2007 Further Mathematics Trial Exam 1 Solutions
Free download and print from www.itute.com

SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
B	E	B	A	A	B	E	C	E	E	B	C	A

SECTION B

Module 1: Number patterns and applications

1	2	3	4	5	6	7	8	9
D	C	B	D	A	A	B	A	B

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
C	A	B	D	D	C	D	B	D

Module 6: Matrices

1	2	3	4	5	6	7	8	9
B	B	A	A	B	B	C	A	B

SECTION A Core: Data analysis

Q1 B

Q2 Arrange data in ascending order:
6 8 13 15 20 21 23 24 27 28 40 41

$\text{Median} = \dfrac{21+23}{2} = 22$ E

Q3 4 people less than 20, percentage $= \dfrac{4}{12} \times 100 \approx 33.3$ B

Q4 To be an outlier it must be $\le Q_1 - 1.5 \times IQR = -6.25$ or $\ge Q_1 + 1.5 \times IQR = 47.75$. A

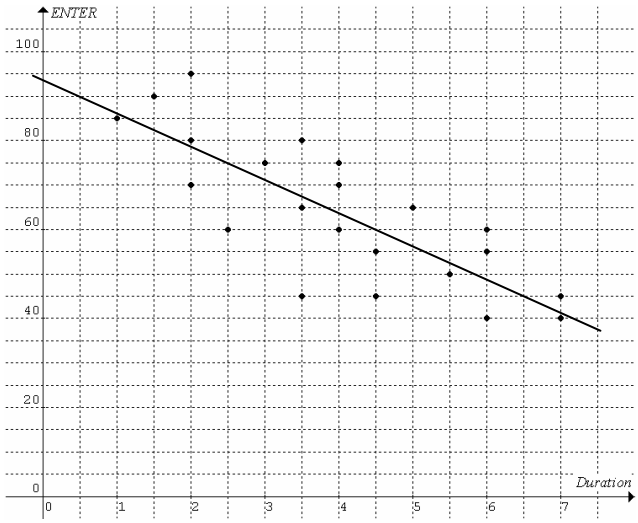
Q5 A

Q6 B

Q7 E

Q8 Three medians: (2, 80), (4, 62.5), (6, 50).

$\text{Gradient} = \dfrac{50-80}{6-2} = -7.5$, y-intercept ≈ 94 (from graph) C



Q9 E
Note: C and D are undefined for $x = 0$.

Q10 No such residual plots exist if all calculations were done correctly. E

Q11 $X = \dfrac{11+27+23}{3} = 20.3$, $Y = 18$. B

Q12 $\text{Average} = \dfrac{1006+956+1210+1352}{4} = 1131$

$\text{Second quarter seasonal index} = \dfrac{956}{1131} = 0.8453$ C

Q13 The deseasonalised figure for each quarter is the average of the four quarters, i.e. 1131. A

SECTION B

Module 1: Number patterns and applications

Q1 $t_1 = 0 = 1^2 - 1$; $t_2 = 3 = 2^2 - 1$; $t_3 = 8 = 3^2 - 1$;
$t_4 = 15 = 4^2 - 1$; $t_5 = 24 = 5^2 - 1$; ……; $t_n = n^2 - 1$ D

Q2 It is an arithmetic sequence with $a = -121$, $d = 4$.
$t_n = a + (n-1)d$, $43 = -121 + (n-1)4$, $n = 42$ C

Q3 Look at the sequence in reverse order. $a = 43$, $d = -4$,
$t_n > 0$, $43 + (n-1)(-4) > 0$, $\therefore n < 11.75$, i.e. $n = 11$ B

Q4 Any rung is 99.9% (i.e. 0.999) of its preceding rung.
Infinite geometric series with $a = 1$ and $r = 0.999$.

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-0.999} = \frac{1}{0.001} = 1000$$ D

Q5 $t_n = 3t_{n-1} + 1$, $\therefore t_6 = 3t_5 + 1 = 3(-2) + 1 = -5$ and
$t_5 = 3t_4 + 1$, i.e. $-2 = 3t_4 + 1$, $\therefore t_4 = -1$ A

Q6 A

Q7 $\frac{t_n}{t_{n-1}} = \frac{3/4}{2/3} = \frac{9}{8}$, $\therefore 8t_n = 9t_{n-1}$ B

Q8 A

Q9 $1 + 2 + 3 + 5 + 8 + 13 + 21 + 34 + 55 + 89 = 231$ B

10^{th} year

Module 5: Networks and decision mathematics

Q1 A bipartite graph is a graph whose vertices can be divided into two disjoint sets such that every edge connects a vertex in one set and a vertex in the other set. There is no edge between two vertices in the same set. C

Q2 A simple graph is an unweighted, undirected graph containing no graph loops or multiple edges. A

Q3 B $V - E + F = 8 - 12 + 6 = 2$

Q4 *WYVXZW* is both a Hamiltonian circuit and an Euler circuit.
An Euler circuit is an Euler path that starts and ends on the same vertex. A Hamiltonian circuit can be converted to a Hamiltonian path by removing one of its edges. ∴ A Hamiltonian circuit is not a Hamiltonian path. D

Q5 D

Q6 a directed edge from *A* to *D*

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$ C

Q7 Maximum flow = minimum cut = $4 + 1 + 2 = 7$ D

Q8 The critical path *ADEFI* gives the shortest time.
$9 + 5 + 0 + 3 + 6 = 23$ B

Q9 *A* from *X*, *B* from *Y*, *C* from *Z* and *D* from *W*.
$1.15 + 0.80 + 1.45 + 0.90 = 4.30$ D

Module 6: Matrices

Q1 *X* is 3×1, $\therefore Y$ is 1×2 so that *XY* is 3×2 consisting of 6 elements. B

Q2 $x + 5 = 2 \times 2$, $x = -1$ B

Q3 A

Q4 $X - Y = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\therefore (X-Y)^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$,
$\therefore (X-Y)^4 = \left((X-Y)^2\right)^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = 8\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
$= 8(X - Y)$ A

Q5 *P* is a singular matrix, $\therefore P^{-1}$ does not exist. B

Q6 $\begin{bmatrix} 0.454 & 0.0284 \end{bmatrix}\begin{bmatrix} 10 & 18 \\ 12 & 5 \end{bmatrix} = \begin{bmatrix} 4.88 & 8.31 \end{bmatrix}$ B

Q7 A transitional matrix is a square matrix with positive elements and the elements in each column sum to 1. C

Q8 $\begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.5 \end{bmatrix}\begin{bmatrix} 400 \\ 600 \end{bmatrix} = \begin{bmatrix} 540 \\ 460 \end{bmatrix}$ A

Q9 $\begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.5 \end{bmatrix}^{12}\begin{bmatrix} 400 \\ 600 \end{bmatrix} \approx \begin{bmatrix} 556 \\ 444 \end{bmatrix}$ B

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

