# Further Mathematics

# 2009

# Trial Examination 2

## Core – Data analysis
## Module 2 – Geometry and trigonometry
## Module 3 – Graphs and relations
## Module 4 – Business-related mathematics

## Instructions:

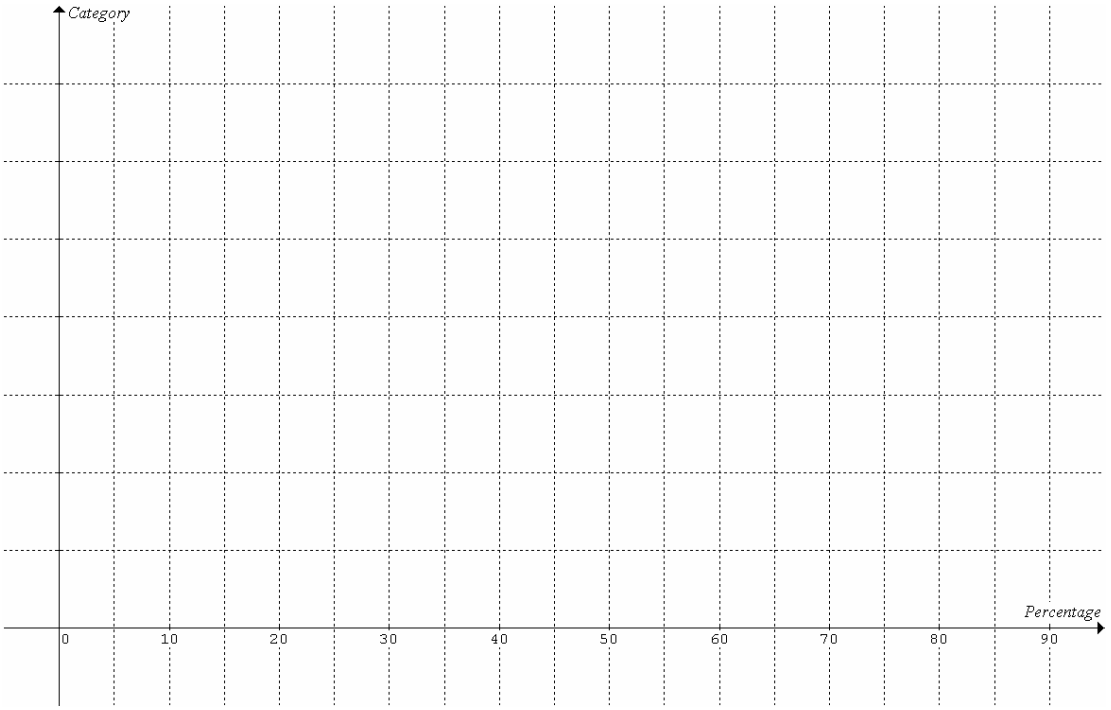
Answer all questions in the core and the three modules.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, $\pi$, surds or fractions.

## Core – Data analysis

### Question 1

There are 100 families in a small town. 35 families have only a dog or dogs as pets; 25 families have only a cat or cats; 20 families have both.

a. Draw a bar chart to display the above information. 1 mark
(Do not include families with no pets in the bar chart)




b. What is the percentage of families in the small town **without** a cat or cats as pets? 1 mark

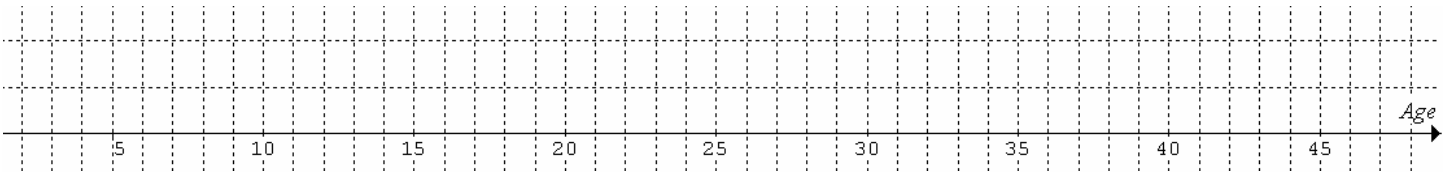
  

**Question 2**
The ages of a group of students entering a tertiary course are shown in the following frequency table.

| Age | Frequency |
|---|---|
| 16 | 1 |
| 17 | 2 |
| 18 | 6 |
| 19 | 1 |
| 20 | 3 |
| 21 | 2 |
| 22 | 1 |
| 26 | 1 |
| 27 | 1 |
| 31 | 1 |
| 39 | 1 |

