MAV Trial Examination Paper 2009
Further Mathematics
Examination 2 – SOLUTIONS

Core: Data Analysis

Question 1

a.

Stem	Leaf
7	4
8	2 5 8
9	2 6 7 8
10	1 5
11	0

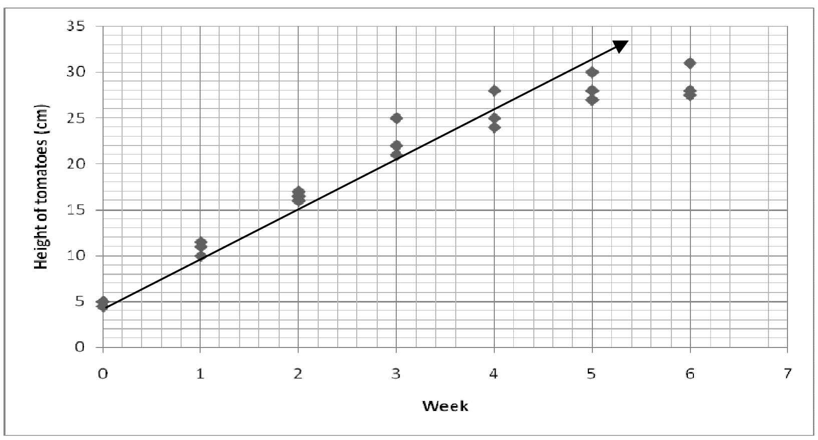
7 | 4 = 74 cm

Correct stem and Correct leaf **[A1]**
Legend included **[A1]**

b. **For shape**: Supermarket tomatoes are normally distributed (or symmetrically distributed) whilst Allen's have a negatively skewed distribution. **[A1]**
For spread: Allen's tomatoes have a wider spread of weights – both in range and IQR. **[A1]**

Question 2

a.



The y-intercept is near 5 cm **[A1]**
A reasonable attempt to balance the points on either side of the line. **[A1]**

b. Y –intercept given as approximately 5. **[A1]**
Gradient, with workings, is in the range of 4 to 5.
For example using the points (0, 5) & (3, 20)

$$m = \frac{(20 - 5)}{(3 - 0)}$$

$$= \frac{15}{3}$$ **[M1]**

$$= 5$$

Gives $y = 5x + 5$
States the equation using the variables names correctly ie.
Plant height = 5 **x** **weeks** + 5 **[A1]**

c. Positive means that as the weeks pass (or weeks increase) the plant will grow taller (height increases) accordingly. **[A1]**

d. Obviously not linear. So as the variable pant height has to be transformed use a y^2 transformation that is (Plant Height)^2. **[A1]**

Question 3

a. The sum of indices of all four seasons is 4, therefore the missing index is 0.98.

$0.90 + 0.98 + 1.14 + x = 4$

$x = 4 - 3.92$ **[A1]**

$= 0.98$

Spring	Summer	Autumn	Winter	
0.90	1.14	0.98	0.98	**[A1]**

b. The initial height of the plant when first planted. **[A1]**

c. For every season that passes (time period) the plant will grow 0.1 metres in height. **[A1]**

TOTAL 15 marks

END OF CORE SOLUTIONS

Module 1: Number Patterns

Question 1

a. $6+1.5=7.5\,\text{m}$ **[A1]**

b. $t_2-t_1=4.5-3=1.5$

$t_3-t_2=6-4.5=1.5$

since $t_2-t_1=t_3-t_2=1.5$ then the sequence is arithmetic **[A1[**

c. using $S_n=\frac{n}{2}[2a+(n-1)d]$ where $n=10,\ a=3,\ d=1.5$

$$S_{10}=\frac{10}{2}[2\times3+(10-1)1.5]$$
$$=5\left[6+9\times1.5\right]$$
$$=97.5$$

[A1]

d. To find *n* when $S_n\geq120$

Enter the general rule in the calculator to generate the sum sequence and then scroll down until the sum first exceeds 120. Read the value of *n*

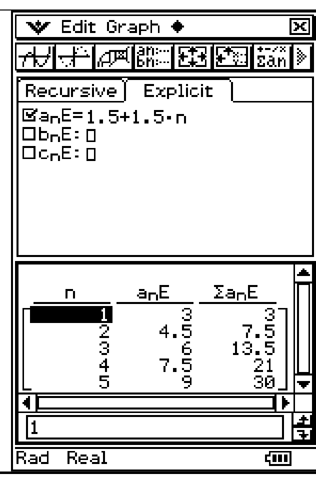
<u>Using the Casio Classpad</u>

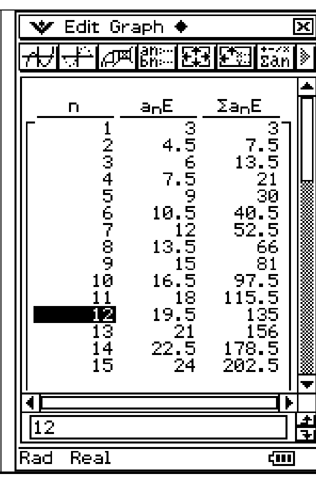
$t_n=a+(n-1)d$

$t_n=3+(n-1)1.5$

$t_n=1.5+1.5n$

Enter the general rule $1.5+1.5n$ and turn the Σ display on.





