Mathematical Association of Victoria
Trial Examination 2009

STUDENT NAME _____

FURTHER MATHEMATICS

Written Examination 1

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A	13	13			13
B	54	27	6	3	27
					Total 40

• Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
• Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
• Question book of 34 pages with a detachable sheet of miscellaneous formulas at the back.
• Answer sheet for multiple-choice questions.
• Working space is provided throughout the book.

Instructions
• Detach the formula sheet from the back of this book during reading time.
• Check that your **name** is printed on your answer sheet for multiple-choice
• Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
• You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Working space

TURN OVER

SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple choice questions.
Choose the response that is **correct** for that question.
A correct answer scores 1, and incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more that one answer is completed.

Core – Data Analysis

Question 1

The height (tall, average or short) of fifty competitors and the distance of their long jump (in cm) can be presented as

A. a back to back stemplot

B. percentaged two way frequency table

C. scatteplot

D. histogram

E. parallel boxplots

Question 2

The ages of the fathers of 17 VCE students are shown in the stemplot below.

stem	leaf
4	2 3 5 5 6 7 7 9
5	0 2 3 5 8
6	1 4 4
7	9

Which statement is **incorrect**?

A. The distribution of ages is positively skewed.

B. The median age is 50.

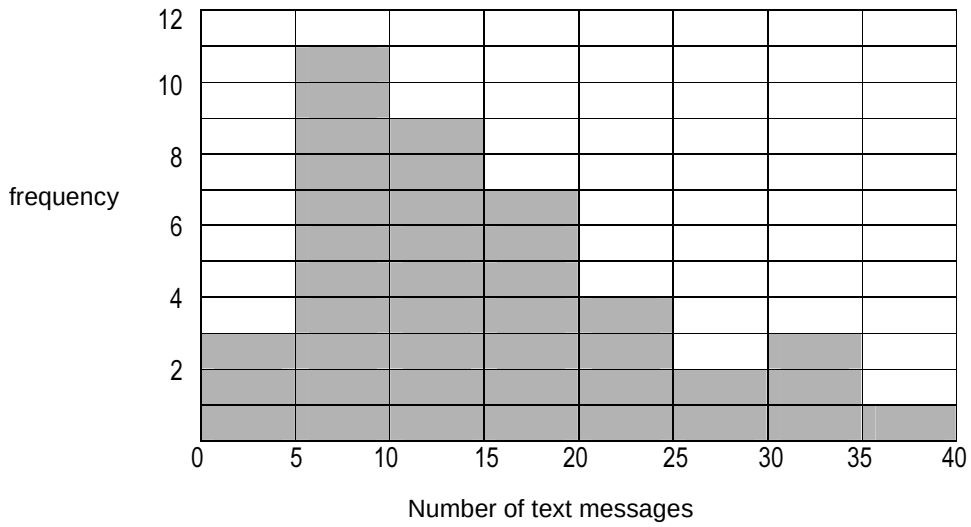
C. The interquartile range is 14 years.

D. There is one outlier at 79.

E. Approximately 75% of the fathers are at least 45.5 years old.

The following information is related to Questions 3 and 4

The distribution of text messages received by a group of students is displayed in the frequency histogram below:



Question 3

The percentage of students who received **at least** 20 text messages is

A. 10%

B. 25%

C. 30%

D. 50%

E. 75%

Question 4

The distribution of text messages is best described as

A. symmetrical with a median at 15

B. negatively skewed with the median between 10 and 15

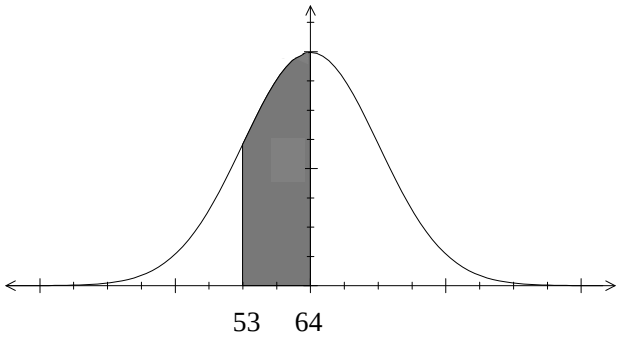
C. negatively skewed with the median between 15 and 20

D. positively skewed with the median between 15 and 20

E. positively skewed with the median between 10 and 15

The following information is related to Questions 5 and 6

The results of a statistics exam were graphed and found to produce the following normal distribution.



The shaded region shown represents the marks obtained for 34% of the student population.

Question 5

2.5% of students achieved a mark greater than

A. 64

B. 75

C. 83

D. 86

E. 97.5

Question 6

A student scored 80 for the statistics exam. The standardised value, z, for this result **is closest to**

A. -1.5

B. -0.5

C. 0.5

D. 1.0

E. 1.5

Question 7

Given that, for a set of bivariate data,

$$\bar{x} = 10,\ \bar{y} = 7,\ r = -0.6,\ s_x = 3,\ s_y = 2$$

The equation of the least squares regression line $y = a + bx$ is

A. $y = 16 - 0.9x$

B. $y = -2 - 0.9x$

C. $y = 11 - 0.4x$

D. $y = 3 - 0.4x$

E. $y = 5 + 0.2x$

Question 8

A least squares regression model for a set of data has the equation: $Weight = -102 + 0.968 \times height$.

A woman who is 160cm tall weighs 58kg. The residual for this observation would be closest to

A. 115

B. 56

C. 53

D. 5

E. 2

Question 9

The relationship between two variables x and y as shown in the scatter plot is non linear.

Which one of the following transformations is not likely to linearise the relationship?

