THE HEFFERNAN GROUP
P.O. Box 1180
Surrey Hills North VIC 3127
ABN 20 607 374 020
Phone 9836 5021
Fax 9836 5025

Student Name……

FURTHER MATHEMATICS

TRIAL EXAMINATION 2

(ANALYSIS TASK)

2004

Reading Time: 15 minutes
Writing time: 90 minutes

Instructions to students

This exam consists of Section A and Section B.
Section A contains a set of extended answer questions from the core, "Data Analysis".
Section A is compulsory and is worth 15 marks.
Section B consists of 5 modules. You should choose 3 of these modules and answer every question in each of your chosen modules. Each of the modules is worth 15 marks.
There is a total of 60 marks available for this exam.
The marks allocated to each of the four questions are indicated throughout.
Students may bring up to two A4 pages of pre-written notes into the exam.
Formula sheets can be found on pages 25-26 of this exam.
Diagrams in this exam are not to scale except where otherwise stated.

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Section A

This section is compulsory.

Question 1

In a Year 11 Maths class, 10 students with their own email address were randomly selected and asked to record on a particular day the number of emails they received from friends and the number they sent to friends. Their responses are recorded in Table 1 below.

Student	Number of outgoing emails	Number of incoming emails
A	6	15
B	8	23
C	9	17
D	6	16
E	7	18
F	5	12
G	10	21
H	7	16
I	9	19
J	9	20

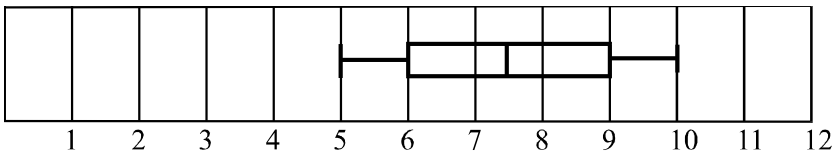
Table 1

a. For the number of incoming emails, find the mean and the standard deviation of the data. Express your answers to one decimal place.

mean = ☐ , standard deviation = ☐

2 marks

b. The number of outgoing emails is displayed on a box plot below.



i. The shape of the data could be described as ☐

ii. The median of this data is ☐

iii. ☐ % of the data lies between 6 and 9.

1 + 1 + 1 = 3 marks

c. The scatterplot in Figure A below has been constructed from the data in Table 1.

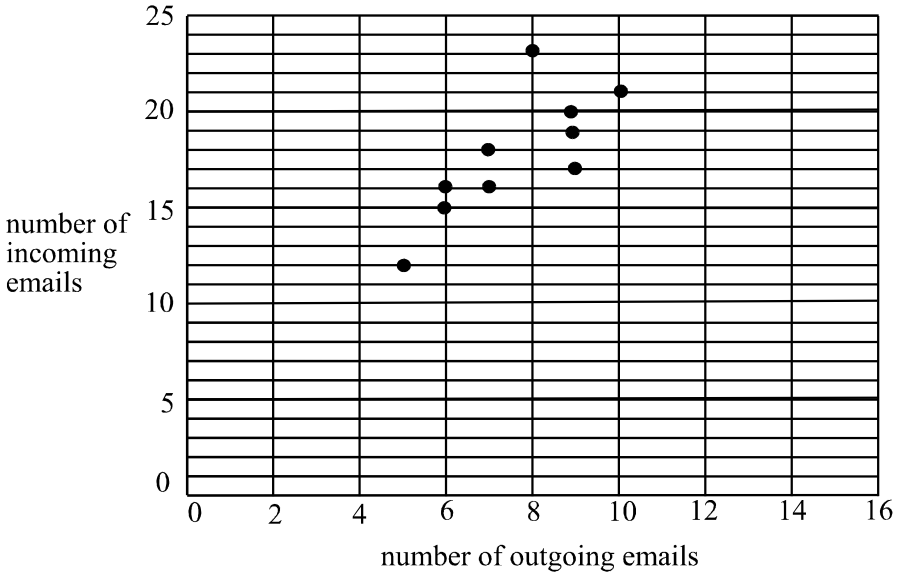


Figure A.