Using the TI 83/84

$S_n \geq 120$

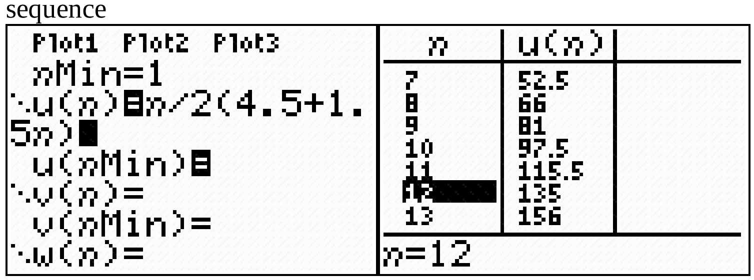
$\frac{n}{2}[2a+(n-1)d] \geq 120$

$\frac{n}{2}[6+(n-1)1.5] \geq 120$

$\frac{n}{2}[6+1.5n-1.5] \geq 120$

$\frac{n}{2}[4.5+1.5n] \geq 120$

Enter $\frac{n}{2}[4.5+1.5n]$ in the calculator to generate the sum sequence



Answer: n=12

Tammy will be eligible after 12 lessons **[A1]**

Question 2

a. $20 \times 0.08 = 1.6$, $20 - 1.6 = 18.4$

OR

decrease by 8% means $r = 1 - 0.08 = 0.92$

therefore during the second lesson a student makes an average of $20 \times 0.92 = 18.4$ errors.

18.4 errors **[A1]**

b.

$E_{n+1} = a \times E_n$

$E_2 = a \times E_1$

$18.4 = a \times 20$

$\therefore a = \frac{23}{25} = 0.92$

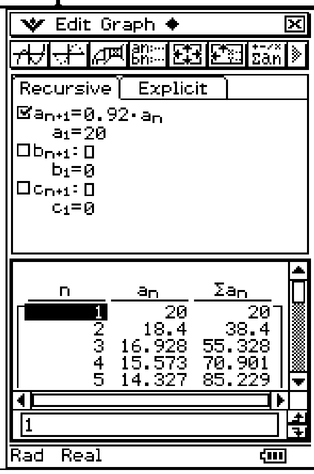
[A1]

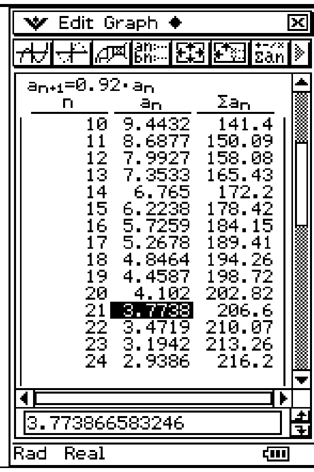
c. Find n when $E_n = 3$

Method 1: Using the calculator

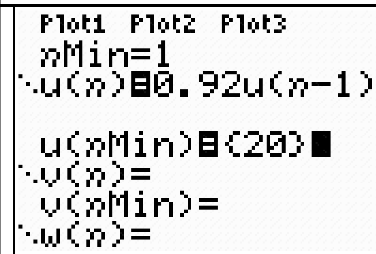
Enter the difference equation $E_{n+1} = 0.92E_n$, $E_1 = 20$ in the calculator to generate the error sequence and scroll where the error first becomes less than 4.

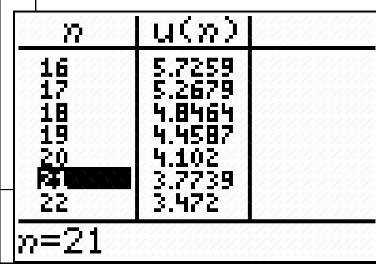
Classpad





TI 83/84





Less than 4 errors would have been made in the 21st lesson. **[A1]**

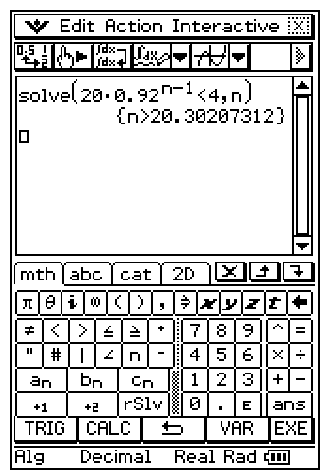
Method 2: Using algebra

Solve the general rule for a geometric sequence

$$a \times r^{(n-1)} < 4$$
$$20 \times 0.92^{n-1} < 4$$

this point you may enter the inequation in the calculator and solve or continue algebraically

$$0.92^{n-1} < \frac{4}{20}$$
$$n - 1 > \frac{\log(0.2)}{\log(0.92)}$$
$$n - 1 > 19.30$$
$$n > 20.30$$
$$\therefore n = 21$$



d. $S_n = \dfrac{a(1-r^n)}{1-r}$, where $a = 20$ and $r = 0.92$

$= \dfrac{20(1-0.92^n)}{1-0.92}$ or $\dfrac{20(1-0.92^n)}{0.08}$ or $250(1-0.92^n)$ [**A1]**

e.

[M1]

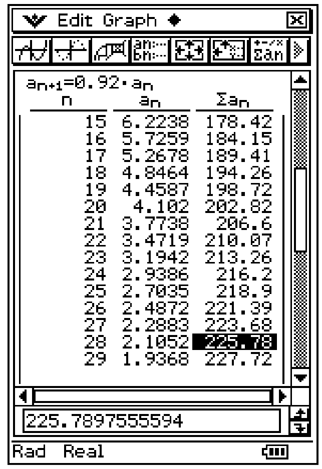
$S_{28} - S_{23}$

$= \dfrac{250(1-0.92^{28})}{1-0.92} - \dfrac{250(1-0.92^{23})}{1-0.92}$

$= 225.79 - 213.267$

$= 12.5$

$=$ 13 errors



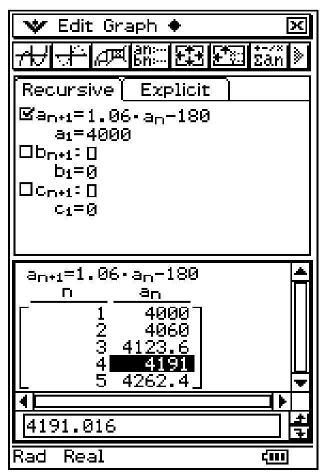
[A1]

Question 3

a.

