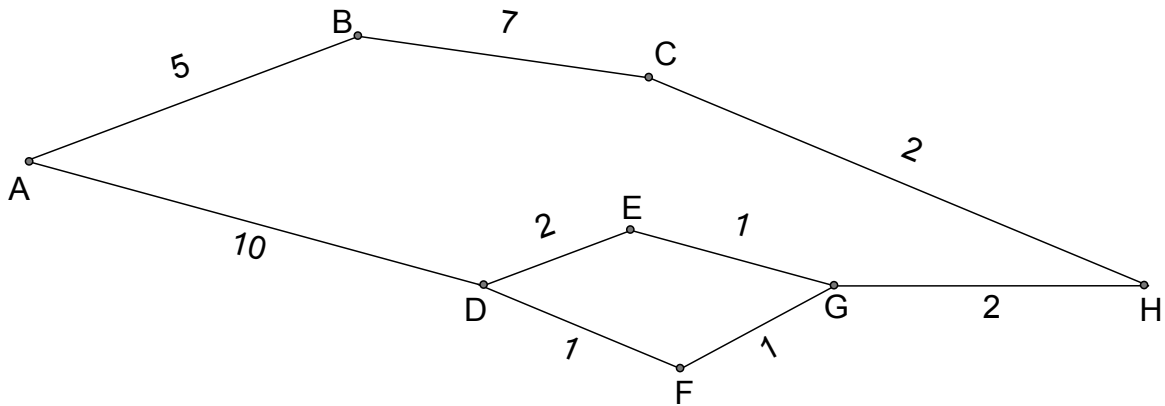
Question 4

The shortest path from A to H is

A. 11

B. 12

C. 13

D. 14

E. 15

Question 5

The minimal spanning tree for this graph has length

A. 14

B. 16

C. 17

D. 18

E. 19

SECTION B – Module 5: Networks and decision mathematics - continued

Question 6
The degree of vertex D is

A. 1

B. 2

C. 3

D. 4

E. 5

Question 7
Which of the following statements is false?

A. a path that includes every edge just once is an Euler path

B. an Euler circuit is an Euler path that starts and finishes at the same vertex

C. a graph with all vertices of odd degree has an Euler path

D. a Hamiltonian path is a path that passes through each vertex exactly once

E. a Hamiltonian circuit is a Hamiltonian path which starts and finishes at the same vertex

Question 8
The minimum number of edges for a connected graph with 6 vertices is

A. 4

B. 5

C. 6

D. 7

E. 8

Question 9
Which of the following is an example of a 2-step dominance?

A. A beat B and B lost to C

B. A lost to B and B beat C

C. A beat B and B beat C

D. A lost to B and C lost to D

E. none of the above

Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple choice questions and write the name of the module in the box provided.

Question 1

Suppose $A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$. If $A - B = \begin{bmatrix} 1 & 4 \\ 3 & 3 \end{bmatrix}$, then the matrix B is

A. $\begin{bmatrix} 1 & -5 \\ 0 & -3 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 5 \\ 0 & -3 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 5 \\ 0 & 3 \end{bmatrix}$

D. $\begin{bmatrix} -1 & 5 \\ 1 & 3 \end{bmatrix}$

E. $\begin{bmatrix} -1 & 5 \\ 1 & -3 \end{bmatrix}$

Question 2

Suppose X is a matrix of order 3×2. Suppose Y is a matrix of order 2×3. Which of the following matrix expressions is defined?

A. $2X + 3Y$

B. YX

C. $XY + 1$

D. $X^{-1}Y$

E. $Y^{-1}X$

SECTION B – Module 6: Matrices- continued

Question 3

The matrix A is the 2×2 matrix $\begin{bmatrix} 1 & 2 \\ x & 8 \end{bmatrix}$. The matrix A does not have an inverse. The value of x is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 4

A tennis club has three classes of membership: Adult, Child and Family.

The cost of a membership is $100 for an adult, $50 for a child and $180 for a family.

The tennis club currently has 100 adult members, 200 children members and 35 family members.

Which of the following matrix expressions gives the **total** revenue for the tennis club from membership fees?

A. $\begin{bmatrix} 100 \\ 50 \\ 180 \end{bmatrix} \begin{bmatrix} 100 & 200 & 35 \end{bmatrix}$

B. $\begin{bmatrix} 100 & 100 \\ 50 & 200 \\ 180 & 35 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 35 & 100 & 200 \end{bmatrix} \begin{bmatrix} 180 \\ 50 \\ 100 \end{bmatrix}$

D. $\begin{bmatrix} 100 & 200 & 35 \end{bmatrix} \begin{bmatrix} 100 \\ 50 \\ 180 \end{bmatrix}$

E. $\begin{bmatrix} 100 \\ 200 \\ 35 \end{bmatrix} \begin{bmatrix} 100 & 50 & 180 \end{bmatrix}$

SECTION B – Module 6: Matrices- continued
TURN OVER

Question 5

You are given that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and that $AB = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$. The matrix B is

A. $\begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$

B. $\begin{bmatrix} -100 & 50 \\ 75 & -25 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 100 & -50 \\ 75 & 25 \end{bmatrix}$

E. $\begin{bmatrix} -2 & 1 \\ 1.5 & 0.5 \end{bmatrix}$

Question 6

You are given that the matrix X is of order $n \times p$, the matrix Y is of order $p \times n$ and that the matrix Z is a column matrix of length p. What is the order of $(XY)^{-1} XZ$?

A. $1 \times p$

B. 1×1

C. $n \times 1$

D. $p \times 1$

E. $p \times n$

SECTION B – Module 6: Matrices- continued

The following information relates to Questions 7, 8 and 9

Companies are given a rating according to their financial strength. There are four rating categories: I, II, III and IV, where I denotes very strong financial position, II denotes satisfactory financial position, III denotes weak financial position and IV denotes extremely weak financial position.

Each January 1 the companies' financial positions are re-assessed by a team of experts and re-rated.

The transition matrix given below shows the proportion of companies in each rating category one year that will be in each particular rating category the following year.

$$
\begin{array}{c}
\text{this year} \\
\begin{array}{cccc} I & II & III & IV \end{array} \\
\begin{bmatrix}
0.8 & 0.2 & 0 & 0 \\
0.1 & 0.6 & 0.2 & 0 \\
0.07 & 0.1 & 0.6 & 0 \\
0.03 & 0.1 & 0.2 & 1
\end{bmatrix}
\begin{array}{l} I \\ II \\ III \\ IV \end{array} \text{ next year}
\end{array}
$$

Question 7

Suppose that in 2009, 80% of companies were rated as category I while the remainder were in category II.

What proportion of companies, correct to 3 decimal places, will be in an extremely weak financial position in 2011?

A. 0.099

B. 0.584

C. 0.160

D. 0.044

E. 0.100

SECTION B – Module 6: Matrices- continued
TURN OVER

Question 8

Suppose now that in 2009, companies were equally split between categories I, II, III and IV.

What proportion of companies, correct to 3 decimal places, were in an extremely weak financial position in 2008?

A. 0.250

B. 0.148

C. 0.333

D. 0.167

E. 0.247

Question 9

In the long run, the model from **Question 8** predicts that

A. 90% of companies will be in a satisfactory financial position

B. All companies will be in an extremely weak financial position

C. All companies will be in a satisfactory financial position

D. 80% of companies will be in a very strong financial position

E. 5% of companies will be in an extremely weak financial position

END OF MULTIPLE-CHOICE QUESTION BOOK