A least squares regression line is to be fitted to this data. In doing this, an assumption has been made that the relationship between the number of outgoing emails and the number of incoming emails is ☐

1 mark

d. Using the data in Table 1, find the equation of the least squares regression line. Express values correct to 2 decimal places.

Number of incoming emails = ☐ + ☐ × number of outgoing emails

2 marks

e. Use the equation you have found to predict the number of incoming emails you would expect if the number of outgoing emails was 15. Express your answer to the nearest whole number.

Predicted number of incoming emails = ☐

1 mark

f. For student *F*, the least squares regression line overestimates the number of incoming emails by ☐ Express your answer correct to 1 decimal place.

1 mark

g. **i.** Find the value of Pearson's product moment correlation coefficient.

$r =$ ☐

Express your answer correct to 3 decimal places.

ii. What percentage of the variation in the number of incoming emails can be explained by the variation in the number of outgoing emails?
Express your answer to the nearest whole percentage ☐ %

1 + 1 = 2 marks

Question 2

The data in Table 2 shows the results of a survey of students with their own email address across all the Year 11 maths classes. The number of students who had received any emails from their friends or sent any emails to their friends was noted for male and for female students on a particular day.

Type of email(s)	Gender of student		
	Male	Female	Total
Outgoing email(s)	17	41	58
Incoming email(s)	32	38	70
Total	49	79	128

Table 2

a. Complete Table 3 below by percentaging the table and finding column percentages. Express your percentages to the nearest whole percentage.

Type of email(s)	Gender of student	
	Male	Female
Outgoing email(s)		
Incoming email(s)		
Total	100%	100%

Table 3

1 mark

b. By percentaging the table in this way, explain whether we are comparing outgoing emails with ingoing emails or comparing males with females.

1 mark

c. Use the data in Table 3 to make a comment about any relationship between the type of email (outgoing or incoming) and the gender of the student.

1 mark

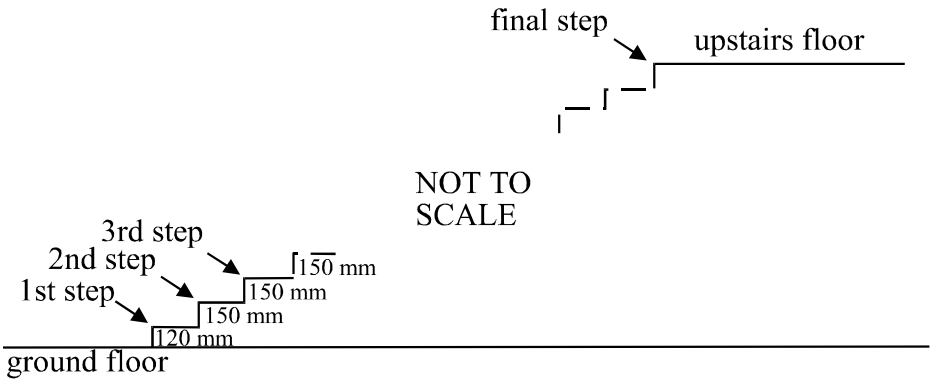
Total 15 marks

Module 1: Number Patterns and Applications

If you choose this module all questions must be answered.

Question 1

Steve owns a company that builds staircases.
The company built a staircase for the Brown family that had its first step a vertical distance of 120 mm above the floor and each step thereafter had a vertical rise of 150mm as indicated in the diagram below.



a. What is the vertical height of the sixth step above the ground floor?

1 mark

b. The vertical distance, t_n, above the ground floor of the n^{th} step in the staircase, forms an arithmetic sequence following the rule

$$t_n = a + (n-1)d$$

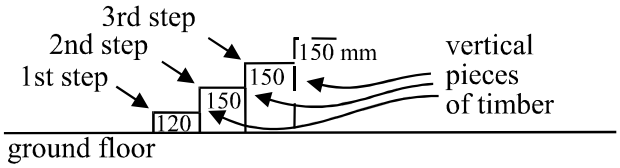
Write down the values of a and d.

2 marks

c. The upstairs floor of the Brown house is 3270mm vertically above the ground floor. How many steps are there in their staircase?

