2009 Further Mathematics Trial Exam 2 Solutions
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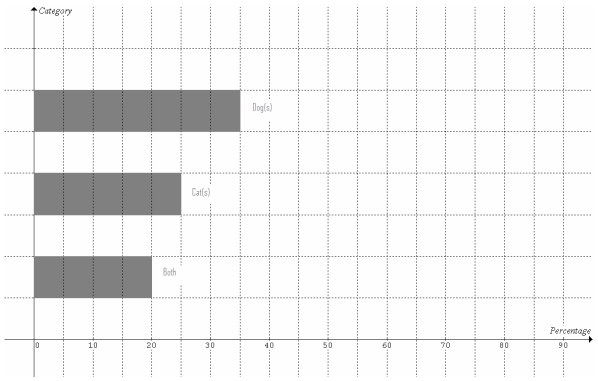
Core – Data analysis

Q1a



Q1b There are 25 + 20 = 45 families with cat(s), $\therefore$ 55 families without cat(s), i.e. 55%.

Q2a

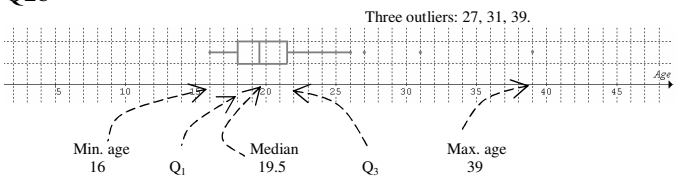
Age	Frequency
16	1
17	2
18	6
19	1
20	3
21	2
22	1
26	1
27	1
31	1
39	1

$Q_1 = 18$

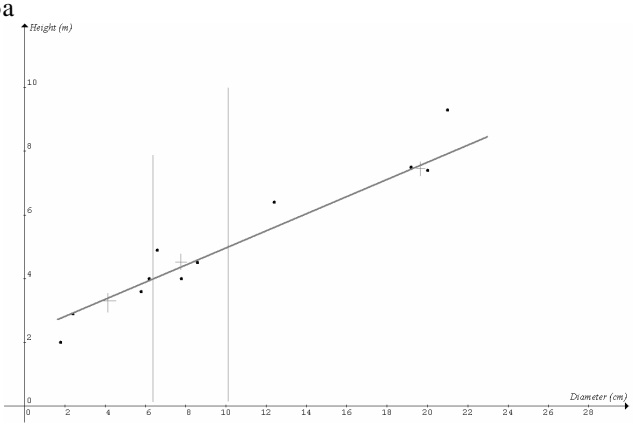
$Q_3 = 21.5$

$IQR = 21.5 - 18 = 3.5$.

Q2b



Q3a



Q3b The y-intercept ≈ 2.25. Use the two outer median points to determine the gradient ≈ 0.26.
$\therefore H = 0.26d + 2.25$.

Q3c When $d = 12.50$, actual $H \approx 6.40$ from graph,
predicted $H \approx 5.50$ from regression line.
Residual = actual value − predicted value $= 6.40 - 5.50 = 0.90$ m.

Q4a $\log_{10} d$ transformation.

Q4b $r^2 = 0.97$ (By calculator).

Q4c $T = -11.66 \log_{10} d + 27.93$ (By calculator).

Alternative to Q4: $\frac{1}{d}$ transformation; $r^2 = 0.94$;

$T = 6.41\left(\frac{1}{d}\right) + 19.90$.

Q5a 2007/08 average $= \frac{sum}{12} \approx 189.42$.

2008/09 average $= \frac{sum}{12} \approx 217.17$.

Seasonal index for November

$= \frac{\frac{194}{189.42} + \frac{218}{217.17}}{2} = 1.0140$.

Q5b Seasonally adjusted figure for November 2008

$= \frac{218}{1.0140} \approx 215$.

Module 2: Geometry and trigonometry

Q1a ΔOPQ: Area $= \frac{1}{2}(40)(30) = 600$.

ΔOPR: Area $= \frac{1}{2}(40)(30) = 600$.

ΔOQR: Area $= \frac{1}{2}(40)(40) = 800$.

ΔPQR: $PQ = PR = \sqrt{30^2 + 40^2} = 50$.

$QR = \sqrt{40^2 + 40^2} = 56.5685$.

$s = \frac{1}{2}(50 + 50 + 56.5685) = 78.2843$.

Area

$= \sqrt{78.2843(78.2843 - 40)(78.2843 - 40)(78.2843 - 56.5685)}$

$= 1578.50$.

Total surface area $= 600 + 600 + 800 + 1578.50 = 3578.50$ cm^2.

Q1b Volume $= \frac{1}{3}$(base area)(height)

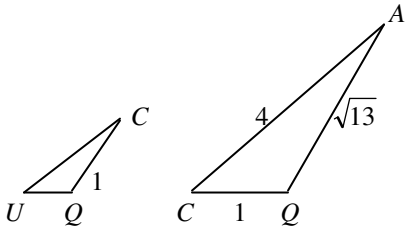
$= \frac{1}{3}(800)(30) = 8000\,\text{cm}^3$.

Q2a ΔCQU , ΔAQC .

Q2b The cosine rule:

$AQ = \sqrt{1^2 + 4^2 - 2(1)(4)\cos 60°} = \sqrt{13}$ m.

Q2ci



$\frac{UQ}{1} = \frac{1}{\sqrt{13}}$, $\therefore UQ = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$ m.

Q2cii $\frac{UC}{1} = \frac{4}{\sqrt{13}}$, $UC = \frac{4}{\sqrt{13}}$, $AS = UC = \frac{4}{\sqrt{13}} = \frac{4\sqrt{13}}{13}$ m.

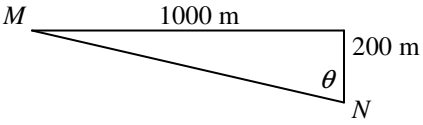
Q2d $SU = \sqrt{13} - \frac{\sqrt{13}}{13} - \frac{4\sqrt{13}}{13} = \frac{8\sqrt{13}}{13}$ m.

Length ratio $AC : SU = 4 : \frac{8\sqrt{13}}{13} = 1 : \frac{2\sqrt{13}}{13}$.

Area ratio $\Delta ABC : \Delta STU = 1^2 : \left(\frac{2\sqrt{13}}{13}\right)^2 = 1 : \frac{4}{13} = 13 : 4$.

$\frac{Area\,\Delta ABC}{Area\,\Delta STU} = \frac{13}{4}$.

Q3a

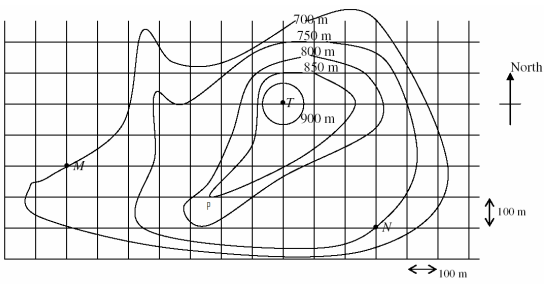


$\tan\theta = \frac{1000}{200}$, $\theta = \tan^{-1}(5) = 78.69°$.

$360 - 78.69 = 281.31 \approx 281$.

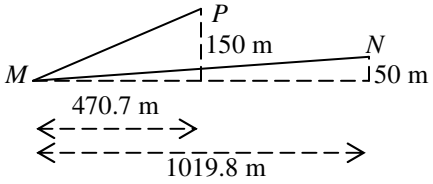
Bearing of *M* from *N* is 281°T.

Q3b



Horizontal distance from *M* to $N = \sqrt{1000^2 + 200^2} \approx 1019.8$ m.

Horizontal distance from *M* to $P = \sqrt{460^2 + 100^2} \approx 470.7$ m.



Slope of $MP = \frac{150}{470.7} \approx 0.32$ is greater than the slope of

$MN = \frac{50}{1019.8} \approx 0.05$.

$\therefore N$ could not be seen by *M* through binoculars.

Q3c 900 + 40 = 940 m.

Q3d

Horizontal distance from *M* to $T = \sqrt{700^2 + 200^2} \approx 728.0$ m.

Horizontal distance from *N* to $T = \sqrt{300^2 + 400^2} = 500$ m.

Angle of elevation of top of tower from *M*

$\theta_M \approx \tan^{-1}\left(\frac{940 - 700}{728.0}\right) \approx 18°$.

Angle of elevation of top of tower from *N*

$\theta_N \approx \tan^{-1}\left(\frac{940 - 750}{500}\right) \approx 21°$.

$\therefore \theta_N > \theta_M$.

Module 3: Graphs and relations

Q1a

Q1b Average tax rate $= \frac{55{,}850 - 0}{180{,}000 - 6{,}000} \approx 0.321$, i.e. 32.1 cents for each dollar over $6,000.

Q1c $Tax\ payable = 0.38(income - 80{,}000) + 17{,}850$.

$Tax\ payable = 0.38 \times income - 12{,}550$.

Q2a

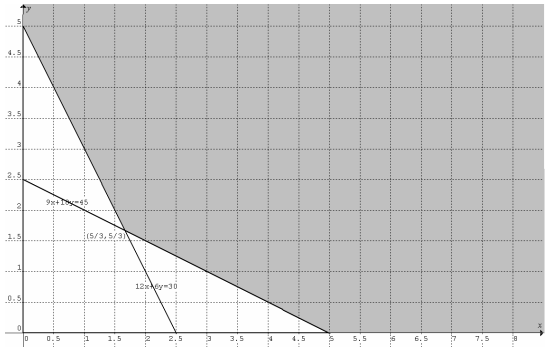
$1/d^2$	0.4444	0.25	0.1111	0.0494	0.0178	0.0069
I	60	34	15	6.7	2.4	0.94

Q2b The points are on a straight line through the origin O.
Constant of proportionality is the gradient of the
line $= \dfrac{60}{0.4444} \approx 135$. $\therefore I = \dfrac{135}{d^2}$.

Q2c When $d = 0.6$ m, $I = \dfrac{135}{0.6^2} = 375$ lm/m^2.

Q3a Either $12x + 6y \geq 30$ or $9x + 18y \geq 45$.

Q3b



Q3ci $\$C = 3x + 4y$.

Q3cii Minimum cost at $\left(\dfrac{5}{3}, \dfrac{5}{3}\right)$. $\$C = 3\left(\dfrac{5}{3}\right) + 4\left(\dfrac{5}{3}\right) = \dfrac{35}{3}$.

Minimum cost per kg $= \dfrac{\frac{35}{3}}{\frac{5}{3} + \frac{5}{3}} = 3.50$, i.e. \$3.50 per kg.

Module 4: Business-related mathematics

Q1a Opening balance (\$) $= 4282.16 + 3146.50 = 7428.66$.

Q1b Amount of Cheque 3211(\$)
$= 4282.16 - (7803.51 - 4500.00) = 978.65$.

Q1c Minimum balance (\$) $= 7803.51 - 4500.00 = 3303.51$

Interest (\$) $= \dfrac{3303.51 \times 1.75 \times \frac{1}{12}}{100} = 4.82$.

Q2a $\dfrac{164.4 - 100.0}{100.0} \times 100\% = 64.4\%$.

Q2b Let x be the 2009 March quarter CPI for Melbourne.
$\dfrac{164.4 - x}{x} \times 100\% = 0.3\%$, $16440 - 100x = 0.3x$,
$100.3x = 16440$, $x = 163.9$.

Q2c $(100\% + 1.1\%)x = 10{,}000.00$, $\dfrac{101.1}{100}x = 10{,}000.00$,
$x = 9891.20$.

Q3a Amount borrowed (\$)
$= 375{,}000 + 12{,}200 - 85{,}000 - 23{,}000 = 279{,}200$.

Q3b Use TVM Solver to find PMT:
N = 240, I = 5.91, PMT = , PV = 279200, FV = 0, P/Y = 12, C/Y = 12
Monthly repayment PMT = \$1985.81

Q3c Use TVM Solver to find FV:
N = 12, I = 5.91, PMT = -1985.81, PV = 279200, FV = ,
P/Y = 12, C/Y = 12
Amount owing at the end of the first year FV = 271669.23.
Loan amount reduced $= 279200 - 271669.18 = 7530.82$.
Amount paid in the first year $= 1985.81 \times 12 = 23829.72$.
Interest paid in the first year $= 23829.72 - 7530.82 = \$16298.90$.

Q3d After a year, the amount owing is \$271669.23, the remaining term is 228 months, new interest rate is 6.41% pa.
Use TVM Solver to find PMT:
N = 228, I = 6.41, PMT = , PV = 271669.23, FV = 0, P/Y = 12, C/Y = 12
Monthly repayment PMT = \$2063.70.
Increase in repayment $= 2063.70 - 1985.81 = \$77.89$.

Q4a Loan amount $= 12{,}500 - 500 = \$12{,}000$.
Total of 36 instalments $= 420 \times 36 = \$15{,}120$.
Interest amount $= 15{,}120 - 12{,}000 = \$3120$.

Q4b Flat rate $r = \dfrac{100I}{PT} = \dfrac{100 \times 3120}{12{,}000 \times 3} = 8.6667$.

Effective rate $= \dfrac{2n}{n+1} \times r = \dfrac{2 \times 36}{37} \times 8.6667 \approx 16.86$.
The effective interest rate p.a. is 16.86%.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

