Year 2004

VCE
Further Mathematics
Trial Examination 2

Suggested Solutions

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Question 1

a. The dependent variable is the number of workers' compensation claims. (1 mark)

b. Use your graphics calculator. Enter values using state, edit. Then use stat, calc. 4, L2, L1 if spending is in L2. 69.7 – 4.3 × spending on safety procedures (1 mark) (1 mark)

c.

$0 = 69.7 - 4.3 \times x$

$\Rightarrow 4.3x = 69.7$

$\Rightarrow x = \dfrac{69.7}{4.3}$

$\Rightarrow x = 16$ thousand dollars (1 mark)

Question 2

a.

$\dfrac{4}{20} \times 100 = 20\%$ (1 mark)

b.

$\dfrac{3}{20} \times 100 = 15\%$ (1 mark)

c.

Electrician		Plumber
9 8 7 6 6 5	0	3 7 8
6 4 2 1 0 0	1	2 4 6 7
8 8 7 4 0	2	0 2
0	3	0 2 4
8	4	1 3 8 9
2	5	3
	6	1 4
	7	
	8	9

(2 marks)

d. Middle number is half way between 3.0 and 3.2. Therefore, 3.1 hours (1 mark)

e. Use calculator or add the values in the Plumber's column and divide by 20. 3.315 hours (1 mark)

f. The median would be better because of the outlier of 8.9 (1 mark for median. 1 mark for outlier)

g. The distribution is positively skewed. (1 mark)

h. Median = 1.3 , Lower Quartile = 0.85, ($\frac{1}{2}$ mark) Upper Quartile = 2.75, ($\frac{1}{2}$ mark)

Interquartile range 2.75 – 0.85 = 1.9 (1 mark)

Total = 15 marks

End of Core

Question 1

a. Arithmetic sequence $t_n = a + (n-1)d$ $a = 12600$ $n = 4$ $d = 1400$ $\Rightarrow t_4 = 12600 + 3 \times 1400$ $\Rightarrow t_4 = 16,800$ (1 mark)	**b.** $t_{n+1} = t_n + 1400 \quad t_1 = 12,600$ (1 mark)
c. $t_2 = 3 \times t_1 - 20000 = 3 \times 12600 - 20000 = 17800$ $t_3 = 3 \times 17800 - 20000 = 33,400$ (1 mark)	**d.** Geometric Sequence $a = 8000$ $n = 7$ $r = 1.03$ $t_7 = ar^6 = 8000(1.03)^6 = 9552$ people (1 mark)

Question2

a. $\%\ \text{water} = \frac{5}{8} \times 100 = 62.5\%$ (1 mark)	**b.** $\frac{3}{8} \times 5600 = 2,100$ litres (1 mark)
c. i. $1000 - 200 = 800$ mL (1 mark)	**c. ii.** $200 : 800 = 1 : 4$

Question 3

a.

Number of days $= \dfrac{8100}{50} = 162$ days (1 mark)

b.

Geometric Sequence

$S_n = 10,000$

$a = 20$

$r = 2$

$S_n = \dfrac{a(r^n - 1)}{r - 1}$

$\Rightarrow S_n = \dfrac{20(2^n - 1)}{2 - 1} = 10,000$

$\Rightarrow 2^n - 1 = 500$ (1 mark)

$\Rightarrow 2^n = 501$

$\Rightarrow n \log_{10} 2 = \log_{10} 501$

$\Rightarrow n = \dfrac{\log_{10} 501}{\log_{10} 2} = 9$ days (1 mark)

c.

$\dfrac{8100}{x} = 9$

$\Rightarrow x = \dfrac{8100}{9} = 900$ (1 mark)

d.