1 mark

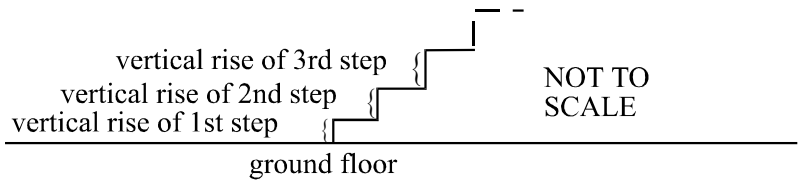
d. If the staircase was constructed so that each vertical rise used a piece of timber that extended vertically down to the ground floor (as indicated in the diagram below), what total length of timber would be required to construct the rises of the first 10 steps of the staircase?



2 marks

Question 2

Steve's company is engaged to build a staircase where the vertical rise of the first step above the ground floor is 100mm and each step thereafter has a vertical rise which is 5% higher than the previous step. The diagram below shows the vertical rise of steps in a staircase.



a. Find the vertical rise of the second step in this staircase.

1 mark

b. Explain why the vertical rise, r_n, of the n^{th} step in the staircase above the previous step is given by

$$r_n = 100(1 \cdot 05)^{n-1}$$

1 mark

c. Building regulations in Victoria require that the vertical rise of a step in a staircase may not be greater than 190mm. How many steps can be legally constructed in this particular staircase?

2 marks

Question 3

For a third staircase constructed by Steve's company, the vertical rise R_n, of the n^{th} step was given by the difference equation

$$R_{n+1} = 0 \cdot 98R_n \qquad \text{where } R_1 = 115$$

and the unit of measurement was millimeters.

a. Find the vertical rise of the third step. Express your answer correct to 3 decimal places.

1 mark

b. State whether the difference equation given above describes an arithmetic sequence, a geometric sequence or neither. Give reasons for your answer.

2 marks

c. What would be the total vertical height of the staircase if the pattern in the step height, described by the difference equation in part a., continued forever?

2 marks

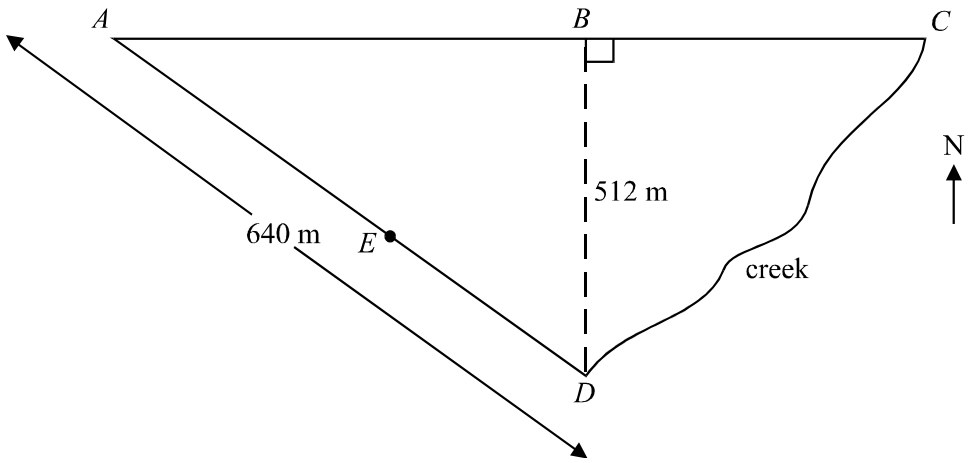
Total 15 marks

Module 2: Geometry and Trigonometry

If you choose this module all questions must be answered.

Question 1

A paddock is bordered by a fence *AC*, running in an east-west direction, a second fence *AD* of length 640m and a creek. Gates are located at point *E* and at point *B*, which is 512m due north of *D*. The bearing of *C* from *D* is 045°.



a. Find *AB* in metres.

1 mark

b. Find the area of ΔABD.

1 mark

c. Show that angle $BAD = 53^\circ 8'$ (to the nearest minute).

1 mark

d. Hence find the bearing of A from D.

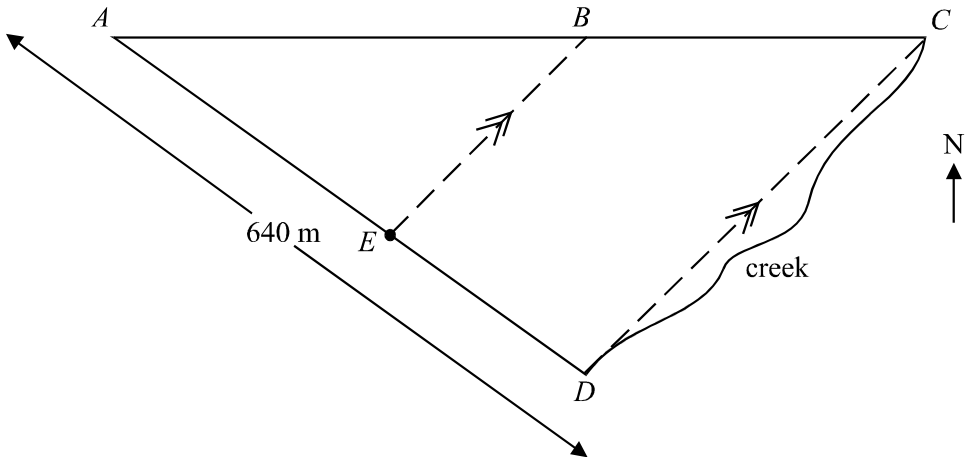
1 mark

e. Find BC in metres.

1 mark

Question 2

A straight track across the same paddock between the gates at B and E runs parallel to the line CD, as indicated in the diagram below.



a. Show that the distance AE is 274m to the nearest metre.

2 marks

b. Find angle *AEB* to the nearest minute.

1 mark

c. Hence find the length of the track *BE* to the nearest metre.

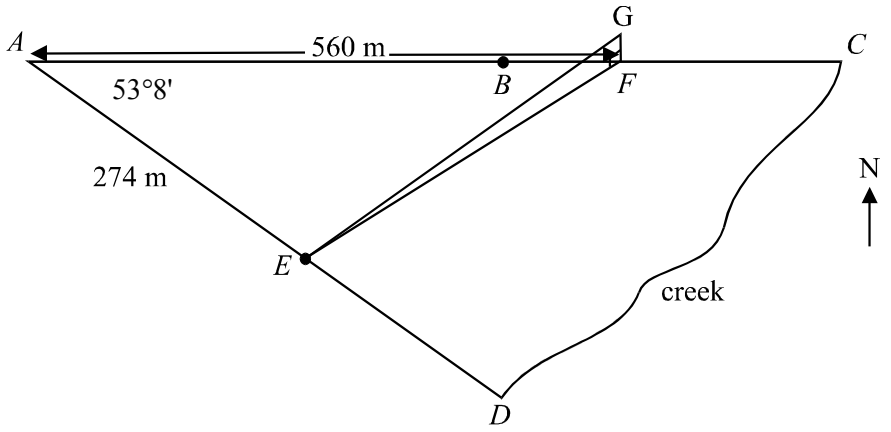
2 marks

d. Using the fact that angle $BAE = 53°8'$, to the nearest minute, find the area of that part of the paddock bordered by *AB*, *EA* and the straight track *BE*.
Express your answer to the nearest square metre.

2 marks

Question 3

An old windmill indicated by *FG*, is located at point *F* which lies on the fence line *AC* 560 m from point *A*.



a. Find the length of *EF*. Express your answer correct to 1 decimal place.

2 marks

b. The distance from the top of the windmill to point *E* is 453 m.
Use your result from part **a.** to find the height of the old windmill in metres correct to 1 decimal place.

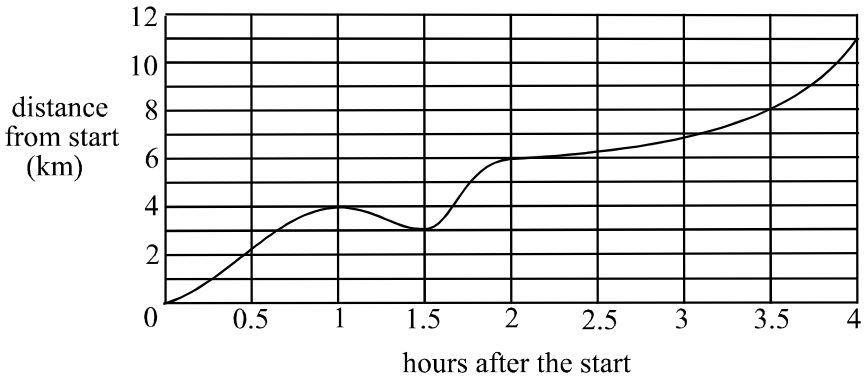
1 mark
Total 15 marks

Module 3: Graphs and relations

If you choose this module all questions must be answered.

Question 1

A school organizes a walkathon to raise money for charity.
Mr Brown, a teacher at the school, rides his bike around the walkathon course checking on the students.
The distance of Mr Brown from the start of the walkathon t hours after he began is shown on the graph below.



a. How long did it take Mr Brown to ride the first 5 km of the course?

1 mark

b. During which hour did Mr. Brown cover the least distance? Give a reason for your answer.

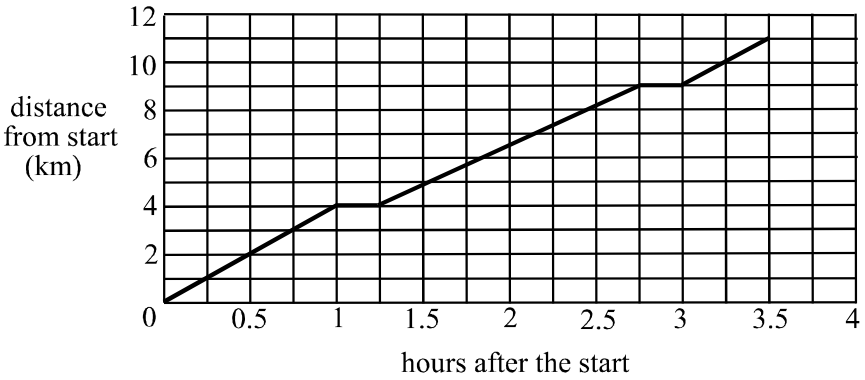
2 marks

c. Explain the direction in which Mr Brown is riding between $t = 1$ and $t = 1 \cdot 5$.

1 mark

Question 2

Kate and her friends participate in the school walkathon and complete it. During the walkathon they take a couple of rests.
The distance time graph for Kate's walk is shown below.



a. What is the total time taken for rests by Kate during the walkathon?

1 mark

b. Between which times is Kate walking at her slowest during the walkathon?

1 mark

c. How fast is Kate walking between $t = 3$ and $t = 3 \cdot 5$?

2 marks

d. Kate has been sponsored a total of \$12 per kilometre for the walkathon. How much does Kate raise?

1 mark

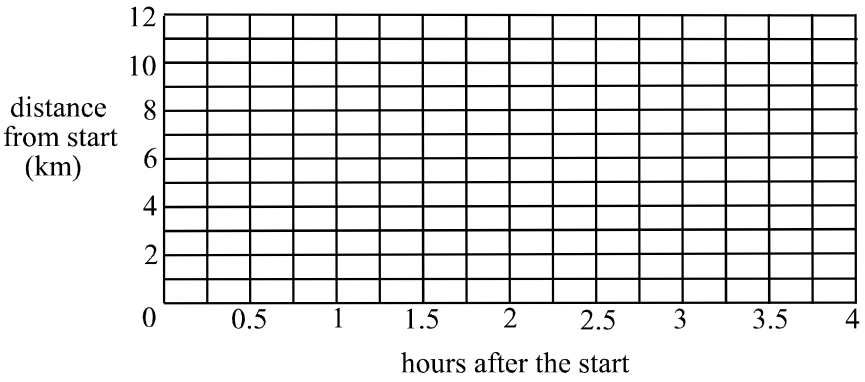
Question 3

Kate's friends, Alexa and Peter decide to jog rather than walk the walkathon course. They each complete it.

a. The distance jogged by Alexa at time t hours after she started jogging is given by

$$d_A = 5t, \qquad 0 \le t \le m$$

i. Draw the graph of d_A against t on the set of axes below.



1 mark

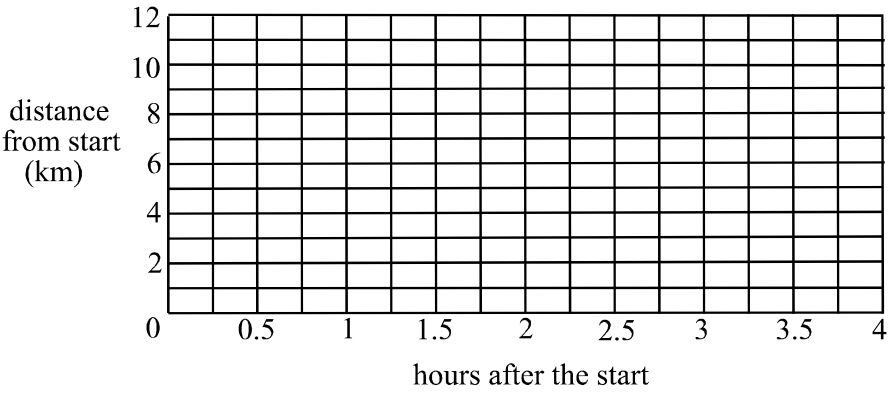
ii. Find the value of m.

1 mark

b. The distance jogged by Peter at time t hours after he started jogging is given by

$$d_P = \begin{cases} 5t & 0 \le t \le 1 \\ 3t + 2 & 1 < t \le n \end{cases}$$

i. Draw the graph of d_P against t on the set of axes below.



2 marks

ii. Find the value of n.

_____

_____

1 mark

c. Given that Kate and Alexa start the walkathon at the same time, by how long does Alexa beat Kate to the finish of the walkathon?

_____

_____

1 mark
Total 15 marks

Module 4: Business-related mathematics

If you choose this module all questions must be answered.

Question 1

Honest Harry owns and runs a car yard, and is renowned for "doing deals".

a. One of the cars in Harry's car yard costs $8500 but Harry will discount this amount by 2.5% if the car is paid for in cash.
How much would you pay for the car if you paid cash?

1 mark

b. Harry offers "pay-later" terms on all his cars. For such a deal, on the car that cost $8500, a buyer would pay $99 a week for two years.

i. What amount of interest would a buyer pay according to this deal?

ii. What annual flat rate of interest does this represent? Express your answer correct to 1 decimal place.

1 + 2 = 3 marks

Question 2

Honest Harry has a courtesy vehicle which can be used by customers who bring their car in for servicing at Harry's service centre. The courtesy vehicle was valued at $20 000 five years ago and in that time it has done 55 000 km.

a. Find the book value of this courtesy vehicle today, if it is depreciated according to the reducing balance method at 10% per annum.

2 marks

b. Find the book value of this courtesy vehicle today if it is depreciated according to the unit-cost method at $0.20 per kilometre.

2 marks

Question 3

Honest Harry has had $75 000 invested in an account which earned interest of 6% per annum that compounded monthly.

a. How much interest would Honest Harry have earned after 4 years?

2 marks

b. How long would it take in months for the amount in the account to first exceed $100 000?

2 marks

Question 4

Honest Harry wants to buy the property next door to enlarge his car yard. To do so he has to borrow $300 000 and has decided to take out a reducing balance loan. Interest on this loan is 9% per annum compounding six-monthly and Harry makes payment of $\$Q$ each six months. The annuities formula below can be used to calculate this loan.

$$A = PR^n - \frac{Q\left(R^n - 1\right)}{R - 1} \qquad \text{where } R = 1 \cdot 045$$

Given that the loan is paid off in 10 years, find the amount owing after 5 years.

3 marks

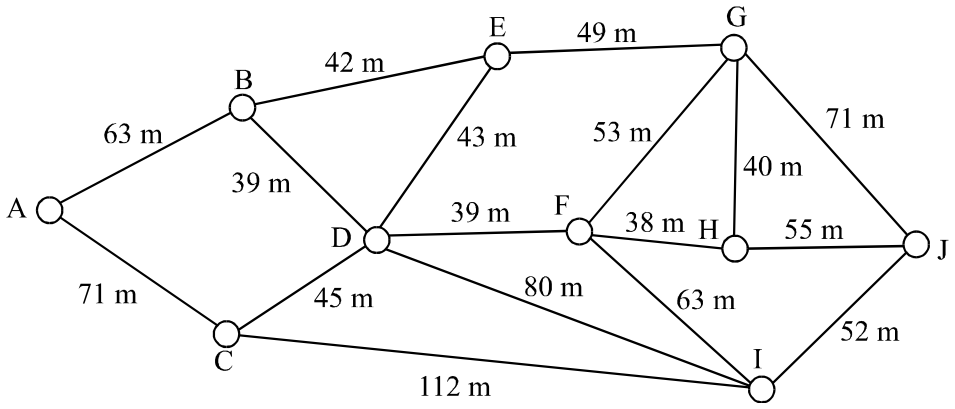
Total 15 marks

Module 5: Networks and decision mathematics

If you choose this module all questions must be answered.

Question 1

A new housing estate is being developed and the network below shows possible routes for underground cabling between house blocks in a section of the estate.

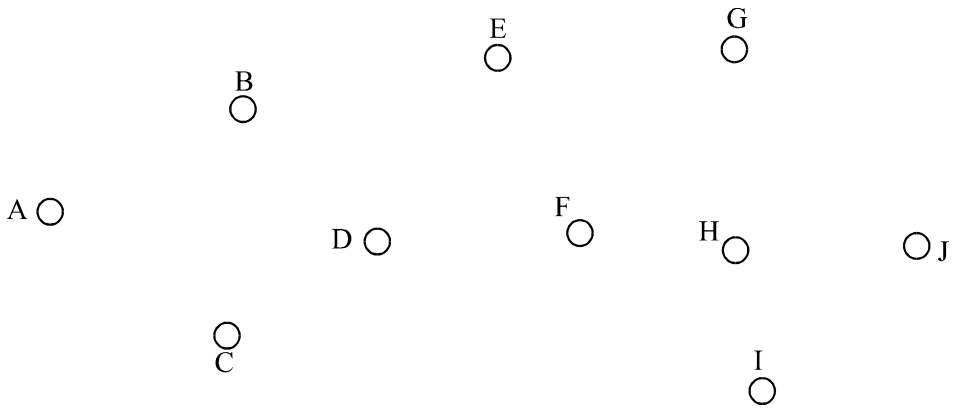


The distance in metres of each of the possible routes is shown in the diagram.

a. Find the length of the shortest path along the possible routes between the house block at A and the one at J.

1 mark

b. Draw the minimal-length spanning tree which will show the shortest length of underground cabling required so that all the house blocks are connected to one another.



2 marks

Question 2

For this housing project, the development company set up a special team in their accounting department. The supervisor of the team estimates the time, in hours, it would take each member to make a report on four different aspects of the project.
His estimations are shown in Table 1 below.

	Construction costing	Financing	Cash-flow analysis	Planning
Kate	6	7	8	6
Monica	8	8	5	7
Gareth	4	4	7	5
Ed	5	4	3	3

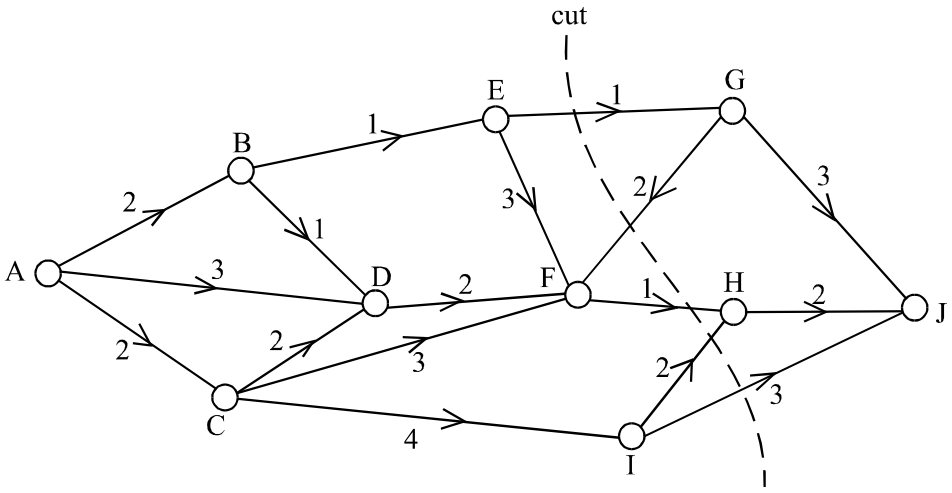
Table 1

Find the allocation of people to writing particular reports so that the time taken is minimised.

3 marks

Question 3

This section of the new housing estate covers some steep areas. The network below shows a proposed drainage system with the amount (in megalitres, mL) of water each section of the drain can cope with in an hour.
Because of the slope of the land the drainage system flows from A to J. A cut has been made in the network.



a. What is the capacity of the cut shown?

_____

1 mark

b. What is the maximum amount of water, in mL, that this drainage system can cope with in an hour?

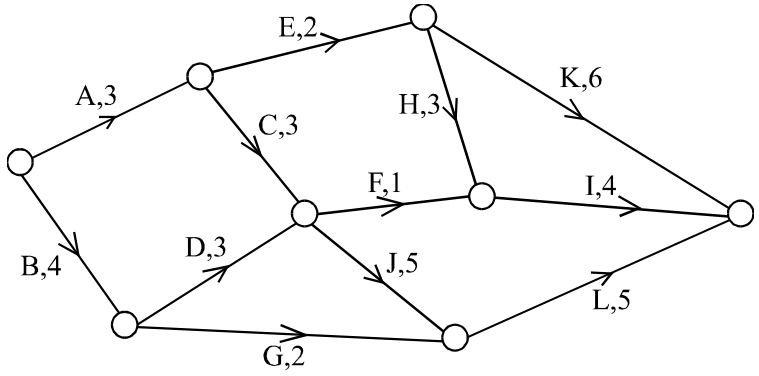
_____

_____

2 marks

Question 4

The construction company has identified the major activities and the time they will take to finish in weeks in order to complete the project.
The network below shows the activities and the time they will take, in weeks, to finish.



The table below shows the activity, together with the immediate predecessor(s) of each activity and the earliest and latest start time for each activity.

Activity	Immediate predecessor(s)	Earliest start time	Latest start time
A	-	0	1
B	-	0	0
C	A	3	4
D	B	4	4
E	A	3	8
F	C,D	7	12
G	B	4	
H	E	5	10
I		8	13
J	C,D	7	7
K	E	5	11
L	G,J		12

a. Use the information on the network to complete the three missing cells in the table above.

3 marks

b. Write down the critical path for the project.

1 mark

c. By comparing the start times of activities *G* and *H*, explain which will be the first to delay the overall project should both activities be indefinitely delayed.

2 marks

Total 15 marks

Further Mathematics Formulas

Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

annuities: $A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$, where $R = 1 + \dfrac{r}{100}$

Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bh$

area of a triangle: $\dfrac{1}{2}bc\sin A$

area of circle: πr^2

volume of sphere: $\dfrac{4}{3}\pi r^3$

volume of cone: $\dfrac{1}{3}\pi r^2 h$

Pythagoras' theorem $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Graphs and relations

Straight line graphs

gradient: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y - y_1 = m(x - x_1)$ gradient-point form

$y = mx + c$ gradient-intercept form

$\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$ two-point form

Number patterns and applications

arithmetic series: $a + (a+d) + \ldots + (a+(n-1)d) = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}, \quad r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r}, \quad |r| < 1$

linear difference equations: $t_n = at_{n-1} + b = a^{n-1}t_1 + b\frac{(a^{n-1}-1)}{a-1}, \quad a \neq 1$

$$= a^n t_0 + b\frac{(a^n - 1)}{a-1}$$

Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Statistics

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$