MAV Trial Examination Paper 2010
Further Mathematics Examination 1
SOLUTIONS

Core: Data Analysis

Answers

1. C	**2.** C	**3.** B	**4.** D	**5.** D	**6.** B	**7.** E	**8.** A	**9.** B	**10.** E	**11.** D	**12.** B	**13.** C

Solutions

Question 1 **Answer C**

For and Against is presented as a *dependent categorical* variable.
Junior and Senior is presented an *independent categorical* variable.
A two way frequency table displays relationship between two categorical variables.

Thus it is Bivariate analysis of the attitude to going co-educational **versus** age (junior/senior) group (y vs x).

Question 2 **Answer C**

38 junior students out of 105 junior students are for going co-educational.

Expressed as a percentage i.e. $\frac{38}{105} \times 100\% = 36.2\%$.

Question 3 **Answer B**

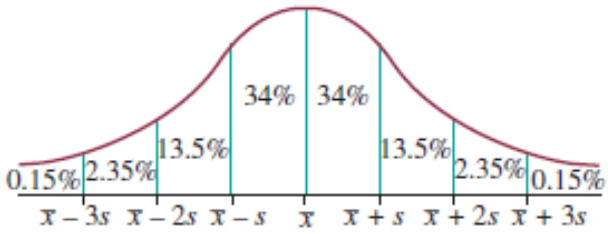
$Minimum = 21$
$Q_1 = 44$
$Median = 48$
$Q_3 = 58$
$Maximum = 79$
$\therefore IQR = 58 - 44 = 14$

$\text{Upper Boundary} = Q_3 + 1.5 \times IQR$
$= 58 + 1.5 \times 14$
$= 79$

$\text{Lower Boundary} = Q_1 - 1.5 \times IQR$
$= 44 - 1.5 \times 14$
$= 23$

So outliers are outside the range of 23 and 79.

Question 4 **Answer D**



Bars that are 145 grams or less are in the bottom 2.5% of a normal distribution or 2 standard deviations away from the mean stated as 150 grams.

Therefore for the distribution described we have established

mean $= \bar{x} = 150$ grams

Standard deviation = $s = 2.5$ grams

Using the reference normal distribution, from 147.5 grams (-1 standard deviation) to 155 grams (+2 standard deviation) the total percentage is 34% + 47.5% = 81.5%.

Question 5 **Answer D**

Physics test	Chemistry test
$z = \frac{x-\bar{x}}{s}$	$z = \frac{x-\bar{x}}{s}$
$x = z \times s + \bar{x}$	$x = z \times s + \bar{x}$
$= -1 \times 5 + 82$	$= 1.5 \times 16 + 62$
$= 77\%$	$= 86\%$

Therefore the student scored 9% lower for his physics test compared to his chemistry test.

Question 6 **Answer B**

This question relates to assumed knowledge from Unit 1&2 on relative and cumulative frequency histograms and polygons. The median for a set of 51 data scores is the 25th score which is two scores out of the 10 scores found in the \$10 to \$15 interval.

Thus the median is

start of interval + proportion of interval range

$\$10 + \frac{2}{10} \times \$5$

$\$10 + \$1 = \$11$

So \$11 is the mean postal charge.

Question 7 **Answer E**

From statement provided, the coefficient of determination is stated as 92% or $r^2 = 0.92$.

The Pearson product-moment correlation coefficient is r or $\sqrt{r^2}$

$r = \sqrt{r^2}$

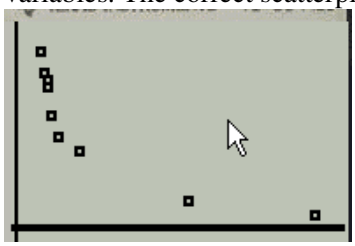
$= \sqrt{0.92}$

$= \pm 0.959$

Remember calculators may not give the negative value which must be considered. In this case the relationship between number of car accidents and driver's age could be either positive or negative relationship.

Question 8 **Answer A**

Winning times is dependent on the number of competitors as stated in the heading of the table given. Use calculator to analyze winning time as dependent variable and no. of competitors as the independent variables. The correct scatterplot is A.



Question 9 **Answer B**

Using a $\frac{1}{y}$ transformation to determine the line of best fit to linearize data set.

Competition	Number of Competitors	Winning time margins (seconds)	$\frac{1}{\text{winning margins}}$
Vancouver 2010	98	0.02	50
Tourin 2006	56	0.042	23.81
World Bobsleigh Cup 2009	21	0.11	9.09
World Luge Cup 2009	14	0.13	7.69
World Bobsleigh Cup 2008	12	0.16	6.25
World Luge Cup 2008	11	0.21	4.76
World Bobsleigh Cup 2007	9	0.22	4.55
USA Trials 2007	8	0.25	4
World Luge Cup 2007	10	0.20	5

The least squares regression gives a gradient = 0.4939 and y-intercept = -0.3222. Thus the equation appropriately stated is option B.

$$\frac{1}{\text{winning margin}} = -0.3222 + 0.4939 \times \text{no. of competitors}$$

Question 10 **Answer E**

There are 15 points so they are grouped Left Middle Right
5 points 5 points 5 points

The three medians are

the median point in the left or lower group (3, 5)
the median point in the middle group (8, 4) and
the median point in the right or upper group (13, 8)

Question 11 **Answer D**

As there is no seasonal pattern, then only one of the three types moving median is suitable. The decision of which of 3, 4 or 5 points is to be used, relate back to nature of the data collected. Given the data is based on sales over 5-day period for each week; 5-point would be the most obvious choice.

Question 12 **Answer B**

The seasonal indices for the quarterly sales have a sum total of 4 with equal unknowns for summer and spring seasons.

$x + 0.8 + 0.35 + x = 4$

$2x + 1.15 = 4$

$2x = 2.85$

$x = 1.425$

Question 13 **Answer C**

Quarter 10 represents Autumn 2010. Substituting 10 in the trend line gives:

$$\text{Deseasonalised no.of golf balls} = 2100 + 125 \times 10$$
$$= 3350$$

Seasonalise this figure to find the actual amount

$$\text{Actual no.of golf balls} = 3350 \times \text{S.I.}$$
$$= 3350 \times 0.8$$
$$= 2680$$

END OF CORE SOLUTIONS

Module 1: Number patterns

Answers

1. E	**2.** B	**3.** C	**4.** C	**5.** D	**6.** D	**7.** A	**8.** B	**9.** A

Solutions

Question 1 — Answer E

Test if arithmetic sequence. It is not arithmetic as common difference shown are not the same.

$d = t_4 - t_3 \qquad = t_3 - t_2$

$= 5\frac{1}{3} - 2\frac{2}{3} \quad = 2\frac{2}{3} - 1\frac{1}{3}$

$= \frac{16-8}{3} \qquad = \frac{8-4}{3}$

$= \frac{8}{3} \qquad = \frac{4}{3}$

Test if geometric sequence which it is with a common ratio, $r = 2$.

$r = \frac{t_4}{t_3} = \frac{t_3}{t_2} = \frac{t_2}{t_1}$

$= \frac{5\frac{1}{3}}{2\frac{2}{3}} \qquad = \frac{2\frac{2}{3}}{1\frac{1}{3}} \qquad = \frac{1\frac{1}{3}}{\frac{2}{3}}$

$= \frac{16}{3} \times \frac{3}{8} \qquad = \frac{8}{3} \times \frac{3}{4} \qquad = \frac{4}{3} \times \frac{3}{2}$

$= \frac{16}{8} \qquad = \frac{8}{4} \qquad = \frac{4}{2}$

$= 2 \qquad = 2 \qquad = 2$

Question 2 — Answer B

Using $r = 1.3$ and $S_4 = 6187$ gives

$S_n = \frac{a(r^n - 1)}{r - 1}$

$6187 = \frac{a(1.3^4 - 1)}{1.3 - 1}$

$6187 = 6.187a$

$a = 1000$

Using $a = 1000$ and $r = 1.3$.

$t_2 = ar$

$= 1000 \times 1.3 = 1300$

Question 3 **Answer C**

Determine the common difference from the first three terms given.

$d = t_3 - t_2 \quad = t_2 - t_1$

$= -5 - 2 \quad = 2 - 9$

$= -7 \quad = -7$

Use arithmetic sequence formula.

$t_n = a + (n-1)d$

$-131 = 9 + (n-1) \times -7$

$-131 = 9 - 7n + 7$

$-131 = -7n + 16$

$-7n = -147$

$n = 21$

Question 4 **Answer C**

Size of vegetable garden increases by 8% means that previous size is multiplied by 1.08
Area of garden initially is $A_0 = 6$. Note option B: $A_n = 1.08 \times A_{n-1}$ where $A_1 = 6$ is incorrect as it states the area after the first year is 6 square metres which is the area it was at the start, $A_0 = 6$.
i.e. $A_n = 1.08 \times A_{n-1}$ where $A_0 = 6$

Question 5 **Answer D**

Use the sequence defined by $t_n = -3n + 1$ where $n = 1, 2, 3, ...$

$t_1 = -3n + 1 \quad t_2 = -3n + 1 \quad t_3 = -3n + 1 \quad t_4 = -3n + 1$

$= -3 \times 1 + 1 \quad = -3 \times 2 + 1 \quad = -3 \times 3 + 1 \quad = -3 \times 4 + 1$

$= -2 \quad = -5 \quad = -8 \quad = -11$

This generates an arithmetic sequence where common difference is -3 and first term is -2. Thus the appropriate difference equation is $t_{n+1} = t_n - 3$, $t_1 = -2$

Question 6 **Answer D**

Find the common ratio

$$r = \frac{t_6}{t_5}$$

$$= \frac{-3.125}{6.25}$$

$$= -0.5$$

Therefore using sum to infinity formula

$$S_\infty = \frac{a}{1-r}$$
$$300 = \frac{a}{1--0.5}$$
$$300 = \frac{a}{1.5}$$
$$300 \times 1.5 = a$$
$$a = 450$$

Question 7 **Answer A**

Using the Fibonacci difference equations $t_{n+2} = t_n + t_{n+1}$
Transpose the equation making the first term the subject gives $t_n = t_{n+2} - t_{n+1}$.

$t_n = t_{n+2} - t_{n+1}$ $t_n = t_{n+2} - t_{n+1}$ $t_n = t_{n+2} - t_{n+1}$
$= -7 - -3$ $= -3 - -4$ $= -4 - 1$
-4 $= 1$ $= -5$

Thus the sequence is –5,1,–4,–3,–7, –10, –17, –27,….. therefore, $t_1 = -5$ and $t_2 = 1$.

Question 8 **Answer B**

Use simultaneous equations to determine *a* and *d.*

$t_n = a + (n-1)d$
$32.5 = a + (8-1)d$
$32.5 = a + 7d$ and

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$187.5 = \frac{10}{2}(2a + (10-1)d)$$
$$187.5 = 5(2a + 9d)$$
$$187.5 = 10a + 45d$$

$10a + 45d = 187.5$ -1 $10a + 45d = 187.5$
$a + 7d = 32.5$ -2×10 $\underline{10a + 70d = 325}$
$-25d = -137.5$
$d = 5.5$

Given the 8th term is 32.5 then the 10th term is 32.5 + 5.5 + 5.5 = 43.5

Question 9 **Answer A**

An increase of 5% means that previous size is multiplied by 1.05 thus $r = 1.05$.
Using the geometric sequence formula

$t_n = ar^{n-1}$

$2 = 0.5 \times 1.05^{n-1}$

$4 = 1.05^{n-1}$

$\log 4 = (n-1) \times \log 1.05$

$n - 1 = \dfrac{\log 4}{\log 1.05}$

$n - 1 = 28.41$

$n = 29.41$

Thus it will take the 30th beam to exceed a length of 2 metres.

END OF MODULE 1 SOLUTIONS

Module 2: Geometry and trigonometry

1. E	**2.** B	**3.** E	**4.** D	**5.** D	**6.** E	**7.** B	**8.** C	**9.** C

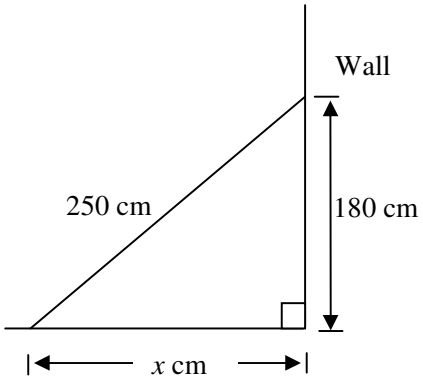
Solutions

Question 1 **Answer E**

Let $x =$ distance from the bottom of the ladder to the base of the wall

Using Pythagoras

$$x^2 + 180^2 = 250^2$$
$$x^2 = 250^2 - 180^2$$
$$x^2 = 30100$$
$$\therefore x = \sqrt{30100}$$
$$x = 173.4935...$$

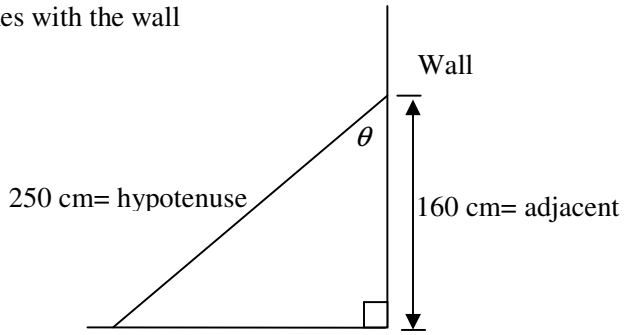


Question 2 **Answer B**

The ladder slips 20cm down the wall, so the top of the ladder is now 160cm.

Let $\theta =$ the angle that the top of the ladder makes with the wall

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$= \frac{160}{250}$$
$$\therefore \theta = \cos^{-1}\left(\frac{160}{250}\right)$$



Question 3 **Answer E**

$$\text{Interior angle} = 180 - \frac{360}{n}$$

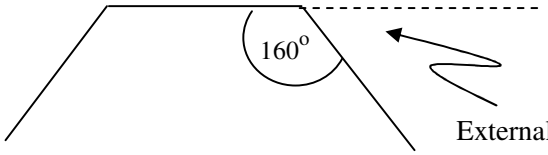
$$160 = 180 - \frac{360}{n}$$

$$\frac{360}{n} = 180 - 160$$

$$\frac{360}{n} = 20$$

$$\therefore n = \frac{360}{20}$$

$$n = 18$$



External angle $= 20^\circ$

$$\frac{360}{n} = 20$$

$$\therefore n = \frac{360}{20}$$

$$n = 18$$

Question 4 **Answer D**

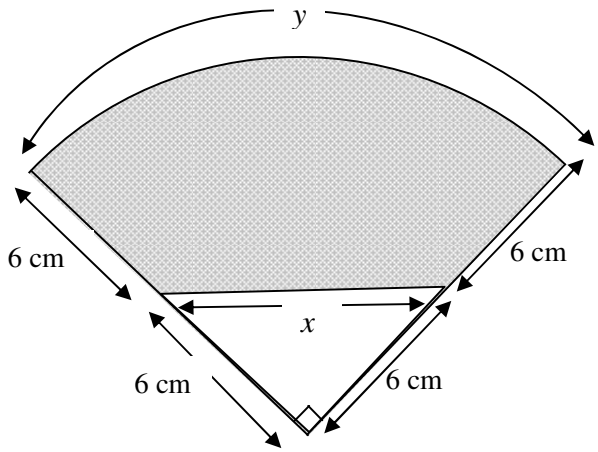
Total length of wire

$$= 4 \times 6 + x + y$$

$$= 24 + \sqrt{6^2 + 6^2} + \frac{1}{4} \times 2\pi \times 12$$

$$= 24 + \sqrt{72} + 6\pi$$

$$= 51.334....$$



Question 5 **Answer D**

$$\text{Shaded Area} = \frac{1}{4} \times \text{Area of circle} - \text{Area of Triangle}$$

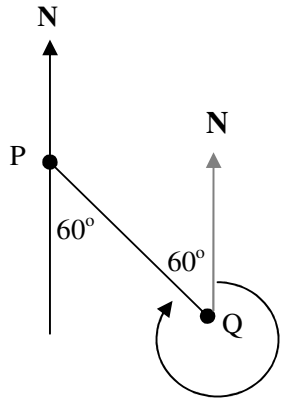
$$= \frac{1}{4} \times \pi r^2 - \frac{1}{2} \times bh$$

$$= \frac{1}{4} \times \pi \times 12^2 - \frac{1}{2} \times 6 \times 6$$

$$= 36\pi - 18$$

Question 6 **Answer E**

Bearing from Q to P is $360^O - 60^O = 300^O T$



Question 7 **Answer B**

Given that AB=BC = 40 cm then Triangle ABC is isosceles.

Therefore $\angle BAC = \angle ACB$

$A + B + C = 180^o$

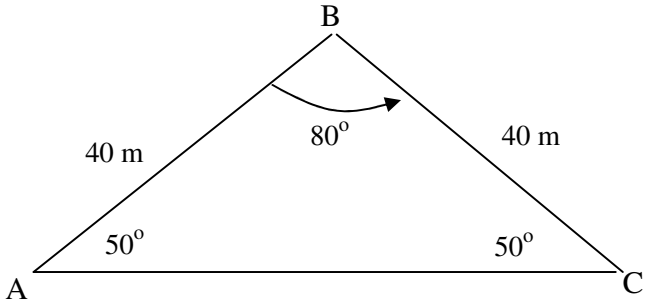
$x + 80^o + x = 180^o$

$2x + 80^o = 180^o$

$2x = 100^o$

$\therefore x = 50^o$

$\angle BAC = \angle ACB = 50^o$

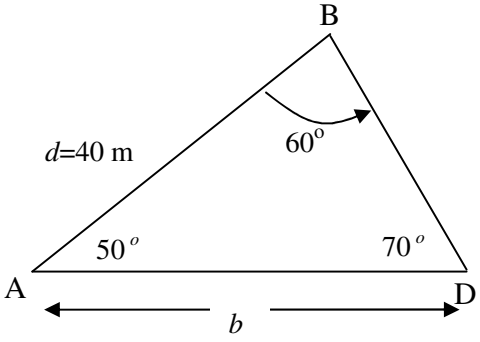


$A + B + D = 180^o$

$50^o + 60^o + D = 180^o$

$110^o + D = 180^o$

$\therefore D = 70^o$



Using the Sine Rule

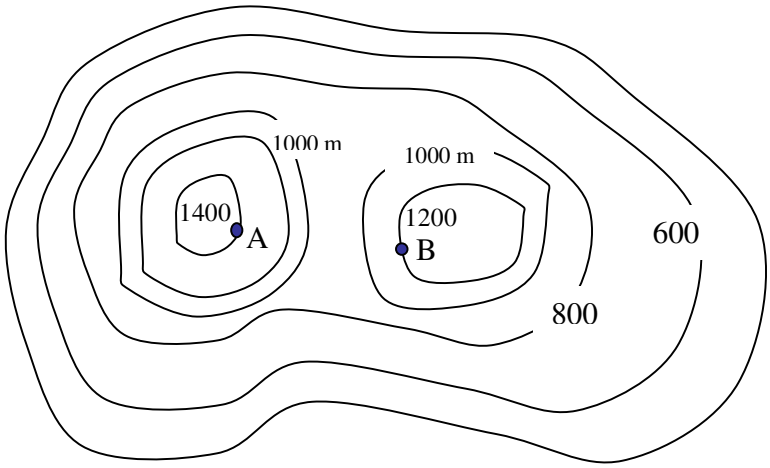
$$\frac{b}{\sin B} = \frac{d}{\sin D}$$

$$\frac{b}{\sin 60^o} = \frac{40}{\sin 70^o}$$

$$b = \frac{40 \sin 60^o}{\sin 70^o}$$

$$b = 36.864...$$

Question 8 **Answer C**



Location A is on the 1400 m contour line.
Location B is on the 1200 m contour line.

This means that the vertical height of A above B is 200 m.

The horizontal distance is 2.5 cm on the map.
Given that the scale ratio 1: 50 000

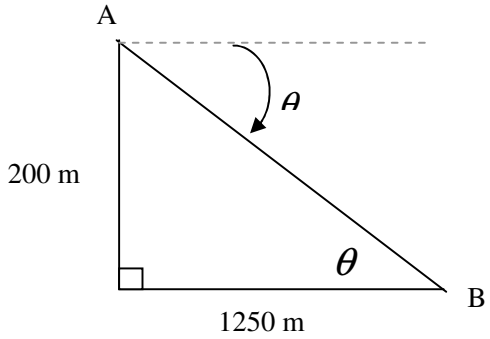
Map : Real

2.5 : x
1 : 50 000

$$x = 2.5 \times 50000$$
$$= 125\ 000 \text{ cm}$$
$$= 1250 \text{ m}$$

Angle of depression from A is the same as the angle of elevation from B

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{200}{1250}$$
$$\therefore \theta = \tan^{-1}\left(\frac{200}{1250}\right)$$
$$= 9.0902...$$



Question 9 **Answer C**

An increase of 20% means that the radius is 1.20 times the original length.

The length ratio is 1 : 1.2

This means that the area ratio is $1^2 : 1.2^2 = 1 : 1.44$

1.44 means that the surface area increases by 44%

END OF MODULE 2 SOLUTIONS

Module 3: Graphs and relations

1. C	**2.** D	**3.** B	**4.** A	**5.** D	**6.** E	**7.** B	**8.** B	**9.** C

Solutions

Question 1 **Answer C**

Estimating the time

0.5 + 0.5 + 0.4 + 0.3 + 0.5 = 2.2 hours

Question 2 **Answer D**

Find equation $y = mx + c$

To find gradient, use (3, 12) and (0, 16)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{16-12}{0-3}$$
$$= \frac{4}{-3}$$

From the graph, the y-intercept, $c = 16$

$$\therefore y = \frac{-4}{3}x + 16$$

When $x = b, \quad y = 0$

Substituting into the equation gives

$$0 = \frac{-4b}{3} + 16$$
$$\frac{4b}{3} = 16$$
$$\therefore x = \frac{16 \times 3}{4} = 12$$

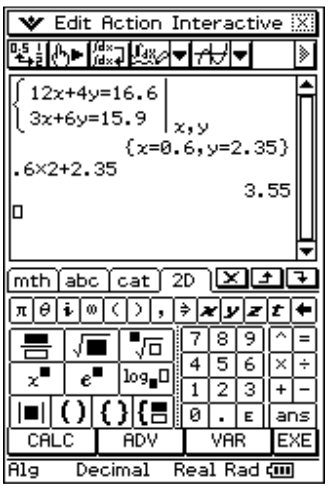
Question 3 **Answer B**

Let x = the cost of one Easter egg and y = the cost of one hot cross bun

Betty's purchase gives the equation $12x + 4y = 16.60$ (1)

Ebony's purchase gives the equation $3x + 6y = 15.90$ (2)

At this point the equations can be entered into the calculator to solve simultaneously.



This means that one Easter egg will cost 60 cents and one hot cross bun will cost \$2.35

So one hot cross bun and two Easter eggs will cost 2.35 +1.20 = \$3.55

Alternatively, using algebra

Eliminate x by

multiplying equation (2) by 4 $\quad 12x + 24y = 63.60$

and then subtracting equation (1) $\quad 12x + 4y = 16.60$

this gives $\quad 20y = 47.00$

$$y = \frac{47.00}{20} = 2.35$$

Substitute $y = 2.35$ in equation (1) will give

$$12x + 4 \times 2.35 = 16.60$$
$$12x + 9.40 = 16.60$$
$$12x = 7.20$$
$$\therefore x = 0.6$$

So the total cost of one hot cross bun and 2 Easter eggs is 2.35 +1.20 = \$3.55

Question 4 **Answer A**

This graph can have two possible equations: $y = \frac{k}{x}$ or $y = \frac{k}{x^2}$

If the graph has an equation of the form $y = \frac{k}{x}$ then $k = x \times y = \frac{1}{2} \times 12 = 6$

So one possible equation is $y = \frac{6}{x}$ (not listed as one of answers)

If the graph has an equation of the form $y = \frac{k}{x^2}$ then $k = x^2 \times y = \left(\frac{1}{2}\right)^2 \times 12 = \frac{1}{4} \times 12 = 3$

So the other possible equation is $y = \frac{3}{x^2}$

Question 5 **Answer D**

Equation given is $2x - 3y = -3$
Make y the subject in order to determine the values of *m* and *c*

$-3y = -2x - 3$

$y = \frac{-2}{-3}x - \frac{3}{-3}$

$y = \frac{2}{3}x + 1$

This means that $m = \frac{2}{3}$ and $c = 1$

- The gradient is positive (i.e. as x increases, y increases) ***∴A is false***
- The gradient is $\frac{2}{3}$ ***∴B is false***.
- The *y*-intercept is 1 ***∴C is false***
- To find *x*-intercept sub $y = 0$ in $2x - 3y = -3$ This gives $2x = -3$ i.e. $x = \frac{-3}{2}$,

  so the *x*-intercept is $\frac{-3}{2}$. This value is greater than -2 ***∴D is true.***
- When $x = 3$, $6 - 3y = -3$

  $-3y = -9$

  $y = 3$

  This means the line $2x - 3y = -3$ passes through the point $(3, 3)$, not (3, -3) ***∴E is false***.

Question 6 **Answer E**

The equation of the revenue graph is given by

$\text{Revenue} = m \times n$

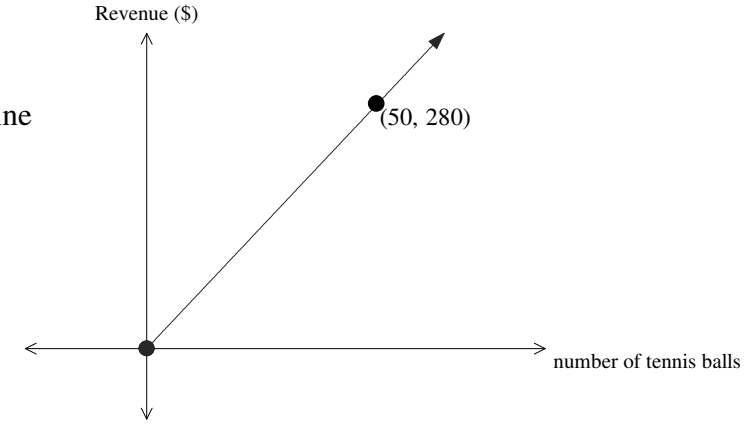
where *m* is the gradient of the revenue line

$m = \frac{280}{50} = 5.6$

so $\text{Revenue} = 5.6n$

This means that one tennis ball is sold for

$\text{Revenue} = 5.6 \times 1 = \$5.60$



Question 7 **Answer B**

$\text{Profit} = \text{Revenue} - \text{Cost}$

$\text{Profit} = 5.6n - \text{Cost}$

When $n = 250$ Profit = 180

$180 = 5.6 \times 250 - \text{Cost}$

$\text{Cost} = 5.6 \times 250 - 180$

$= \$1220$

Cost consists of a fixed cost of \$600 plus the cost of each tennis ball.
Let t = cost of one tennis ball.

Cost = $t \times n + 600$

When $n = 250$ the Cost = 1220

$\text{Cost} = t \times n + 600$

$1220 = t \times 250 + 600$

$620 = 250t$

$t = \frac{620}{250} = \$2.48$

Question 8 **Answer B**

If x = the number of rap songs and y = the number of R'n'B songs

At ***least twice*** as many rap songs as R'n'B songs.

A table of possible values gives

x	***y***
2 or more	1
4 or more	2
6 or more	3
8 or more	4
10 or more	5

To equate, double y. The x value can be more than double the y so $x \geq 2y$

Question 9 **Answer C**

k	$Z = 2kx + y$	(10, 5)	(10, 40)	(15, 25)	Maximum point
1	$Z = 2x + y$	25	60	55	(10, 40)
-1	$Z = -2x + y$	-15	20	-5	(10, 40)
2	$Z = 4x + y$	45	80	85	(15, 25)
-1	$Z = -4x + y$	-35	0	-35	(10, 40)
0	$Z = y$	5	40	25	(10, 40)

The point (15, 25) is a maximum only when $k = 2$

END OF MODULE 3 SOLUTIONS

Module 4: Business related mathematics

Answers

1. D	**2.** D	**3.** C	**4.** B	**5.** C	**6.** C	**7.** A	**8.** B	**9.** E

Solutions

Question 1 **Answer D**

Number of interest bearing periods = number of years × number of periods per year
= 5 years × monthly
= 5 × 12
= 60

Question 2 **Answer D**

Time (years)	Bookvalue ($)	Annual Depreciation $
0	260 000	
1	234 000	26 000
2	210 600	23 400
3	189 540	21 060
4	170 586	18 954

As annual depreciation is not constant and therefore it is not straight line depreciation but reducing balance depreciation.

$$\%\ \text{reducing balance depreciation} = \frac{\$26000 - \$23400}{\$26000} \times \frac{100}{1}\%$$
$$= 10\%$$

Question 3 **Answer C**

Date	Transaction	Debit $	Credit $	Balance $
5th May	Salary		420.50	841.00
12th May	ATM withdrawal	70.00		**771.00**
23rd May	EFTPOS purchase	150.70		620.30
25th May	Salary		200.00	**820.30**
31st May	ATM withdrawal	300.00		**520.30**

Need to also consider the balance on 1st May or the start of the month = $841.00 - $420.50 = **$420.50**
Therefore the minimum monthly balance is $420.50.

Question 4 **Answer B**

The perpetuity pays out quarterly and thus 5% pa is 1.25% per quarter

$$Q = \frac{Pr}{100}$$
$$= \frac{\$220000 \times 1.25\%}{100}$$
$$= \$2750 \text{ per quarter}$$

or

Question 5 **Answer C**

$A = PR^n$ where

$$R = 1 + \frac{8}{100}$$
$$= 1.08$$

$A = 5000 \text{ x } 1.08^3$

$= \$6298.56$ (using compound interest)

$\therefore$ Interest earned= $ 1298.56

$$\text{Simple Interest} = \frac{Pr T}{100}$$
$$= \frac{5000 \times 9 \times 3}{100}$$
$$= \$1350$$

$\therefore$ Simple interest earns an extra $51.44

Question 6 **Answer C**

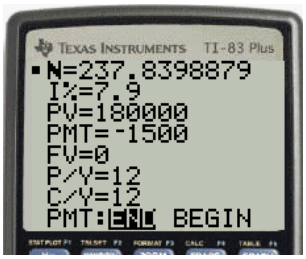
From the stamp duty schedule table, the duty for a new vehicle under $57009 is $8 per $200 or part thereof.

$$\text{Multiples of } \$200 = \frac{\$37900}{200}$$
$$= 194.5$$

Therefore there are 195 multiples as part thereof means to round up.

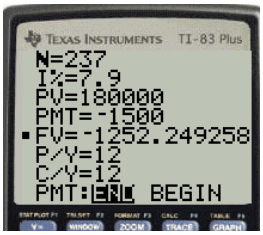
$$\text{Stamp duty} = 195 \times \$8$$
$$= \$1560$$

Question 7 **Answer A**

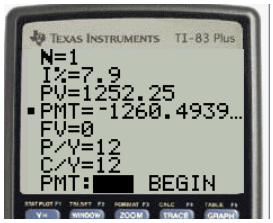


Use **TVM solver** to determine the term of the loan and whether the final payment is the full or part amount of $1500.

The term of loan is calculated as 237.839 thus requiring 237 full payments and the 238th payment is part of $1500.



Use TVM solver to find the balance after 237 full payments so set N=237 and the balance is $1252.25



The final payment required needs to also include interest on the $1252.25.
Set N=1 for the final payment.

The final payment is $1260.49

Question 8 **Answer B**

Using the quick method to convert simple Interest to effective interest rate is a *little less than double.* Therefore to convert from effective interest rates to simple interest do the opposite or a *little more than half.* Half of 7.5% is 3.75% and little more would suggest 3.8%. Test answer using the effective interest rate formula.

$$\text{Effective rate} = \frac{2n}{n+1} \times \text{flat rate}$$
$$= \frac{2 \times 60}{60+1} \times 3.8\%$$
$$= 7.475\% \approx 7.5\%$$

Alternatively

$$\text{Effective rate} = \frac{2n}{n+1} \times \text{flat rate}$$
$$7.5\% = \frac{2 \times 60}{60+1} \times \text{flat rate}$$
$$7.5\% = 1.9672 \times \text{flat rate}$$
$$\text{flat rate} = \frac{7.5\%}{1.9672}$$
$$\text{flat rate} = 3.8125\%$$

Question 9 **Answer E**

Using TVM solver, the terms of her superannuation scheme are
Total number of payments for 12 years at monthly instalments is 144 payments.
Her employer provides 10% of $60 000 or $6000 per year or $500 per month.

The monthly contribution required to achieve goal from TVM solver is $1796.87.

Therefore
Extra contribution = $1796.87 - $500 = $1296.87 or $1300.

END OF MODULE 4 SOLUTIONS

Module 5: Networks and decision mathematics

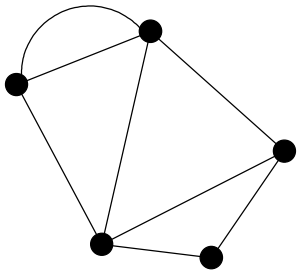
Answers

1. A	**2.** C	**3.** A	**4.** C	**5.** B	**6.** E	**7.** E	**8.** A	**9.** B

Solutions

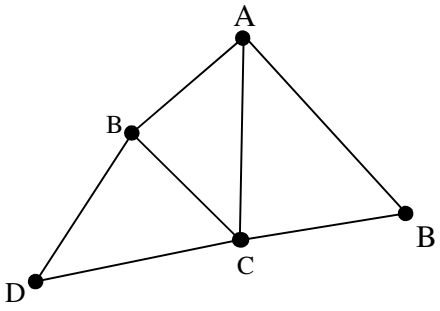
Question 1 **Answer A**

Each vertex can reach all other vertices, therefore this graph **is connected**
No edges intersect, therefore this graph **is planar**.
Every vertex is not connected to every other vertex, so this graph is not complete.
This graph has more than three circuits- i.e. paths that start and end at the same vertex.
This graph is not a tree because it contains multiple circuits.
This graph is not simple because there are multiple edges between two vertices.



Question 2 **Answer C**

An Euler Path exists since there are only two vertices with an odd degree. The Euler path will start and end at an odd degree vertex. Vertices B, C and D have even degrees, 2, 4, 2 respectively, but A and E each have a degree of 3, therefore an Euler path can start at E and finish at A.



Question 3 **Answer A**

Using Euler's rule $v + f = e + 2$ where $v = 7$

$7 + f = e + 2$

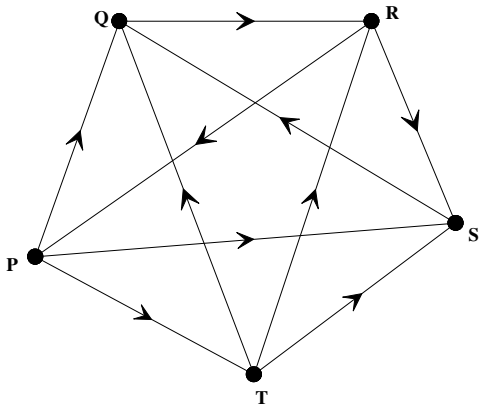
$5 = e - f$

when $e = 9$ and $f = 4$ $\quad 9 - 4 = 5$

Question 4 **Answer C**

The adjacency matrix for the competition is given by

$$M=\begin{bmatrix}0&1&0&1&1\\0&0&1&0&0\\1&0&0&1&0\\0&1&0&0&0\\0&1&1&1&0\end{bmatrix}$$



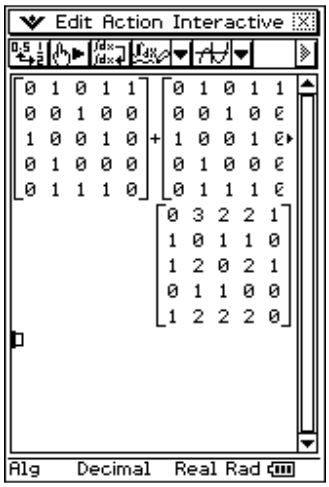
The dominance vector is given by

$$D=\begin{bmatrix}3\\1\\2\\1\\3\end{bmatrix}$$

T is equal first with P with 3 one step wins.

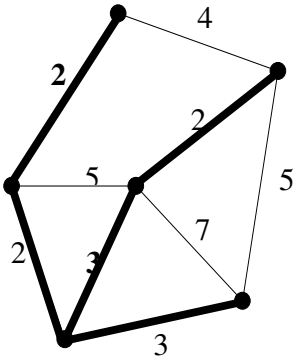
Question 5 **Answer B**

$$\begin{bmatrix}0&1&0&1&1\\0&0&1&0&0\\1&0&0&1&0\\0&1&0&0&0\\0&1&1&1&0\end{bmatrix}+\begin{bmatrix}0&1&0&1&1\\0&0&1&0&0\\1&0&0&1&0\\0&1&0&0&0\\0&1&1&1&0\end{bmatrix}^2=\begin{bmatrix}0&3&2&2&1\\1&0&1&1&0\\1&2&0&2&1\\0&1&1&0&0\\1&2&2&2&0\end{bmatrix}$$



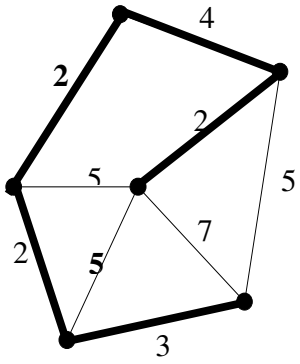
Question 6 **Answer E**

When $a = 2 \quad b = 3$



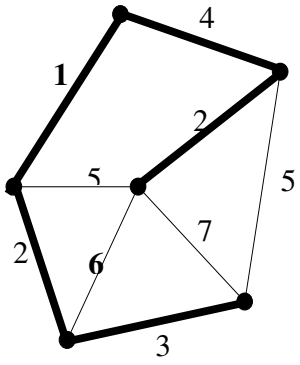
The minimum spanning tree has weighting 12

When $a = 2 \quad b = 5$



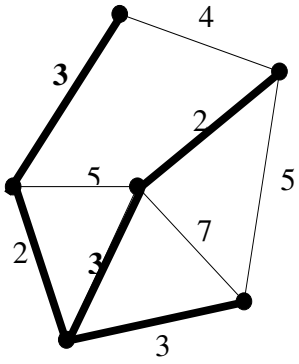
The minimum spanning tree has weighting 13

When $a = 1 \quad b = 6$



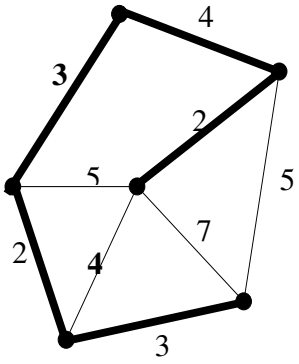
The minimum spanning tree has weighting 12

When $a = 3 \quad b = 3$



The minimum spanning tree has weighting 13

When $a = 3 \quad b = 4$



The minimum spanning tree has weighting 14

Question 7 **Answer E**

Using the Hungarian Algorithm

Circle the minimum time from each row.

$$\begin{bmatrix} (2) & 5 & 10 & 3 \\ 4 & (3) & 8 & 9 \\ 7 & 8 & 9 & (2) \\ 6 & 8 & 8 & (4) \end{bmatrix}$$

Subtract the minimum row time from all elements in the row.

$$\begin{bmatrix} (0) & 3 & 8 & 1 \\ 1 & (0) & 5 & 6 \\ 5 & 6 & 7 & (0) \\ 2 & 4 & 4 & (0) \end{bmatrix}$$

Cover zeros with the minimum of lines

$$\begin{bmatrix} (0) & 3 & 8 & 1 \\ 1 & (0) & 5 & 6 \\ 5 & 6 & 7 & (0) \\ 2 & 4 & 4 & (0) \end{bmatrix}$$

Subtract the minimum time in the 3rd column from all other elements in the column.

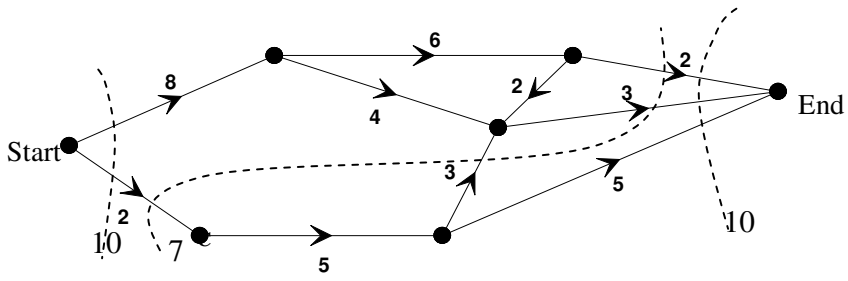
$$\begin{bmatrix} (0) & 3 & 4 & 1 \\ 1 & (0) & 1 & 6 \\ 5 & 6 & 3 & (0) \\ 2 & 4 & (0) & (0) \end{bmatrix}$$

To allocate the task look for the column or row that has only one zero.

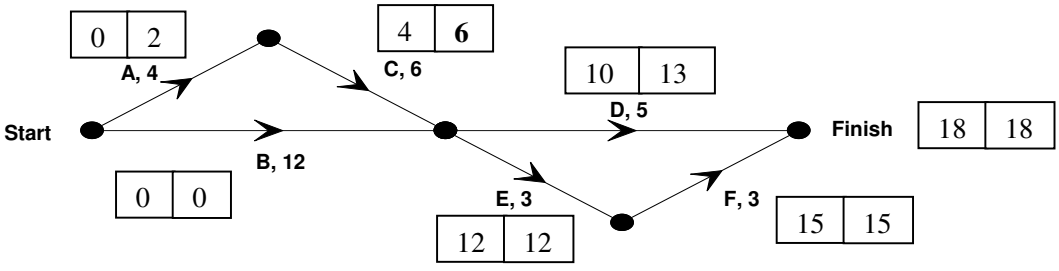
	Task H	R	C	S
A	[0]	3	4	1
B	1	[0]	1	6
C	5	6	3	[0]
D	2	4	[0]	0

Anne hurdles, Brendan runs, Cindy swims and Dan cycles.

Question 8 **Answer A**



Question 9 **Answer B**



The critical path is BEF = 12 +3 + 3 = 18 hours. EST for C is 4 hours and the longest path from the start of C to the end is CEF = 6 + 3 + 3 = 12 hours. So the LST is 18 – 12 = 6 hours.

END OF MODULE 5 SOLUTIONS

Module 6: Matrices

Answers

1. D	**2.** A	**3.** E	**4.** B	**5.** C	**6.** D	**7.** D	**8.** B	**9.** A

Solutions

Question 1 **Answer D**

Testing each for multiplication using order of matrix gives

$A\times B\times C\times D$ order of matrix for $A\times B$ is $3\times 2\times 1\times 3$ which cannot be multiplied.

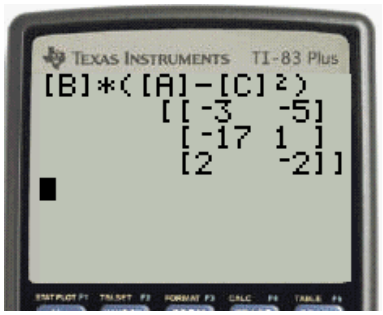
$C\times A$ order of matrix is $2\times 2\times 3\times 2$ which cannot be multiplied.

$A\times D$ order of matrix is $3\times 2\times 3\times 3$ which cannot be multiplied.

$D\times A$ order of matrix is $3\times 3\times 3\times 2$ which **can be multiplied**.

$A\ \times A\times C$ order of matrix for $A\times A$ is $3\times 2\times 3\times 2$ which cannot be multiplied.

Question 2 **Answer A**



Question 3 **Answer E**

$$\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}\times\begin{bmatrix}2 & 5\\ 3 & 6\end{bmatrix}=\begin{bmatrix}r_1c_1 & r_1c_2\\ r_2c_1 & r_2c_2\end{bmatrix}$$

$$=\begin{bmatrix}2\times 2+3\times 3 & 2\times 5+3\times 6\\ 1\times 2+4\times 3 & 1\times 5+4\times 6\end{bmatrix}$$

Remember to multiply the rows of the first matrix with the columns of the second matrix as shown.

Question 4 **Answer B**

$$\begin{bmatrix} 10 & 50 \\ 13 & 85 \\ 23 & 125 \end{bmatrix} \times \begin{bmatrix} 0.92 & 0 \\ 0 & 1.05 \end{bmatrix} \Rightarrow \begin{bmatrix} 10\times 0.92 & 50\times 1.05 \\ 13\times 0.92 & 85\times 1.05 \\ 23\times 0.92 & 125\times 1.05 \end{bmatrix}$$

Reduce by 8% - use 0.92.
Increase by 5% - use 1.05
So **E is** out.
C and D are not defined.
A gives the combined new prices.
B gives matrix showing new prices separate.

Question 5 **Answer C**

For each option

A. $k(A-B)=kA-kB$ Distributive law is true

B. $A\times A^{-1}=A^{-1}\times A$ Only for a matrix and its inverse does the order of multiplication not matter

C. $A-B=B-A$ Order of subtraction is important.

D. $A(B-C)=AB-AC$ Distributive law holds

E. $A+(B+C)=(A+B)+C$ Associative law for addition holds

Therefore the incorrect statement is $A-B=B-A$

Question 6 **Answer D**

$$\begin{array}{c} \\ Wet \\ Dry \end{array} \begin{array}{c} \begin{array}{cc} Wet & Dry \end{array} \\ \begin{bmatrix} 0.40 & 0.35 \\ 0.60 & 0.65 \end{bmatrix} \end{array}$$

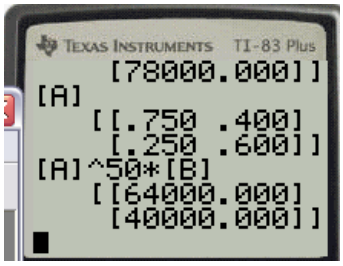
A, B and D are not suitable transition matrices as the columns do not sum to 1.
40% chance is 0.40 for wet day stays wet day which is row 1 column 1.
65% chance is 0.65 for dry day stays dry day which is row 2 column 2.

Question 7 **Answer D**

A. A diagonal matrix has to be a square matrix. TRUE
B. Only square matrices can be raised to a power. TRUE
C. Finding the inverse of a matrix can only be applied to a square matrix. TRUE
D. When multiplying square matrices, order is not important. FALSE **Order is important otherwise there are two possible outcomes.**
E. All transition matrices are square matrices. TRUE

Question 8 **Answer B**

Use a graphics calculator to evaluate by setting n to a very large value such as 50 as follows.



Question 9 **Answer A**

The sum of two numbers is 87 expressed as an equation is $1x + 1y = 87$.

The difference is 27 expressed as an equation is $1x - 1y = 27$.

The coefficients are $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

END OF SOLUTIONS