Enter the difference equation in the calculator

$M_4 = 4191$



Alternatively, using algebra

$M_2 = 1.06 \times M_1 - 180$

$= 1.06 \times 4000 - 180$

$= 4060$

$M_3 = 1.06 \times M_2 - 180$

$= 1.06 \times 4060 - 180$

$= 4123.6$

$M_4 = 1.06 \times M_3 - 180$

$= 1.06 \times 4123.6 - 180$

$= 4191$

[A1]

b. $M_2 - M_1 = 4060 - 4000 = 60$ and $M_3 - M_2 = 4123.6 - 4060 = 63.6$

$M_2 - M_1 \neq M_3 - M_2$ therefore the sequence is not arithmetic.

$\frac{M_2}{M_1} = \frac{4060}{4000} = 1.015$ and $\frac{M_3}{M_2} = \frac{4123.6}{4060} = 1.016$

$\frac{M_2}{M_1} \neq \frac{M_3}{M_2}$ therefore the sequence is not geometric.

Both must be shown **[A1]**

c. If $M_{n+1} = M_n = 4000$ then

$M_{n+1} = x \times M_n - 180$

$4000 = x \times 4000 - 180$

$4180 = 4000x$

$x = \frac{4180}{4000} = 1.045$

A 4.5% increase

Alternatively increase by the amount that is removed,

$x = \frac{180}{4000} \times \frac{100}{1}\% = 4.5\%$ **[A1]**

d.

Enter the difference equation in the calculator

$S_4 = 4474$

Alternatively, using algebra

$S_3 = 1.10 \times S_2 - 0.12 S_1 - 80$

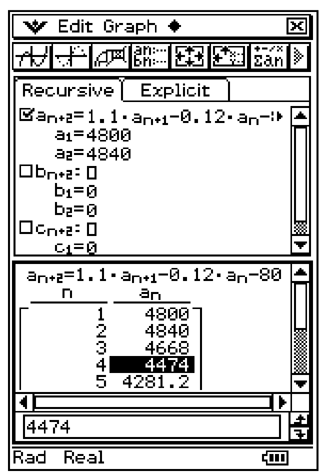
$= 1.10 \times 4840 - 0.12 \times 4800 - 80$

$= 4668$

$S_4 = 1.10 \times S_3 - 0.12 S_2 - 80$

$= 1.10 \times 4668 - 0.12 \times 4840 - 80$

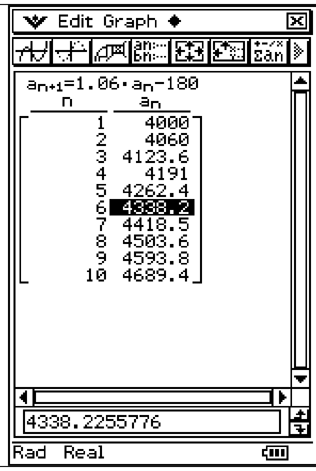
$= 4474$

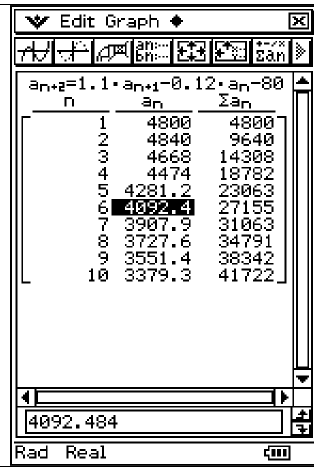


[A1]

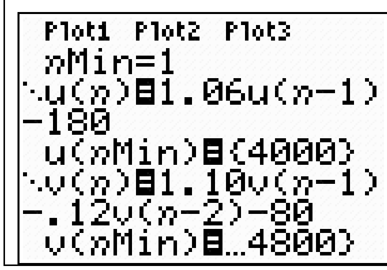
e. Enter both difference equations in the calculator to find when $M_n > S_n$

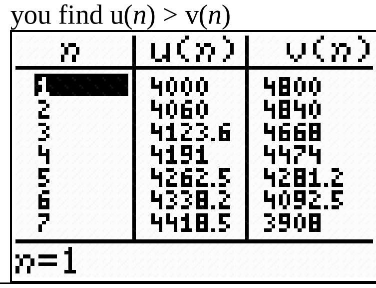
$M_{n+1} = 1.06 \times M_n - 180,\ M_1 = 4000$	$S_{n+2} = 1.10S_{n+1} - 0.12S_n - 80,$ $S_1 = 4800,\ S_2 = 4840$





Enter the difference equations in *y*=	Go to the table and scroll down until you find u(*n*) > v(*n*)





This occurs in the sixth month ie. When $n = 6$ or June 2009 [**A1**]

Total 15 Marks

END OF MODULE 1 SOLUTIONS

Module 2: Geometry and Trigonometry

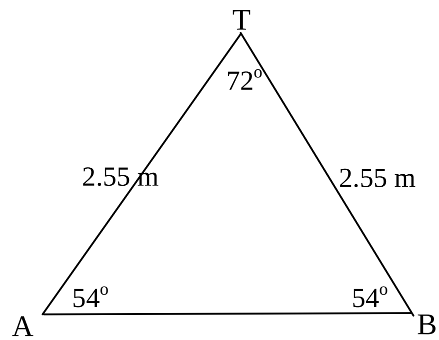
Question 1

a. The angle at the centre of a regular polygon is given by $\frac{360}{n}$ where n is the number of sides.