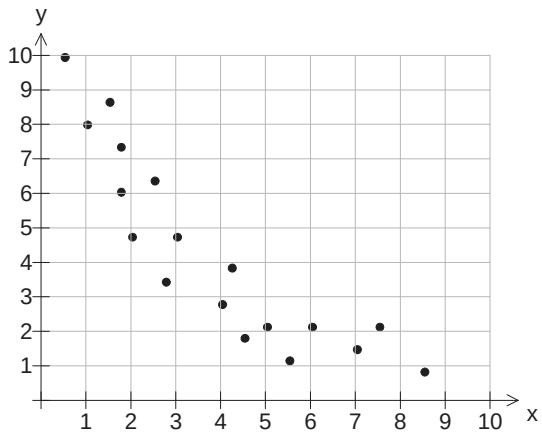
A. a $\frac{1}{x}$ transformation

B. a y^2 transformation

C. a $\log y$ transformation

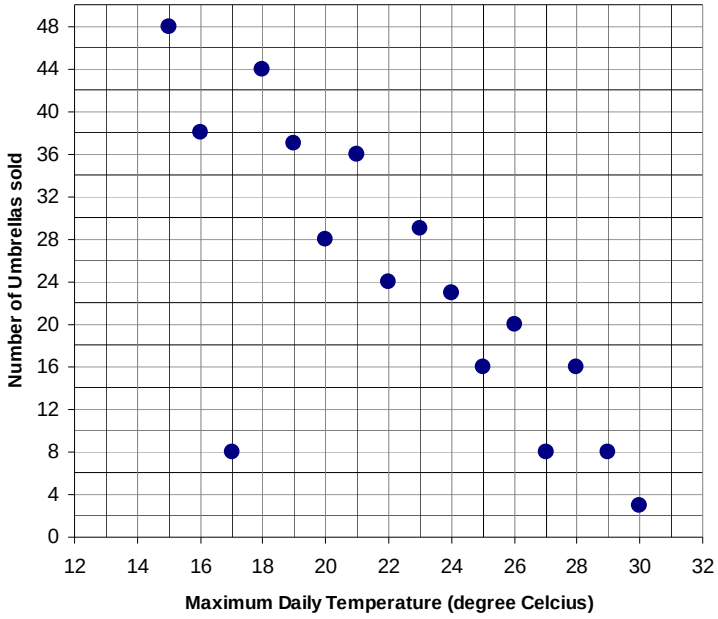
D. a $\frac{1}{y}$ transformation

E. a $\log x$ transformation



The following information is related to Questions 10 and 11

The association between the number of umbrellas sold is known to be negatively associated with the maximum daily temperature. To investigate this association, a scatterplot is constructed as shown below.



While there is a strong negative relationship between the maximum temperature and umbrella sales, there is a clear outlier. Because of this, it is decided to model the relationship by fitting a 3-median line to the data displayed in the scatterplot.

Question 10

The slope of this 3-median line is closest to

A. – 3.2

B. – 2.7

C. – 1.6

D. – 0.4

E. – 0.3

Question 11

When a least squares regression line is used to model this data, Pearson's product moment correlation coefficient is found to be -0.7650. If the outlier is removed, and a least squares regression line is refitted to the remaining data, the value of the correlation coefficient will

A. decrease.

B. increase.

C. remain the same.

D. be halved.

E. be doubled.

The following information relates to Questions 12 and 13

Quarter	Summer	Autumn	Winter	Spring
Seasonal Index		0.8	0.35	1.2

The table shows the seasonal indices for the quarterly sales of air conditioners.

Question 12
The seasonal index for summer is missing from the table.
The value of the missing seasonal index for Summer is

A. 0.65

B. 0.95

C. 1.45

D. 1.55

E. 1.65

Question 13
A trend line that can forecast the deseasonalised number of air conditioners sold is given by

$$\textit{Deseasonalised number of air conditioners} = 2300 + 2.215 \times \textit{Quarter}$$

where Quarter 1 is Summer 2006, Quarter 2 is Autumn 2006, and so on.

The **actual** number of air conditioners sold for Spring 2009 is closest to

A. 2803

B. 2792

C. 2335

D. 2327

E. 1946

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.
Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Module 1: Number Patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1
The first and third terms of an arithmetic sequence are 38 and 10 respectively. The sum of the first six terms of the sequence is

A. 18
B. 24
C. 50
D. 68
E. 126

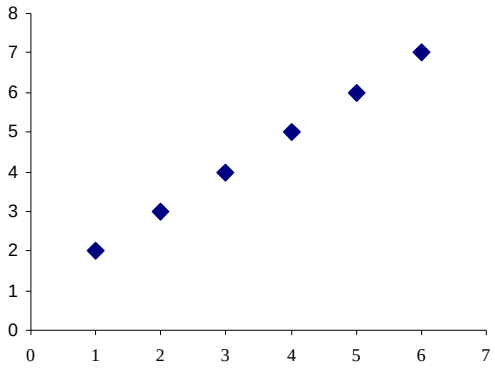
Question 2
The first term of a geometric sequence is 5 and the fifth term has the value of 80.
The third term of this sequence is

A. 4
B. 10
C. 16
D. 20
E. 40

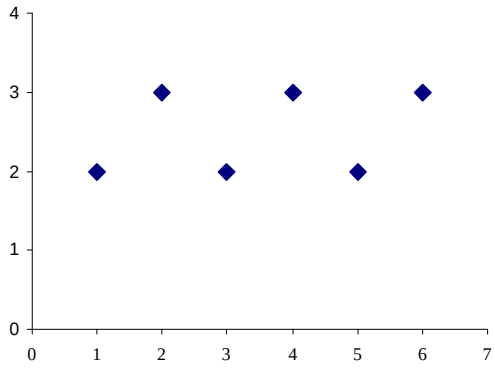
Question 3

Which one of the following graphs would be generated by the difference equation $t_{n+1} = 5 - t_n$; $t_1 = 2$?

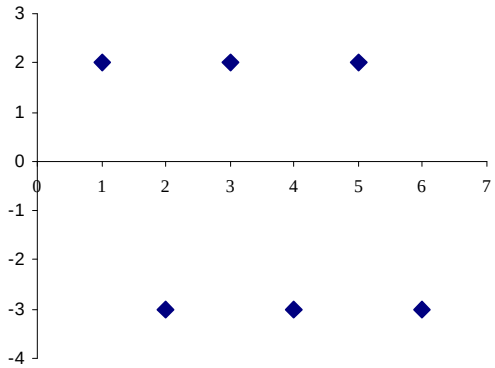
A.



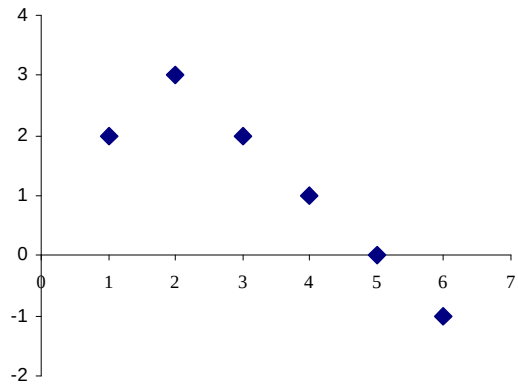
B.



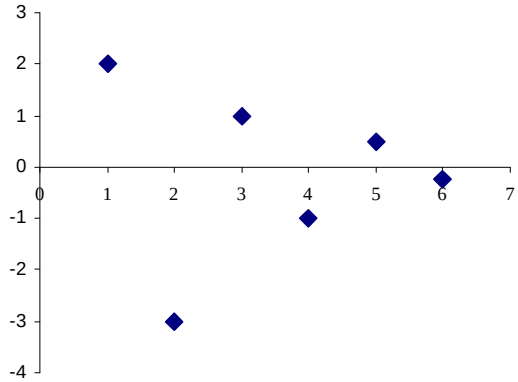
C.



D.



E.



Question 4

Chris the mechanic notices an oil leak soon after filling the motor with 2000 ml of oil. The car loses 50 ml of oil in the first hour, 45 ml of oil in the second hour, 40.5 ml in the third hour, and so on. If this continues indefinitely, the amount of oil that will **remain** in the car is

A. 0 ml.

B. 300 ml.

C. 500 ml.

D. 1500 ml.

E. 1800 ml.

The following information relates to Questions 5 and 6

A retail company is monitoring the number of people shopping in a supermarket for a period of *n* hours. During this time it has found that the number of people shopping increases by 8% each hour. There were 500 people shopping when first counted.

Question 5

Let P_n be the number of people shopping in the supermarket *n* hours after first counting.

A difference equation describing this situation is

A. $P_n = 0.08 \times P_{n-1}$ where $P_0 = 500$

B. $P_n = P_{n-1} + 0.08$ where $P_0 = 500$

C. $P_n = 1.08 \times P_{n-1}$ where $P_0 = 500$

D. $P_n = 1.8 \times P_{n-1}$ where $P_0 = 500$

E. $P_n = 0.92 \times P_{n-1}$ where $P_0 = 500$

Question 6

The number of people shopping at the supermarket will double after

A. 2 hours.

B. 3 hours.

C. 9 hours.

D. 10 hours.

E. 11 hours.

Question 7

A sequence is defined by the difference equation $t_{n+1} = -\frac{1}{2}t_n + 3$. If the third term of the sequence is −8, the first term is

A. −50

B. −47

C. −38

D. −29

E. −0.5

Question 8

The number of items of junk mail, J_n, delivered in the n^{th} week is defined by the second order difference equation $J_{n+2} = 2J_n + J_{n+1}$, where $J_1 = 2$ and $J_2 = 5$.

The **total** number of items of junk mail delivered after 5 weeks is

A. 72

B. 75

C. 85

D. 118

E. 147

Question 9

The first order difference equation is defined by $t_{n+1} = 0.8t_n + k$ where $t_1 = 10$

The sequence will produce constant terms when

A. $k < 2$

B. $-2 < k < 2$

C. $k > 2$

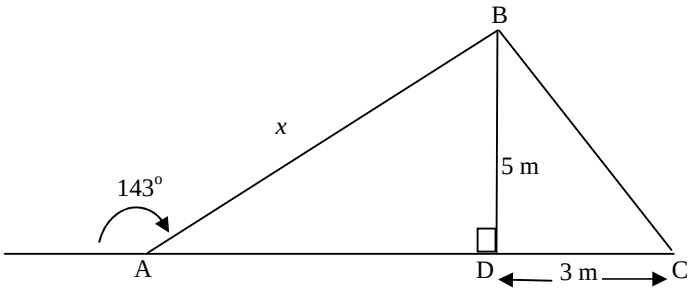
D. $k = 2$

E. $k = -2$

Module 2: Geometry and Trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

The following information relates to Questions 1 and 2



Question 1

The length of x is given by

A. $5\sin 37^0$

B. $5\cos 37^0$

C. $\dfrac{5}{\sin 53^0}$

D. $\dfrac{5}{\cos 53^0}$

E. $\dfrac{\tan 37^0}{5}$

Question 2

The area of the triangle ABC is

A. 7.5 m^2

B. 16.6 m^2

C. 24.1 m^2

D. 33.2 m^2

E. 48.2 m^2

Question 3

The perimeter of the triangle shown is

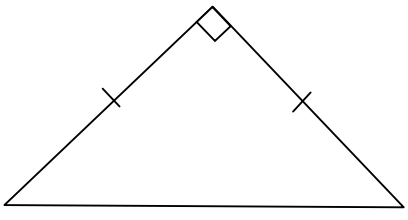
A. $15(2+\sqrt{2})$

B. $45\sqrt{2}$

C. $45+\sqrt{2}$

D. $30\sqrt{2}$

E. 30



Question 4

The angle of depression of a canoe sighted from the top of a bridge is 27^o. The bridge is 28.6 metres above the water line.
The horizontal distance, in metres, of the canoe to a point directly under the bridge is

A. 14.6

B. 31.8

C. 32.1

D. 56.1

E. 63.0

Question 5

The smallest angle in the triangle shown is closest to

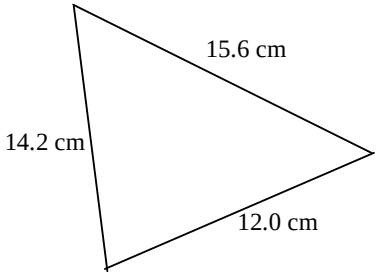
A. 28^o

B. 36^o

C. 47^o

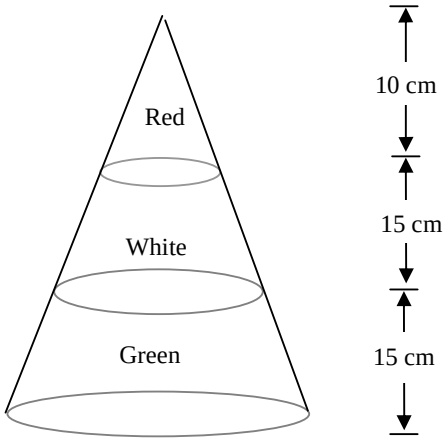
D. 60^o

E. 72^o



The following information relates to Questions 6 and 7

A 40 cm high Christmas cake, in the shape of a cone, is decorated with red, white and green icing as shown in the diagram below



Question 6

If the volume of the cake is found to be 8000 cm^3, then the diameter of the base is closest to

A. 43 cm

B. 28 cm

C. 26 cm

D. 16 cm

E. 14 cm

Question 7

The ratio of area covered in green icing to the total area covered in icing is

A. 3:8

B. 9:64

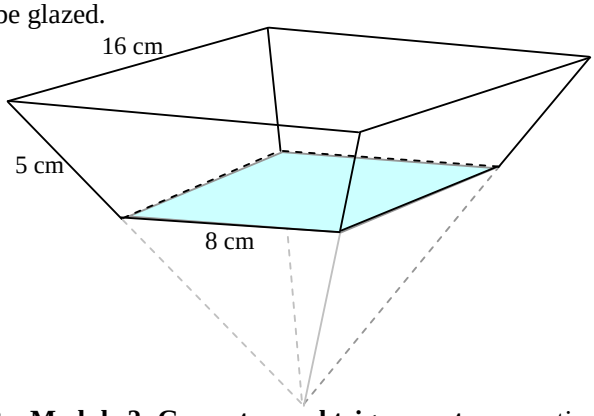
C. 9:25

D. 25:64

E. 39:64

Question 8

A ceramic bowl, in the shape of a truncated square based pyramid, is shown in the diagram below.
The *entire* surface of the bowl (inside and out) is to be glazed.
The area, in square centimetres, to be glazed is



A. 208

B. 304

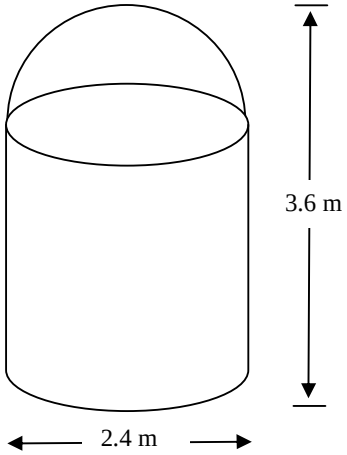
C. 416

D. 464

E. 608

Question 9

A rainwater tank is in the shape of a cylinder with a hemispherical top. The diameter of the base is 2.4 metres and the height of the tank is 3.6 metres.



When completely full, the volume of rainwater the tank can hold, in cubic metres, is **closest to**

A. 14.5

B. 13.9

C. 13.1

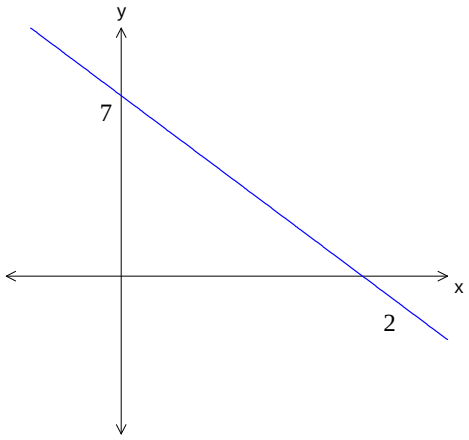
D. 16.3

E. 20.7

Module 3: Graphs and Relations

Before answering these questions you must **shade** the Graphs and Relations box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1
The graph shown has equation



A. $y = 7x + 2$

B. $y = 2x + 7$

C. $2y + 7x = 7$

D. $7x + 2y = 14$

E. $2x + 7y = 14$

Question 2
For the straight line with equation $3x - 5y = 30$, which statement is true?

A. As x increases, y decreases.

B. The y-intercept is 6.

C. The x-intercept is -10.

D. The line has a gradient of $\frac{5}{3}$.

E. The line passes through the point $(5, -3)$.

Question 3
At the bakery, Peter bought a dozen bread rolls and two loaves of bread for $10.70. Harry bought five bread rolls and three loaves of bread and received $8.50 change from $20.
From this information, ten bread rolls cost

A. $3.25

B. $3.50

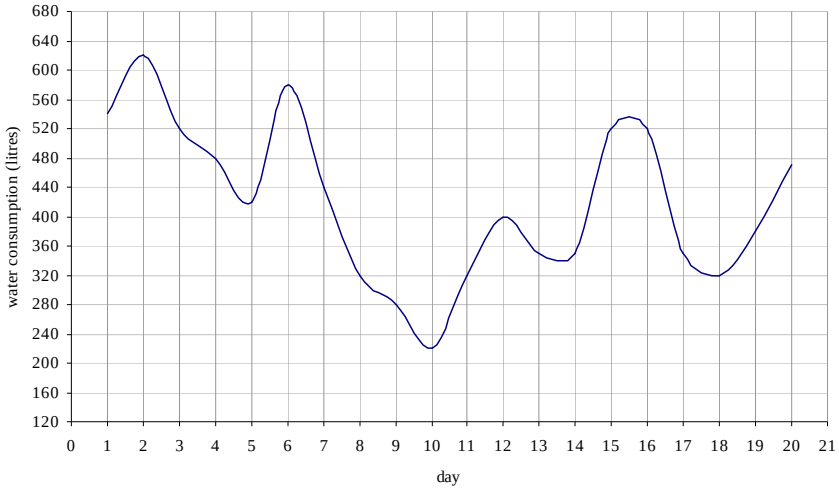
C. $4.50

D. $5.05

E. $5.80

Question 4

The graph below shows the Smith household water consumption (in litres) over a period of 20 days.



The maximum recommended water consumption is 155 litres per person per day. Three people live in the Smith household.

The number of days the Smith household exceeds the maximum limit is

A. 20 days

B. between 15 and 19 days

C. between 10 and 14 days

D. between 5 and 9 days

E. between 0 and 4 days

Question 5

Two types of donuts are sold at the Delicious Donut store. It takes 10 minutes to prepare a dozen jam donuts and 8 minutes to prepare a dozen iced donuts.

Two people are employed to prepare the donuts and 4 hours of preparation time is allocated to each person.

The inequality that expresses the time constraint for preparing x dozen jam donuts and y dozen iced donuts is given by

A. $10x + 8y \leq 240$

B. $10y + 8x \leq 240$

C. $10x + 8y \leq 480$

D. $10y + 8x \leq 480$

E. $0.10x + 0.08y \leq 4$

Question 6

The Delicious Donut store has running costs of $300 each day. The cost of making a dozen donuts is $1.50. If the donuts are sold at $6.00 a dozen, how many donuts must be sold in a day to break even?

A. 40

B. 67

C. 50

D. 200

E. 800

Question 7

The local library charges a late fee for the late return of each book or DVD.

$$\text{Late book fine} = \begin{cases} \$2.00 & 0 < days \leq 3 \\ \$7.00. & 3 < days \leq 7 \\ \$10.00, & days > 7 \end{cases}$$

$$\text{Late DVD fine} = \$3.00 \times days$$

Mark returned a book that was one week late and Melanie returned one book and two DVDs that were three days late.

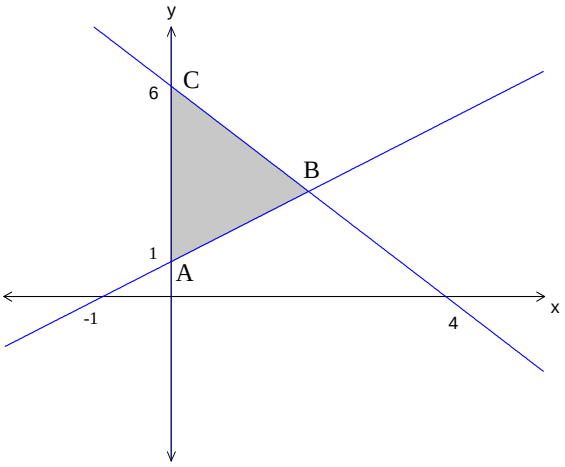
The total late fee charged is

A. \$15
B. \$18
C. \$20
D. \$27
E. \$30

Question 8

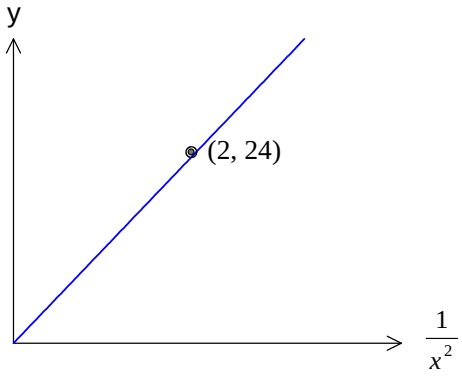
The maximum value occurs at the point B for which one of the following objective functions?

A. $M = x + 3y$
B. $M = x - 3y$
C. $M = x + y$
D. $M = x + 2y$
E. $M = 2x + y$



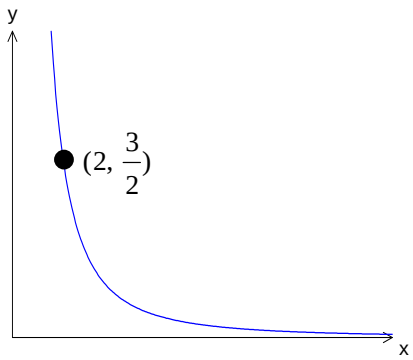
Question 9

The graph of y versus $\frac{1}{x^2}$ is shown

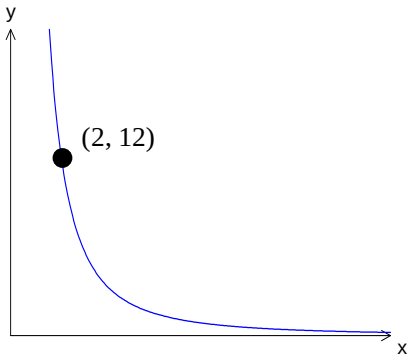


The graph of y versus x is:-

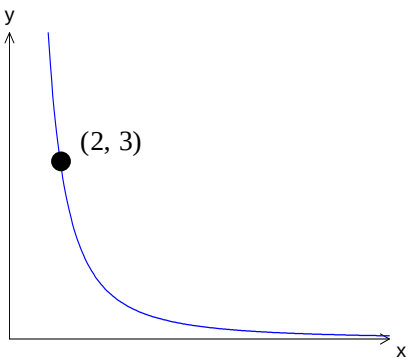
A.



