The Mathematical Association of Victoria

FURTHER MATHEMATICS

Trial written examination 2

(Extended Answer)

2008

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ....

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2008 Further Mathematics Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

Published by The Mathematical Association of Victoria
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Working space

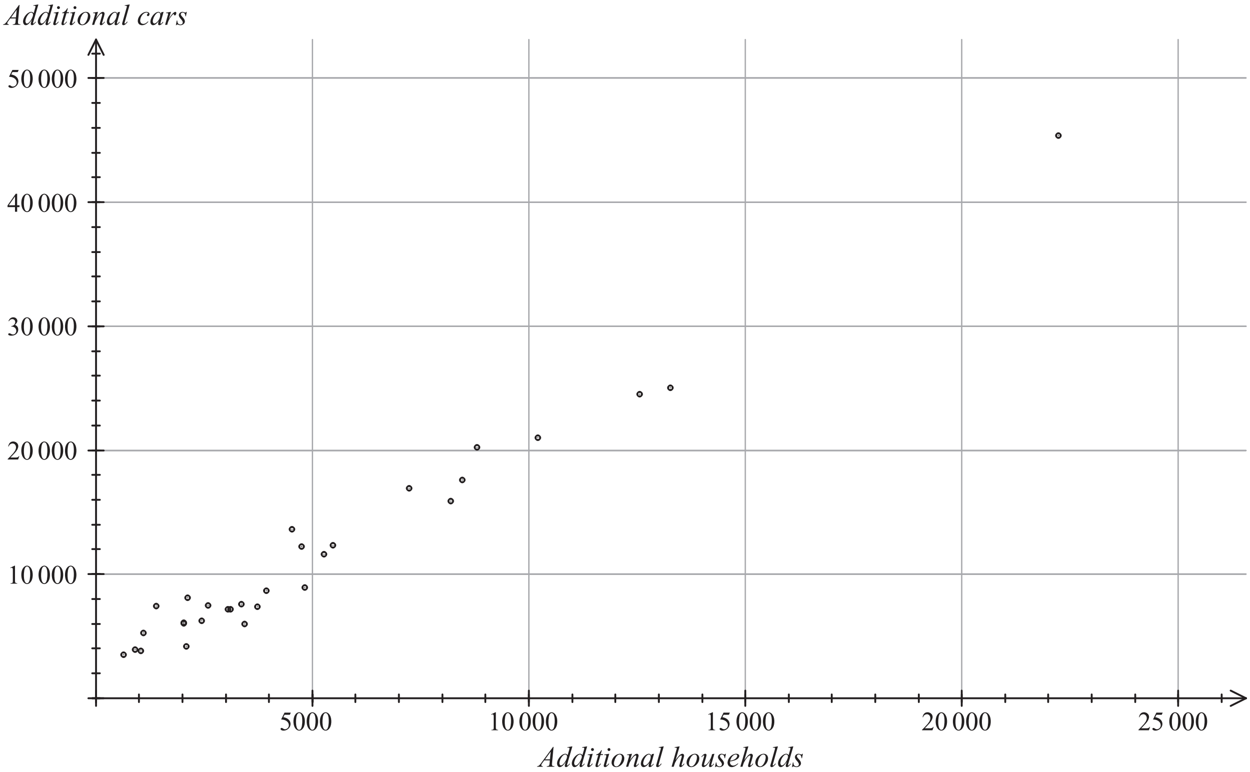
Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Core

A study, over the period 1996 – 2006, into the increase in households and car ownership for council areas in Melbourne, produced the following scatterplot.



Question 1

a. Use the scatterplot to describe the relationship between *additional households* and *additional cars* in terms of

i. direction

____

____

ii. strength

____

____

iii. form

____

____

1 + 1 + 1 = 3 marks

The least-squares regression line for the data, graphed above, has a slope of 1.87 and an intercept of 1966.

b. Write the equation for the least-squares regression line in terms of the variables, *additional households* and *additional cars*.

2 marks

c. Interpret the slope of the least-squares regression line.

1 mark

The data for Melbourne City has not been included on the graph above. In the City of Melbourne there were 15 786 *additional households* and 11 221 *additional cars*.

d. On the scatterplot above, plot the data point for the City of Melbourne. Label the point MC.

1 mark

e. Describe the effect that the inclusion of the data point for the City of Melbourne would have on the slope of the least squares regression line.

1 mark

The following information relates to Question 2

In 2006 there were 154 driver deaths on Victorian roads, of which young drivers, aged 18 to 25 years, were a significant proportion.
It is generally thought that a larger proportion of male driver deaths were young male driver deaths when compared to the proportion of female driver deaths that were young female driver deaths.

The following two-way frequency table gives the data for driver deaths on Victoria's roads for 2006 categorised by age group and gender.

Table 1

Driver deaths 2006		***Gender***		
		Male	Female	Total
Age Group	18 – 25years	26	9	35
	> 25 years	87	32	119
	Total	113	41	154

Question 2

a. Percentage **Table 1** by columns, by filling in the percentages in **Table 2** below. Round your figures to the nearest percentage.

Table 2

Driver deaths 2006		***Gender***	
		Male	Female
Age Group	18 – 25 years		
	> 25 years		
	Total	100	100

1 mark

b. Use the figures in **Table 2** to comment on the statement:
"It is generally thought that a larger proportion of male driver deaths were young male driver deaths when compared to the proportion of female driver deaths that were young female driver deaths."