Arithmetic Sequence

$S_n = 7425$

$a = 55$

$d = 5$

$7425 = \dfrac{n}{2}\left[2 \times 55 + (n-1)5\right]$

$\Rightarrow 7425 = \dfrac{n}{2}\left[110 + 5n - 5\right]$

$\Rightarrow 14850 = n[105 + 5n]$ (1 mark)

$\Rightarrow 5n^2 + 105n - 14850 = 0$

$\Rightarrow n^2 + 21n - 2970 = 0$

$\Rightarrow n = \dfrac{-21 \pm \sqrt{21^2 + 11880}}{2}$

$\Rightarrow n = \dfrac{-21 \pm 111}{2}$

$n > 0$

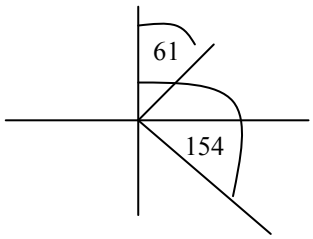
$\Rightarrow n = \dfrac{-21 + 111}{2} = \dfrac{90}{2} = 45$ days (1 mark)

Total = 15 marks

End of Module 1

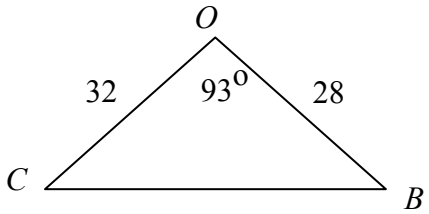
Question 1

a.



$\angle BOC = 154 - 61 = 93^{o}$ (1 mark)

b.



Area

$= \frac{1}{2} bc \sin\theta$ (1 mark)

$= \frac{1}{2} \times 32 \times 28 \sin 93°$

$= 447.386 \text{ m}^2$

c.

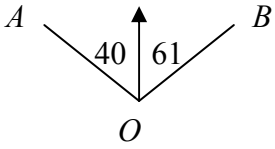
$BC^2 = 28^2 + 32^2 - 2 \times 28 \times 32 \cos 93°$

$BC^2 = 1808 + 93.786$

$BC = \sqrt{1901.786} = 43.61 \text{ m}$

(1 mark)

d.



$\angle AOB = 40 + 61 = 101^{o}$ (1 mark)

e.

$\frac{30}{\sin 101°} = \frac{28}{\sin \angle BAO}$

$\Rightarrow \sin \angle BAO = \frac{28 \sin 101°}{30}$ (1 mark)

$\Rightarrow \angle BAO = \sin^{-1}\left(\frac{28 \sin 101°}{30}\right)$

$\Rightarrow \angle BAO = 66°$ (1 mark)

Question 2

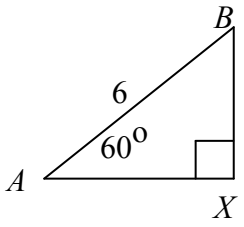
a.
ABC is an equilateral triangle (all angles = 60^o)
Therefore, $AB = AC = 6$ m (1 mark)

b.
$\tan 60° = \frac{3}{x}$ (1 mark)

$\Rightarrow x = \frac{3}{\tan 60°} = 1.73$ m (1 mark)

c.
Width of room = $6 - (2 \times 1.73) = 2.54$ m
(1 mark)

d.



$\sin 60° = \frac{BX}{6}$ (1 mark)

$\Rightarrow BX = 6 \sin 60° = 5.20$ (1 mark)

e.
Area of ceiling = $2.54 \times 10 = 25.4$ m^2
(1 mark)

f.
Perpendicular height of triangular end
$= 5.20 - 3 = 2.20$ (1 mark)
Volume of space above ceiling
= area of base × height
= area of triangular end × 10
$= \frac{1}{2} \times 2.54 \times 2.2 \times 10$
$= 27.94$ m^3 (1 mark)

Total = 15 marks

End of Module 2

Question 1

a. 90 – 30 = 60 minutes or 1 hour (1 mark)

b.

80 km in 2 hours

80 ÷ 2 = 40 km/hr (1 mark)

Question 2

a.

$C = 30x + 15y$ (1 mark)

b.

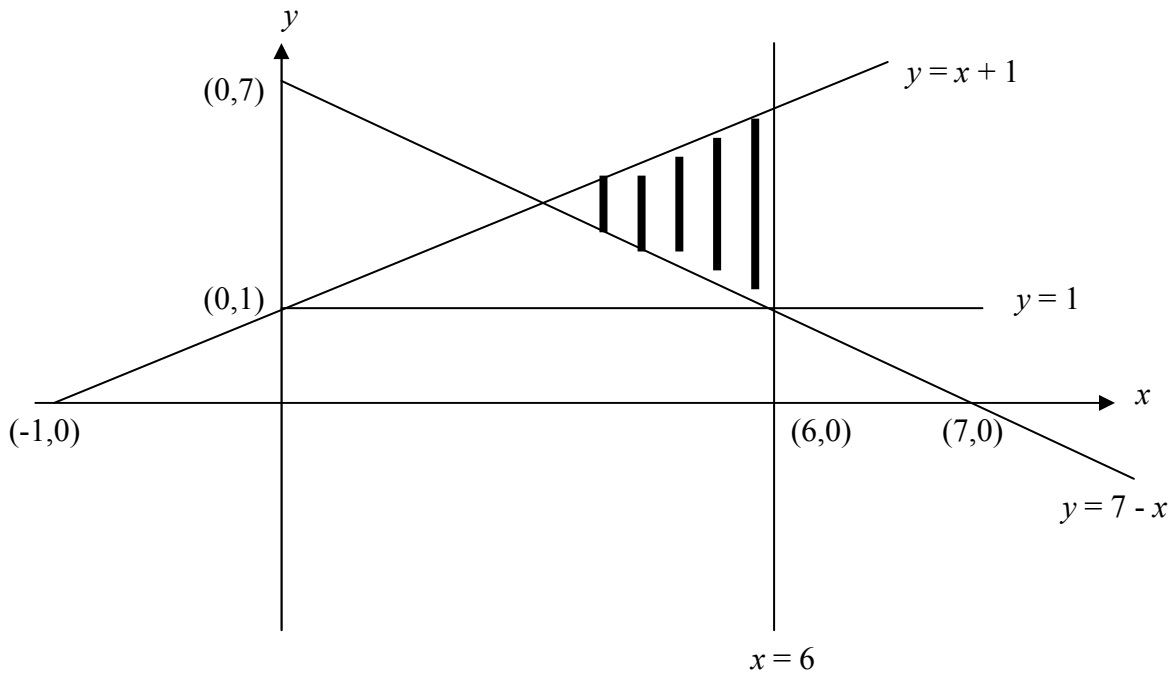
(1) $x \leq 6$ (1 mark)

(2) $x + y \geq 7$ (1 mark)

(3) $x \geq y - 1$ or $y \leq x + 1$ (1 mark)

(4) $y \geq 1$ (1 mark)

c.



The shaded region is the required region.

- $\frac{1}{2}$ mark for the line $y = 1$
- $\frac{1}{2}$ mark for the line $x = 6$
- 1 mark for the line $y = x + 1$
- 1 mark for the line $y = 7 - x$
- 1 mark for the correct area identified

Question 2 (continued)

d.

Intersection of $y = x + 1$ and $y = 7 - x$

$y - x = 1 \quad (1)$

$y + x = 7 \quad (2)$

$(1) + (2)$

$2y = 8$

$y = 4$

From (1), when $y = 4,\ x = 3 \quad (3,4)$ (1 mark)

$\left.\begin{array}{l}\text{Intersection of } x = 6, y = 1 \quad (6,1) \\ \text{Intersection of } x = 6, y = x + 1 \\ y = 6 + 1 = 7 \qquad\qquad (6,7)\end{array}\right\}$ (1 mark)

Cost at (3,4)
$= 30 \times 3 + 15 \times 4 = 90 + 60 = \$150$

Cost at (6,1)
$= 30 \times 6 + 15 \times 1 = 180 + 15 = \$195$

Cost at (6,7)
$= 30 \times 6 + 15 \times 7 = 180 + 105 = \$285$

Minimum cost = \$150 (1 mark)

e.

3 art books and 4 science books (1 mark)

Total = 15 marks

End of Module 3

Question 1

a.

$\frac{15}{100} \times 15,700 = \$2,355$ (1 mark)

b.

$P = 15700 - 2355 = 13,345$ (1 mark)

$I = \frac{PRT}{100}$

$I = \frac{13345 \times 12.5 \times 6}{100}$

$I = \$10,008.75$ (1 mark)

c.

Amount to be repaid
= 13345 + 10008.75 = 23,353.75 (1 mark)

Number of repayments = 6 × 12 = 72

Amount of each repayment
= 23353.75 ÷ 72 = \$324.36 (1 mark)

d.

Effective interest rate

$= \frac{2n}{n+1} \times 12.5$

$= \frac{2 \times 72}{73} \times 12.5$

= 24.7% (1 mark)

Question 2

a.

$\frac{14}{12} = 1.17\%$ (1 mark)

b. i.

$R = 1 + \frac{1.16667}{100} = 1.0117$ (1 mark)

b. ii.

$n = 6 \times 12 = 72$ (1 mark)

c.

$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$

$0 = 15700(1.0117)^{72} - \frac{Q(1.0117^{72} - 1)}{0.0117}$ (1 mark)

$\Rightarrow \frac{Q(1.0117^{72} - 1)}{0.0117} = 15700(1.0117)^{72} = 36276.52279$

$\Rightarrow Q(1.0117^{72} - 1) = 424.4353$

$\Rightarrow Q = \$323.85$ (1 mark)

d.

Total amount paid
= 323.85 × 72 = 23,317.20
Interest
= 23,317.20 – 15,700
= \$7,617.20 (1 mark)

e.

$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$

$A = 15700(1.0117)^{24} - \frac{323.85(1.0117^{24} - 1)}{0.0117}$

$\Rightarrow A = 20753.90 - 8913.68$

$\Rightarrow A = \$11,842.22$ (1 mark)

f.

$0 = 11842.22(1.0117)^n - \frac{323.85(1.0117^n - 1)}{0.0117}$

$11842.22(1.0117)^n = 20110.77(1.0117^n)$

$1.58885 = 1.0117^n$

$\log_{10} 1.58885 = n \log_{10} 1.0117$

$n = 39.8$

$n = 40$ months (1 mark)

40 months = 3 years and 4 months

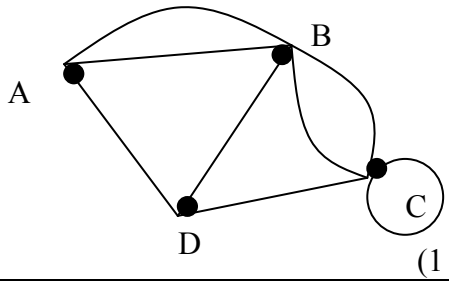
Total time = 2 years + 3 years 4 months

= 5 years and 4 months (1 mark)

End of Module 4 **Total = 15 marks**

Question 1

a.



(1 mark)

b.

The network has no loops

(1 mark)

c.

It represents a directed graph.

(1 mark)

Question 2

a.

- Immediate predecessor for F is B (1 mark)
- Immediate predecessors for I are G and F (1 mark)
- Immediate predecessors for J are H and I (1 mark)
- Earliest starting time for F is the earliest time B finishes = 5 (1 mark)
- Earliest starting time for I depends on when F and G finish. Earliest time for F to finish is 5 + 6 = 11. Earliest time for G to finish is 12 + 2 = 14. I cannot start until both F and G finish so earliest starting time for I = 14. (1 mark)
- Earliest starting time for J depends on when H and I finish. Earliest time for H to finish = 10 + 7 = 17. Earliest time for I to finish = 14 + 4 = 18. Therefore, earliest starting time for J is 18. (1 mark)
- Latest starting time for J = total time for the whole job which is 21 less 3 = 18. (1 mark)
- Latest starting time for H = 18 – 7 = 11 (1 mark)
- To get the latest starting time for F, backtrack from the end along J , I and F = 21 – 3 – 4 – 6 = 8 (1 mark)

b.

From the Table, for activity E, float time
= latest start time – earliest start time
= 8 – 5 = 3 hours. (1 mark)

c.

The critical path is the longest path which is
C G I J (1 mark)

d. The length of the critical path = 12 + 2 + 4 + 3 = 21 hours (1 mark)

End of Module 5 Total = 15 marks

End of suggested solutions
2004 Further Mathematics Trial Examination 2

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