Therefore $\frac{360}{5} = 72^o$ **[M1]**

b. **i.** ATB is an isosceles triangle.
Therefore AT=BT=2.55

and $= \angle TAB = \angle TBA = \frac{180^o - 72^o}{2} = 54^o$



Using Cosine rule

$$AB^2 = 2.55^2 + 2.55^2 - 2 \times 2.55 \times 2.55 \cos 72^o$$
$$= 8.986$$
$$AB = \sqrt{8.986} = 3 \text{ m}$$

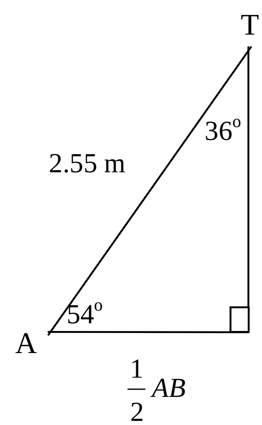
or

Using the Sine rule

$$\frac{AB}{\sin 72^o} = \frac{2.55}{\sin 54^o}$$
$$AB = \frac{2.55}{\sin 54^o} \times \sin 72^o = 3 \text{ m}$$ **[M1]**

or Using Trig Ratios

$$\sin 36^o = \frac{\frac{1}{2}AB}{2.55}$$
$$\frac{1}{2}AB = 2.55 \times \sin 36^o$$
$$AB = 2 \times 2.55 \sin 36^o = 3\text{m}$$



[A1]

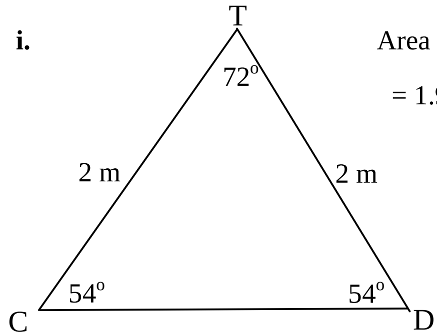
b. **ii.** set up ratio AD : AT

11 : 51
x : 2.55

$$51x = 11 \times 2.55$$
$$x = \frac{11 \times 2.55}{51}$$
$$x = 0.55$$

Therefore AD = 0.55 m **[A1]**

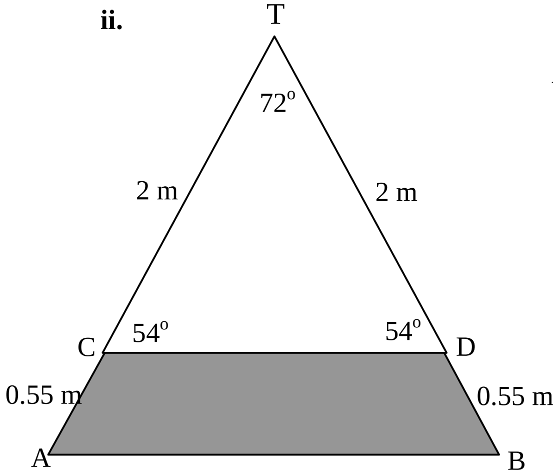
c. **i.**



$\text{Area } = \frac{1}{2} \times 2 \times 2 \sin 72^o$ [**M1**]

$= 1.902 \text{ m}^2$ **[A1]**

ii.



$\text{Area ABT } = \frac{1}{2} \times 2.55 \times 2.55 \sin 72^o$

$= 3.092 \text{ m}^2$

$\text{Area ABCD} = 3.092 - 1.902 = 1.19 \text{ m}^2$ **[A1]**

So the area of the bench top is

$= 5 \times 1.19 = 5.95 \text{ m}^2$ **[A1]**

Question 2

a.

$\angle RBQ = 58^0 + (360 - 318)^0$

$= 58^0 + 42^0$ [**M1**]

$= 100^0$

b. Angle at R is $\angle BRQ = 90 - 58 = 32^o$

Using the sine rule

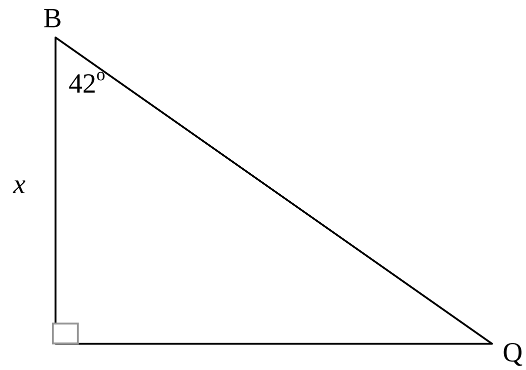
$$\frac{QB}{\sin 32^0} = \frac{12}{\sin 100^o}$$

$$QB = \frac{12}{\sin 100^o} \times \sin 32^0$$ [**M1**]

$= 6.5$ m

[A1]

c. The direct distance from B to the riverbank creates a right angle.

