Student Name: _____

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2

2008 Trial Examination

Reading Time: 15 minutes
Writing Time: 1 hour and 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
1	1	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one approved graphics calculator or CAS (memory DOES NOT have to be cleared) and, if desired, one scientific calculator, one bound reference (may be annotated). The reference may be typed or handwritten (may be a textbook).
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials Supplied

- Question book of 35 pages.
- Working space provided throughout the book.

Instructions

- Print your **name** in the space provided at the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected. You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

TURN OVER

Core

Curtis Board and Taylor Kent run Curtail Boarding Kennels. They board dogs, cats and sometimes other pets for owners who are travelling and can't take their beloved animals with them.

Question 1

Taylor is in charge of keeping their records and has been making a list of the amounts spent on dog food during the 2005/2006 financial year.

July '05	Aug '05	Sept '05	Oct '05	Nov '05	Dec '05	Jan '06	Feb '06	Mar '06	Apr '06	May '06	June '06
$272	$357	$875	$647	$285	$342	$524	$110	$603	$775	$532	$486

a. Calculate the mean amount that they spent on dog food during this time?

_____

_____

1 mark

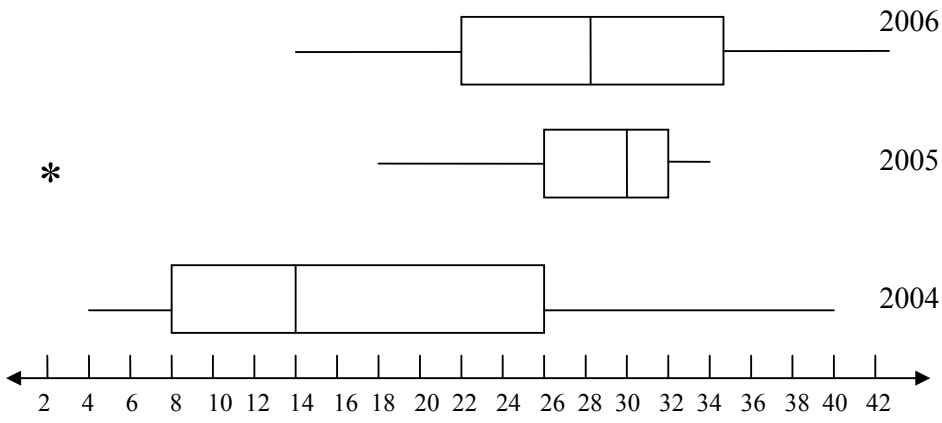
b. Taylor discovered that the amount they spent on dog food in January 2007 when compared with their 2005/2006 costs had a z score of -0.3. Calculate how much this actually represents to the nearest whole number of dollars.

_____

_____

1 mark

The Kennels opened in 2004. Taylor has produced parallel box plots showing the numbers of dogs boarding each month of the year over the last three years.



Core – Question 1 - continued

c. In which year was the data positively skewed?

_____

1 mark

d. Taylor indicated an outlier in his 2005 box plot. Prove that Taylor was correct when she graphed this outlier.

_____

_____

_____

1 mark

Curtail Boarding Kennels also board cats for their owners. Taylor is concerned that they are not charging enough in this section of the business. She decides to analyse figures for the number of cats boarded each month for the last ten months compared with the costs of running the cattery.

No of cats	11	14	15	17	20	12	8	3	15	18
Costs ($)	2200	2200	3200	4600	5700	2000	1600	800	4000	5300

e. Calculate Pearson's product-moment correlation coefficient and comment on the strength of the correlation. Give your answer to two decimal places.

_____

_____

1 mark

f. Use the least squares regression line to predict the costs associated with boarding 22 cats. Give your answer to the nearest $100.

_____

_____

_____

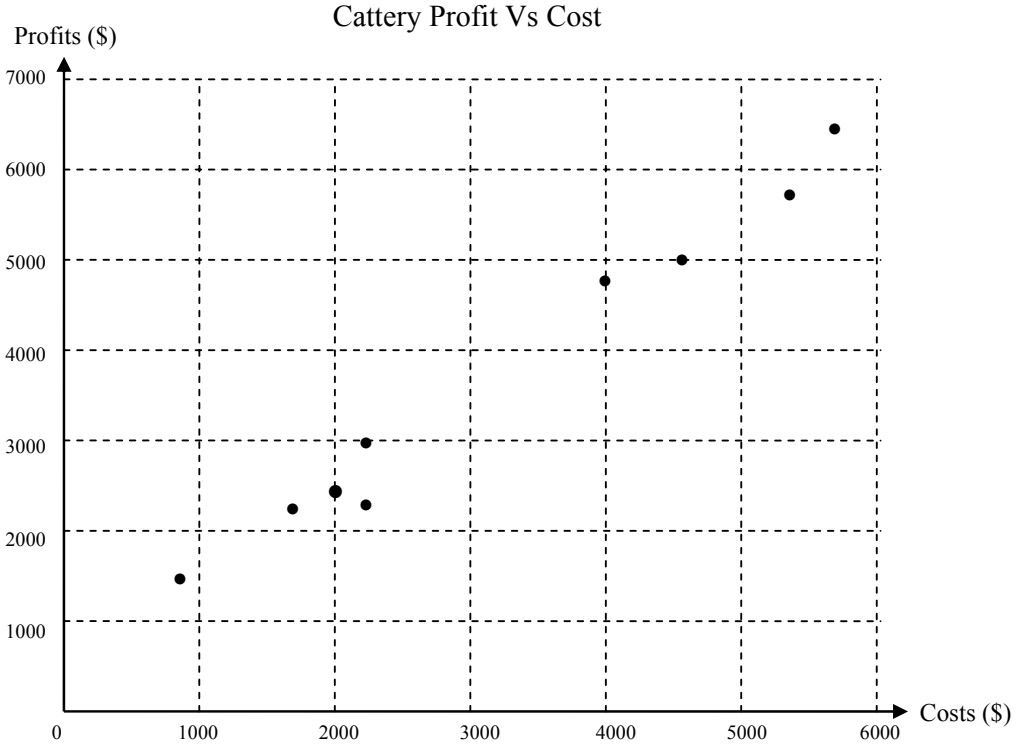
1 mark

Core – Question 1 - continued
TURN OVER

Taylor now compares the monthly costs of running the cattery with the monthly profit.

Costs ($)	2200	2200	**3200**	4600	5700	2000	1600	800	4000	5300
Profit ($)	2250	3000	**3500**	5000	6300	2400	2200	1500	4800	5700

She constructs a scatter plot of the data.



g. The point highlighted in bold in the table has been left off the scatterplot. Plot this point in the appropriate place on the scatterplot above.

1 mark

The least squares regression line for this graph has the equation

$$P = 533.8 + 0.99C$$

h. Use this equation to predict the costs if the cattery made a profit of $4000. Give your answer to the nearest $100.

_____

_____

1 mark

i. If their costs were only $500, predict how much profit they would make. Give your answer to the nearest $100.

_____

_____

1 mark

Core – Question 1 - continued

j. Is your answer to the previous question an example of extrapolation, interpolation or interpretation?

_____

1 mark

k. Describe the strength of the association between the costs of running the cattery and the profits obtained.

_____

1 mark

l. Calculate the proportion in the variation of the profit that can be explained by the variation in the running costs. Give your answer to the nearest whole percent.

_____

1 mark

m. The boarding kennel's monthly income statement for the 2005/2006 financial year is given in the table below.

July '05	Aug '05	Sept '05	Oct '05	Nov '05	Dec '05	Jan '06	Feb '06	Mar '06	Apr '06	May '06	June '06
$6500	$4500	$2800	$500	$2400	$3400	$5900	$11400	$5800	$7700	$600	$4300

Complete the three point moving mean calculation for this December and fill in the blank space in the table below.

1	2	3	4	5	6	7	8	9	10	11	12
July '05	Aug '05	Sept '05	Oct '05	Nov '05	Dec '05	Jan '06	Feb '06	Mar '06	Apr '06	May '06	June '06
$6500	$4500	$2800	$500	$2400	$3400	$5900	$11400	$5800	$7700	$600	$4300
	$4600	$2600	$1900	$2100		$6900	$7700	$8300	$4700	$4200	

1 mark

n. Determine the trend of the data from this smoothed information.

_____

1 mark

o. Why should three point median smoothing be used instead of three point mean smoothing for the data above?

_____

_____

1 mark

Total 15 marks

END OF CORE

TURN OVER

Module 1: Number patterns

Question 1
As part of the Curtail Boarding Kennels business, Taylor runs a dog training school. She has been working with a particularly difficult corgi named Bubo. This week she is trying to teach Bubo to stay as commanded. The maximum length of time Bubo stays still each day as the training proceeds has been recorded in seconds for the first four days.

2, 6, 18, 54

a. Write the equation for t_n for this sequence

_____

_____

1 mark

b. Taylor believes that Bubo will have learnt this command if he stays in place for at least 10 minutes. How many days training will it take for Bubo to demonstrate that he can stay in place for 10 minutes or more?

_____

_____

_____

1 mark

c. What is the total length of time (in seconds) that Bubo has obeyed the command after seven days of training?

_____

_____

1 mark

Question 2
Taylor also runs a puppy obedience class open to the public on Saturday mornings. Although she stresses the need for consistency, the number of puppies (and their masters) turning up each week is inconsistent. Recently the numbers have started to show an improvement and she realised that there was a Fibonacci pattern involved in the number of attendances over the last 5 weeks.

$$f_{n+2} = f_{n+1} + 2f_n \qquad f_1 = 1 \qquad f_2 = 1$$

a. Assuming this pattern continues, how many puppies will turn up in week 6?