a. Calculate the interquartile range of the set of data. 1 mark

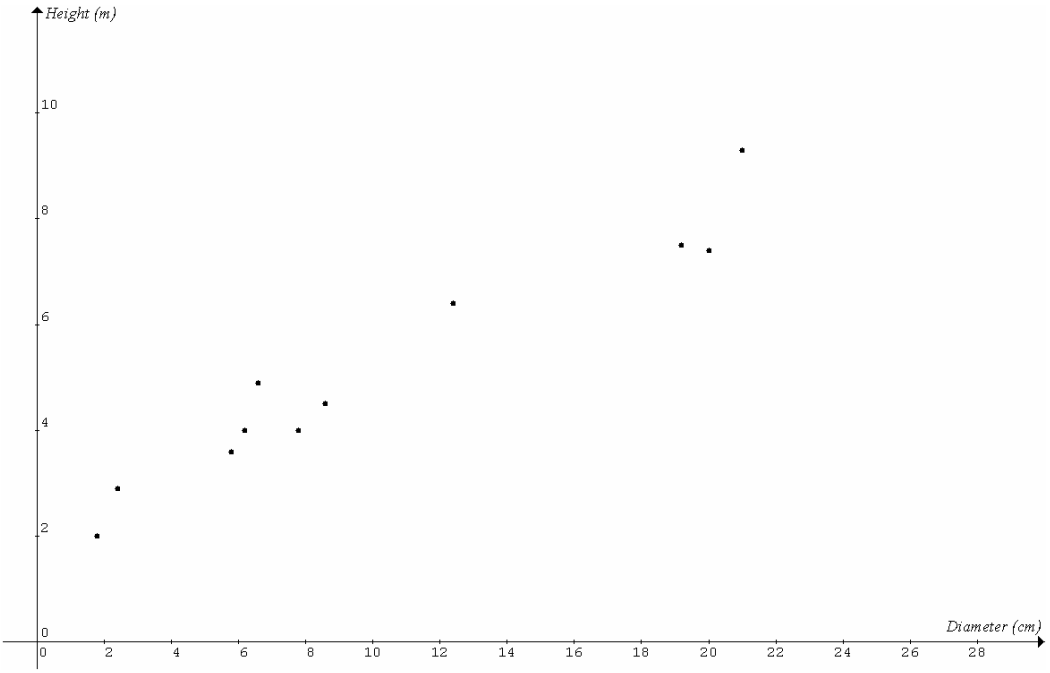
b. Construct a boxplot showing the relevant statistics and the outlier(s). 2 marks



**Question 3**

A researcher seeking information on the growth of certain tree species constructed the following scatterplot of height versus diameter of 11 tree trunks 1.50 m above the ground. She wishes to determine if the diameter measurement can be used to predict tree height. Let $H$ be the height in metres and $d$ the diameter in cm.




a. Draw the three median line to fit the set of data. 1 mark

b. Determine the equation (with the coefficients correct to 2 decimal places) of the three median line in terms of $H$ and $d$.

1 mark

c. Use the three median line to determine the residual (2 decimal places) when $d = 12.50$ cm. 1 mark

**Question 4**

Air temperature $T$ at distance $d$ in front of a radiant heater is measured at different positions. The set of data is shown in the following table.

| $d$ (m) | 0.50 | 1.2 | 2.0 | 2.5 | 4.3 | 5.5 |
|---|---|---|---|---|---|---|
| $T$ (°C) | 32 | 27 | 24 | 22 | 21 | 20 |

a. Name a suitable transformation of the $d$-axis to linearise the set of data. 1 mark

b. Determine the coefficient of determination (2 decimal places) of the linearised data. 1 mark

c. Write down the equation (with the coefficients correct to 2 decimal places) that shows the relationship between $d$ and $T$.

2 marks

**Question 5**

In a city the monthly demand for rental houses over a 24-month period starting from July 2007 is shown in the following table.

| Month | J | A | S | O | N | D | J | F | M | A | M | J |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Demand | 168 | 193 | 187 | 198 | 194 | 193 | 183 | 200 | 179 | 172 | 198 | 208 |

| J | A | S | O | N | D | J | F | M | A | M | J |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 208 | 228 | 225 | 230 | 218 | 211 | 239 | 245 | 200 | 189 | 215 | 198 |

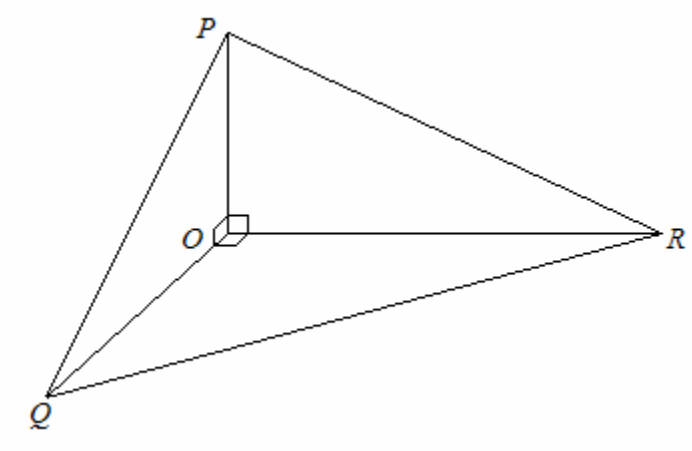
a. Calculate the seasonal index for November. 2 marks

b. Hence calculate the seasonally adjusted figure (nearest whole number) for November 2008. 1 mark

## Module 2: Geometry and trigonometry

**Question 1** $OPQR$ is a corner solid cut out from a rectangular solid. $OP = 30$ cm, and $OQ = OR = 40$ cm.




a. Calculate the total surface area ($cm^2$, 2 decimal places) of the corner solid cut out from the rectangular solid.

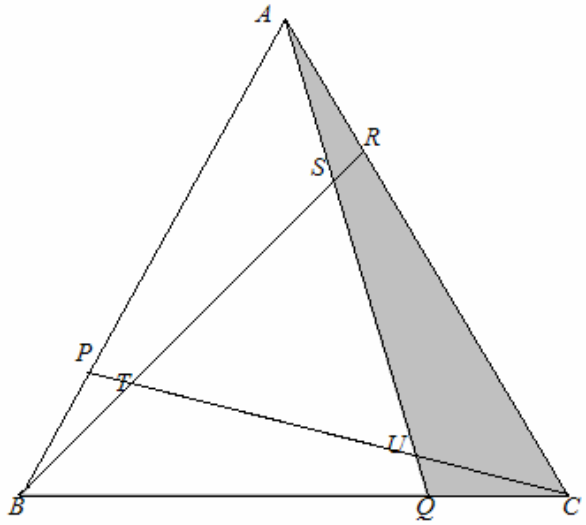
2 marks

b. Calculate the volume ($cm^3$, 2 decimal places) of the corner solid cut out from the rectangular solid.

1 mark

**Question 2** $ABC$ and $STU$ are equilateral triangles. $AP = 3PB$, $BQ = 3QC$ and $CR = 3RA$. $RA = 1$ m.




a. In the shaded region there are three scalene triangles. Name a pair of triangles that are similar but not congruent. 1 mark

b. Calculate the exact length (m) of line segment $AQ$. 1 mark

c. Calculate the exact length (m) of

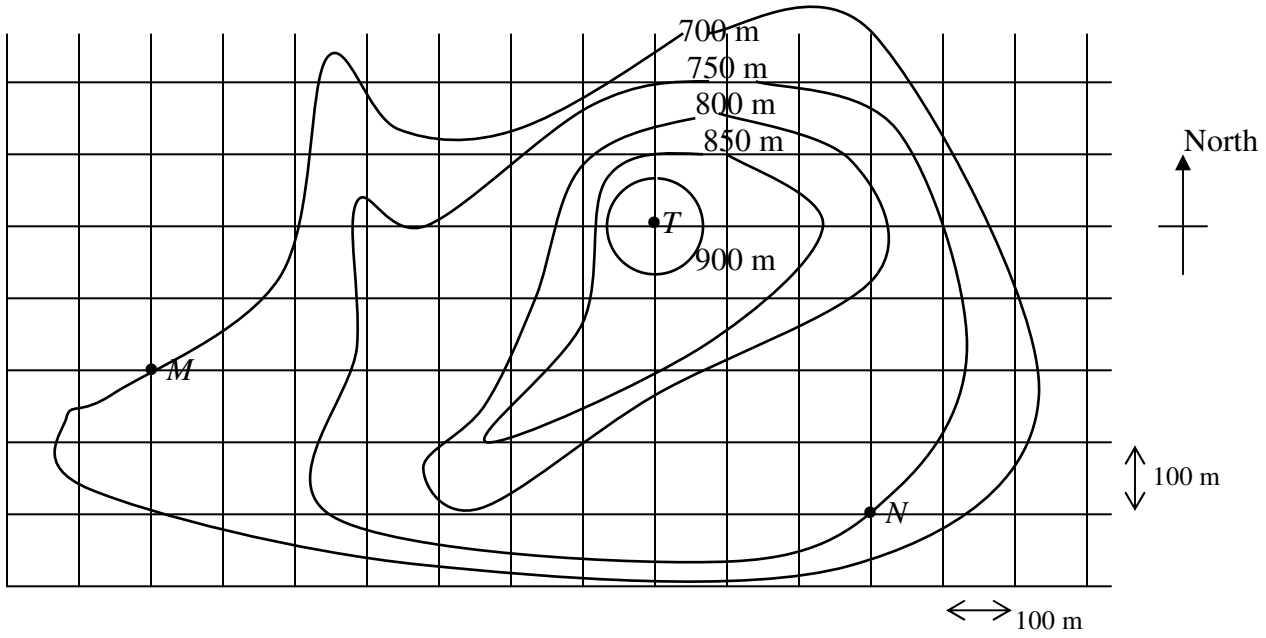
i line segment $UQ$ 1 mark

ii line segment $AS$. 1 mark

d. Determine the exact value (in fraction form) of the ratio $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta STU}$. 2 marks

**Question 3** The following diagram is a contour map. Martha is at location *M* and Nora is at location *N*. A 40-metre tall TV tower is at location *T*. The region within the circular 900-m contour is flat.




a. Determine the true bearing (nearest degree) of Martha from Nora. 1 mark

b. Can Nora be seen by Martha through binoculars? Explain with calculations. 2 marks

c. How high above the sea level is the top of the TV tower? 1 mark

d. Calculate and compare the angles (nearest degree) of elevation of the top the TV tower from *M* and *N*. 2 marks

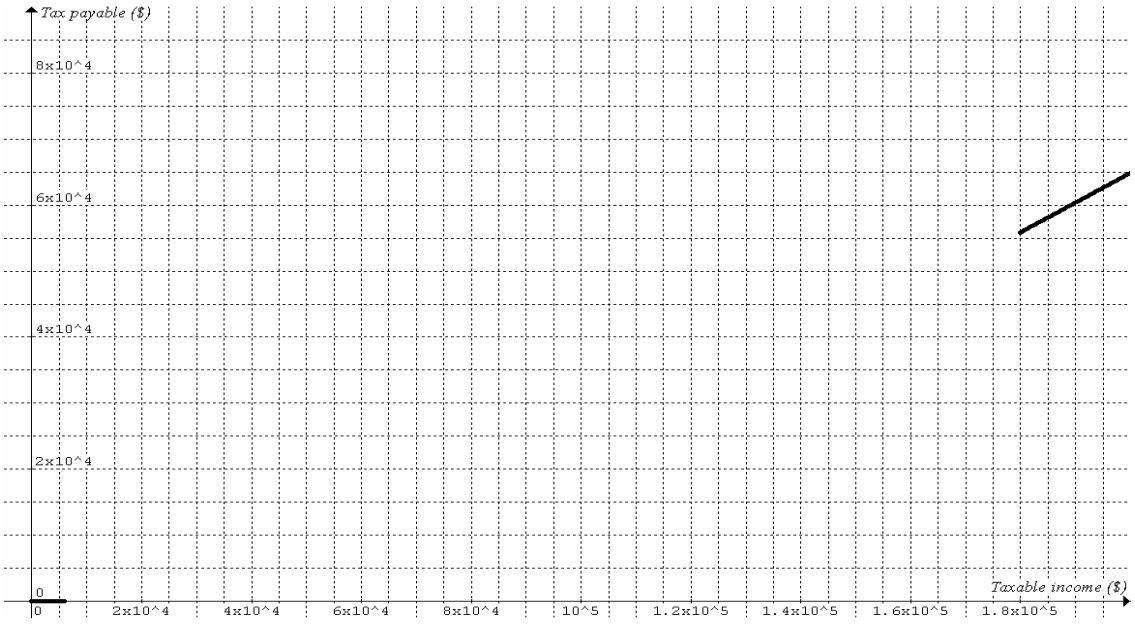
## Module 3: Graphs and relations

**Question 1** Tax rates for 2009 – 2010 are shown in the following table.

| Tax Rates 2009-2010 | |
|---|---|
| **TAXABLE INCOME** | **TAX ON THIS INCOME** |
| $0 – $6,000 | Nil |
| $6,001 – $35,000 | 15c for each $1 over $6,000 |
| $35,001 – $80,000 | $4,350 plus 30c for each $1 over $35,000 |
| $80,001 – $180,000 | $17,850 plus 38c for each $1 over $80,000 |
| Over $180,000 | $55,850 plus 45c for each $1 over $180,000 |

http://www.taxcalc.com.au/

a. Use the table to complete the following graph of *Tax payable* against *Taxable income*. 2 marks




b. Calculate *average* tax rate for income between $6,001 and $180,000. 1 mark

c. Determine the equation used to calculate *Tax payable* ($) for *Taxable income* between $80,001 and $180,000. 2 marks

**Question 2** Light intensity $I$ (lm/m$^2$) changes with distance $d$ (m) from its source. The relationship is shown in the following table.

| $d$ | 1.5 | 2.0 | 3.0 | 4.5 | 7.5 | 12 |
|---|---|---|---|---|---|---|
| $I$ | 60 | 34 | 15 | 6.7 | 2.4 | 0.94 |

a. Plot the graph of $I$ against $\frac{1}{d^2}$. Label the axes and show scales on the axes. 2 marks


b. Write an equation that shows the relationship between $I$ and $d$. Write the constant of proportionality correct to the nearest whole number.

2 marks

c. Predict the light intensity in lm/m$^2$ at a distance of 0.6 m from the source. Write your answer correct to the nearest whole number.

1 mark

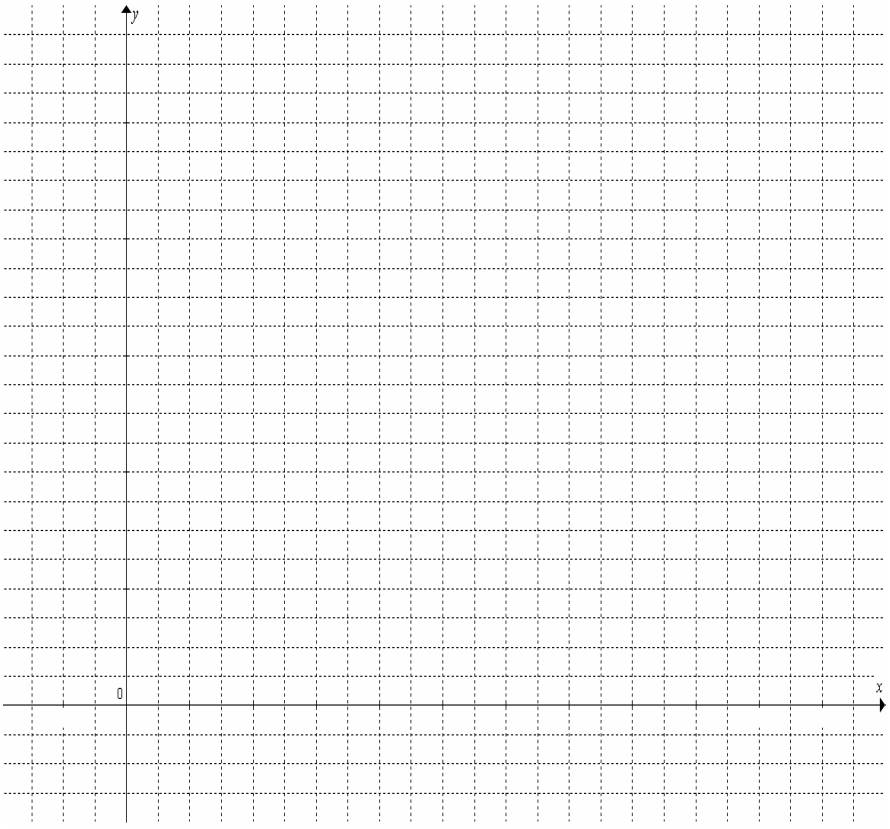
**Question 3** Two types of food, X and Y, are to be blended so that the mixture has at least 30 units of soluble fibre and 45 units of insoluble fibre. The fibre content in units per kg of the two types of food is shown in the following table.

| Food | Soluble fibre | Insoluble fibre |
|---|---|---|
| **X** | 12 | 9 |
| **Y** | 6 | 18 |

X costs \$3.00 per kg and Y costs \$4.00 per kg.
Let $x$ kg of X and $y$ kg of Y be the amounts to be blended, where $x \geq 0$ and $y \geq 0$.
Let \$$C$ be the cost of the mixture.

a. Write an inequality to represent the constraints on $x$ and $y$. 1 mark

b. On the graph below clearly shade the feasible region for $x$ and $y$. Show scales on the axes. 2 marks




c i Express $C$ in terms of $x$ and $y$. 1 mark

ii Find the minimum cost *for each kg* of the mixture. 1 mark

## Module 4: Business-related mathematics

**Question 1** The following bank statement for July 2009 has some missing entries (shaded).

| | Transaction | Debit | Credit | Balance |
|---|---|---|---|---|
| 01/07/2009 | Opening balance | | | |
| 06/07/2009 | Cheque 3210 | 3146.50 | | 4282.16 |
| 13/07/2009 | Cheque 3211 | | | |
| 13/07/2009 | Deposit | | 4500.00 | 7803.51 |
| 23/07/2009 | eTransfer | 180.00 | | |
| 31/07/2009 | Interest | | | |

a. Calculate the opening balance. 1 mark

b. Calculate the amount of Cheque 3211. 1 mark

c. Interest is paid monthly at a rate of 1.75% p.a. on the minimum monthly balance. Calculate the amount of interest for July.

2 marks

**Question 2** The **Consumer Price Index (CPI)** is regarded as Australia's key measure of inflation. It is designed to provide a general measure of price inflation for the Australian household sector as a whole. The CPI measures changes over time in the prices of a wide range of consumer goods and services acquired by Australian metropolitan households. The CPI rose in all capital cities in the June quarter 2009. Darwin registered the highest increase with a rise of 1.1%, while Melbourne registered the lowest increase with a rise of 0.3%. The base of all indices is 100.0 in 1989-90.

a. For Melbourne the CPI at the end of the 2009 June quarter is 164.4. Since 1989-90 what is the overall percentage increase in the prices of a wide range of consumer goods and services acquired by Melbourne metropolitan households at the end of the 2009 June quarter?

1 mark

b. What is the 2009 March quarter CPI for Melbourne? 1 mark

c. In Darwin the value of \$10,000.00 at 1st July 2009 is equal to the value of \$ $x$ at 1st April 2009. Find the value of $x$ (correct to **2** decimal places).

1 mark

**Question 3** Jac has $85,000 savings for deposit and costs in buying his first home. He wants to buy a brand new house in a fast growing outer suburb of Melbourne. The negotiated price of the house is $375,000. He obtains a 20-year term home loan at an interest rate of 5.91% p.a from a bank. The total cost of borrowing and buying is $12,200. He receives from the government the first home owner grant, bonus and boost totaling $23,000.

a. How much does Jac borrow from the bank? 1 mark

b. Determine the monthly repayment. 1 mark

c. Calculate the amount of interest to be paid in the first year. 1 mark

d. The interest rate is expected to increase by 0.50% after the first 12 months. Calculate the increase in the monthly repayment.

2 marks

**Question 4** The cash price of a used car is $12,500. A deposit of $500 and 36 monthly instalments of $420 each are required using a hire-purchase agreement.

a. Determine the total amount of interest over the 36-month period. 1 mark

b. Calculate the effective interest rate p.a. (correct to 2 decimal places). 2 marks

## End of Exam 2