1 mark

c. Use **Table 1** to calculate the overall percentage of driver deaths that were young driver deaths in 2006. Give your answer correct to the nearest percentage.

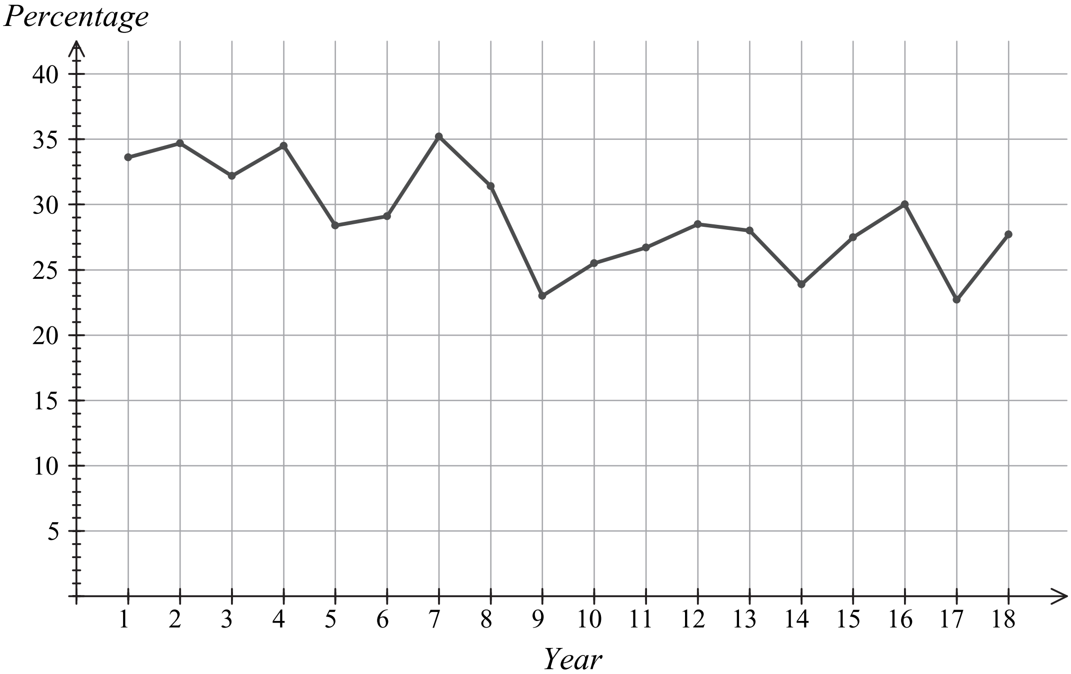
1 mark

d. In Victoria in 2006 young drivers were 15% of all the drivers. Comment on this statistic compared to your percentage found in part **c.**

1 mark

The following time-series plot shows the young driver deaths as a percentage of driver deaths, in Victoria, for 1990 to 2007 (Year 1 is 1990…. Year 18 is 2007)

Young driver deaths as a percentage of driver deaths



Question 3

a. Describe the time-series plot above in terms of trend.

1 mark

A regression line is fitted to the time-series data graphed above.
The equation of the line is

Percentage of driver deaths that are young drivers* = 33.7 – 0.49 × *Year

b. Use the regression line to predict the percentage of driver deaths that are young drivers in 2010.
Give your answer correct to 1 decimal place.

2 marks

Total 15 marks

END OF CORE

Module 1: Number patterns

It has been recorded that in 1995 the percentage of the Australian population with access to the internet was 7% but in 2007 the percentage had risen to 91%.

Question 1

a. If an arithmetic sequence was used to model the yearly change in internet access, calculate the yearly increase using $t_1 = 7$ and $t_{13} = 91$

1 mark

b. Use the arithmetic sequence model to predict the percentage of the population that would have internet access in 2009.

1 mark

A more feasible model for the yearly change in internet access is a geometric sequence.

Question 2

If the yearly change in internet access from 7% to 91% in 13 years followed a geometric sequence:

a. Write down an equation for the 13^{th} term of the sequence, in terms of the first term, $\boldsymbol{a} = 7$ and the common ratio, $\boldsymbol{r}$.

1 mark

b. Show that the common ratio, $\boldsymbol{r}$, for this geometric increase is 1.238, correct to three decimal places.

1 mark

c. Use the geometric sequence to predict the percentage of the population that would have internet access in 2008.

1 mark

Question 3

State a reason why neither an arithmetic nor a geometric sequence would be suitable to model the increase in internet access.

1 mark

A mathematician has suggested that a rule for a suitable sequence to model the internet access increase is the difference equation

$$t_{n+1} = 0.8t_n + 19.5;\ t_1 = 7$$

Question 4

a. Calculate the 2nd term of this sequence showing your working. Give your answer correct to one decimal place.

2 marks

b. Calculate t_{13} for this sequence giving your answer correct to two decimal places.