_____

_____

1 mark

Module 1: Number patterns – Question 2 - continued

b. The number of puppies that actually turn up in week 6 was 30. For the rest of the puppy obedience school term, Taylor noticed a different trend occurring.

$$t_{n+1} = 2t_n - 31 \quad t_6 = 30$$

Assuming this trend continues, calculate in which week Taylor will have no puppies turn up for obedience school.

_____

_____

_____

1 mark

Question 3

Taylor invests part of the profit from her puppy school into an investment account.
At the start of this year she had $12000 in her account.
Her investment is earning an interest rate of 6.0 % pa compounded monthly.

a. Write an equation to show the total amount (A) in Taylor's investment account for the n^{th} month of the year.

_____

_____

_____

1 mark

b. Calculate the amount of interest Taylor earns in 6 months.

_____

_____

_____

1 mark

c. Taylor needs $15000 to upgrade her puppy school. Calculate the number of months she will need to continue her investment account until she has over $15000.

_____

_____

_____

1 mark

Question 4

Curtis Board is the other partner in the Curtail Boarding Kennels. He is responsible for the health of the animals. One of their customers, an Old English sheepdog called Shep is riddled with fleas. He decides to use a new flea product that claims to reduce the number of fleas on the dog by 12.5% each week. Curtis estimates that Shep has 250 fleas annoying him.

a. Calculate the number of fleas on Shep destroyed after three weeks of applying the product. Give your answer to the nearest whole number of fleas.

_____

_____

1 mark

b. Curtis is hoping that Shep will have fewer than 20 fleas by the end of the 3 months (13 weeks) that Shep is staying at the kennels.

i. Calculate the number of fleas that will still be annoying Shep after the 13 weeks.

_____

ii. Calculate how many more weeks the product needs to be applied until there are fewer than 20 fleas annoying Shep.

_____

_____

_____

_____

1 + 1 = 2 marks

Question 5

As the Curtail Boarding kennels have become more successful, Curtis has noticed a pattern in the number boxes of worming tablets used each month last year.

5, 6.5, 8, 9.5, 11, ...

a. Calculate the number of boxes of tablets used in the 8^{th} month of the year.

_____

1 mark

b. Assuming this trend continues for the year, calculate the total number of boxes of tablets used in 12 months.

_____

_____

1 mark

Module 1: Number patterns- continued

Question 6

Curtis has been asked to investigate the breeding of mice for a local snake handler who wants live food for his snakes. He has estimated that a single breeding pair will produce 8 offspring every 20 days. This produces a pattern showing the total number of mice every 20 days as

2, 10, 18, 26, ...

a. Assuming that this pattern is correct, calculate the total number of mice (including the initial pair) that will have been produced in 360 days.

_____

_____

_____

1 mark

b. Curtis forgot that newly bred mice are able to reproduce within 20 days of being born. He realises that a better estimate of the number of offspring from a single breeding pair will follow the pattern

2, 8, 32, 128, ...

If this pattern is correct, calculate the number of days Curtis will need to breed mice before he has over 2000 mice.

_____

_____

_____

_____

_____

_____

_____

1 mark

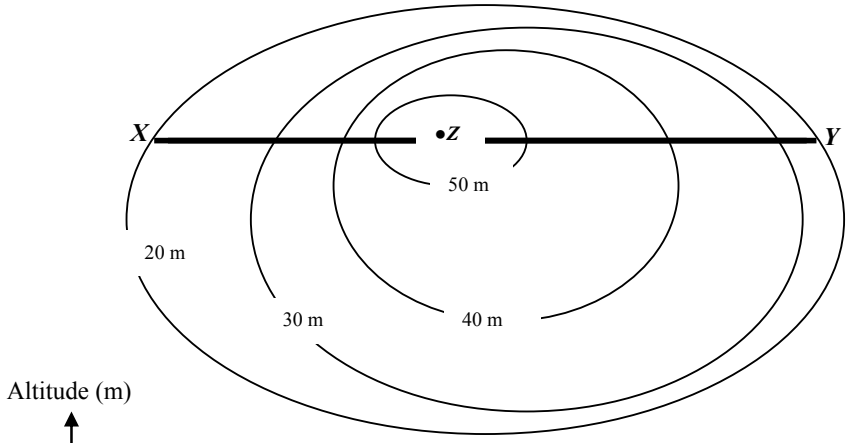
Total 15 marks

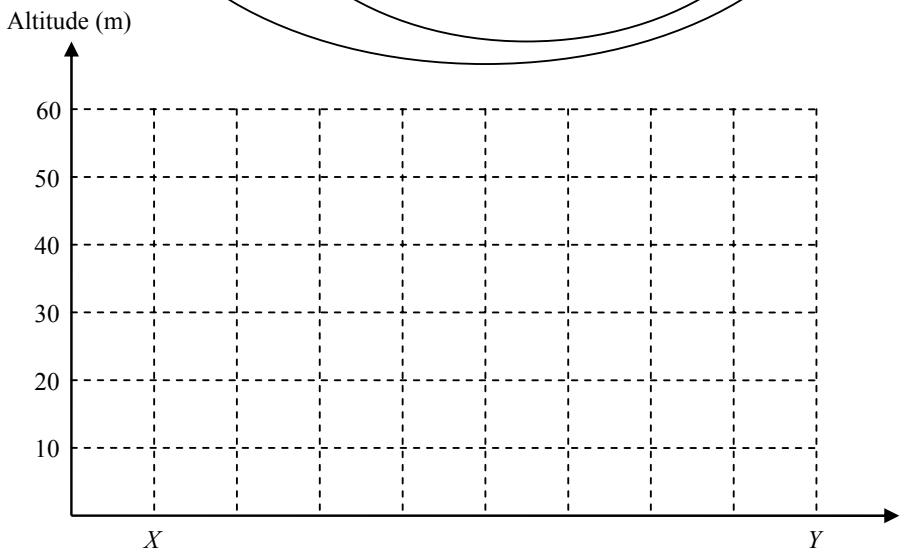
END OF MODULE 1
TURN OVER

Module 2: Geometry and trigonometry

Question 1

a. The contour map below shows the topography around the Curtail Boarding kennels. Use this map to construct a graph of altitude verses horizontal distance along the path labelled XY using the same scale as in the diagram. Point Z is at the highest point of 60 metres.





1 mark

b. The measured distance from *X* to *Y* is 8 cm and the horizontal scale on the contour map is 1:2000. *Z* is 3.5 cm from point *X*. Calculate the distance (to the nearest metre) along the path *XY* via *Z*.

_____

_____

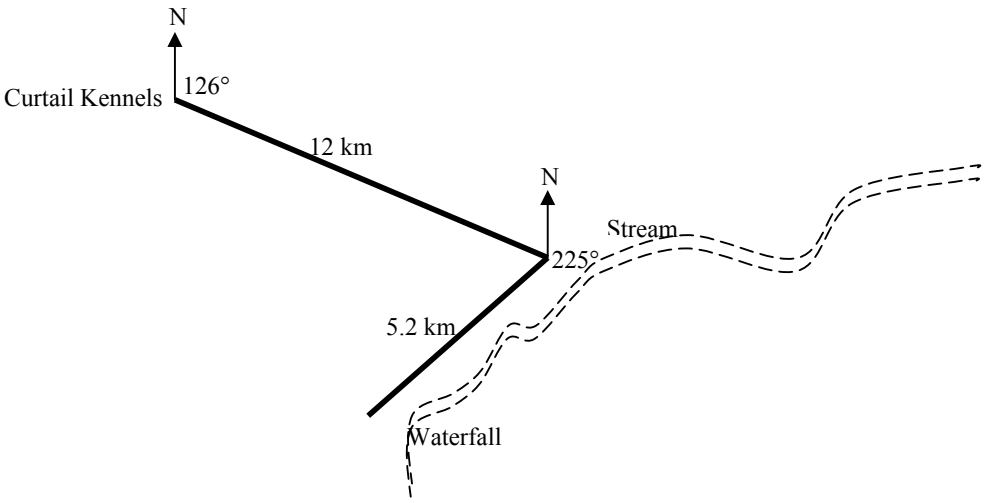
_____

_____

2 marks

Module 2: Geometry and trigonometry- continued

Question 2
Taylor and Curtis decide to take their favourite dogs, two kelpies called Butcher and Birdie and go for a cross country hike. They set out from the kennels on a bearing of 126° and walk for 12 km until they reach a stream. They then follow the stream for 5.2 km on a bearing of 225° until they reach a waterfall where they stop and have some lunch.



a. Calculate the direct line distance back to their kennels from the waterfall. Give your answer to the nearest 0.1 km.

_____

_____

_____

_____

1 mark

b. Butcher and Taylor have to get back to the kennels after lunch and they take the direct route back. Calculate the bearing they should take to return home to the nearest whole number of degrees.

_____

_____

_____

_____

_____

1 mark

Module 2: Geometry and trigonometry- Question 2 - continued
TURN OVER

c. Curtis and Birdie are going to continue their trek from the waterfall. They set off due west until they meet the road that runs due north to their home. Calculate how far west they have travelled. Give your answer to the nearest 0.1 km.

_____

_____

_____

1 mark

d. Curtis and Birdie walk home along the north bound road. What is the total distance they have travelled on their hike? Give your answer to the nearest 0.1 km.

_____

_____

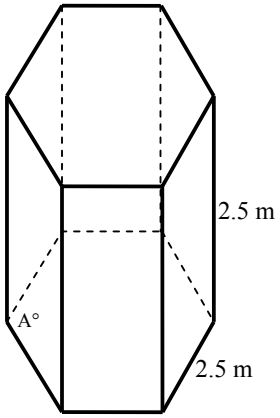
_____

_____

1 mark

Question 3

Curtis and Taylor have decided that their cattery needs a new cat playground. Taylor designs the new playground in the shape of a regular hexagonal prism. All the lengths are 2.5 metres.



a. Calculate the size of the interior angle of A° of the hexagon.

_____

_____

_____

_____

1 mark

b. Calculate the area of the hexagonal base. Give your answer in m^2 to two decimal places.

_____

_____

1 mark

Module 2: Geometry and trigonometry – Question 3 - continued

c. Calculate the total surface area of the playground. Give your answer in m^2 to two decimal places.

_____

_____

_____

1 mark

d. Calculate the volume of the playground. Give your answer in m^3 to two decimal places.

_____

_____

_____

_____

1 mark

Curtis builds a $\frac{1}{10}^{th}$ scale version of the playground.

e. Calculate the surface area of the scale version. Give your answer to the nearest cm^2.

_____

_____

_____

_____

1 mark

f. Curtis also builds a $\frac{1}{10}^{th}$ scale version as a pyramid rather than a prism. Calculate the volume of this pyramid. Give your answer to the nearest cm^3.

_____

_____

_____

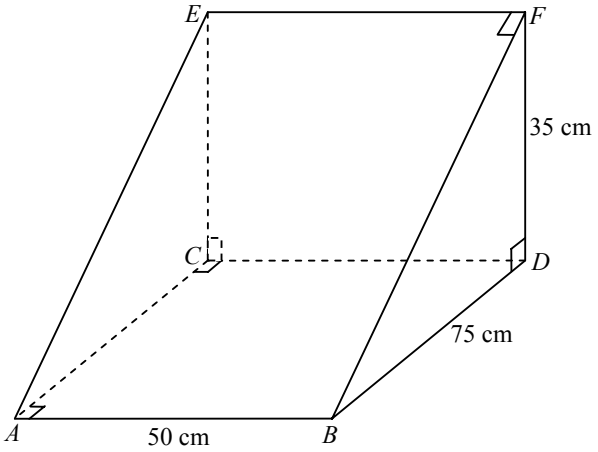
_____

1 mark

Module 2: Geometry and trigonometry- continued
TURN OVER

Question 4

Taylor wants to include a climbing apparatus in the cat playground. It is in the shape of a rectangular inclined plane.



a. Calculate the size of angle *FBD*. Give your answer to the nearest whole number of degrees.

_____

_____

_____

_____

1 mark

b. Calculate the size of angle *EBC*. Give your answer to the nearest whole number of degrees.

_____

_____

_____

_____

1 mark

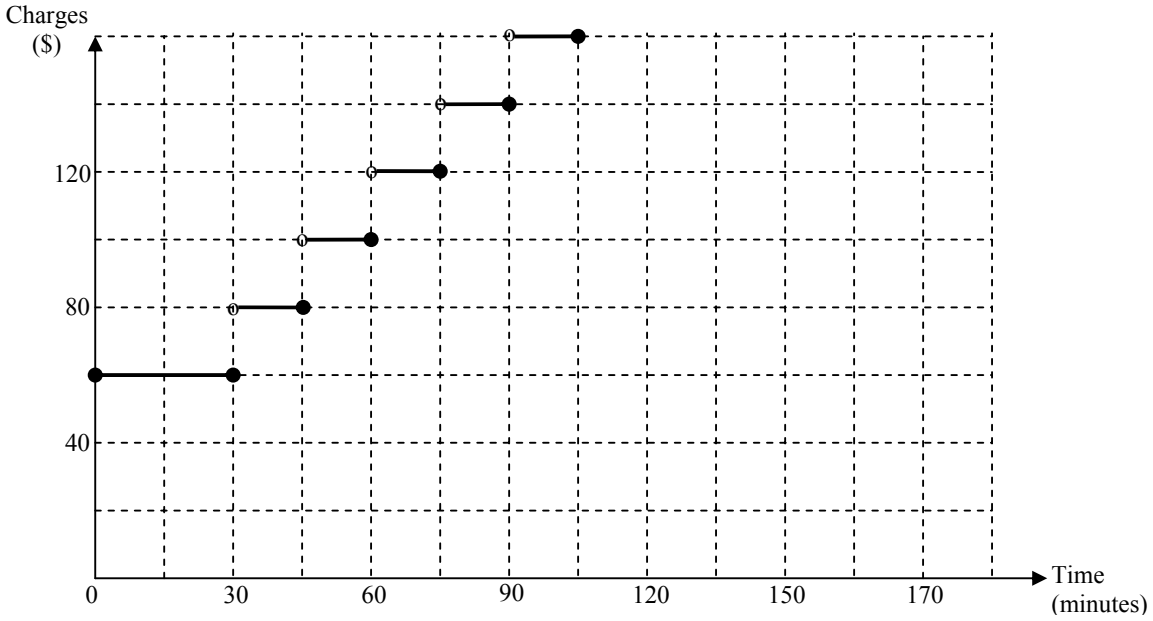
Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1

Attached to the Curtail boarding kennels business is a veterinary clinic run by Curtis Board. When he is called out to a neighbouring farm his charges are given in the following graph.



a. Curtis is called out to a nearby farm to help with a difficult carving. He is there for 1 hour and 15 minutes. How much does he charge for this visit?

_____

1 mark

b. Over the last month, Curtis has visited the same farm on four occasions to check on some sick horses. His visit each time has been fifteen minutes long. As a discount he offers to accumulate the charges as if he had visited only once for sixty minutes. Calculate by how much Curtis has discounted the charges.

_____

_____

_____

_____

_____

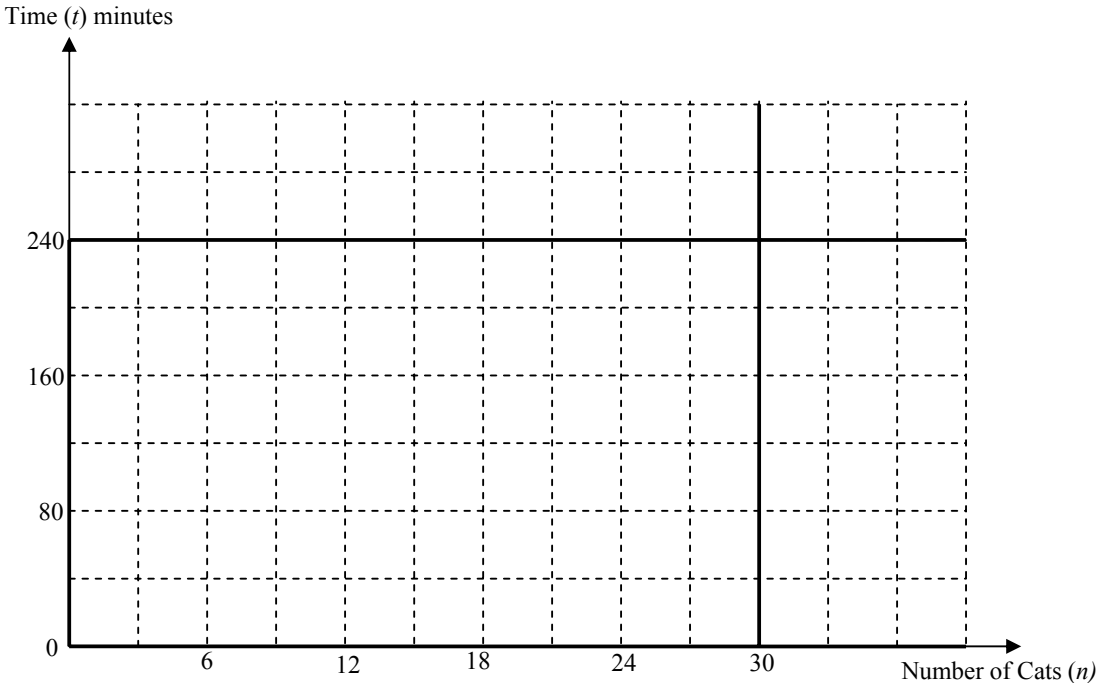
1 mark

Question 2
Taylor Kent is concerned about her cattery business. The maximum number of cats (n) she can accommodate is 30. The maximum amount of time (t) she can devote to the cat's welfare is 240 minutes. This time is taken up with 40 minutes general maintenance and at least 8 minutes cleaning and grooming per cat.

These constraints can be written as three linear inequations:

$$0 \le n \le 30 \qquad 0 \le t \le 240 \qquad t \ge 8n + 40$$

The lines representing two of these constraints have been shown on the graph.



a. Graph the third line, $t \ge 8n + 40$ and indicate the solution region on the graph.

1 + 1 = 2 marks

b. Calculate the maximum number of cats that Taylor can manage given her time constraints.

_____

_____

_____

1 mark

Module 3: Graphs and relations – Question 2 - continued

c. Calculate the amount of time needed by Taylor to look after 30 cats.

_____

_____

_____

1 mark

d. The profit that Taylor can make each month can be calculated by

$$P = 28n - 2t\ .$$

What is the maximum profit that Taylor can make given the constraints of the cattery?

_____

_____

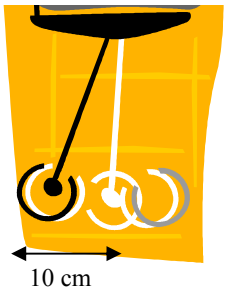
_____

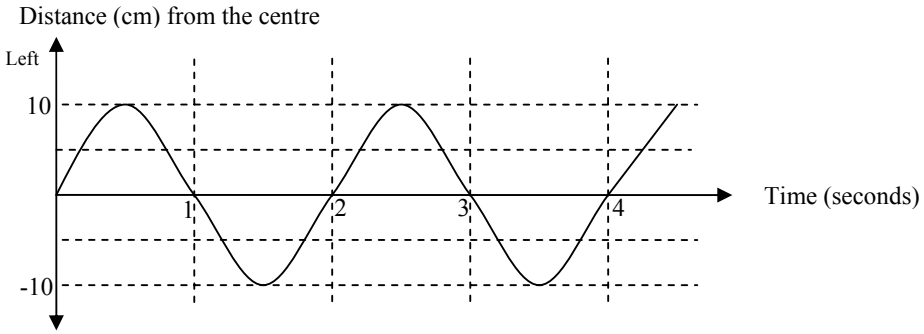
_____

1 mark

Question 3

The cats at the cattery are amused by a pendulum bob that swings back and forth when hit by an inquisitive paw.





a. Calculate the distance the pendulum travels in 2 seconds.

_____

_____

_____

_____

1 mark

b. Predict the position of the pendulum after 7.5 seconds.

_____

_____

_____

_____

1 mark

Module 3: Graphs and relations- continued

Question 4

Taylor Kent is listening to Mrs Cosset as she describes her recent trip to Mount Baw Baw. She has come to collect her cat Fluffykins who has spent a week in the cattery.

"Mr Cosset doesn't believe in stoping to sightsee so we drove for 2 hours at a speed of 80km per hour before we reached more windy roads.. In the second part of the journey we covered the remaining 75 km in 1.5 hours. I was pleased to finally arrive I can tell you."

a. Construct a graph to show the Cosset's journey.

1 + 1 = 2 marks

b. Find the gradient for the second part of the journey.

_____

1 mark

c. Find the equation that relates time (t) and distance (d) for the second part of their journey.

_____

_____

1 mark

d. Calculate the total distance of the journey.

_____

1 mark

e. Find the average speed for the journey. Give your answer in km/h to two decimal places.

_____

1 mark

Total 15 marks

END OF MODULE 3

TURN OVER

Module 4: Business-related mathematics

Question 1
Curtis Board is having a stocktaking sale of all items in the retail side of the veterinary clinic. He has decided to discount all items by 12.5%

a. If a cat carrier was priced at $85.40 before the sale, calculate its sale price.

_____

_____

1 mark

b. Curtis has worked out that the sale price on a 200 g container of gold fish food is $49.98. Calculate the original cost of the gold fish food.

_____

_____

1 mark

c. Curtis has just received a delivery of worming tablets. He has worked out that the cost of the tablets is $4.25 per packet. Curtis wishes to add his 30% profit, then 10% GST and then discount the item by 12.5%. Calculate the selling price of the discount tablets.

_____

_____

1 mark

Question 2
Taylor Kent is looking over a summary of her bank statements for the last few months. She has discovered that the interest on her savings account is calculated at 0.3% per month (3.6% pa) on the minimum monthly balance. She has listed the transactions for the last three months in the following table.

Date	Transaction Details	Debit	Credit	Balance
01 Jan 07	Balance brought forward			2560.00
10 Jan 07	Withdrawal	508.00		2052.00
19 Jan 07	Deposit		735.00	2787.00
08 Feb 07	Withdrawal	694.00		2093.00
19 Feb 07	Deposit		735.00	2828.00
26 Feb 07	Withdrawal	1000.00		1828.00
03 Mar 07	Withdrawal	250.00		1578.00
11 Mar 07	Withdrawal	368.00		1210.00
19 Mar 07	Deposit		735.00	1945.00

a. In which month did Taylor earn the most interest?

_____

1 mark

b. Calculate the total amount of interest earned by Taylor in the three months.

_____

_____

1 mark

Question 3
The Curtail boarding kennels and veterinary clinic recently purchased a new computer to use for record keeping and accounting purposes. The original cost of the system was $12 500. Curtis has worked out that the system should be depreciated at a rate of 15% per year.

a. Calculate the value of the computing system after three years of use. Give your answer to the nearest whole number of dollars.

_____

_____

1 mark

b. Curtis decides to scrap the system when its value drops below $5000. Calculate how many years before Curtis will need to replace the computer system.

_____

_____

1 mark

Module 4: Business-related mathematics- continued
TURN OVER

Question 4
Taylor and Curtis are thinking of updating their kennels. They are looking at taking out a loan for $50000.

a. Taylor's father (Mr Kent) has offered them a loan but he wants them to pay it back with 12% simple interest.

i. Calculate the amount of money Taylor and Curtis will have to pay back to Mr. Kent.

_____

_____

ii. Mr Kent is happy to receive monthly payments, but wants the entire loan paid back over three years. Calculate the monthly repayments that Taylor and Curtis will have to make to pay back the loan. Give your answer to the nearest whole number of dollars.

_____

_____

_____

1 + 1 = 2 marks

b. Curtis is concerned that they will not be able to make these repayments and approaches a bank for a loan. The bank will lend them the money at 9% pa interest calculated monthly on the reducing balance.

i. How much will Taylor and Curtis have to pay each month if they want to pay off the loan in three years? Give your answer to the nearest whole number of dollars.

_____

_____

1 mark

ii. Curtis and Taylor have decided that they can only afford to pay the loan back at a rate of $1150 per month. How long will it take them to pay back the loan at this rate? Give your answer in years and months.

_____

_____

1 mark

Module 4: Business-related mathematics – Question 4 - continued

c. Mrs Board who is Curtis' mother has offered to give Curtis and Taylor $8000 towards their upgrade. This means that they will only need to borrow $42000 either from Mr Kent or the bank.

i. Calculate how much more they will have to pay back the bank over a three year loan when compared with the total amount they will have to pay Mr Kent over the three years. Give your answer to the nearest whole number of dollars.

_____

_____

_____

_____

_____

ii. Curtis and Taylor have reluctantly decided to take out the bank loan for $42000. Paying it back at a rate of $1150 per month at a reducing balance interest rate of 9%. How many payments will they make until they owe $5000 or less?

_____

_____

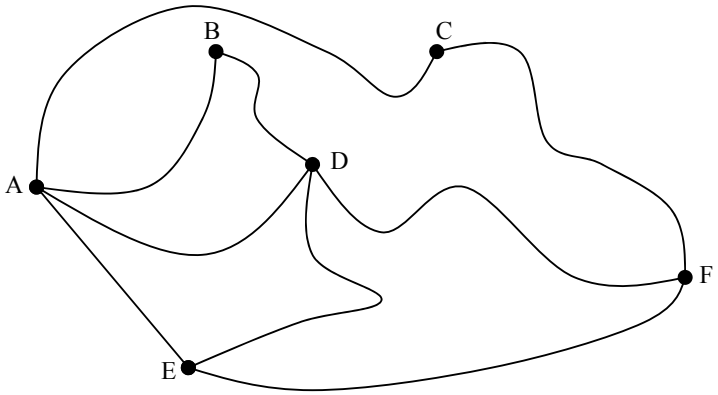
$3+1=4$ marks
Total 15 marks

END OF MODULE 4
TURN OVER

Module 5: Networks and decision mathematics

Question 1

The dog training yard at the Curtail boarding kennels has six features. They are connected with various paths as shown in the diagram.



Taylor Kent's dog Butcher is trained to travel along each path once only.

a. List the features in order that Butcher must visit so that he travels once along every path.

_____

1 mark

b. How many times does Butcher visit each of the six features on an Euler path of the dog training yard?

A	B	C
D	E	F

6 x 0.5 = 3 marks

c. Give the adjacency matrix for this network.

_____

_____

_____

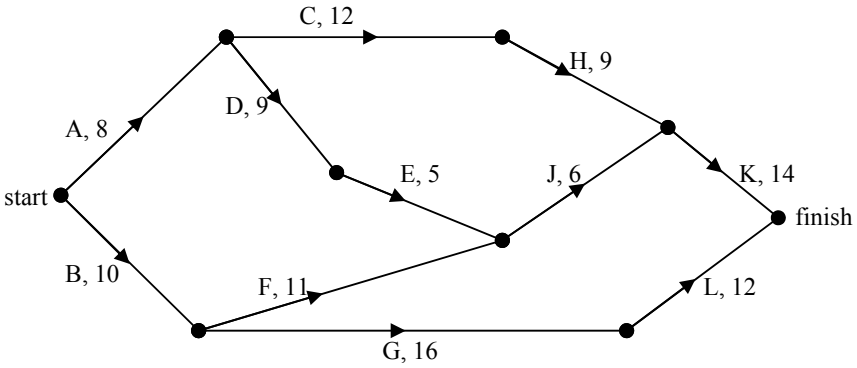
_____

_____

_____

1 mark

Question 2
Curtis Board and Taylor Kent have taken out a loan to upgrade the Curtail boarding kennels. They have constructed a network diagram for this project. All times shown are in days.



a. List all the activities which must be completed before activity *J* can commence.

_____

1 mark

b. Perform a forward scan to find the minimum project time.

_____

_____

_____

_____

_____

_____

_____

_____

_____

1 mark

Module 5: Networks and decision mathematics – Question 2 – continued
TURN OVER

c. What is the latest time in which activity *F* can begin?

_____

_____

_____

_____

1 mark

d. By how long can activity *G* be delayed without affecting the overall project time?

_____

_____

_____

1 mark

e. Due to bad weather activity *D* was delayed by 4 days. Calculate the minimum project time as caused by this delay.

_____

_____

_____

1 mark

Question 3

Taylor Kent has employed four new dog handlers to help with the new kennels
They are Al Sation, B Eagle, Col Lee and D L Mation. Over the last week the four new staff have each been tackling four different tasks: cleaning kennels, exercising the dogs, feed the dogs and working in reception. Taylor has ranked each one out of 10 for their performance in each task.

	Cleaning	**Exercising**	**Feeding**	**Reception**
Al Sation	6	7	4	3
B Eagle	8	7	5	2
Col Lee	8	8	6	4
D L Mation	7	8	5	5

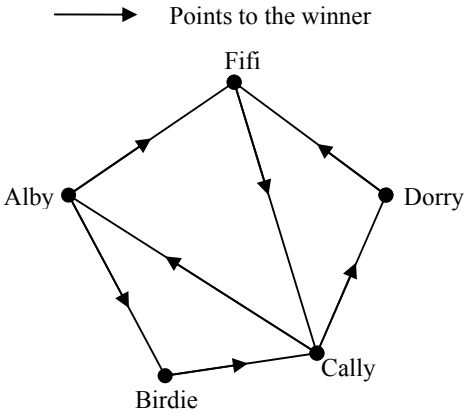
Find the task that Taylor should give each person so that they give a maximum performance according to their ratings. Show all your working.

3 marks

Module 5: Networks and decision mathematics- continued
TURN OVER

Question 4

Taylor is hosting a dog obedience competition on her new dog training course. Different pairs of dogs compete on the training ground and are rated to see which of the pair is better at completing the course. Five dogs, Alby, Birdie, Cally, Dorry and Fifi have competed and the results have been shown in a graph.



a. Show the dominance matrix **D** for this competition.

_____

_____

_____

_____

_____

1 mark

b. By finding $\mathbf{D} + \mathbf{D}^2$ rank the five dogs from best to worst.

_____

_____

_____

_____

_____

_____

1 mark

Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1
Curtis Board is stocktaking the medicines available for sale in his veterinary practice. He has noticed that he has purchased worming tablets and flea powders from the same three manufacturers. He has recorded his results in a matrix W.

$$W = \begin{array}{c} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 5 & 2 & 8 \\ 6 & 1 & 12 \end{bmatrix} \end{array} \begin{matrix} tablets \\ powder \end{matrix}$$

a. What is the order of matrix W?

_____

1 mark

b. Curtis has also constructed a price matrix P which shows how much he charges per packet for each product.

$$P = \begin{array}{c} \begin{matrix} Tablets & Powder \end{matrix} \\ \begin{bmatrix} \$5.40 & \$3.25 \\ \$6.90 & \$2.50 \\ \$12.75 & \$4.50 \end{bmatrix} \end{array} \begin{matrix} A \\ B \\ C \end{matrix}$$

Why did Curtis write this matrix round the other way?

_____

1 mark

c. Find the matrix product WP. Show all your working.

_____

_____

_____

_____

_____

_____

_____

2 marks

d. Curtis is only interested in two numbers from *WP*. Choose the correct two numbers to find the total amount of money Curtis expects to earn from the sale of all these products.

_____

1 mark

Question 2

During the summer months, the Curtail boarding kennels offer three types of accommodation for their canine customers: Luxury (l), Pampered (p) and Regular (r).

The numbers of customers for each type of accommodation during the first three weeks of summer, together with the total income earned from them is given in the three equations:

$$l + 5p + 3r = 1510$$
$$2l + p + 6r = 1400$$
$$p + 3r = 540$$

a. Write the equations in matrix form using the following template.

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

1 mark

b. Find the determinant of the coefficient matrix.

_____

_____

1 mark

Module 6: Matrices – Question 2 - continued

c. Write down an inverse that may be used to solve this equation. Give your answer in fraction form.

1 mark

d. Calculate the amount that the Curtail boarding kennels charge per week for each of their three types of canine accommodation.

1 mark

Question 3
Curtis is looking at the work of his veterinary practice. He has noticed the following information about the animals he sees regularly.

92% of the healthy animals that he sees in one month are healthy the next month. 7.5% of healthy animals are ill the following month and unfortunately 0.5% will probably die in the next month.

Fortunately 75% of ill animals are healthy in the next month but 5 % will probably die in the next month.

It is pleasing to note that of the animals that Curtis predicts are likely to die within the month, 30% are likely to be ill the following month, rather than die. Unfortunately, none are likely to completely recover

a. Enter this information into a transition matrix with percentages entered as proportions. (That is 60% becomes 0.6.)

$$T = \begin{array}{c} \begin{matrix} \textit{Healthy} & \textit{Ill} & \textit{Dying} \end{matrix} \\ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array} \begin{matrix} \textit{Healthy} \\ \textit{Ill} \\ \textit{Dying} \end{matrix}$$

1 mark

b. Last month Curtis saw 40 healthy animals, 52 ill animals and 12 animals he thought were dying. Write this information as a column matrix P_0.

$$P_0 = \begin{bmatrix} \\ \\ \end{bmatrix} \begin{matrix} \textit{Healthy} \\ \textit{Ill} \\ \textit{Dying} \end{matrix}$$

1 mark

c. Calculate the number of animals that Curtis expects to be ill this month.

_____

_____

_____

1 mark

Module 6: Matrices – Question 3 - continued

d. Calculate the number of animals that Curtis expects to be ill in two months time. (Calculate P_3).

1 mark

e. By showing at least two calculations, calculate the long term proportions of the fitness of animals in Curtis' veterinary practice. Give your answer in percentages to two decimal places.

2 marks
Total 15 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2

2008 Trial Examination

SOLUTIONS

Core

Curtis Board and Taylor Kent run Curtail Boarding Kennels. They board dogs, cats and sometimes other pets for owners who are travelling and can't take their beloved animals with them.

Question 1

a.

$\bar{x} = \frac{5808}{12} = 484$

A1

b.

$z = \frac{x - \bar{x}}{s_x}$ $\quad x = z \times s_x + \bar{x} = -0.3 \times 222.2 + 484 = \$417.34$

Answer: \$417

M1

c. 2004 data a shows positive skew

A1

d.

$IQR = 32 - 26 = 6$

$1.5 \times IQR = 1.5 \times 6 = 9$

$Q_1 - 1.5 \times IQR = 26 - 9 = 17$

The point is less than $Q_1 - 1.5IQR$ so the point at 2 is an outlier.

M1

e. $r = 0.92$ This is a strong positive correlation.

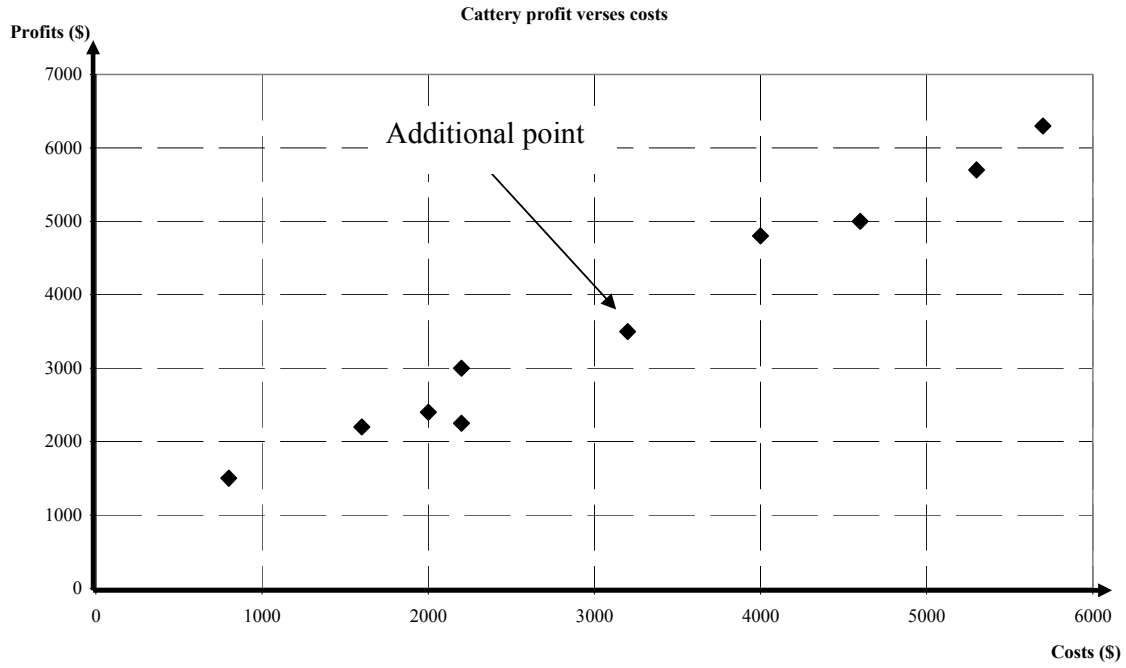
A1

f. $C = -893.56 + 304.78n = -893.56 + 304.78 \times 22 = 5811.6$

Answer: \$5800

M1

g.



A1

h. $C = \dfrac{P - 533.8}{0.99} = \dfrac{4000 - 533.8}{0.99} = 3501.21$

Answer: \$3500

M1

i. $P = 533.8 + 0.99C = 533.8 + 0.99 \times 500 = 1028.80$

Answer: \$1000

M1

j. This is extrapolation as a cost of \$500 is less than any of the set data.

A1

k. There is a strong positive linear association.

A1

l. $r^2 = 98\%$ A1

m. $\dfrac{2400 + 3400 + 5900}{3} = \dfrac{11700}{3} = 3900$

1	2	3	4	5	6	7	8	9	10	11	12
July '05	Aug '05	Sept '05	Oct '05	Nov '05	Dec '05	Jan '06	Feb '06	Mar '06	Apr '06	May '06	June '06
\$6500	\$4500	\$2800	\$500	\$2400	\$3400	\$5900	\$11400	\$5800	\$7700	\$600	\$4300
	\$4600	\$2600	\$1900	\$2100	\$3900	\$6900	\$7700	\$8300	\$4700	\$4200	

M1

n. The data shows a moderate positive trend r = 0.51.
Also accept as an answer: The data shows a seasonal trend.

A1

o. The data for February '06 is an outlier and median smoothing deals better with outliers than mean smoothing.

A1
Total 15 marks

Module 1: Number patterns

Question 1

a. This is a geometric sequence with $a = 2$ and $r = 3$

$$t_n = 2 \times 3^{n-1}$$

A1

b.

$10 \times 60 = 600$ seconds

$2 \times 3^{n-1} \geq 600 \qquad 3^{n-1} \geq 300$

$(n-1)\log 3 \geq \log 300 \qquad n - 1 \geq \dfrac{\log 300}{\log 3}$

$n - 1 \geq 5.19 \qquad n \geq 6.19$

Bubo will be able to demonstrate that he can stay in place for 10 minutes or more by day 7.

M1

c. $S_7 = \dfrac{2(3^7 - 1)}{3 - 1} = 2186$ seconds

M1

Question 2

a. The pattern is

1, **1**, 1 + 2 = **3**, 3 + 2 = **5**, 5 + 6 = **11**, 11 + 10 = **21**

So the number of puppies that turn up in week 6 is 21.

A1

b.

$t_7 = 2 \times 30 - 31 = 29 \qquad t_8 = 2 \times 29 - 31 = 27$

$t_9 = 2 \times 27 - 31 = 23 \qquad t_{10} = 2 \times 23 - 31 = 15$

$t_{11} = 2 \times 15 - 31 = -1$

So there will be no puppies in the school by week 11

A1

Question 3

a. $R = 1 + \dfrac{6}{1200} = 1.005$

$A = 12000 \times 1.005^n$

M1

b. $I = A - 12000 = 12000 \times 1.005^6 - 12000$

$= 12364.53 - 12000 = \$364.53$

M1

c. $1.005^n = \dfrac{15000}{12000} = 1.25 \qquad n = \dfrac{\log 1.25}{\log 1.005} = 44.7$

Taylor will need 45 months until she has over \$15000

M1

Question 4

a. $a = 250 \qquad r = 1 - 0.125 = 0.875$

Number destroyed $= 250 - 250 \times 0.875^3 = 82.51$

The number of fleas destroyed after three weeks would be 83.

A1

b.

i. $t_{13} = 250 \times 0.875^{13} = 44.06;$ ie 44 fleas

A1

ii. $250 \times 0.875^n \le 20 \qquad 0.875^n \le \dfrac{20}{250} \le 0.08$

$n \ge \dfrac{\log 0.08}{\log 0.875} \ge 18.9$

There will be fewer than 20 fleas annoying Shep after week 19. This is an additional 6 weeks of treatment.

M1

Question 5

a. $a = 5 \;\; d = 1.5$

$t_8 = a + 7d = 5 + 7 \times 1.5 = 15.5$ boxes

A1

b. $S_{12} = \dfrac{12}{2}(2 \times 5 + 11 \times 1.5) = 6 \times 26.5 = 159$

A1

Question 6

a. $a = 2 = t_1 \;\; d = 8 \;\; n = \dfrac{360}{20} = 18$ intervals , 19 terms required

$S_{19} = 9.5(4 + 18 \times 8) = 1406$

A1

b. $a = 2, r = \dfrac{32}{8} = \dfrac{8}{2} = 4$

$$2000 = \frac{2(4^n - 1)}{4 - 1}$$

$2000 = 4^n - 1 = 2000 \times 1.5 = 3000$

$$n = \frac{\log 3001}{\log 4} = 5.8$$

6^{th} term

5 lots of 20 days in between 6 terms

You will need 5 lots of 20 days = 100 days

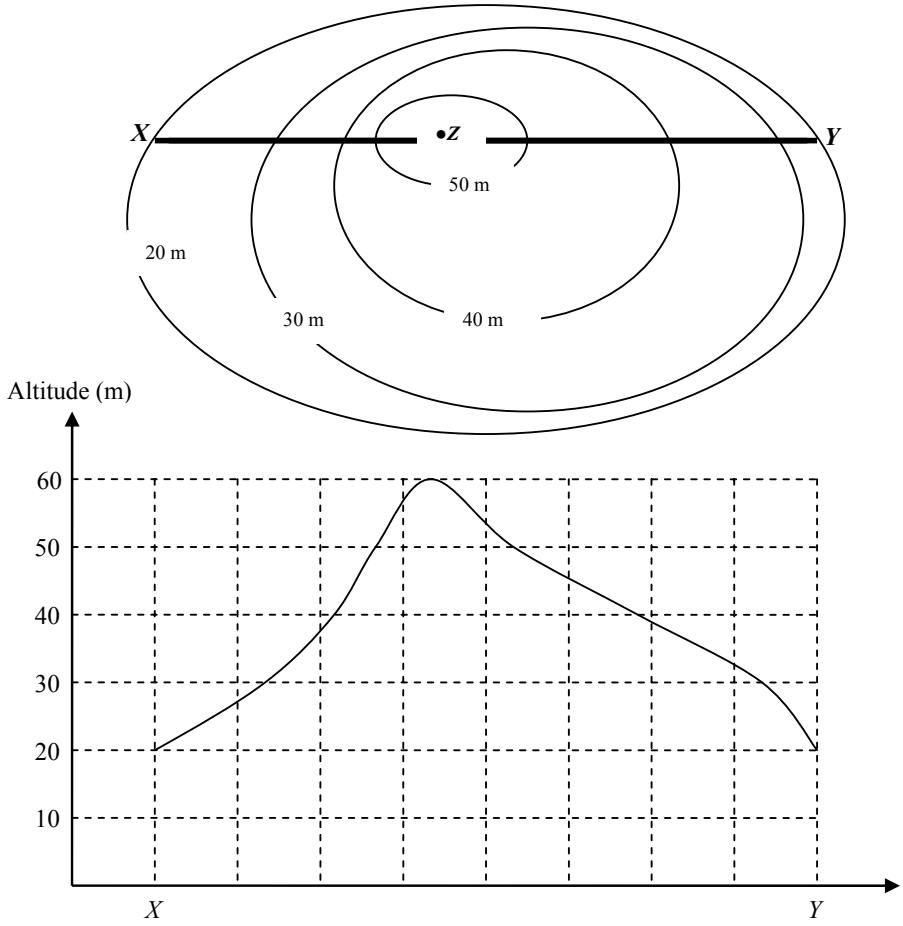
M1

Total 15 marks

Module 2: Geometry and trigonometry

Question 1

a.



M1

b.

1 cm = 2000 cm = 20 m

3.5 cm = 70 m

4.5 cm = 90 m

$XZ = \sqrt{70^2 + 40^2} = 80.63$ m

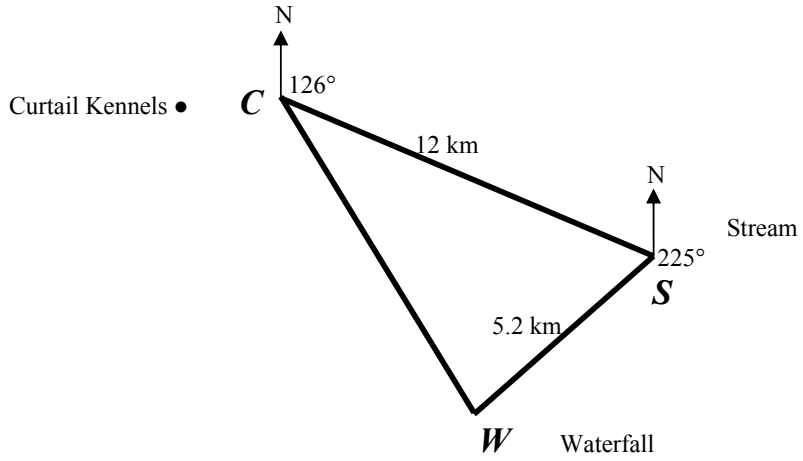
$ZY = \sqrt{90^2 + 40^2} = 98.49$ m

M1

The distance from X to Y via Z to the nearest metre is 179 metres.

A1

Question 2



a.

$\angle S = (360 - 225) - (180 - 126) = 81^\circ$

$CW = \sqrt{12^2 + 5.2^2 - 2 \times 12 \times 5.2 \cos 81} = 12.31 \text{ km}$

The distance is 12.3 km

M1

b. $\angle C = \sin^{-1}\left(\dfrac{5.2 \sin 81}{12.31}\right) = 24.66^\circ$

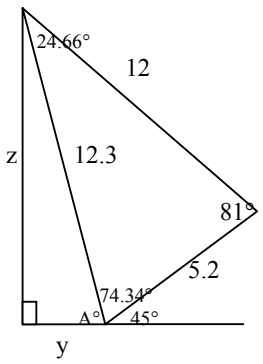
The bearing of the waterfall from the kennels is $126 + 24.66 = 150.66^\circ$

The bearing of the kennels from the waterfall is $360 - (180 - 150.66) = 330.66$

Answer is 331°

M1

c.



$\angle W = 180 - 81 - 24.66 = 74.34^\circ$

$A^\circ = 180 - 74.34 - 45 = 60.66^\circ$

$y = 12.31 \cos 60.66 = 6.03 \text{ km}$

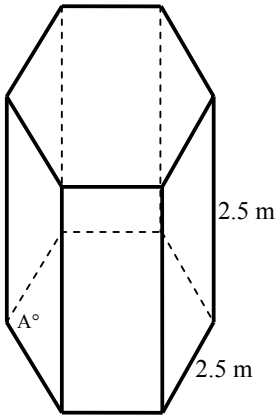
Answer: They travelled 6.0 km west.

A1

d. Total distance = $12 + 5.2 + 6.03 + 12.31 \sin 60.66 = 33.96 \approx 34.0 \text{ km}$

A1

Question 3



a.

$$A = 180 - \frac{360}{6}$$
$$= 180 - 60$$
$$= 120^\circ$$

A1

b. Using Heron's formula

$$s = \frac{2.5 \times 3}{2} = 3.75$$
$$Area = 6\sqrt{3.75 \times 1.25^3} = 16.238 \text{ m}^2$$

or Use $Area = \frac{1}{2}bc\sin A$

$$A = 0.5 \times 2.5 \times 2.5 \times \sin 60$$
$$\text{Area} = 16.238 \text{ m}^2$$

Answer to two decimal places = 16.24 m^2

M1

c. The total surface area consists of two hexagons and six squares.

$$TSA = 2 \times 16.24 + 6 \times 2.5^2$$
$$= 69.98 \text{ m}^2$$

A1

d.

Volume = Area of the base X height

$$V = 16.24 \times 2.5$$
$$= 40.60 \text{ m}^3$$

A1

e.

$$TSA_{scale} = \frac{1}{10^2} TSA_{full\ size}$$

$$= \frac{69.98}{100} = 0.6998 \text{ m}^2$$

$$= 6.998 \times 100^2 = 6998 \text{ cm}^2$$

A1

f.

$$V_{scale\ \ pyramid} = \frac{1}{10^3} \times \frac{1}{3} \times V_{prism}$$

$$V_{scale} = \frac{40.6}{3000} = 0.013533 \text{ m}^3$$

$$= 0.013533 \times 100^3 = 13533 \text{ cm}^3$$

A1

Question 4

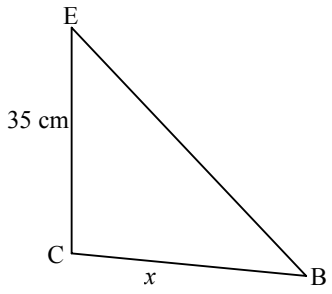
a.

$$\angle FBD = \tan^{-1}\left(\frac{35}{75}\right)$$

$$= 25^\circ$$

A1

b.



$$x = \sqrt{75^2 + 50^2} = 90.14$$

$$\angle EBC = \tan^{-1}\frac{35}{90.14} = 21.22^\circ$$

Answer: 21°

M1

Module 3: Graphs and relations

Question 1

a. Charge is \$120

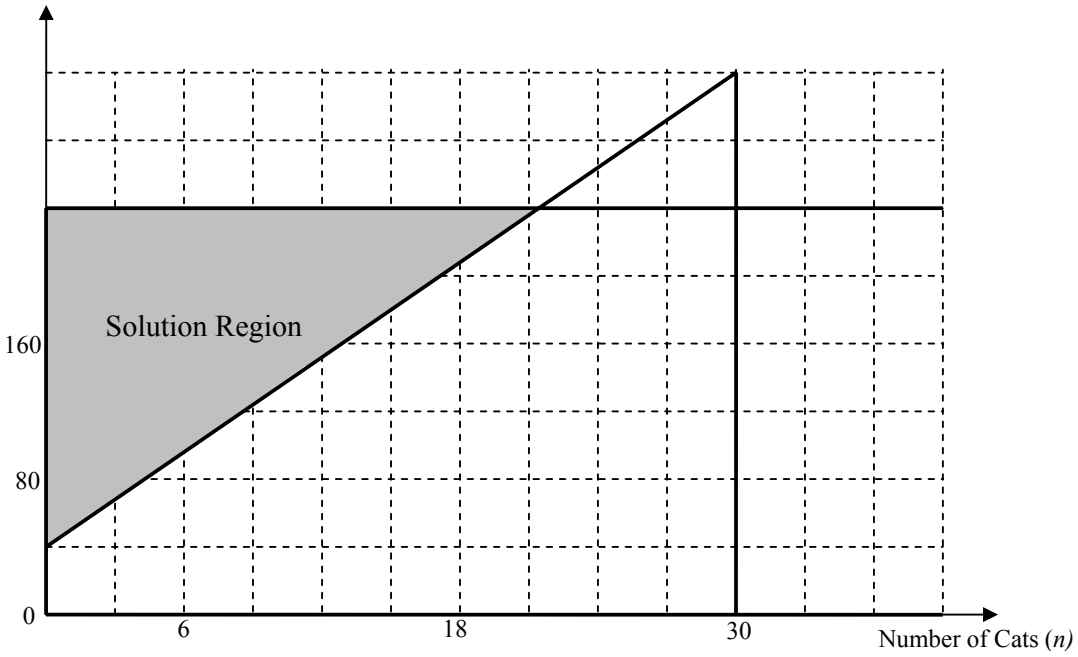
A1

b. Discount = $4 \times 60 - 100 = \$140$

A1

Question 2

a.



Inequation A1
Region A1

b.

$8n + 40 \le 240$

$8n \le 200$ The maximum number of cats is 25.

$n \le 25$

A1

c.

$t \ge 8 \times 30 + 40$ 30 cats need at least 280 minutes.

$t \ge 280$

A1

d.

$P = 28 \times 25 - 2 \times 240 = 220$

The maximum profit is \$220.

A1

Question 3

a. In 2 seconds the pendulum travels $4 \times 10 = 40cm$

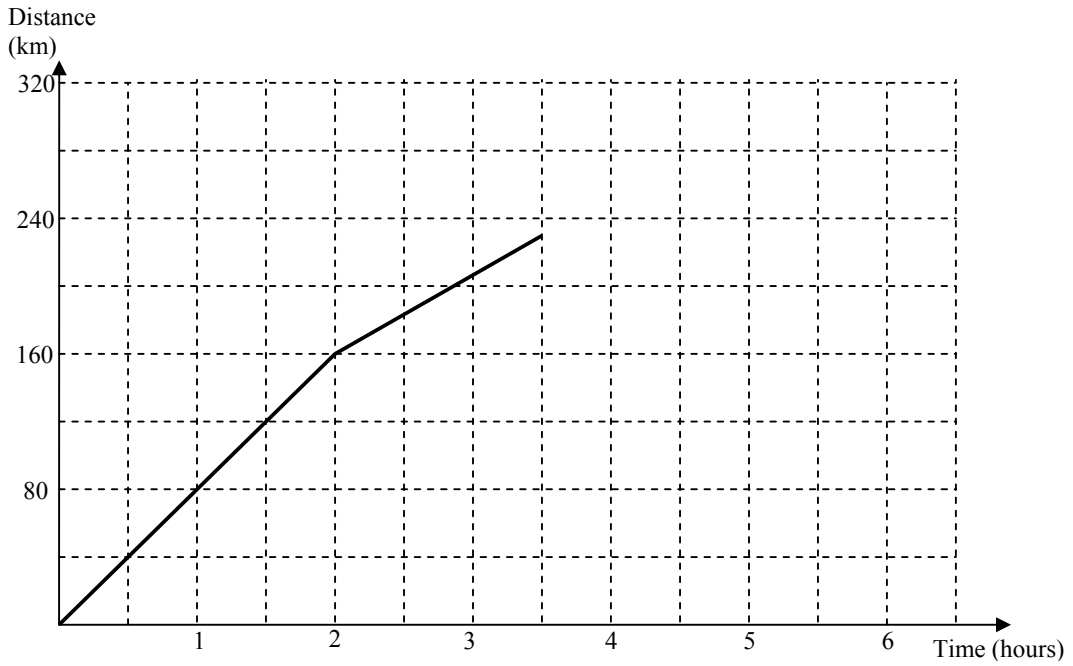
A1

b. Because of the repetitive nature of the pendulum the position at 7.5 seconds will be the same as the position at 1.5 seconds: 10 cm to the right (or -10 cm)

A1

Question 4

a. The Cosset's journey.



(1 mark for each part of the journey) A2

b.

$$m = \frac{75}{1.5} = 50 \quad km/h$$

A1

c.

$m = 50$ (2,160)
$d - 160 = 50\,(t - 2)$
$d = 50\,t + 60$

M1

d.

160 + 75 = 235 km

A1

e.

$$speed = \frac{d}{t} = \frac{235}{3.5} = 67.14 \quad km/h$$

M1
Total 15 marks

Module 4: Business-related mathematics

Question 1

a. $SP = 85.4 \times (1 - 0.125) = 85.4 \times 0.875 = 74.725$
The Sales Price should be \$74.73

A1

b. $Original \quad \Pr ice = \dfrac{49.98}{0.875} = 57.12$
The Original Price of the gold fish food was \$57.12

A1

c. $SP = 4.25 \times 1.3 \times 1.1 \times 0.875 = 5.3178$
The Selling Price should be \$5.32

A1

Question 2

a. Minimum monthly balance: Jan 2052, Feb 1828, Mar 1210 Therefore Taylor earned the most interest in January.

A1

b. $I = 0.003(2052 + 1828 + 1210) = 15.27$

M1

Question 3

a. $book \quad value = 12500 \times 0.85^3 = 7676.5625$
After three years the value of the system is \$7677.

M1

b. $12500 \times 0.85^n \leq 5000$
$0.85^n \leq 0.4$
$n \geq \dfrac{\log 0.4}{\log 0.85} \geq 5.64$ The computer system will be scrapped after 6 years.

OR using TVM solver:

N = 0
I = -15
PV = -12500
PMT = 0
FV = 5000
P/Y = C/Y = 1
N = 5.638

M1

Question 4

a.

i. $A = 50000 \times 1.12 = \$56000$

The amount they have to pay back is \$56000

A1

ii. Monthly repayments $= \dfrac{56000}{36} = \$1555.56$

They must pay back \$1556 per month.

A1

b.

i. Using the TVM solver

N = 36 I% = 9 PV = 50000 FV = 0 P/Y = 12 C/Y = 12 PMT End

PMT = -1589.99

They will have to pay back \$1590 per month.

M1

ii. Using the TVM solver

I% = 9 PV = 50000 FV = 0 PMT = -1150 P/Y = 12 C/Y = 12 PMT End

N = 52.82

They will have 4 years and 5 months to pay back the loan.

M1

c.

i. Payment to Mr Kent = $42000 \times 1.12 = \$47040$

A1

Payment to the bank: Use the TVM solver to work out the monthly payment over 3 years.

N = 36 I% = 9 PV = 42000 FV = 0 P/Y = 12 C/Y = 12 PMT End

PMT = -1335.59 Total Payment to the bank = $36 \times 1335.59 = 48081.24$ M1

Taylor and Curtis will pay the bank \$1041 more than to Mr Kent.

A1

ii. Using the TVM solver

I% = 9 PV = 42000 FV = -5000 PMT = -1150 P/Y = 12 C/Y = 12 PMT End

N = 38.40

In 39 payments they will have reduced their loan to under \$5000. M1

Total 15 marks

Module 5: Networks and decision mathematics Solutions

Question 1

a. There are many possible answers, but all must begin at *E* (or *F*) and end at *F* (or *E*).

One answer is:

EDABDFCAEF

A1

b.

A 2	*B* 1	*C* 1
D 2	*E* 2	*F* 2

6 x 0.5 = A3

c.

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

A1

Question 2

a. A, B, D, E and F

A1

b.

There are four possible paths:

A C H K = 43 days

A D E J K = 42 days

B F J K = 41 days

B G L = 38 days

So the critical path is *A C H K* and the minimum project time is 43 days.

M1

c. 43 – 14 – 6 – 11 = day 12

M1

d. Delay time = latest start time – earliest start time = 15 – 10 = 5 days

A1

e. As *D* has a slack time of 1 day, the project has been delayed by 3 additional days making a minimum project time of 46 days.

M1

Question 3

To maximise the performance we must first organise a minimising matrix by taking each value away from 10.

$$\begin{bmatrix} 4 & 3 & 6 & 7 \\ 2 & 3 & 5 & 8 \\ 2 & 2 & 4 & 6 \\ 3 & 2 & 5 & 5 \end{bmatrix} \xrightarrow{row\ \ reduction} \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 2 & 4 \\ 1 & 0 & 3 & 3 \end{bmatrix}$$

M1

From here it is not possible to allocate the result

.

$$Column\ reduction \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

M1

From this result it is possible to allocate the results:

Al

Sation to exercising, B Eagle to cleaning, Col Lee to feeding and D L Mation to reception.

A1

Question 4

a. The dominance matrix **D** for this competition is:

$$\begin{array}{c} \\ \text{Loss} \end{array} \begin{array}{c} \text{Win} \\ \begin{array}{ccccc} A & B & C & D & F \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ F \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

A1

b.

$$\mathbf{D} + \mathbf{D}^2 = \begin{bmatrix} 0 & 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$ This gives a dominance vector of: $\begin{bmatrix} 4 \\ 3 \\ 5 \\ 2 \\ 3 \end{bmatrix}$

Order: *C, A, B* & *F, D*

This ranks the dogs from best to worst as:
Cally, Alby, Birdie and Fifi, Dorry.

M1

Total 15 marks

Module 6: Matrices Solutions

Question 1

a. 2×3

A1

b. The matrix was written this way so that it would be possible to perform matrix multiplication. The number of rows from P = the number of columns from W and vice versa.

[For full marks this answer must refer to numbers of rows and columns.]

A1

c.

$$\begin{bmatrix} 5 & 2 & 8 \\ 6 & 1 & 12 \end{bmatrix} \times \begin{bmatrix} 5.4 & 3.25 \\ 6.9 & 2.5 \\ 12.75 & 4.5 \end{bmatrix} = \begin{bmatrix} 5\times5.4+2\times6.9+8\times12.75 & 5\times3.25+2\times2.5+8\times4.5 \\ 6\times5.4+1\times6.9+12\times12.75 & 6\times3.25+1\times2.5+12\times4.5 \end{bmatrix}$$

$$= \begin{bmatrix} \$142.80 & \$57.25 \\ \$192.30 & \$76 \end{bmatrix}$$

M1 + A1

d. $Total = \$142.80 + \$76 = \$218.80$

A1

Question 2

a.

$$\begin{bmatrix} 1 & 5 & 3 \\ 2 & 1 & 6 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} l \\ p \\ r \end{bmatrix} = \begin{bmatrix} 1510 \\ 1400 \\ 540 \end{bmatrix}$$

A1

b.

$$\text{Determinant of matrix } \begin{bmatrix} 1 & 5 & 3 \\ 2 & 1 & 6 \\ 0 & 1 & 3 \end{bmatrix} \text{ is -27}$$

A1

c.

$$\frac{1}{27}\begin{bmatrix} 3 & 12 & -27 \\ 6 & -3 & 0 \\ -2 & 1 & 9 \end{bmatrix}$$

Also accept:

$$\begin{bmatrix} \frac{1}{9} & \frac{4}{9} & -1 \\ \frac{2}{9} & \frac{-1}{9} & 0 \\ \frac{-2}{27} & \frac{1}{27} & \frac{1}{3} \end{bmatrix}$$

A1

d.

$$\begin{bmatrix} l \\ p \\ r \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 3 & 12 & -27 \\ 6 & -3 & 0 \\ -2 & 1 & 9 \end{bmatrix}\begin{bmatrix} 1510 \\ 1400 \\ 540 \end{bmatrix} = \begin{bmatrix} 250 \\ 180 \\ 120 \end{bmatrix}$$

Luxury is charged at \$250 per week,
Pampered is charged at \$180 per week
Regular is charged at \$120 per week. M1

Question 3

a.

$$T = \begin{array}{c} \begin{matrix} \textit{Healthy} & \textit{Ill} & \textit{Dying} \end{matrix} \\ \begin{bmatrix} 0.92 & 0.75 & 0 \\ 0.075 & 0.2 & 0.3 \\ 0.005 & 0.05 & 0.7 \end{bmatrix} \end{array} \begin{matrix} \textit{Healthy} \\ \textit{Ill} \\ \textit{Dying} \end{matrix}$$

A1

b.

$$P_0 = \begin{bmatrix} 40 \\ 52 \\ 12 \end{bmatrix} \begin{matrix} \textit{Healthy} \\ \textit{Ill} \\ \textit{Dying} \end{matrix}$$

A1

c.

$$P_1 = TP_0 = \begin{bmatrix} 0.92\times 40 + 0.75\times 52 + 0\times 12 \\ 0.075\times 40 + 0.2\times 52 + 0.3\times 12 \\ 0.005\times 40 + 0.05\times 52 + 0.7\times 12 \end{bmatrix}$$

$= \begin{bmatrix} 75.8 \\ 17 \\ 11.2 \end{bmatrix}$ Curtis expects that 17 animals will be ill this month

M1

d. $P_3 = T^3P_0 = \begin{bmatrix} 85.22 \\ 11.40 \\ 7.38 \end{bmatrix}$ Curtis expects that 11 animals will be ill in two months time.

M1

e.

$$T_{40} = \begin{bmatrix} 0.8763 & 0.8763 & 0.8763 \\ 0.0935 & 0.0935 & 0.0935 \\ 0.0302 & 0.0302 & 0.0302 \end{bmatrix} \quad T_{50} = \begin{bmatrix} 0.8763 & 0.8763 & 0.8763 \\ 0.0935 & 0.0935 & 0.0935 \\ 0.0302 & 0.0302 & 0.0302 \end{bmatrix}$$

Thus 87.63 healthy, 9.35 ill and 3.02% dying.

M1 + A1

Total 15 marks