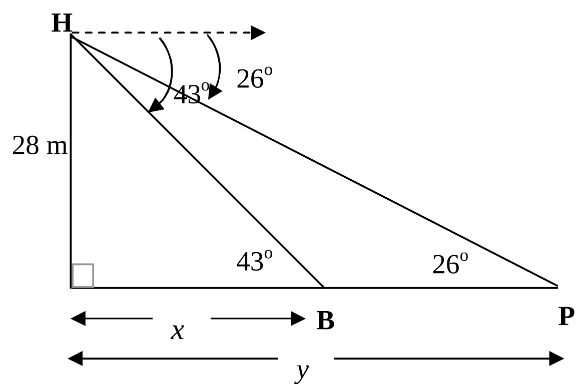


$$\cos 42^o = \frac{x}{BQ}$$

$$x = BQ \times \cos 42^o$$
$$= 6.5 \times \cos 42^o$$
$$= 4.83 \text{ m}$$

[A1]

d. **i.**



$\tan 43^0 = \frac{28}{x}$ $\tan 26^0 = \frac{28}{y}$

[**M1]**

$\Rightarrow x = \frac{28}{\tan 43^0}$ $\Rightarrow y = \frac{28}{\tan 26^0}$

$$BP = y - x$$
$$= \frac{28}{\tan 43^0} - \frac{28}{\tan 26^0}$$
$$= 57.41 - 30.03$$
$$= 27.38 \text{ m}$$

[A1]

d. **ii.** B is 4.83 m to the riverbank and 27.38 m to the opposite side of the riverbank
This means that the river is $27.38 - 4.83 = 22.55$ m .

[A1]

Total 15 Marks

END OF MODULE 2 SOLUTIONS

Module 3: Graphs and Relations

Question 1

a. $R = 55n$ **[A1]**

b. **i.** $C = 240 + 5n$ **[A1]**

ii. To break even

$$R = C$$
$$55n = 240 + 5n$$
$$50n = 240$$
$$n = \frac{240}{50}$$
$$n = 4.8$$

5 teams to break even **[A1]**

c.

$$\text{Profit} = \text{Revenue} - \text{Cost}$$
$$= 55n - (240 + 5n)$$
$$= 50n - 240$$

If the Profit = $910 then

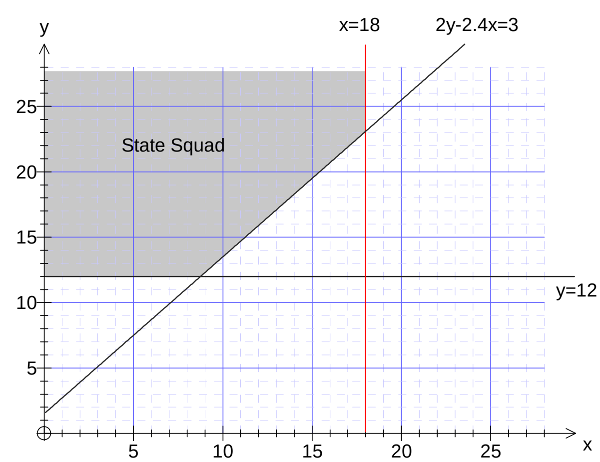
$$50n - 240 = 910$$
$$50n = 1150$$
$$n = \frac{1150}{50} = 23$$

23 teams **[A1]**

Question 2

a. The soccer competitors need to be able to run 100m in 18 seconds or less. **[A1]**

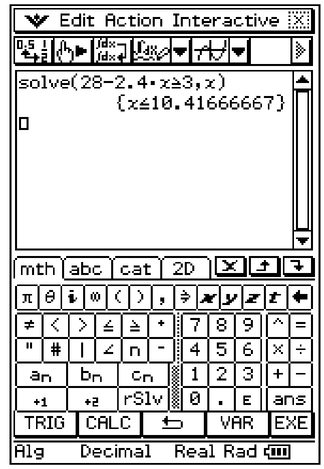
b.



Each line correctly drawn and labelled (1 mark each) **[A3]**
Shaded region correct **[A1]**

c. The competitor must satisfy the inequality $2y - 2.4x \geq 3$ where y=14

$$2y - 2.4x \geq 3$$
$$2 \times 14 - 2.4x \geq 3$$
$$28 - 2.4x \geq 3$$
$$-2.4x \geq -25$$
$$x \leq \frac{-25}{-2.4}$$
$$x \leq 10.42 \quad \text{seconds}$$



The minimum time is 10.42 seconds **[A1]**

d. To determine whether (6, 23) satisfies all inequalities substitute x=6 and y=23.

Inequality 1; $x \leq 18$ is true since $x = 6$

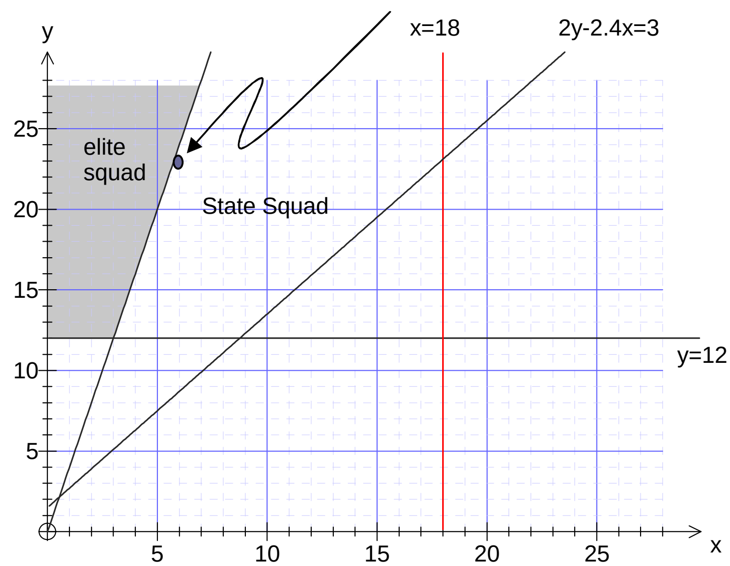
Inequality 2: $y \geq 12$ is true since $y = 23$

Inequality 3: $2y - 2.4x \geq 3$ is true since $2 \times 23 - 2.4 \times 6 = 31.6 \geq 3$

Inequality 4: $y - 4x \geq 0$ is not true since $23 - 4 \times 6 = -1 < 0$

Therefore Zoe is in the state squad but is not in the elite squad **[A1]**

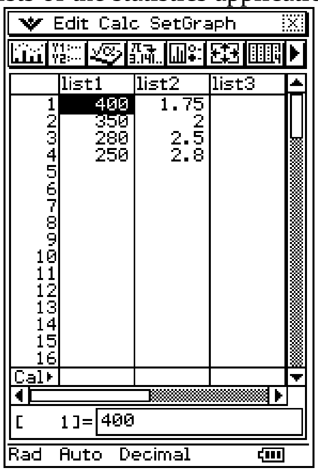
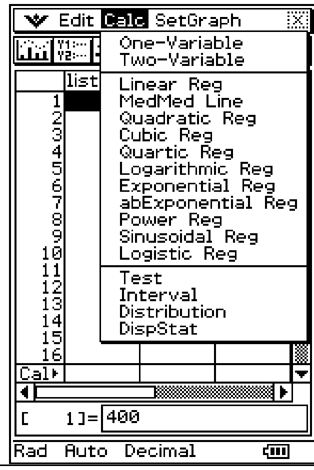
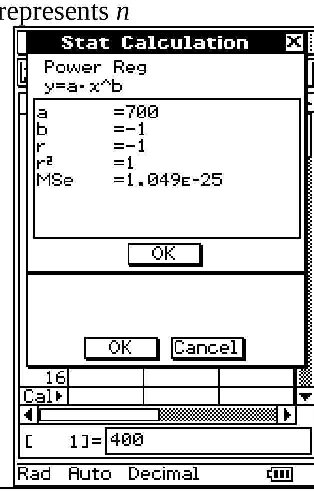
Zoe's score

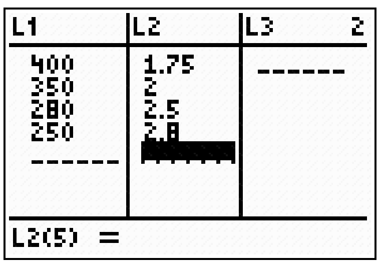


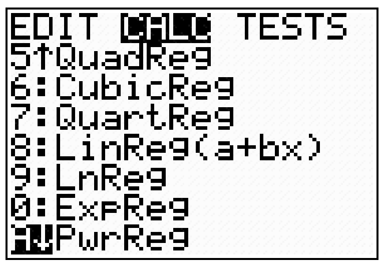
Question 3

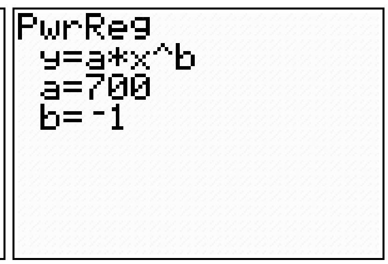
a.

Method 1: Using the Calculator

Step 1: Enter the values in the Lists of the statistics application	Step 2: Go to Calc and select the Power Reg option	Step 3: The value of a represents k and the value of b represents n
		







$k = 700$ **[A1]**

$n = -1$ **[A1]**

Method 2: Using Algebra

Step 1**:** Use two points to set up equations

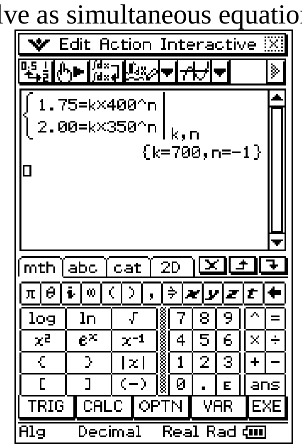
Substituting (400, 1.75) gives

$1.75 = k \times 400^n$ *equation 1*

substituting (350, 2.00) gives

$2.00 = k \times 350^n$ *equation 2*

At this stage you can enter these into the calculator and solve as simultaneous equations



or continue algebraically to step 2

Step 2**:** Eliminate k

equation 2 $\div$ equation 1

$$\frac{2.00}{1.75} = \frac{k \times 350^n}{k \times 400^n}$$

$$\frac{2.00}{1.75} = \frac{350^n}{400^n}$$

$$\frac{2.00}{1.75} = \left(\frac{350}{400}\right)^n$$

$$n = \frac{\log\left(\frac{2}{1.75}\right)}{\log\left(\frac{350}{400}\right)} \quad \therefore n = -1$$

Step 3**:** sub into equation 2 gives

$$2.00 = k \times 350^{-1}$$

$$k = 2.00 \times 350 = 700$$

b. Using the rule $F = 700S^{-1}$ or $F = \frac{700}{S}$

substituting S = 195 gives $F = \frac{700}{195} = \$3.60$ **[A1]**

c. Using the rule $F = 700S^{-1}$ or $F = \frac{700}{S}$

substituting F = 4 gives

$$4 = \frac{700}{S}$$

$$S = \frac{700}{4}$$

$$= 175 \text{ spectators}$$

[A1]

Total 15 Marks

END OF MODULE 3 SOLUTIONS

Module 4: Business related mathematics

Question 1

a. $110\% \rightarrow 3800 + x$

$100\% \rightarrow x$

$$\frac{110\%}{100\%} = \frac{3800 + x}{x}$$

$$x = (3800 + x) \times \left(\frac{100}{110}\right)$$ **[A1]**

$$11x = 38\,000 + 10x$$

$$x = \$38\,000$$

b. Cost price = $33 000
Deposit = $3000
Balance = $30 000
Hire purchase loan (simple interest)

$$I = \frac{\Pr T}{100}$$

$$= \frac{30\,000 \times 3 \times 3}{100}$$ **[M1]**

$$= \$2700$$

$$\text{Monthly} = \frac{(2700 + 30\,000)}{36}$$ **[A1]**

$$= \$908.33$$

c.

$$\text{Effective rate} = \frac{2n}{n + 1} \text{ x flat rate}$$

$$= \frac{72}{37} \times 3\%$$ **[A1]**

= 5.84% p.a.

d.

$$\text{Stamp Duty} = \frac{33\,000}{200} \times 5$$

$$= 165 \times 5$$ **[A1]**

$$= \$825$$

Question 2

a. Because effectively the 3% simple interest rate is higher (effective rate of 5.84%) than the compound rate of 5%. **[A1]**

b.

$$A = PR^n$$

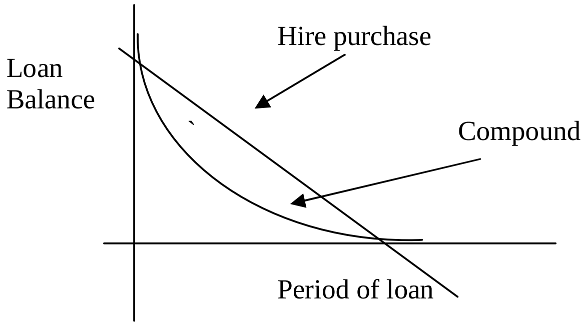
$$= 33\,000 \times 1.004116666^{36}$$

$$= \$38\,328.58$$ **[A1]**

$$\text{Equal mthly instal.} = \frac{38\,328}{36}$$

$$= \$1064.68$$

c. Hire purchase straight line. **[A1]**
Compound curved. **[A1]**
NB. Deduct 1 mark if they do not start at the same initial loan balance (same *y*-intercept)



Question 3

a.

$$BV = P\left(1-\frac{r}{100}\right)^n$$

$$8000 = 33\,000\times\left(1-\frac{r}{100}\right)^5$$ **[M1]**

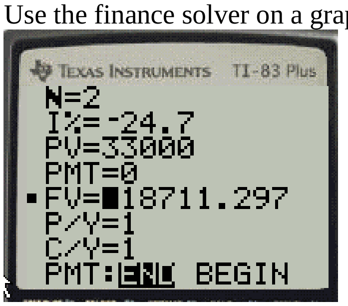
Or show the table of values similar to those on a finance solver.

The reducing balance rate of depreciation is 24.7% **[A1]**

b. After 2 yrs

$$\text{bookvalue} = 33\,000\times\left(1-\frac{24.7}{100}\right)^2$$

Use the finance solver on a graphics calculator to evaluate.



V = \$18 711 **[A1]**

c. Dist. Travelled= 80 000 kms

Depreciation = \$33 000 - \$8000 **[A1]**

= \$25 000 for 80 000 kms

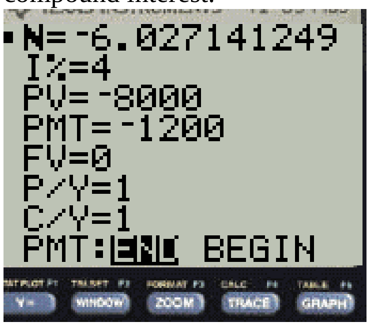
= x for 100kms

$$x = 25\,000 \times \frac{100}{80\,000}$$

$$= \$31.25 / 100\text{kms}$$ **[A1]**

Question 4

Use finance solver as this is an example of an annuity where there is a regular payment with a compound interest.



Six years of scholarships before the funds run out. **[A1]**

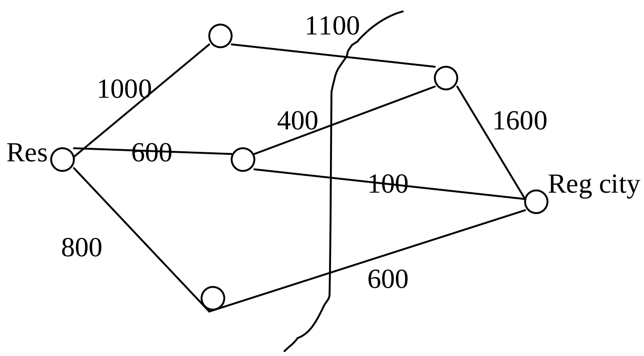
Total 15 Marks

END OF MODULE 4 SOLUTIONS

Module 5: Networks and decision mathematics

Question 1

a.



[A1] [A1]

b. Using count back method 2100 max flow **[A1]**

c. 2100 megalitres **[A1]**

Question 2

a. 9 weeks **[A1]**

b. C, F, G, H, J **[A1]**

c. A - float time 1.5 weeks **[A1]**
D - float time 1.5 weeks **[A1]**

Question 3

a. Euler Path – odd vertices. $\therefore$ D & E **[A1]**

b. The main criteria for an Euler path is the need to use each edge once only. Starting position is not specific so either D or E is appropriate. **[A1]**

c. D – C – A – B – D – E – F – G – B – E or
D – C – A – B – D – E – B – G – F – E **[A1]**

d. D $\rightarrow$ E All vertices would then be even. **[A1]**

e. Hamiltonian Path is the most appropriate path because he needs to pass through each worksite exactly once. **[A1]**
He has no need to start and finish at the same worksite; therefore it needn't be a circuit. **[A1]**
He doesn't need to travel each road, therefore not Euler. **[A1]**

Total 15 Marks

END OF MODULE 5 SOLUTIONS

Module 6: Matrices

Question 1

a.

$$\frac{1}{2}\begin{bmatrix} 3200 & 2600 & 1700 & 1400 \\ 2800 & 2400 & 1600 & 1300 \\ 2400 & 2100 & 1400 & 1200 \end{bmatrix} = \begin{bmatrix} 1600 & 1300 & 850 & 700 \\ 1400 & 1200 & 800 & 650 \\ 1200 & 1050 & 700 & 600 \end{bmatrix}$$ **[A1]**

b.

$$\begin{bmatrix} 0.9 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$ ` **[A1]**

c.

$$\begin{bmatrix} 3200 & 2600 & 1700 & 1400 \\ 2800 & 2400 & 1600 & 1300 \\ 2400 & 2100 & 1400 & 1200 \end{bmatrix}\begin{bmatrix} 0.9 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} = \begin{bmatrix} 2880 & 2080 & 1020 & 700 \\ 2520 & 1920 & 960 & 650 \\ 2160 & 1680 & 840 & 600 \end{bmatrix}$$

Use a graphics calculator to evaluate. **[A1]**

d. The element's position is row 2, column 3 or $e_{2,3}$ for a discounted price of $960. **[A1]**

Question 2

a.

$$\begin{aligned} 2x + 5y + 10z &= 44000 \\ 4y + 5z &= 22000 \\ 3x + 4y + z &= 27500 \end{aligned}$$

All three correct **[A2]**
Only 2 correct **(1 mark)**

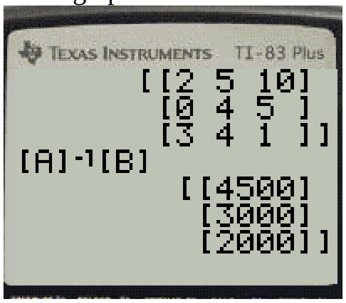
b.

$$\begin{bmatrix} 2 & 5 & 10 \\ 0 & 4 & 5 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 44000 \\ 22000 \\ 27500 \end{bmatrix}$$

$$A \quad X = B$$ **[M1]**

$$X = A^{-1}B$$

Use a graphics calculator to evaluate.



$$\begin{bmatrix} 4500 \\ 3000 \\ 2000 \end{bmatrix}$$ **[A1]**

Stateroom	$4500
Balcony	$3000
Inner Cabin	$2000

Student needs to interpret each of the elements of the solution appropriately. **[A1]**

Question 3

a.

$$\begin{bmatrix} 9/10 & 1/5 & 1/10 \\ 8/100 & 7/10 & 3/10 \\ 2/100 & 1/10 & 6/10 \end{bmatrix} or \begin{bmatrix} 0.9 & 0.2 & 0.1 \\ 0.08 & 0.7 & 0.3 \\ 0.02 & 0.1 & 0.6 \end{bmatrix}$$ **[A1]**

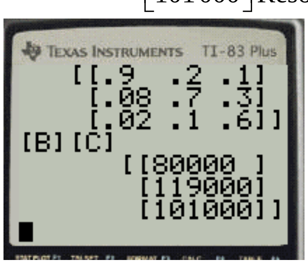
b.

$$H_{2008} = \begin{bmatrix} 50\,000 \\ 100\,000 \\ 150\,000 \end{bmatrix}$$ **[A1]**

c.

$$T \times H_{2008} = \begin{bmatrix} 0.9 & 0.2 & 0.1 \\ 0.08 & 0.7 & 0.3 \\ 0.02 & 0.1 & 0.6 \end{bmatrix} \begin{bmatrix} 50\,000 \\ 100\,000 \\ 150\,000 \end{bmatrix}$$

$$= \begin{bmatrix} 80\,000 \\ 119\,000 \\ 101\,000 \end{bmatrix} \begin{matrix} \text{Cruise} \\ \text{Tour} \\ \text{Resort} \end{matrix}$$ **[A1]**

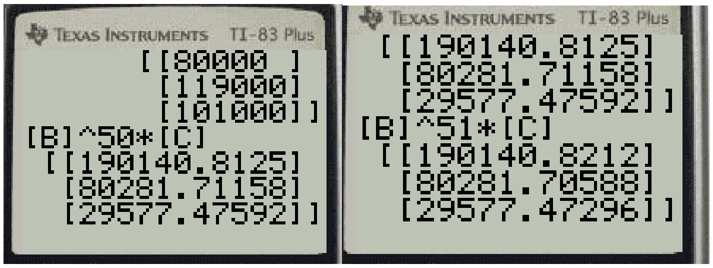


80 000 for cruise, 119 000 for tours and 101 000 for resort **[A1]**

d.

$$T \times H_{2008} = \begin{bmatrix} 0.9 & 0.2 & 0.1 \\ 0.08 & 0.7 & 0.3 \\ 0.02 & 0.1 & 0.6 \end{bmatrix}^{50} \begin{bmatrix} 50\,000 \\ 100\,000 \\ 150\,000 \end{bmatrix}$$

$$= \begin{bmatrix} 190141 \\ 80\,282 \\ 29\,577 \end{bmatrix} \begin{matrix} \text{Cruise} \\ \text{Tour} \\ \text{Resort} \end{matrix} \quad (\textit{rounded})$$



$$T \times H_{2008} = \begin{bmatrix} 0.9 & 0.2 & 0.1 \\ 0.08 & 0.7 & 0.3 \\ 0.02 & 0.1 & 0.6 \end{bmatrix}^{51} \begin{bmatrix} 50\,000 \\ 100\,000 \\ 150\,000 \end{bmatrix}$$

$$= \begin{bmatrix} 190\,141 \\ 80\,282 \\ 29\,577 \end{bmatrix} \begin{matrix} \text{Cruise} \\ \text{Tour} \\ \text{Resort} \end{matrix}$$

Setting n to a large number say 50. **[A1]**
Showing the next transition eg n=51 **[A1]**

Total 15 Marks

END OF EXAMINATION 2 SOLUTIONS