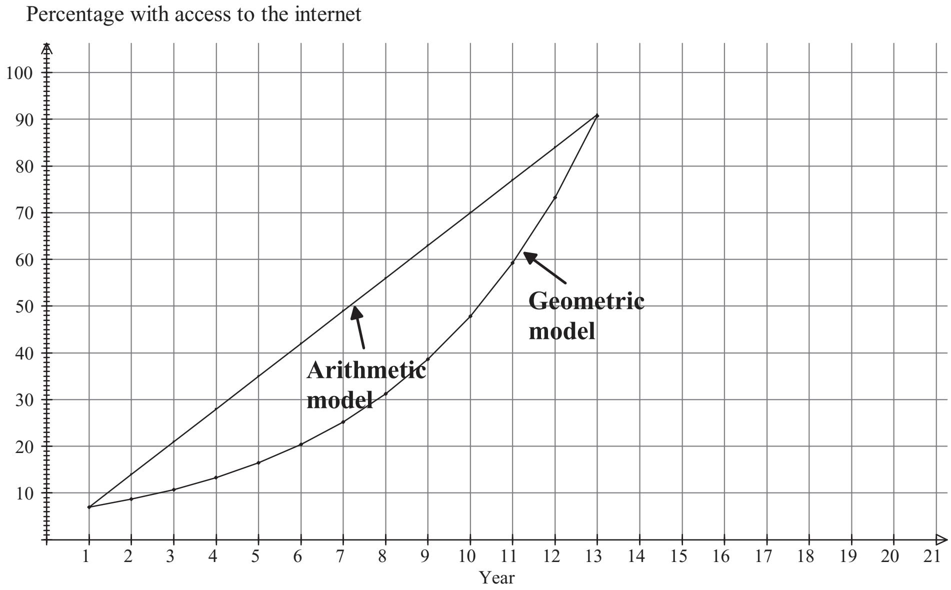
1 mark

c. Using this model state the maximum value possible for internet access in Australia.

1 mark

On the set of axes below, the terms for the increase in internet access over the 13 years for both the arithmetic and geometric models have been graphed.

d. On the same set of axes show a graph of the increase in internet access for the difference equation model for the 20 year period from 1995.



2 marks

An internet access provider is offering a good deal for internet access and is attracting new customers.
In the first week they attract 1800 new customers.
In the second week they attract 1620 new customers.
In the third week they attract 1458 new customers.

Question 5

a. Calculate the total number of new customers that they will have attracted after 10 weeks. Give your answer to the nearest whole number.

2 marks.

b. Calculate the maximum number of customers that they will attract in total if the same patern continues.

1 mark

Total 15 marks

END OF MODULE 1

Working space

Module 2: Geometry and trigonometry

In order to preserve an historical kiln it is proposed to construct a hexagonal glass pyramid over it, as shown in **Figure 1,** at right.

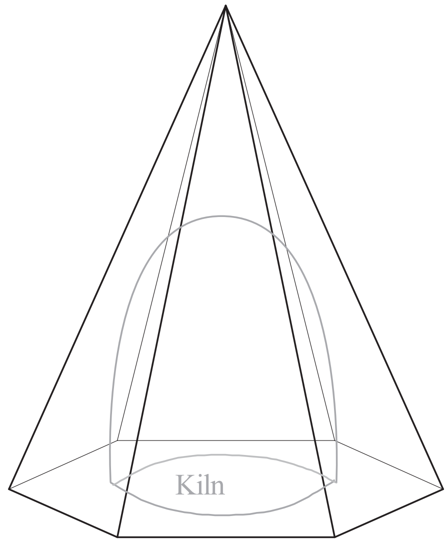
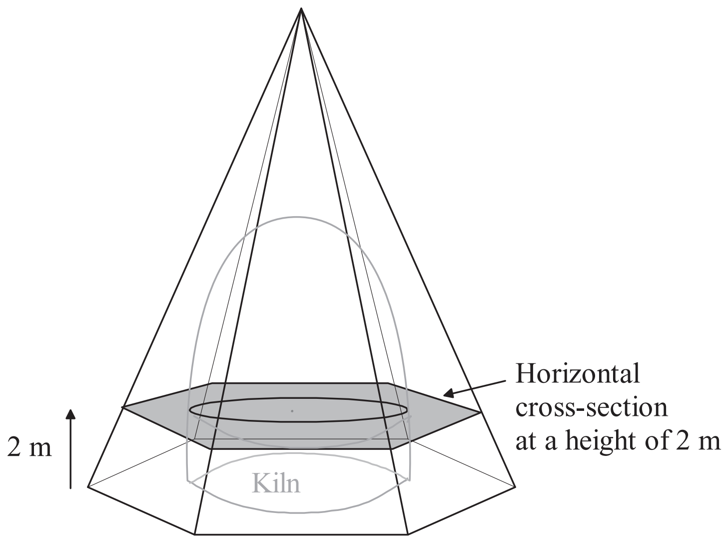


Figure 1

To find the measurements required for the pyramid it is decided that, at the height of two metres, the nearest point of the pyramid should be at a distance of two metres from the kiln.

A horizontal cross-section of the pyramid and kiln , **at the height of two metres**, is shown in **Figure 2** below.

The kiln has a circular cross-section with a radius of 2 m at this level and the cross-section of the pyramid is a regular hexagon.



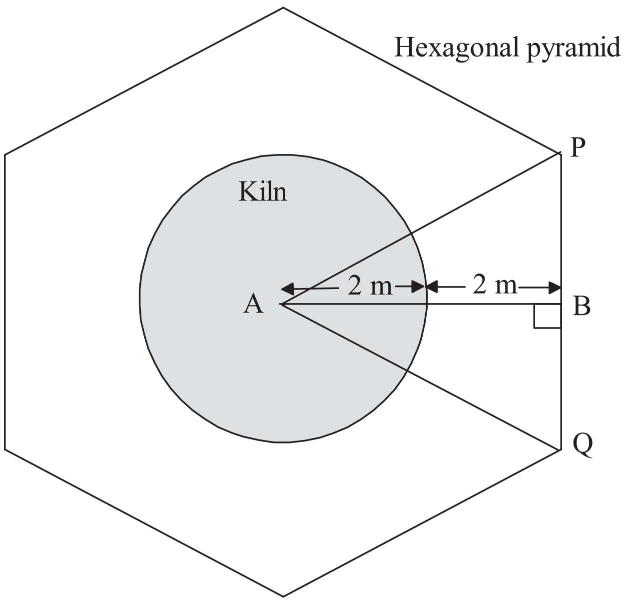


Figure 2

Question 1

a. Explain why the size of angle PAQ, in **Figure 2** above, is equal to 60°.

1 mark

b. Calculate the length PQ, giving your answer in metres correct to three decimal places.

2 marks

A vertical cross-section of the pyramid and kiln is shown in **Figure 3** below. The height of the pyramid is designed to be 10 metres.

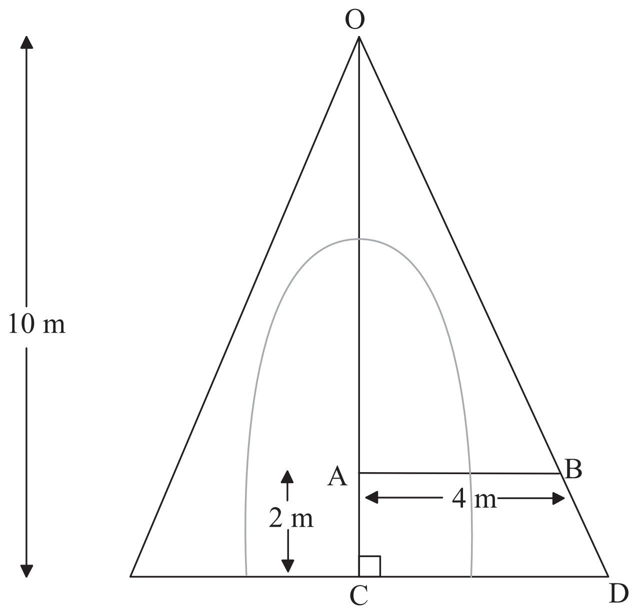


Figure 3

Question 2

a. Use similar triangles, or otherwise, to calculate the length CD shown on **Figure 3** above.

2 marks

A cross-section of the pyramid **at ground level** is shown in **Figure 4** below.

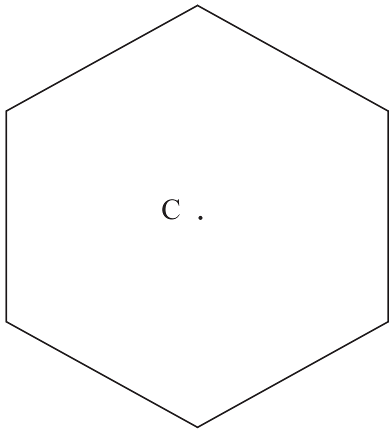


Figure 4

b. On **Figure 4** draw in a possible position for the line CD.

1 mark

c. Calculate the length of one of the sides of the regular hexagon in **Figure 4**. Give your answer correct to 3 decimal places.

1 mark

d. Calculate the area of the hexagon shown in **Figure 4** giving your answer in square metres correct to two decimal places.

2 marks

e. Calculate the area of one of the triangular glass faces of the pyramid, shown shaded in **Figure 5** below. Give your answer in square metres correct to two decimal places.

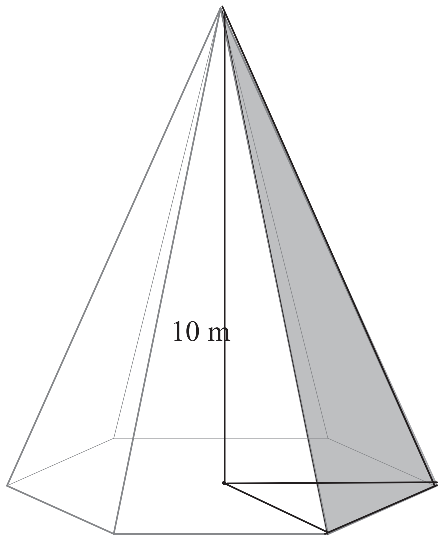


Figure 5

3 marks

Question 3

The kiln and the proposed covering are located in a triangular park, XYZ, as shown in **Figure 6** below.
The lengths of two of the park boundaries are known. XY = YZ = 70 m.
The angle YZX has magnitude 50°

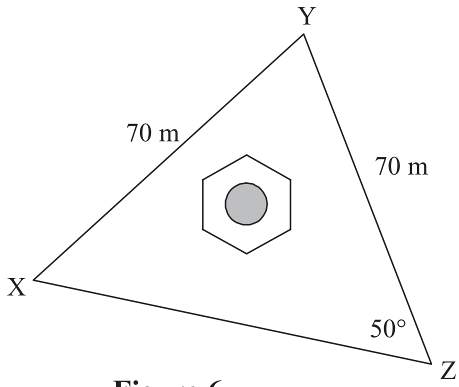


Figure 6

a. Calculate the length of the side XZ. Give your answer in metres correct to two decimal places.

2 marks

b. If the bearing of point Y from point X is 062° calculate the bearing of point Z from point X.

1 mark

Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1

An 18Watt compact fluorescent light globe (CFL globe) costs \$5.90 to buy and \$0.0027 per hour to run. An equivalent 100Watt incandescent light globe costs \$0.98 to buy and \$0.015 per hour to run.
The equation for the costs (\$$A$) involved in buying and running a CFL globe for x hours is

$$A = 5.90 + 0.0027x$$

a. Write down an equation that gives the cost (\$$B$) involved in buying and running an incandescent globe for x hours.

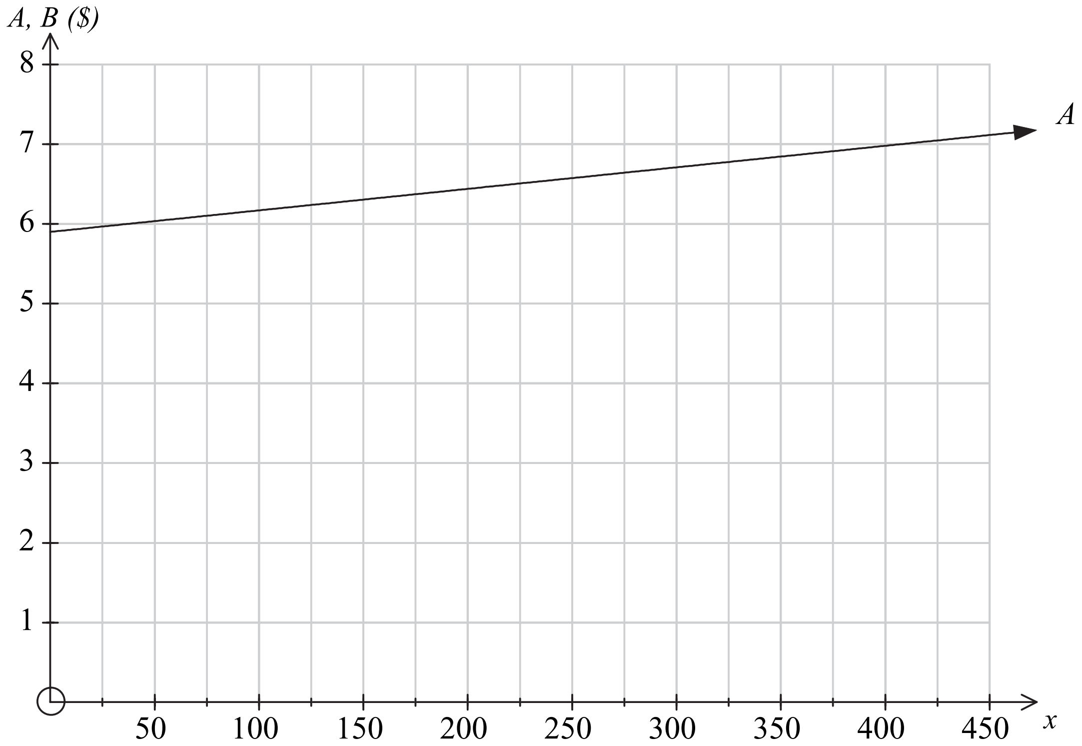
$B =$

1 mark

b. Calculate the number of hours before the cost will be the same for both types of globe.

2 marks

The equation $A = 5.90 + 0.0027x$ has been graphed on the set of axes below.



c. On the same set of axes graph your equation B from **part a**. Label the graph with the co-ordinates of the intercept(s) with the axes and the point of intersection with the graph of A.

2 marks

A CFL globe lasts for about 8000 hours but an incandescent globe only lasts for 1000 hours.

d. Calculate the total cost of buying a globe and running a light containing an 18Watt CFL globe for 8000 hours.

1 mark

e. Calculate the total cost of buying globes and running a light that contains a 100Watt incandescent globe for 8000 hours.

1 mark

A light globe manufacturer produces both incandescent light globes and CFL globes. The number and type of globe that are manufactured are subject to the following constraints.

i. The factory has the capacity to produce a maximum of 40 000 globes in one day.

ii. Demand on globes means that at least 10 000 of each type of globe need to be produced each day.

iii. To satisfy the changing demand for types of light globe the manufacturer has found that for every 3 incandescent globes that he makes he must produce at least 2 CFL globes.

Question 2

If x is the number of incandescent globes made each day and y is the number of CFL globes made each day

a. Write down, in terms of x and y, the constraint associated with statement **i** above.