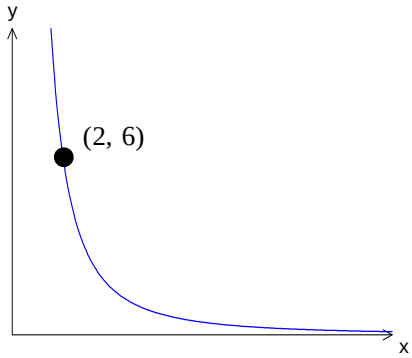
B.



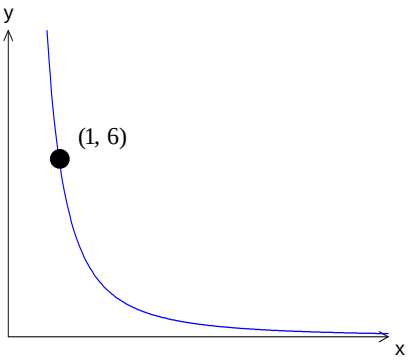
C.



D.



E.



SECTION B – continued

Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business related mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Consider the following transactions on a bank account.

Date	Details	Credit $	Debit $	Balance $
1 April	Forward			360.22
4 April	Wages	1450.60		
11 April	ATM		2000.00	
17 April	ATO transfer	900.00		
30 April	Bank fee		26.50	
30 April				

The final balance on 30th of April is

A. $36.10
B. $36.12
C. $684.30
D. $684.32
E. $710.82

Question 2

The total bill for a car service was $550. The GST that was included in this bill was

A. $55.00
B. $52.50
C. $50.00
D. $60.00
E. $60.50

Question 3

The price of a 2-litre container of milk has increased from $0.90 to $2.65 over the past 8 years. The average annual inflation rate is closest to

A. 14.45%
B. 16.68%
C. 1.94%
D. 24.31%
E. 36.81%

Question 4

A charity fund wants to set up a $2000 annual scholarship. The charity is able to get a financial institution to offer a perpetuity with a long-term interest rate of 5% per annum. The amount that needs to be raised and invested by the charity to provide for the scholarship is

A. $4000
B. $20 000
C. $40 000
D. $50 000
E. $80 000

Question 5

The following are two options for investments of $5000 **or more**.

Option 1: Compound interest at 7.6% p.a. compounded quarterly
Option 2: Simple interest at a flat rate of interest of 9% p.a.

If a **$10 000** investment for **8 years** is made, the best option and amount extra in interest earned is given by

A. Compound Interest loan is best by $11 063.07
B. Compound Interest loan is best by $1063.07
C. Compound Interest loan is best by $1132.36
D. Simple Interest loan is best by $1063.07
E. Simple Interest loan is best by $7200.00

Question 6

For tax purposes a landlord chooses to depreciate household items at 12% per year. If the items initially cost $7000, how long would it take for the items to have a scrap value below $1000?

A. 7 years
B. 8 years
C. 10 years
D. 15 years
E. 16 years

Question 7

Jeremy wishes to repay a loan of $15 000 over 6 years at 6% p.a. compounded monthly. His monthly repayment is calculated using the equation $Q = \frac{PR^n(R-1)}{R^n-1}$ and is given by

A. $Q = \frac{15000 \times 1.06^6(1.06-1)}{1.06^6-1}$

B. $Q = \frac{15000 \times 1.005^{72}(1.005-1)}{1.005^{72}-1}$

C. $Q = \frac{15000 \times 1.005^{60}(1.005-1)}{1.005^{60}-1}$

D. $Q = \frac{2500 \times 1.06^6(1.06-1)}{1.06^6-1}$

E. $Q = \frac{2500 \times 1.05^{12}(1.05-1)}{1.05^{12}-1}$

Question 8

A school elected to purchase a photocopier worth $25 000 and chose to depreciate the machine by 12 cents per ream of paper used (1 ream = 500 sheets). If in the first year 20 million sheets were used, the book value of the photocopier after the first year was

A. $4800
B. $19 000
C. $20 000
D. $20 200
E. $22 600

Question 9

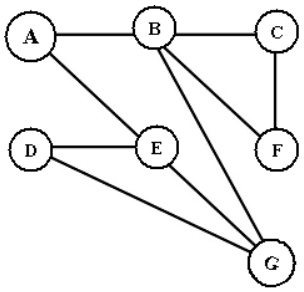
The local car yard advertises a second-hand car for $9900. The terms are $1500 deposit plus $425 per month for 3 years. The **effective rate** of interest per annum is

A. 27.38%
B. 35.38%
C. 41.87%
D. 53.28%
E. 54.76%

Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

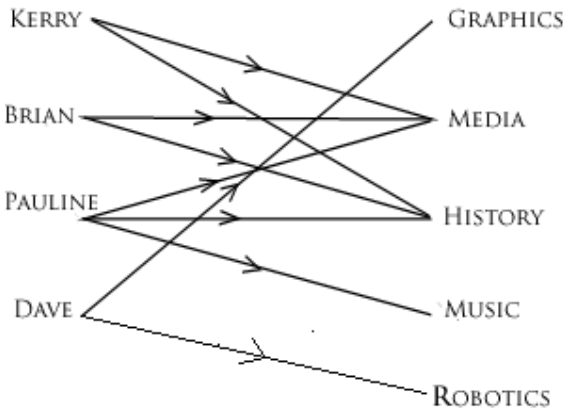


The number of faces in the planar graph shown above is

A. 3
B. 4
C. 5
D. 6
E. 7

Question 2

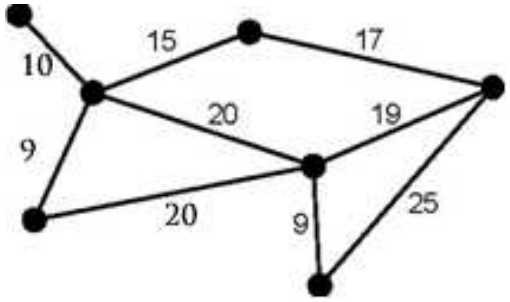
The bipartite graph below shows the preferences of four students for various subjects offered to them at a small regional college.



Which one of the following statements is supported by the graph?

A. Kerry has more preferences than other students.
B. Media and History are the most popular subjects.
C. Dave has fewer preferences than anyone.
D. Music and Robotics only are the least popular subjects.
E. Kerry and Brian have different interests in subjects.

Question 3



For the graph above, the total weight of the minimum weight spanning tree is

A. 6
B. 78
C. 79
D. 80
E. 82

Question 4

In a league of four soccer teams, the following were the results of 6 games played over the past two weeks.

Team A 2 vs. Team B 0
Team C 0 vs. Team D 2
Team B 2 vs. Team D 1
Team C 2 vs. Team A 2 (Team A won a penalty shootout.)
Team C 1 vs. Team B 0
Team D 0 vs. Team A 1

A suitable dominance matrix is

A.

	A	B	C	D
A	0	2	2	1
B	0	0	0	2
C	2	0	0	0
D	0	1	2	0

B.

	A	B	C	D
A	1	2	2	1
B	0	0	0	2
C	2	0	0	0
D	0	1	2	0

C.

	A	B	C	D
A	1	1	1	1
B	0	1	0	1
C	0	1	1	0
D	0	0	1	1

D.

	A	B	C	D
A	0	1	1	1
B	0	0	0	1
C	0	1	0	0
D	0	0	1	0

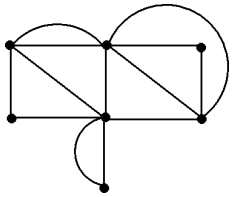
E.

	A	B	C	D
A	3	0	0	0
B	0	1	0	0
C	0	0	1	0
D	0	0	0	1

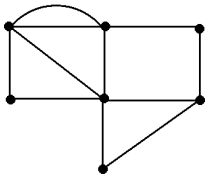
Question 5

Which one of the following graphs contains an Euler circuit?

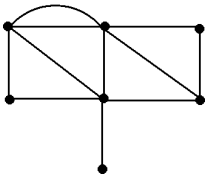
A.



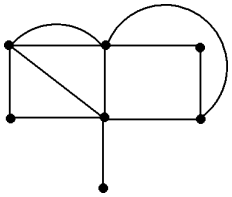
B.



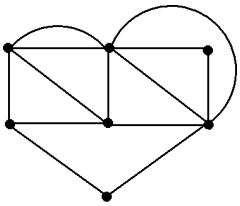
C.



D.



E.



SECTION B – Module 5: Network and decision mathematics - continued

Question 6

A manager is assigning four projects to four employees. The table below shows the typical time in days taken by each of the four employees to complete similar projects in the past.

	Project			
Employee	A	B	C	D
Jade	2	4	5	5
Alisha	3	4	7	6
Rhiannon	4	6	6	3
Wendy	4	3	2	3

To minimise the time taken to complete the four projects, the manager should assign

A. Jade to project A
Alisha to project B
Rhiannon to project C
Wendy to project D

B. Jade to project D
Alisha to project A
Rhiannon to project B
Wendy to project C

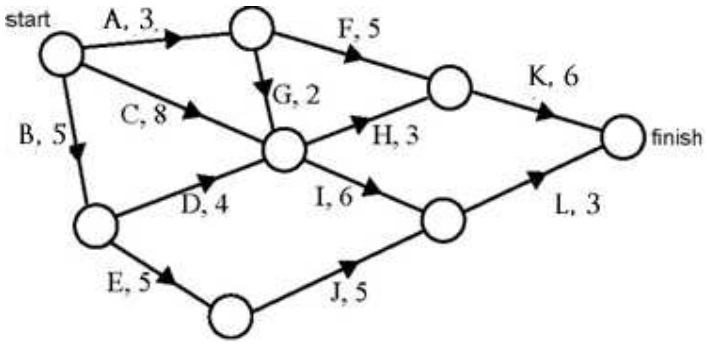
C. Jade to project C
Alisha to project D
Rhiannon to project A
Wendy to project B

D. Jade to project B
Alisha to project C
Rhiannon to project D
Wendy to project A

E. Jade to project A
Alisha to project B
Rhiannon to project D
Wendy to project C

The following information relates to questions 7 to 8

The network diagram shown below is for the construction of a bushfire charity concert stage. Activity times are shown in hours.



Question 7

The minimum time in which this building project can be completed is

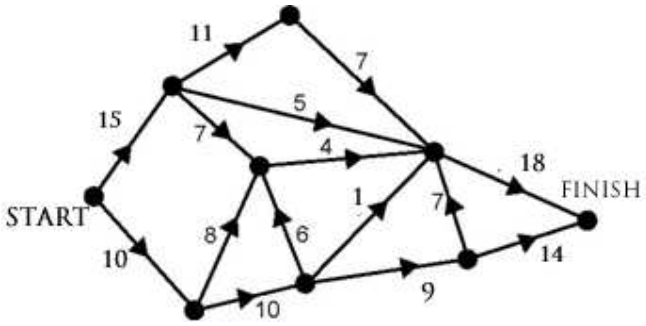
A. 12 hours
B. 14 hours
C. 16 hours
D. 18 hours
E. 20 hours

Question 8

By how many hours can activity F be increased without delaying the building of the stage?

A. 3 hours
B. 4 hours
C. 5 hours
D. 6 hours
E. 0 hours (It is on the critical path)

Question 9



The maximum flow through the network above is

A. 32
B. 27
C. 26
D. 25
E. 24

Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

If $A = \begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 & 3 \end{bmatrix}$,

then a possible product of the two matrices would result in a matrix with the order of

A. 2×1
B. 1×2
C. 3×1
D. 3×2
E. 2×3

Question 2

If $\begin{bmatrix} 2 & -9 \\ a & 0 \end{bmatrix} + \begin{bmatrix} -5 & b \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix}$,

then the values of a and b are

A. $a = -2, b = 7$
B. $a = -2, b = 11$
C. $a = 0, b = -11$
D. $a = 2, b = -11$
E. $a = -1, b = -9$

Question 3

Which of these matrices is singular?

A. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 5 & 6 \\ 4 & 5 \end{bmatrix}$

C. $\begin{bmatrix} 4 & 8 \\ 6 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 6 & -8 \\ 3 & 4 \end{bmatrix}$

E. $\begin{bmatrix} 6 & -3 \\ -10 & 5 \end{bmatrix}$

Question 4

A sports store has three types of games machines priced at \$550, \$750 and \$990 and three types of game control units priced at \$60, \$120 and \$150. The owner of the store wishes to

- reduce the prices of the games machines by 10% and
- increase the prices of the control units by 10%

To calculate the new prices (to the nearest dollar) the matrix equation used is **best represented** by

A. $\begin{bmatrix} 550 & 60 \\ 750 & 120 \\ 990 & 150 \end{bmatrix} \times \begin{bmatrix} -0.1 & 0 \\ 0 & +0.1 \end{bmatrix}$

B. $\begin{bmatrix} 550 & 750 & 990 \\ 60 & 120 & 150 \end{bmatrix} \times \begin{bmatrix} -0.1 & 0 \\ 0 & +0.1 \end{bmatrix}$

C. $\begin{bmatrix} 550 & 750 & 990 \\ 60 & 120 & 150 \end{bmatrix} \times \begin{bmatrix} 0.9 & 0 \\ 0 & 1.1 \end{bmatrix}$

D. $\begin{bmatrix} 550 & 60 \\ 750 & 120 \\ 990 & 150 \end{bmatrix} \times \begin{bmatrix} 0.9 & 0 \\ 0 & 1.1 \end{bmatrix}$

E. $\begin{bmatrix} 550 & 60 \\ 750 & 120 \\ 990 & 150 \end{bmatrix} \times \begin{bmatrix} 0.9 \\ 1.1 \end{bmatrix}$

Question 5

Which of the following matrix equation statements is true?

A. $A - B = B - A$

B. $A \times B = B \times A$

C. $A \div B = B \div A$

D. $A - (B - C) = (A - B) - C$

E. $A + (B + C) = (A + B) + C$

Question 6

The steady state matrix, given the transition matrix $\begin{bmatrix} 0.8 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.7 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}$ is **best** represented as a matrix by

A. $\begin{bmatrix} 0.68 & 0.32 & 0.32 \\ 0.21 & 0.52 & 0.51 \\ 0.11 & 0.16 & 0.17 \end{bmatrix}$

B. $\begin{bmatrix} 0.500 & 0.500 & 0.500 \\ 0.364 & 0.364 & 0.364 \\ 0.136 & 0.136 & 0.136 \end{bmatrix}$

C. $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{4}{11} & \frac{4}{11} & \frac{4}{11} \\ \frac{3}{22} & \frac{3}{22} & \frac{3}{22} \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} \frac{4}{3} & \frac{-1}{3} & \frac{-1}{3} \\ -1 & -1 & 9 \\ \frac{2}{3} & \frac{7}{3} & \frac{-23}{3} \end{bmatrix}$

Question 7

For the following simultaneous equations

$$2x - y + z = 13$$
$$3y - 2z = 0$$
$$x + 4y - 5z = -19$$

The solution is

A. $x = 4 \quad y = -2 \quad z = 3$

B. $x = 3.68 \quad y = -5.52 \quad z = 0.12$

C. $x = 4.8 \quad y = 6.8 \quad z = 10.2$

D. $x = -2 \quad y = -1 \quad z = 1$

E. All or some lines are parallel thus NO solution.

The following information relates to Questions 8 and 9

In a country regional college of **400 students**, the students have to choose three Further Maths modules from 4 modules offered as follows.

- Module 1 - Number applications
- Module 2 - Geometry and trigonometry
- Module 5 - Networks and decision mathematics and
- Module 6 - Matrices

From research of the past 3 years the following transition matrix can be used to predict the number of students studying each of the four modules in future years.

$$T = \begin{array}{c} \\ \text{to} \\ 1 \\ 2 \\ 5 \\ 6 \end{array} \begin{array}{c} \text{from} \\ \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{bmatrix} 0.7 & 0.15 & 0.1 & 0.05 \\ 0.1 & 0.7 & 0.15 & 0.05 \\ 0.15 & .1 & 0.7 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.85 \end{bmatrix} \end{array}$$

Question 8

If in 2008 the number of students doing each of the modules was

Module	Number of students
Module 1 - Number applications	330
Module 2 - Geometry and trigonometry	400
Module 5 - Networks and decision mathematics	350
Module 6 - Matrices	120

The expected number of students in **total** doing Module **1 - Number applications** and Module **6 - Matrices** in **2010** is expected to be

A. 400
B. 450
C. 488
D. 515
E. 534

Question 9

The transition matrix predicts, in the **long term**, that

A. It will never reach steady state.
B. The Matrices module is the most popular module.
C. There will be an equal number of students in each module.
D. All students will do the Matrices module.
E. All modules will have a decrease in the number of students.

END OF MULTIPLE CHOICE QUESTION BOOK

Exam 1 & 2 Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n-1)d) = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\frac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET