# FURTHER MATHEMATICS

## Units 3 & 4 – Written examination 2

## 2009 Trial Examination

## ***SOLUTIONS***

**SECTION A: Core**

**Question 1**

**a.** Negatively skewed. A1

**b.**

**i.** 11+12+11+4+1=39 A1

**ii.** $\frac{29}{60} \times 100\% = 48.3\%$ A1

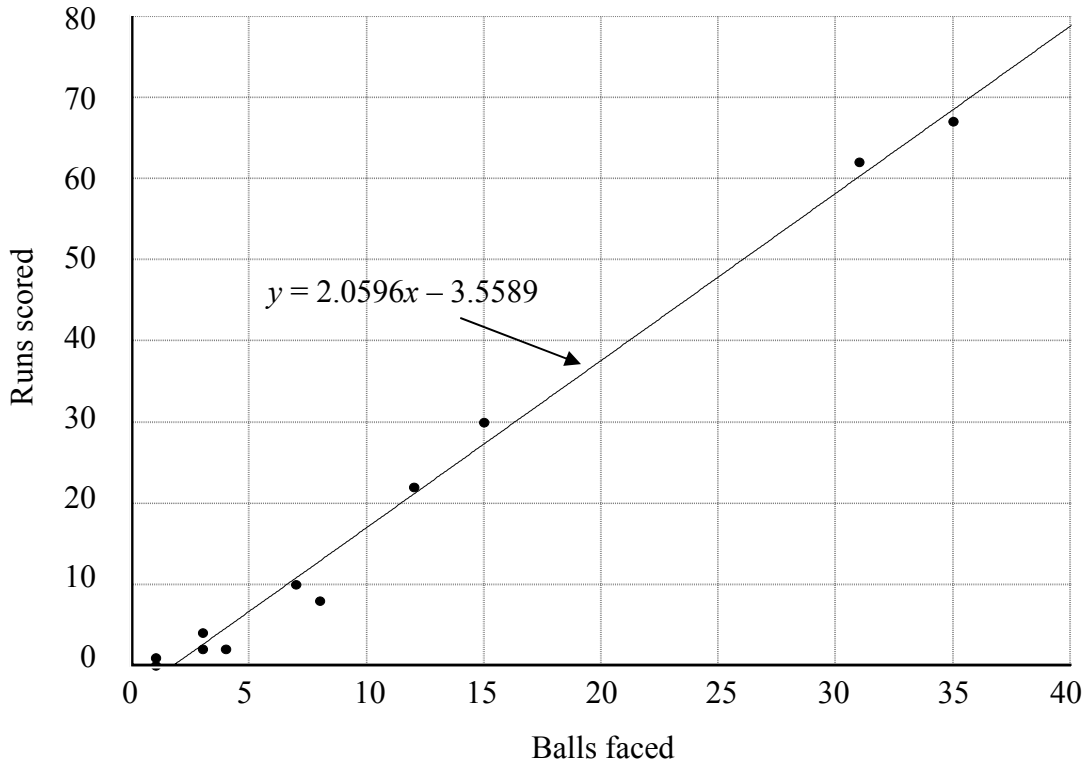
**Question 2**

**a.** Bar chart, segmented bar chart, pie chart. A1

These graphs allow the display of categorical data with less than 6 options. A1

**b.** Runs scored A1

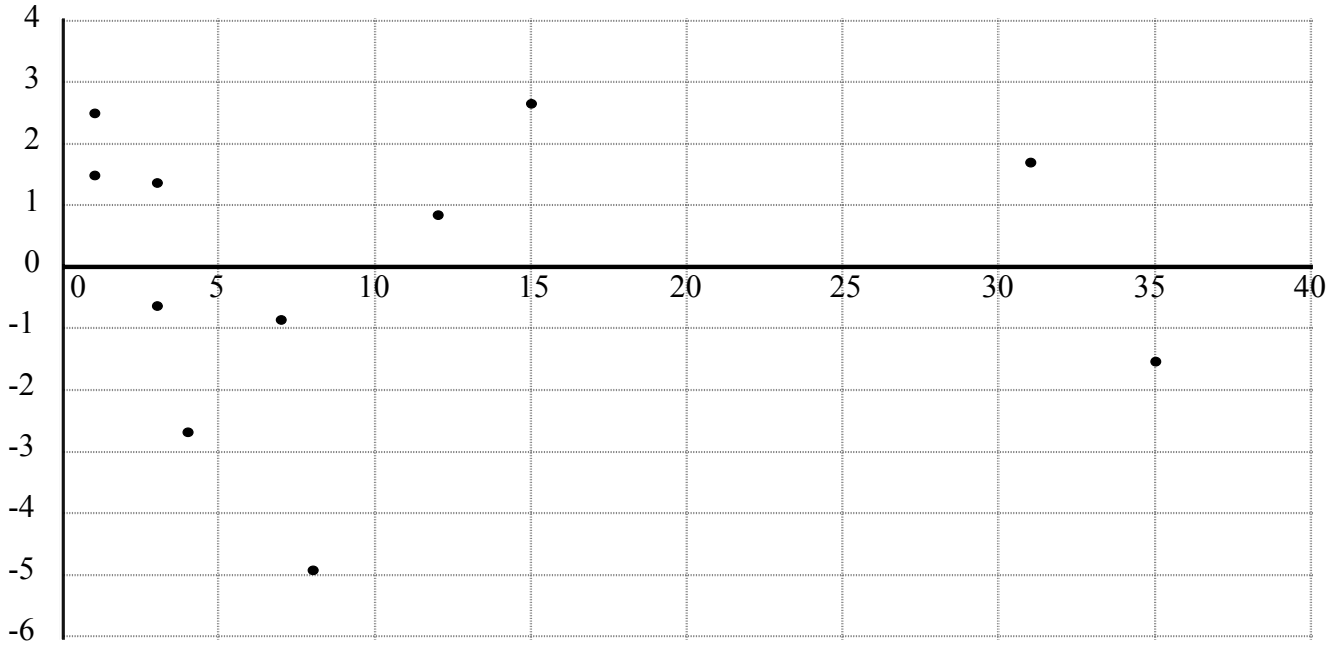
**c.** Appropriately labelled axes A1
Correct choice of $x$ and $y$ axes A1
Appropriate placement of data points M1




**d.** Least squares regression line: number of runs = 2.06 × number of balls faced – 3.56 A1
Appropriate placement of regression line A1

**e.** Appropriate scale on graph A1

Appropriate placement of residuals A1




**f.** Yes, it does suggest linearity. A1

There is a relatively random distribution of the residuals. A1

## SECTION B: Modules

### Module 1: Number Patterns

### Question 1

**a.** $3 \times 140 = 420km$ A1

**b.** $a = 140$ A1

**c.**

$3430 = 140n$

$\therefore n = \dfrac{3430}{140}$ A1

$n = 24.5days$

**d.** $140n = \dfrac{n}{2}\left[2 \times 100 + (n-1)10\right]$ M1

$\dfrac{140n}{n} = \dfrac{\frac{n}{2}\left[2 \times 100 + (n-1)10\right]}{n}$

$140 = \dfrac{1}{2}\left[2 \times 100 + (n-1)10\right]$

$140 = \dfrac{1}{2}\left[200 + 10n - 10\right]$

$140 = \dfrac{1}{2}\left[190 + 10n\right]$ M1

$140 = 95 + 5n$

$45 = 5n$

$\therefore n = 9days$ A1

## Question 2

**a.**

$D_4 = 0.8 \times 350 + 100 = 380$

$D_5 = 0.8 \times 380 + 100 = 404km$ A1

**b.**

$\frac{350}{380} = 0.921$

$\frac{380}{404} = 0.941$

$\therefore$ *not geometric*

$380 - 350 = 30$

$404 - 380 = 24$

$\therefore$ *not arithmetic* M1

**c.**

$D_n = \frac{D_{n+1} - 100}{0.8}$ M1

$D_2 = \frac{350 - 100}{0.8} = 312.5$

$D_1 = \frac{312.5 - 100}{0.8}$

$D_1$=265.625km A1

## Question 3

**a.** $d_n = 0.9d_{n-1}$ A1

**b.** $d_5 = 400(0.9)^4$

$= 262.44km$ A1

**c.** $3430 = \frac{a\left(1 - 0.9^{10}\right)}{1 - 0.9}$ M1

$3430 = \frac{a\left(1 - 0.9^{10}\right)}{0.1}$

$3430 \times 0.1 = \frac{a\left(1 - 0.9^{10}\right)}{0.1} \times 0.1$

$343 = a\left(1 - 0.9^{10}\right)$ M1

$343 = 0.65132a$

$\frac{343}{0.65132} = \frac{0.65132a}{0.65132}$

$a = \frac{343}{0.65132}$

$\therefore a = 526.62km$ A1

**Module 2: Geometry and Trigonometry**

**Question 1**

**a.** $\tan\theta = \frac{O}{A}$

$\therefore \tan 18^\circ = \frac{x}{8.5}$

$\therefore x = 8.5\tan 18^\circ$ A1

$= 2.76\text{ m}$

**b.** Yes, total height = 2.4 + 2.76 + 0.15 = 5.31*m* A1

**c.** The maximum allowed value of *x* is 5.5-2.4-0.15 = 2.95. Therefore

$\theta = \tan^{-1}\left(\frac{2.95}{8.5}\right) = 19.14^\circ$ A1

**Question 2**

**a.** $C = \sqrt{a^2 + b^2 - 2ab\cos C}$

$= \sqrt{28^2 + 42^2 - 2\times 28\times 42\times\cos 70}$

$= 41.76\text{ m}$ A1

**b.** $A = \frac{1}{2}ac\sin B$

$A = \frac{1}{2}\times 28\times 42\times\sin 70$

$=552.54\text{ m}^2 \approx 553\text{ m}^2$ A1

**c.** Scale factor 1:500, Area scale factor $1^2 : 500^2 = 1 : 25000$ M1

$552.54\,\text{m}^2 = 5525400\,\text{cm}^2$

$\therefore \dfrac{5525400}{25000} = 221\,\text{cm}^2$ A1

**d.** Volume scale factor $1^3 : 500^3 = 1 : 25000000$

$16 \times 20 \times 2.4 = 768m^3 \times 100^3 = 768000000$

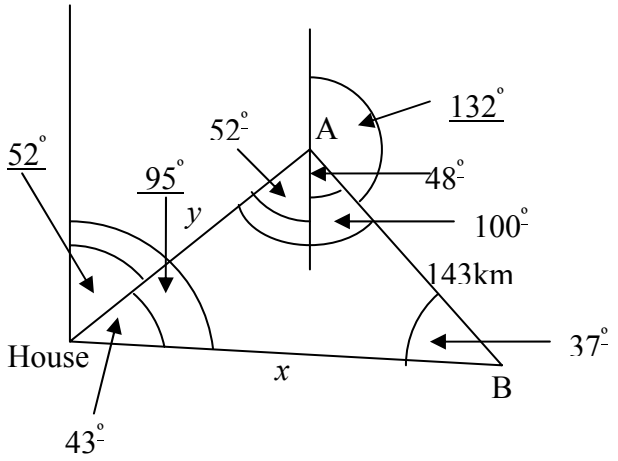
$\dfrac{768000000}{125000000} = 6cm^3$ M1

A1

**Question 3**

**a.** Appropriately drawn diagram of points A1

Underlined angles included A1




**b.** **i.** $\angle BAH = 232^\circ - 132^\circ = 100^\circ$

$\angle ABH = 180^\circ - 43^\circ - 100^\circ = 37^\circ$ M1

**ii.** $\dfrac{135}{\sin 43} = \dfrac{x}{\sin 100} = \dfrac{y}{\sin 37}$

$x = \dfrac{135 \sin 100}{\sin 43}$ $\quad$ $y = \dfrac{135 \sin 37}{\sin 43}$ A2

$x = 194.94km$ $\quad$ $y = 119.13km$

**c.** $052°T$ A1

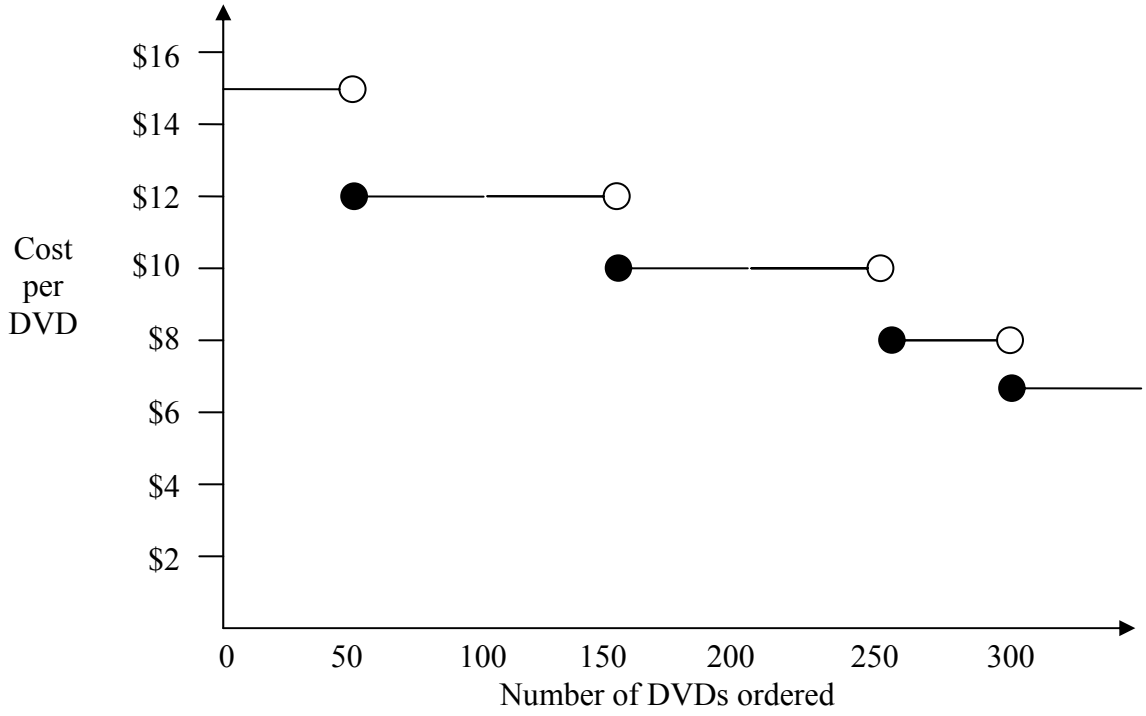
**Module 3: Graphs and Relations**
**Question 1**

**a.** $12 A1

**b.** 150 x $10 = $1500 A1

**c.** 50 copies A1

**d.** Correct graph A1



**Question 2**

**a.** $C = 20000 + 3x$ A1

**b.** $R = 20x$ A1

**c.** $20x = 20000 + 3x$ M1

$20x - 3x = 20000$

$17x = 20000$

$x = \dfrac{20000}{17}$

$x = 1176.47$

$\therefore 1177$ DVDs A1

**Question 3**

**a.** $x \geq 10$ A1

$y \geq 8$ A1

$2x + 2.5y \leq 50$ A1

**b.** Correct positioning of lines A1

Correct calculation of corner points A1

Intersect of $x \geq 10$ and $y \geq 8$ is (10, 8)

Intersect of $x \geq 10$ and $2x + 2.5y \leq 50$ is $2 \times 10 + 2.5y = 50$

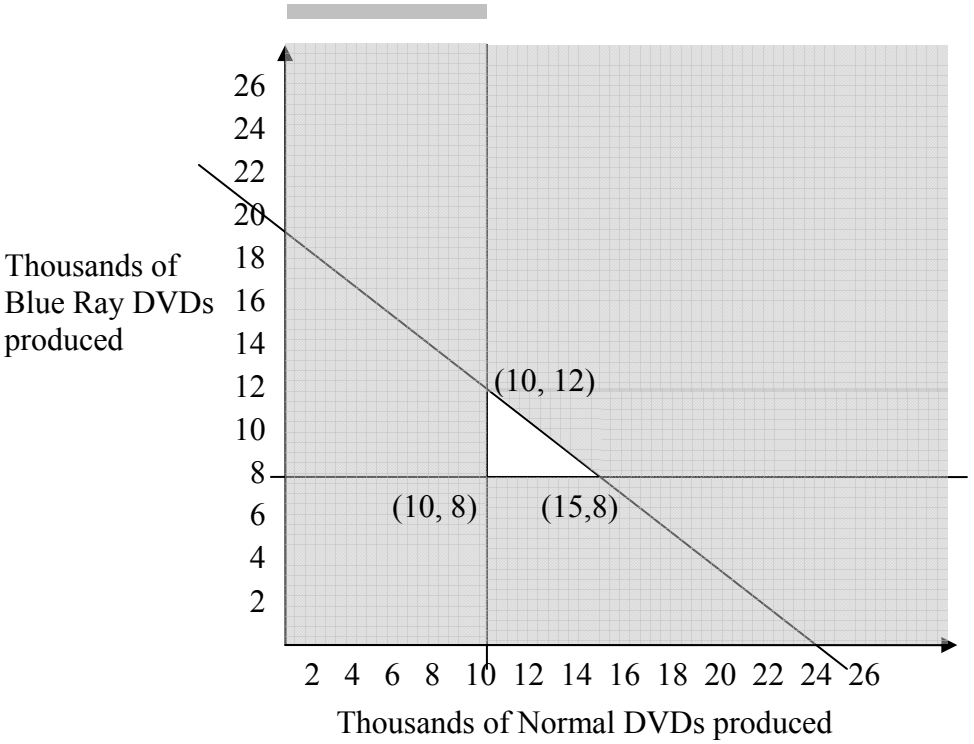
$2.5y = 30$

$y = 12$ (10, 12)

Intersect of $y \geq 8$ and $y \geq 8$ is $2x + 2.5 \times 8 = 50$

$2x = 30$

$x = 15$ (15, 8)




**c.** $(10, 8) = 10 \times 15 + 8 \times 25 = \$350$

$(10, 12) = 10 \times 15 + 12 \times 25 = \$450$

$(15, 8) = 15 \times 15 + 8 \times 25 = \$425$ M1

Maximum profit of \$450 by producing 10000 Normal and 12000 Blue Ray DVDs. A1

**Module 4: Business-Related Mathematics**
**Question 1**

**a.**

**i.** $\$2800 \times 20\% = \$560$ A1

**ii.** $2800 - 560 = 2240$

$\frac{2240}{12} = \$186.67$ A1

**b.** $500 + 52 \times 50 = \$3100$

Better to go to the first store. A1

Will save $3100 – 2800 = \$300$ A1

**c.** $2950 \times 0.825 = \$2433.75$ A1

**d.** $2433.75 \times \frac{100}{110} = 2212.5$

$2433.75 - 2212.5 = \$221.25$ A1

**Question 2**

**a.** \$341.80 using TVM solver A1

**b.** TVM solver gives weekly payments of \$78.73

$78.73 \times 364 = \$28657.72$ total payment by weekly instalments M1

$341.80 \times 84 = \$28711.20$ total payment by monthly instalments

$28711.20 - 28657.72 = \$53.48$ saving A1

**c.** Using TVM solver

Balance after 2 years (104 weeks) = \$16899.62

After \$5000 is paid he then owes \$11899.62 M1

Using TVM solver this will be paid off in 171.84 weeks

Total time is 171.84 + 104 = 275.84 weeks $\rightarrow$ 276 weeks = 5 years 16 weeks A1

**d.** $\$28657.72 - (275.84 \times 78.73 + 5000) = \$1935.84$ A1

**e.** Option 1: $5000 = 22000 - (0.065 \times 22000)x$

$5000 - 22000 = 22000 - 1430x - 22000$

$-17000 = -1430x$

$x = 11.89$

$x$ = 12 years A1

Option 2: $y = 22000 \times (0.89)^x$

Using Table function on graphics calculator

Value first gets below 5000 after 13 years A1

Answer: Option 1 is best

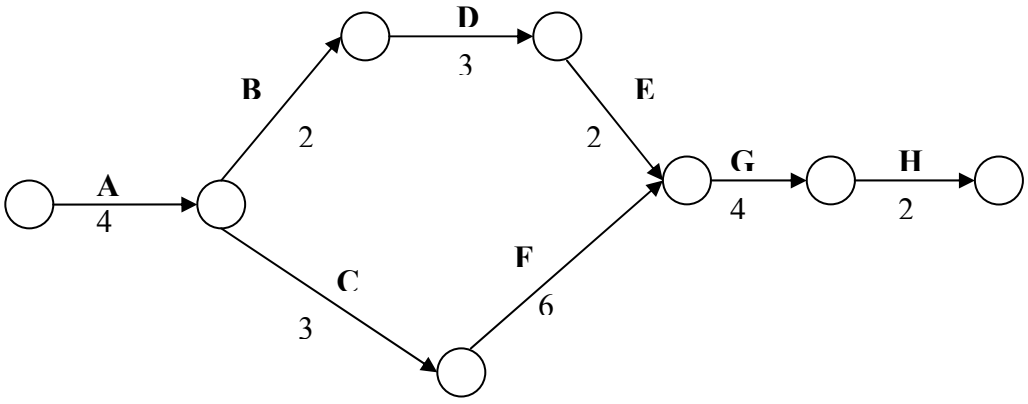
A1

**Module 5: Networks and Decision Mathematics**

**Question 1**

**a.** Correct placement of nodes and activities A1

Appropriate placement of times A1




**b.** A, C, F, G, H. Project takes 19 weeks. A1

**c.** A, B, D, E, G, H $= 4+2+3+2+4+2 = 17$ weeks

20-17 weeks = 3 weeks A1

**d.**

**i.** A, C, F, G, H=19 weeks

Cheapest to crash on critical path is C =\$2000 A1

**ii.** There are two paths to complete this task A, C, F, G, H = 19 weeks

and A, B, D, E, G, H = 17 weeks

The only task available to be crashed in both paths is task G = 1 week = \$3000

$\therefore$ need to reduce A, C, F, G, H by 3 more weeks ➔ crash C 1 week (\$2000) and F 2 weeks (\$10000)

Also, need to reduce A, B, D, E, G, H by 1 week ➔ crash E by 1 week (\$3000)

Total = \$3000 + \$2000 + \$10000 + \$3000 = \$18000 A1

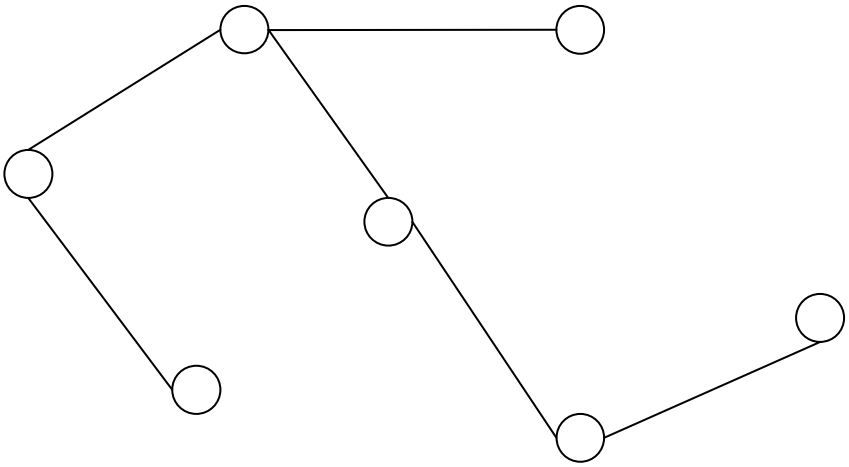
**iii.** Complete in 18 weeks crashing C. This gets a profit of 5000 – 2000 = \$3000

versus crashing to 15 weeks gets a profit of 20000 – 18000 = \$2000 A1

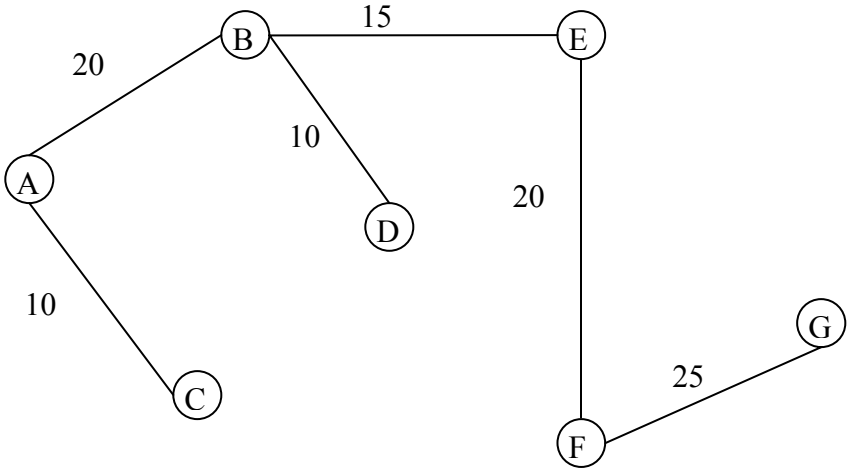
**Question 2**

**a.** $A - C - F - G = 65\,\text{min}$ A1

**i**. Appropriate choice and drawing of minimum spanning tree. A1



OR



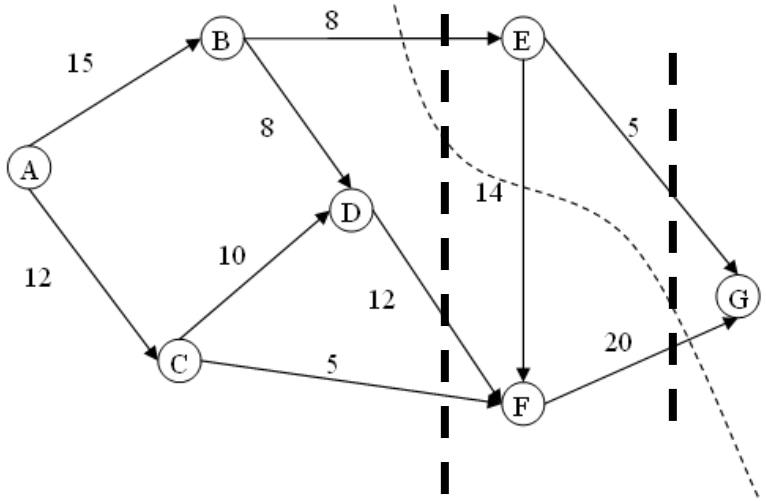

**ii**. 10+20+10+15+20+25 = 100 A1

**Question 3**

**a.** 8+20=28 people A1

**b.** 20+5=25 people
or 8+12+5=25 people A1

Appropriate placement of either cut A1




**c.** No, it doesn't have an effect. A1
8+12+5=25 people is still the minimum cut A1

**Module 6: Matrices**

**Question 1**

**a.** $\begin{bmatrix} 12 \\ 20 \\ 8 \end{bmatrix}$ A1

**b.** $\begin{bmatrix} 2 & 3 & 2 \\ 1 & 3 & 3 \\ 4 & 6 & 6 \\ 0 & 2 & 2 \end{bmatrix}$ A1

**c.** $OC = \begin{matrix} 2\times12+3\times20+2\times8 \\ 1\times12+3\times20+3\times8 \\ 4\times12+6\times20+6\times8 \\ 0\times12+2\times20+2\times8 \end{matrix} = \begin{bmatrix} 100 \\ 96 \\ 216 \\ 56 \end{bmatrix}$ A1

Bill 1 =\$100, bill 2 =\$96, bill 3 =\$216 and bill 4 =\$56 A1

**d.** The number of columns of *C* does not match the number of rows of *O*. A1

**Question 2**

**a.** $\begin{bmatrix} 3 & 2 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 90 \\ 204 \end{bmatrix}$ A1

**b.** $A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \dfrac{1}{3\times6-2\times6}\begin{bmatrix} 6 & -2 \\ -6 & 3 \end{bmatrix}$ M1

$= \dfrac{1}{6}\begin{bmatrix} 6 & -2 \\ -6 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -0.33 \\ -1 & 0.5 \end{bmatrix}$ A1

**c.** $\begin{bmatrix} 3 & 2 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 90 \\ 204 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -0.33 \\ -1 & 0.5 \end{bmatrix} \begin{bmatrix} 90 \\ 204 \end{bmatrix}$ M1

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ 12 \end{bmatrix}$

Main meal costs \$22 and dessert costs \$12. A1

**Question 3**

**a.** $\begin{bmatrix} 0.72 & 0.14 & 0.12 \\ 0.16 & 0.8 & 0.04 \\ 0.12 & 0.06 & 0.84 \end{bmatrix}$ A1

**b.** $\begin{bmatrix} 2000 \\ 1500 \\ 800 \end{bmatrix}$ A1

**c.** $\begin{bmatrix} 0.72 & 0.14 & 0.12 \\ 0.16 & 0.8 & 0.04 \\ 0.12 & 0.06 & 0.84 \end{bmatrix}^3 \begin{bmatrix} 2000 \\ 1500 \\ 800 \end{bmatrix}$ M1

$= \begin{bmatrix} 1504.0 \\ 1549.7 \\ 1246.3 \end{bmatrix}$ A1

Therefore, after three weeks, 1504 diners are expected to dine at Applebox, 1550 at Baskerville and 1246 at The Club. A1