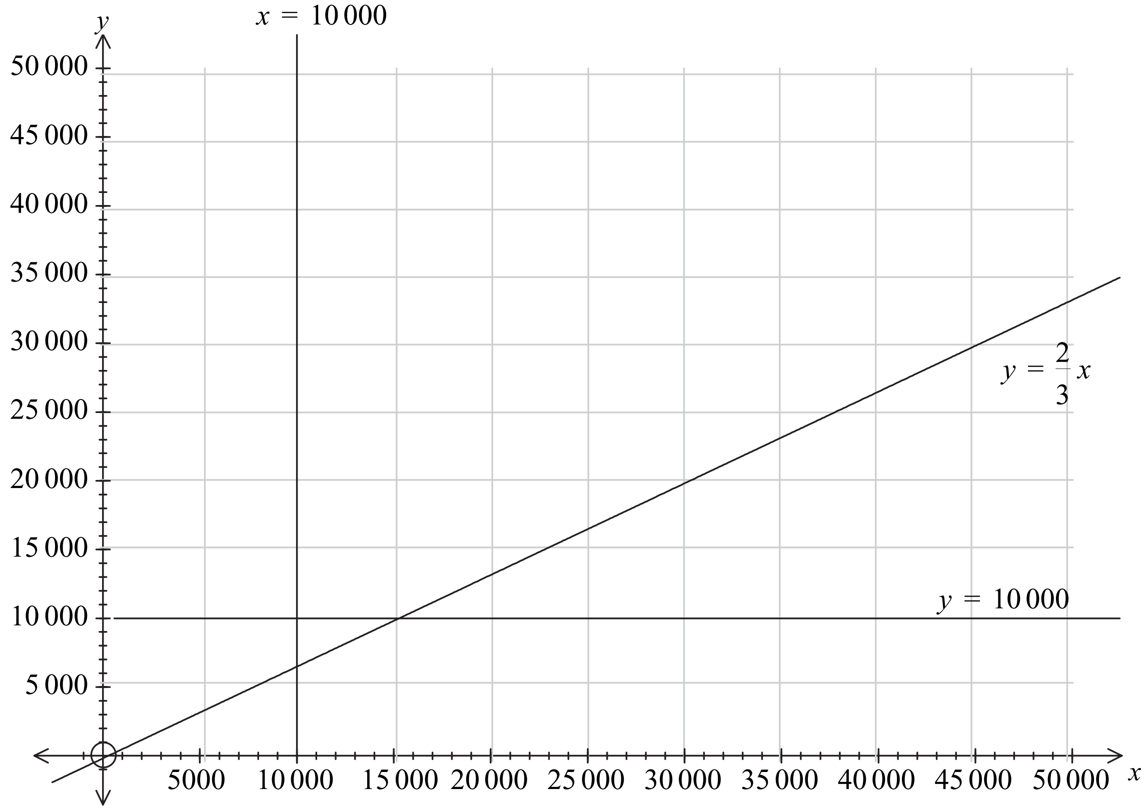
1 mark

The constraints associated with statements **ii.** and **iii**. are,

ii. $x \geq 10\,000$; $y \geq 10\,000$

iii. $y \geq \frac{2}{3}x$

The boundary lines for these constraints are graphed on the set of axes below:



b. On the same set of axes draw in the boundary line associated with the constraint that you have written down in part **a**.

1 mark

c. On the same set of axes indicate clearly the region that satisfies all the constraints.

1 mark

d. If the manufacturer makes 16 cents profit on an incandescent globe and 15 cents profit on a CFL globe write down an objective function for the profit, in dollars, made on selling the globes.

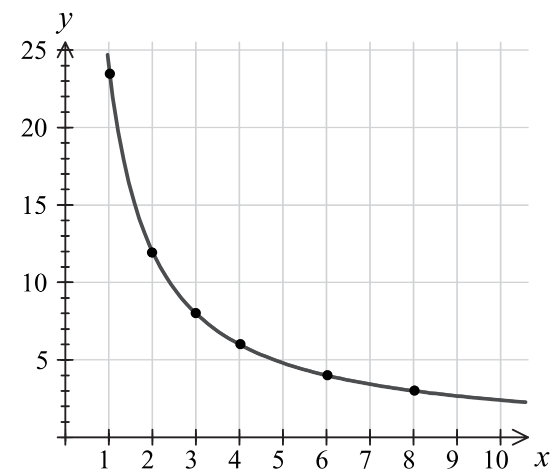
1 mark

e. Find the number of each type of globe that the manufacturer needs to make to maximise his profit.

2 marks

The Government is keen to phase-out the use of incandescent light globes so the manufacturer has produced the following table and graph that shows his proposed phase-out:

Year, ***x***	1	2	3	4	6	8
Number of incandescent globes made/week. (1000's), ***y***	24	12	8	6	4	3



Question 3

The model for the phase-out of incandescent globes is of the form $y = kx^n$ where k and n are constants.
Find the values of k and n

2 marks

Total 15 marks

END OF MODULE 3

Module 4: Business related mathematics

Crazy Auto has a used car for sale, priced at $12 500 including GST. It is available on hire purchase agreement with 24% deposit and equal monthly instalments over 3 years at 9.5% pa interest rate.

Question 1

a. Calculate the pre-GST (wholesale) price of the car to the nearest dollar.

1 mark

b. Determine the deposit amount to the nearest dollar.

1 mark

The stamp duty on used cars in Victoria is $8 per $200 for cars up to $35 000.

c. Calculate the stamp duty on the car.

1 mark

Scott is a young motorist who is able to pay in cash the deposit only and will have to borrow the rest including the stamp duty.

d. Calculate the amount of the loan.

1 mark

e. Calculate the monthly instalment to the nearest dollar.

3 marks

Back in 1994 the price of petrol was 85 cents per litre. Now in 2008 (fourteen years later) the price of petrol has increased to $1.45 per litre.

Question 2

Calculate the average percentage inflation rate for the past fourteen years for petrol.

3 marks

Question 3

Scott has taken up an apprenticeship and uses his car to carry his work tools to job sites. He is able to claim on his tax return the depreciation on his \$12 500 car. He chooses to use the reducing balance model which is currently set at 22½%pa.

a. Calculate the amount of depreciation to the nearest dollar in the first two years.

2 marks

b. How long will it take for the car to reach a scrap value of \$5000?

2 marks

c. Is it possible for the car to be written off (book value of zero dollars) if the reducing balance depreciation model is used? Justify your answer.

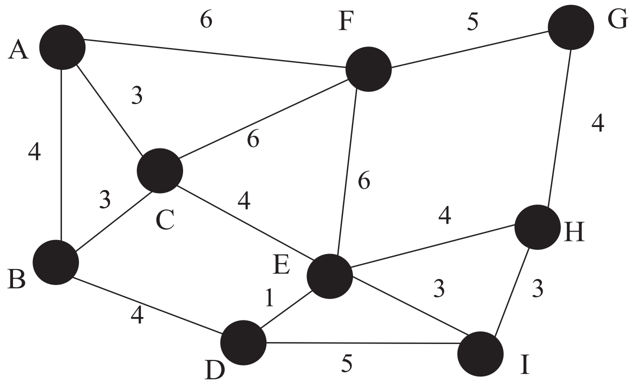
1 mark

Total 15 marks

END OF MODULE 4

Module 5: Networks & decision mathematics

A tour of 9 wineries has been organized. The diagram below shows the network of roads joining the wineries.



The distances indicated in the above figure are in kilometres.

Question 1

The tour starts at winery B and follows a Hamiltonian circuit.

a. Write down a possible order in which the wineries might be visited.

____

____

1 mark

b. Write down the length of the shortest route from winery A to winery I.

____

____

1 mark

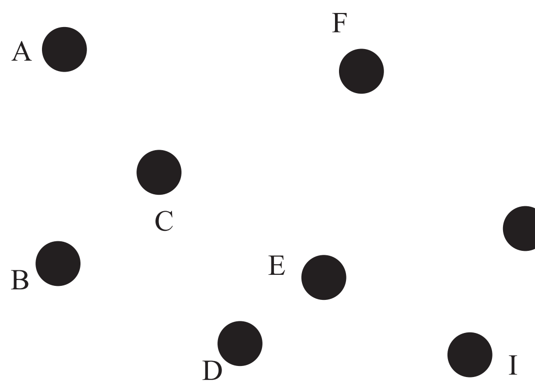
c. The 9 wineries share an ambulance service that is stationed at winery E. The roads connecting the wineries need to form a minimal-length spanning-tree. The purpose of this spanning tree is to map the shortest length of roads that need to be upgraded to a sealed surface from their current gravel surface.

i. State why the choice for the location of the ambulance service at winery E was the correct choice.

___

___

ii. On the diagram below, draw this minimal-length spanning-tree.



iii. What is the minimum length of roads that need to be sealed for the ambulances to travel on safely?

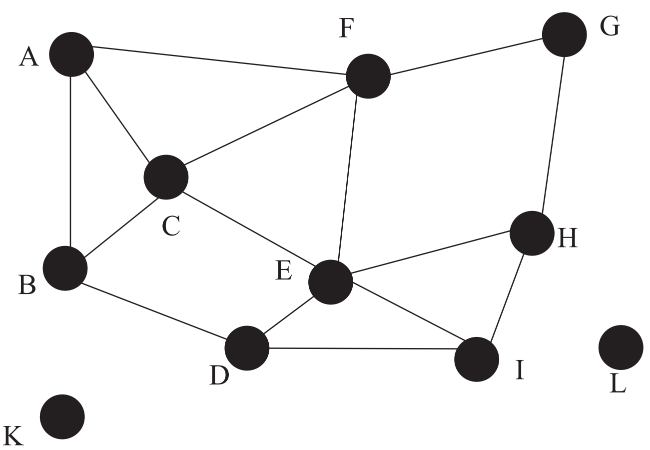
___

___

1 + 1 + 1 = 3 marks

d. The region wishes to expand the tourist industry by building another two wineries, K and L as shown in the diagram below. The construction of a number of new roads connecting the two new wineries is done so that an Euler path through the wine district can be established.

i. On the diagram below add the minimum number of new roads connecting the new wineries directly that would be required to establish an Euler path.



ii. State which wineries will need to be the start and/or finish of the winery tour.

___

___

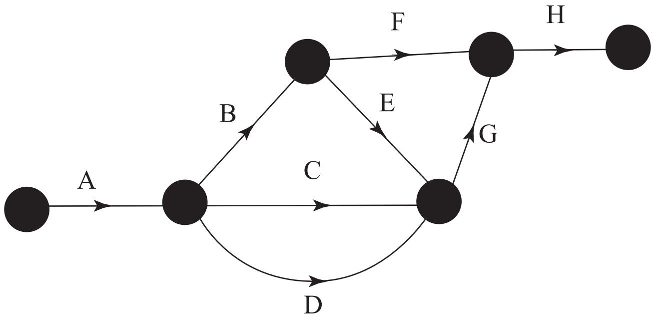
1 + 1 = 2 marks

Question 2

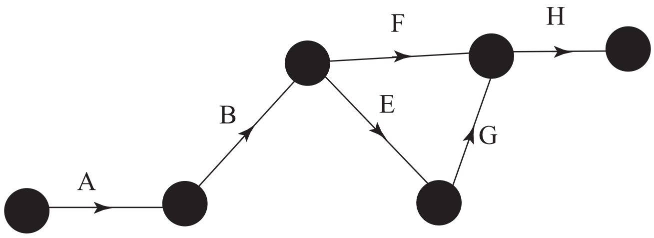
The construction of the cellars at winery K was a project involving 8 major activities. The table below gives the completion time and the immediate predecessors for each of these activities.

Activity	Immediate predecessor	Time taken to complete (weeks)
A	–	3
B	A	2
C	A	5
D	A	3
E	B	7
F	B	6
G	C,D,E	3
H	F,G	5

A directed graph that can be used to show the activities involved in the construction of the cellars is shown in the following diagram.



a. In the directed graph below add a dummy activity so that the parallel activities C and D can be redrawn for the purpose of network analysis.



2 marks

b. Complete the table below.

Activity	Earliest start time	Latest start time
A	0	0
B	3	3
C	3	7
D	3	9
E		5
F	5	
G		12
H	15	15

3 marks

c. Write down the critical path of the network.

1 mark

d. Write down the completion time for the project.

1 mark

e. Write down the slack (float time) for activity C.

1 mark

Total 15 marks

END OF MODULE 5

Module 6: Matrices

Two Australian Rules football teams both from the same Victorian country town have completed an 18 game season. The season results were

Eaglehawks: 10 wins, 1 draw and 7 losses
Magpiedemons: 8 wins, 2 draws and 8 losses

For a win they received 4 points, a draw 2 points and a loss 1 point.

Question 1

a. Represent the season results as a 2×3 matrix

1 mark

b. Represent the points awarded as a 3×1 matrix

1 mark

c. Using matrices, calculate the total premiership points for each team.

2 marks

Historical records of the two teams' first season in 1975 showed they played each other three times and the number of tickets sold for those matches was summarised as follows.

Match	Adults	Children	Pensioners	Total Match Takings $
Round 2	245	310	76	720.40
Round 9	120	44	0	311.00
Round 16	321	410	102	945.80

However no records of ticket prices were found.

Question 2

a. Three simultaneous equations were set up to find the ticket prices charged back in 1975. Complete the final two equations given

x = adult ticket prices

y = children ticket prices

z = pensioner ticket prices

First Equation: $245x + 310y + 76z = \$720.40$

Second Equation:

Third Equation:

2 marks

b. Represent the simultaneous equations as matrices and state the prices of the tickets in 1975.

3 marks

The town folks were constantly changing allegiance for their two local teams when clearly one team performed far better than the other team in a season. Over the years the probability of change was determined and summarised as follows.

If Eaglehawks was the better performing team

- 90% of Eaglehawks' supporters remained Eaglehawks' supporters the following season.
- 40% of Magpiedemons' supporters became Eaglehawks' supporters the following year.

If Magpiedemons was the better performing team

- 95% of Magpiedemons' supporters remained Magpiedemons' supporters the following season.
- 30% of Eaglehawks' supporters became Magpiedemons' supporters the following year.

Question 3

In the town of 20 000 supporters, assume the supporters are equally divided between the two teams. Now consider that Eaglehawks were to perform better than Magpiedemons for a three year period.

a. State the transition matrix for when Eaglehawks is the better side.

1 mark

b. Set up an appropriate matrix equation and find the number of supporters for the two teams after 3 years of the Eaglehawks performing better than the Magpiedemons.

2 marks

c. Find the supporter base distribution if Magpiedemons were to have a three year period of outperforming the Eaglehawks.

1 mark

d. Which team would in the long term have the largest supporter base if they continued to perform better? Justify your answer with calculations.

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK

MULTIPLE CHOICE ANSWER SHEET

Student Name: ....

Circle the letter that corresponds to each correct answer

Section A	Section B		
Compulsory	Answer three different modules. Show each module selected by ticking the appropriate box.		
	Module:	Module:	Module:
	❑ Number patterns ❑ Geometry and trigonometry ❑ Graphs and relations ❑ Business related mathematics ❑ Networks and decision mathematics ❑ Matrices	❑ Number patterns ❑ Geometry and trigonometry ❑ Graphs and relations ❑ Business related mathematics ❑ Networks and decision mathematics ❑ Matrices	❑ Number patterns ❑ Geometry and trigonometry ❑ Graphs and relations ❑ Business related mathematics ❑ Networks and decision mathematics ❑ Matrices
1. A B C D E	1. A B C D E	1. A B C D E	1. A B C D E
2. A B C D E	2. A B C D E	2. A B C D E	2. A B C D E
3. A B C D E	3. A B C D E	3. A B C D E	3. A B C D E
4. A B C D E	4. A B C D E	4. A B C D E	4. A B C D E
5. A B C D E	5. A B C D E	5. A B C D E	5. A B C D E
6. A B C D E	6. A B C D E	6. A B C D E	6. A B C D E
7. A B C D E	7. A B C D E	7. A B C D E	7. A B C D E
8. A B C D E	8. A B C D E	8. A B C D E	8. A B C D E
9. A B C D E	9. A B C D E	9. A B C D E	9. A B C D E
10. A B C D E			
11. A B C D E			
12. A B C D E			
13. A B C D E			

FURTHER MATHEMATICS

Trial written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time This formula sheet is provided for your reference

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1-r^n)}{1-r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1-r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a+b+c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET